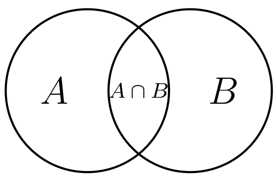
SRMJEEE Mathematics Sample Paper – 9

Duration: 47 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **SRMJEEE** (SRM Joint Engineering Entrance Examination).
- Each correct answer carries **+1 mark**. There is **no negative marking**; an unattempted or wrong answer scores 0.
- Only **one** option is correct. Choose carefully.
- The actual SRMJEEE is a **computer-based test** conducted in remote-proctored online mode, with all sections sharing a common time window and no per-section limit.
- Personal calculators, mobile phones, log tables and other electronic gadgets are strictly prohibited.

Q1. For two sets A and B, $n(A \cup B) = 42$, $n(A) = 27$ and $n(A \cap B) = 10$, as shown in the Venn diagram. The value of $n(B)$ is:



(A) 32

(B) 25

(C) 17

(D) 52

Q2. On the set of natural numbers $\mathbb{N}$, the relation R defined by "$a\,R\,b$ if and only if a divides b" is reflexive, transitive and:

(A) symmetric only

(B) neither symmetric nor antisymmetric

(C) antisymmetric (a partial order)

(D) not reflexive

Q3. If a set A has 3 elements and a set B has 4 elements, then the total number of functions from A to B is:

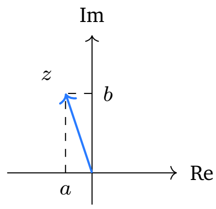
(A) 64

(B) 81

(C) 12

(D) 24

Q4. The complex number $z = \dfrac{1+2i}{1-i}$, plotted on the Argand plane, has real part a and imaginary part b. The ordered pair (a, b) is:



(A) $\left(\dfrac{3}{2}, \dfrac{1}{2}\right)$

(B) $\left(\dfrac{1}{2}, \dfrac{3}{2}\right)$

(C) $\left(\dfrac{1}{2}, -\dfrac{3}{2}\right)$

(D) $\left(-\dfrac{1}{2}, \dfrac{3}{2}\right)$

Q5. The value of $i + i^2 + i^3 + i^4$, where $i = \sqrt{-1}$, is:

(A) 0

(B) 1

(C) -1

(D) $2i$

Q6. For what value of k does the quadratic equation $x^2 - 6x + k = 0$ have equal roots?

(A) 3

(B) 6

(C) 9

(D) 36

Q7. If α and β are the roots of $x^2 - 8x + 15 = 0$, then the arithmetic mean $\frac{\alpha+\beta}{2}$ of the roots is:

(A) 15

(B) 4

(C) 8

(D) $\frac{15}{2}$

Q8. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, then A^2 equals:

(A) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$

Q9. If A is a 3×3 matrix with $|A| = 5$, then the value of $|2A|$ is:

(A) 10

(B) 40

(C) 20

(D) 80

Q10. If $\begin{vmatrix} x & 3 \\ 2 & x \end{vmatrix} = 10$, then a possible value of x is:

(A) 4

(B) 2

(C) $\sqrt{10}$

(D) 16

Q11. The system of equations $x + y = 3$ and $2x + 2y = 7$ is:

(A) consistent with a unique solution

(B) consistent with infinitely many solutions

(C) inconsistent (no solution)

(D) dependent

Q12. The value of ${}^7P_2 = \dfrac{7!}{(7-2)!}$ is:

(A) 14

(B) 21

(C) 49

(D) 42

Q13. A committee of 3 is to be formed by choosing 2 men from 4 men and 1 woman from 3 women. The number of ways of doing this is:

(A) 18

(B) 12

(C) 35

(D) 24

Q14. Six friends are to be seated around a circular table. If one particular friend's seat is fixed, the number of ways of seating the remaining five is:

(A) 720

(B) 120

(C) 24

(D) 600

Q15. One root of $x^3 - 7x + 6 = 0$ is $x = 1$. The sum of the other two roots is:

(A) 7

(B) 6

(C) -1

(D) 1

Q16. If α and β are the roots of $x^2 - 5x + 6 = 0$, then the quadratic equation whose roots are $\alpha + 1$ and $\beta + 1$ is:

(A) $x^2 - 5x + 6 = 0$

(B) $x^2 + 7x + 12 = 0$

(C) $x^2 - 5x + 12 = 0$

(D) $x^2 - 7x + 12 = 0$

Q17. The value of $\lim_{x\to\infty} \dfrac{3x^2 + 2x + 1}{5x^2 - x + 4}$ is:

(A) 0

(B) $\dfrac{3}{5}$

(C) ∞

(D) 1

Q18. If $x^2 + y^2 = 25$, then $\dfrac{dy}{dx}$ is:

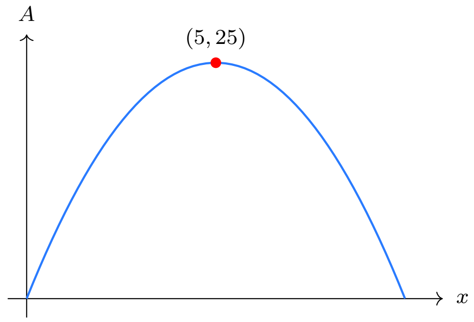
(A) $-\dfrac{x}{y}$

(B) $\dfrac{x}{y}$

(C) $-\dfrac{y}{x}$

(D) $\frac{y}{x}$

Q19. A rectangle has perimeter 20 units. The graph shows its area $A(x) = x(10 - x)$ against one side x. The maximum possible area is:



(A) 20 sq. units

(B) 50 sq. units

(C) 25 sq. units

(D) 100 sq. units

Q20. The position of a particle is $s(t) = t^3 - 2t^2 + 5$. Its velocity $\frac{ds}{dt}$ at $t = 2$ is:

(A) 8

(B) 12

(C) 5

(D) 4

Q21. The general solution of the differential equation $\frac{dy}{dx} = e^x$ is:

(A) $y = e^x + C$

(B) $y = e^x$

(C) $y = xe^x + C$

(D) $y = \frac{e^x}{x} + C$

Q22. $\int (3x + 2)^4 \, dx$ equals:

(A) $\dfrac{(3x+2)^5}{5} + C$

(B) $\dfrac{(3x+2)^5}{15} + C$

(C) $\dfrac{(3x+2)^5}{3} + C$

(D) $3(3x+2)^3 + C$

Q23. Using $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, the value of $\int_0^{\pi/2} \dfrac{\sin x}{\sin x + \cos x}\,dx$ is:

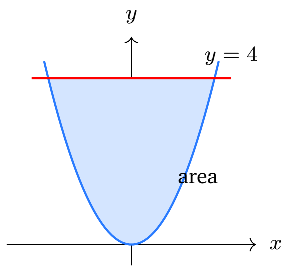
(A) $\dfrac{\pi}{2}$

(B) 1

(C) $\dfrac{\pi}{4}$

(D) 0

Q24. The area of the region bounded by the parabola $y = x^2$ and the line $y = 4$ (shaded) is:



(A) $\dfrac{16}{3}$ sq. units

(B) 8 sq. units

(C) 16 sq. units

(D) $\dfrac{32}{3}$ sq. units

Q25. The value of $\int_1^2 x\,dx$ is:

(A) $\dfrac{3}{2}$

(B) 2

(C) $\frac{1}{2}$

(D) 3

Q26. The foot of the perpendicular drawn from the point $(2, 3)$ to the x-axis is:

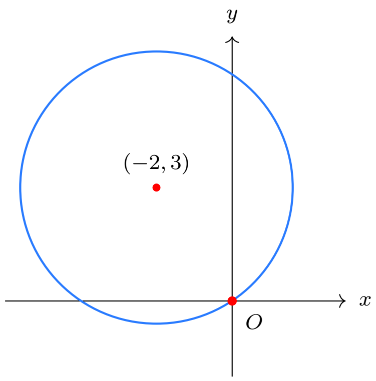
(A) $(0, 3)$

(B) $(2, 0)$

(C) $(3, 2)$

(D) $(0, 0)$

Q27. The circle $x^2 + y^2 + 4x - 6y + c = 0$ passes through the origin, as shown. The value of c is:



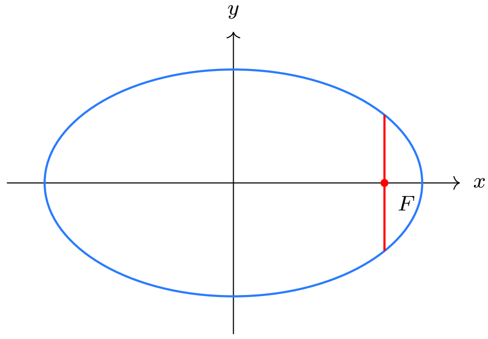
(A) 4

(B) -6

(C) 0

(D) 13

Q28. The length of the latus rectum of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, shown below, is:



(A) $\dfrac{9}{5}$
(B) $\dfrac{25}{3}$
(C) 6
(D) $\dfrac{18}{5}$

Q29. A straight line in the xy-plane makes an angle of 45° with the positive x-axis. Its slope is:

(A) 1
(B) $\dfrac{1}{\sqrt{2}}$
(C) $\sqrt{3}$
(D) 0

Q30. The line $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3} = t$ meets the plane $x + y + z = 12$ at the point:

(A) $(1, 2, 3)$
(B) $(4, 4, 4)$
(C) $(2, 4, 6)$
(D) $(3, 4, 5)$

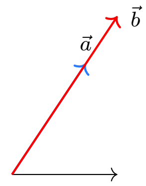
Q31. The direction cosines of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ are:

(A) $(2, 3, 6)$
(B) $\left(\dfrac{2}{7}, \dfrac{3}{7}, \dfrac{6}{7}\right)$

(C) $\left(\frac{2}{11}, \frac{3}{11}, \frac{6}{11}\right)$

(D) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$

Q32. The vectors $\vec{a} = 2\hat{i} + \lambda\hat{j}$ and $\vec{b} = 6\hat{i} + 9\hat{j}$ are parallel (so $\vec{a} \times \vec{b} = \vec{0}$), as shown. The value of λ is:



(A) 6

(B) 9

(C) $\frac{1}{3}$

(D) 3

Q33. The vector triple product $\vec{a} \times (\vec{b} \times \vec{c})$ is equal to:

(A) $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

(B) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$

(C) $(\vec{a} \cdot \vec{b})\vec{a} - (\vec{b} \cdot \vec{c})\vec{c}$

(D) $\vec{0}$

Q34. The mean of n observations is 15. If 4 is added to every observation, the new mean is:

(A) 15

(B) 19

(C) 60

(D) 11

Q35. An urn contains 4 red and 6 black balls. Two balls are drawn one after another without replacement. The probability that both are red is:

(A) $\frac{4}{25}$

(B) $\frac{2}{5}$

(C) $\frac{2}{15}$

(D) $\frac{1}{5}$

Q36. A fair coin is tossed 3 times. The probability of getting at least one head, i.e. $1 - P(X = 0)$, is:

(A) $\frac{1}{8}$

(B) $\frac{3}{8}$

(C) $\frac{1}{2}$

(D) $\frac{7}{8}$

Q37. The maximum value of $3\sin x + 4\cos x$ is:

(A) 5

(B) 7

(C) 1

(D) 25

Q38. If $\sin\theta = \frac{1}{2}$, then the value of $\cos 2\theta = 1 - 2\sin^2\theta$ is:

(A) $\frac{1}{4}$

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 1

Q39. The range of the principal values of $\cos^{-1} x$ is:

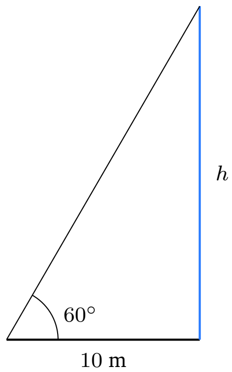
(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(B) $[0, \pi]$

(C) $(0, \pi)$

(D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Q40. From a point on the ground the angle of elevation of the top of a vertical pole is $60°$. If the point is 10 m from the foot of the pole, the height of the pole is:



(A) 10 m

(B) $\frac{10}{\sqrt{3}}$ m

(C) 20 m

(D) $10\sqrt{3}$ m

Detailed Solutions

Q1.

Solution

Concept — Inclusion–exclusion principle: For any two finite sets the size of the union is the sum of the individual sizes minus the size of the overlap, because the elements lying in $A \cap B$ are counted once in $n(A)$ and once again in $n(B)$, so we must subtract them once to avoid double counting. This is captured by

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Rearranging to isolate the unknown gives $n(B) = n(A \cup B) - n(A) + n(A \cap B)$, which is what we need here.

Step 1 — Read off the given data: From the statement and the Venn diagram we have $n(A \cup B) = 42$, $n(A) = 27$ and $n(A \cap B) = 10$. The only unknown is $n(B)$.

Step 2 — Substitute into the rearranged formula:

$$n(B) = n(A \cup B) - n(A) + n(A \cap B) = 42 - 27 + 10.$$

Step 3 — Simplify:

$$n(B) = 42 - 27 + 10 = 15 + 10 = 25.$$

Step 4 — Cross-check with disjoint regions: Split the union into three non-overlapping pieces: only A, only B, and $A \cap B$. Here only $A = n(A) - n(A \cap B) = 27 - 10 = 17$, the overlap is 10, and only $B = n(B) - n(A \cap B) = 25 - 10 = 15$. Their total is

$$17 + 10 + 15 = 42 = n(A \cup B),$$

which confirms the answer.

Why other options are wrong:

- (A) 32 comes from adding instead of subtracting, $42 - 27 + 10$ mishandled as $42 - (27 - 10) = 42 - 17 = 25$ done wrongly, or from $42 - 10 = 32$ (subtracting the overlap from the union and stopping).
- (C) 17 is the "only A" region $27 - 10$, not $n(B)$.
- (D) 52 comes from $42 + 10 = 52$, forgetting to subtract $n(A)$ entirely.

Final Answer: $n(B) = 25 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Properties of relations and partial orders: A relation R on a set is *reflexive* if $a\,R\,a$ for every a, *symmetric* if $a\,R\,b \Rightarrow b\,R\,a$, *antisymmetric* if $a\,R\,b$ and $b\,R\,a$ together force $a = b$, and *transitive* if $a\,R\,b$ and $b\,R\,c \Rightarrow a\,R\,c$. A relation that is simultaneously reflexive, antisymmetric and transitive is called a *partial order*. The divisibility relation on the natural numbers is the classic example.

Step 1 — Reflexivity: Every natural number divides itself, since $a = 1 \cdot a$, so $a \mid a$ holds for all $a \in \mathbb{N}$. Hence R is reflexive.

Step 2 — Transitivity: If $a \mid b$ and $b \mid c$, write $b = ka$ and $c = mb$ for integers k, m. Then

$$c = mb = m(ka) = (mk)a,$$

so $a \mid c$. Thus R is transitive.

Step 3 — Antisymmetry: Suppose $a \mid b$ and $b \mid a$ with $a, b \in \mathbb{N}$. Then $b = ka$ and $a = mb$ give $a = mka$, so $mk = 1$; since m, k are positive integers this forces $m = k = 1$, hence $a = b$. So R is antisymmetric.

Step 4 — Conclude: Divisibility on $\mathbb{N}$ is reflexive, antisymmetric and transitive, i.e. a partial order.

Why other options are wrong:

- (A) "symmetric only" fails: $2 \mid 4$ but $4 \nmid 2$, so R is not symmetric.
- (B) "neither symmetric nor antisymmetric" is false because Step 3 proves antisymmetry.
- (D) "not reflexive" contradicts Step 1, where $a \mid a$ always holds.

Final Answer: antisymmetric (a partial order) $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q2

Q3.

Solution

Concept — Counting functions: A function from A to B assigns to *each* element of the domain A exactly one image in the codomain B. The assignments for different domain elements are made independently, so by the multiplication principle the total count is $(\text{number of images})^{(\text{number of inputs})}$. If $|A| = m$ and $|B| = n$, the number of functions is n^m.

Step 1 — Identify the sizes: Here the domain A has $m = 3$ elements and the codomain B has $n = 4$ elements.

Step 2 — Count choice by choice: The first element of A can map to any of the 4 elements of B; so can the second and the third, independently. By the multiplication principle the number of functions is

$$4 \times 4 \times 4 = 4^3.$$

Step 3 — Evaluate:

$$4^3 = 64.$$

Why other options are wrong:

- (B) $81 = 3^4$ swaps the roles of domain and codomain (it counts functions from B to A).
- (C) $12 = 4 \times 3$ simply multiplies the two set sizes, which counts ordered pairs, not functions.
- (D) $24 = 4!$ counts arrangements/permutations and is irrelevant since the map need not be injective.

Final Answer: $4^3 = 64 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Dividing complex numbers via the conjugate: To write a quotient $\frac{z_1}{z_2}$ in the standard form $a + bi$ we multiply numerator and denominator by $\overline{z_2}$, the complex conjugate of the denominator. This works because $z_2\,\overline{z_2} = |z_2|^2$ is a real number, which clears the imaginary part from the denominator without changing the value of the fraction.

Step 1 — Multiply by the conjugate of the denominator: The conjugate of $1 - i$ is $1 + i$, so

$$z = \frac{1+2i}{1-i} \cdot \frac{1+i}{1+i} = \frac{(1+2i)(1+i)}{(1-i)(1+i)}.$$

Step 2 — Simplify the denominator: Using $(1-i)(1+i) = 1^2 - i^2 = 1 - (-1) = 2$,

$$(1-i)(1+i) = 2.$$

Step 3 — Expand the numerator: With $i^2 = -1$,

$$(1+2i)(1+i) = 1 + i + 2i + 2i^2 = 1 + 3i - 2 = -1 + 3i.$$

Step 4 — Combine and read off:

$$z = \frac{-1+3i}{2} = -\frac{1}{2} + \frac{3}{2}i,$$

so the real part is $a = -\frac{1}{2}$ and the imaginary part is $b = \frac{3}{2}$, giving $(a, b) = \left(-\frac{1}{2}, \frac{3}{2}\right)$. This matches the diagram, where z sits in the second quadrant (negative real, positive imaginary).

Why other options are wrong:

- (A) $\left(\frac{3}{2}, \frac{1}{2}\right)$ swaps the real and imaginary parts and drops the minus sign.
- (B) $\left(\frac{1}{2}, \frac{3}{2}\right)$ keeps a positive real part, i.e. it forgets the -1 in the numerator.
- (C) $\left(\frac{1}{2}, -\frac{3}{2}\right)$ uses the conjugate $1 - i$ in the wrong place, flipping both signs.

Final Answer: $\left(-\frac{1}{2}, \frac{3}{2}\right) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Cyclic powers of i: The powers of the imaginary unit repeat with period 4, because $i^4 = 1$. Concretely $i^1 = i$, $i^2 = -1$, $i^3 = i^2 \cdot i = -i$ and $i^4 = (i^2)^2 = (-1)^2 = 1$. A useful fact that follows is that the sum of any four consecutive powers of i is zero.

Step 1 — Write out each power:

$$i^1 = i, \qquad i^2 = -1, \qquad i^3 = -i, \qquad i^4 = 1.$$

Step 2 — Add them, grouping like parts:

$$i + (-1) + (-i) + 1 = (i - i) + (-1 + 1) = 0 + 0 = 0.$$

The two imaginary terms cancel and the two real terms cancel.

Step 3 — Cross-check via geometric series: The sum is $i\,\dfrac{i^4 - 1}{i - 1} = i\,\dfrac{1 - 1}{i - 1} = 0$, confirming the result.

Why other options are wrong:

- (B) 1 results from forgetting the $i^2 = -1$ term, leaving $i - i + 1 = 1$.
- (C) -1 results from mis-evaluating i^4 as -1 instead of 1.
- (D) $2i$ results from taking $i^3 = +i$ (wrong sign), giving $i + i = 2i$ after the reals cancel.

Final Answer: $i + i^2 + i^3 + i^4 = 0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — Discriminant and the nature of roots: For a quadratic $ax^2 + bx + c = 0$ the discriminant is $D = b^2 - 4ac$. If $D > 0$ the roots are real and distinct, if $D = 0$ they are real and equal (a repeated root), and if $D < 0$ they are a complex-conjugate pair. "Equal roots" therefore translates exactly into the single condition $D = 0$.

Step 1 — Identify the coefficients: For $x^2 - 6x + k = 0$ we have $a = 1$, $b = -6$ and $c = k$.

Step 2 — Form the discriminant:

$$D = b^2 - 4ac = (-6)^2 - 4(1)(k) = 36 - 4k.$$

Step 3 — Impose equal roots: Set $D = 0$:

$$36 - 4k = 0 \;\Rightarrow\; 4k = 36 \;\Rightarrow\; k = 9.$$

Step 4 — Verify: With $k = 9$ the equation is $x^2 - 6x + 9 = (x-3)^2 = 0$, a perfect square with the repeated root $x = 3$, confirming equal roots.

Why other options are wrong:

- (A) $k = 3$ gives $D = 36 - 12 = 24 > 0$, so distinct real roots, not equal.
- (B) $k = 6$ gives $D = 36 - 24 = 12 > 0$, again distinct roots.
- (D) $k = 36$ gives $D = 36 - 144 = -108 < 0$, so complex roots, not equal real ones.

Final Answer: $k = 9 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

Q7.

Solution

Concept — Vieta's formulas: For a monic quadratic $x^2 - sx + p = 0$ with roots α and β, the sum of the roots equals s (the negative of the coefficient of x) and the product equals p (the constant term): $\alpha + \beta = s$ and $\alpha\beta = p$. The arithmetic mean of the two roots is simply half their sum, so it depends only on s and not on the product.

Step 1 — Extract sum and product: Comparing $x^2 - 8x + 15 = 0$ with $x^2 - sx + p = 0$ gives

$$\alpha + \beta = 8, \qquad \alpha\beta = 15.$$

Step 2 — Compute the arithmetic mean:

$$\frac{\alpha + \beta}{2} = \frac{8}{2} = 4.$$

Step 3 — Verify with the explicit roots: Factoring, $x^2 - 8x + 15 = (x-3)(x-5)$, so $\alpha = 3$, $\beta = 5$. Their mean is $\frac{3+5}{2} = 4$, matching Step 2 (and indeed $3 + 5 = 8$,

$3 \cdot 5 = 15$).

Why other options are wrong:

- (A) 15 is the product $\alpha\beta$, not the mean.
- (C) 8 is the sum $\alpha + \beta$; the question asks for half of it.
- (D) $\frac{15}{2}$ halves the product instead of the sum.

Final Answer: arithmetic mean $= 4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q7

Q8.

Solution

Concept — Matrix multiplication (row by column): The square of a matrix is $A^2 = A \cdot A$. The (i, j) entry of the product is obtained by taking the dot product of the i-th row of the first factor with the j-th column of the second. For a 2×2 matrix this means four such dot products.

Step 1 — Set up the product:

$$A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}.$$

Step 2 — Compute each entry:

$$\begin{aligned} (1,1) &= 1 \cdot 1 + 2 \cdot 0 = 1, \quad (1,2) = 1 \cdot 2 + 2 \cdot 1 = 4, \\ (2,1) &= 0 \cdot 1 + 1 \cdot 0 = 0, \quad (2,2) = 0 \cdot 2 + 1 \cdot 1 = 1. \end{aligned}$$

Step 3 — Assemble:

$$A^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}.$$

Step 4 — Cross-check (shear pattern): A is an upper-triangular shear $\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$ with $t = 2$, and such matrices satisfy $A^n = \begin{pmatrix} 1 & nt \\ 0 & 1 \end{pmatrix}$. For $n = 2$ this gives off-diagonal $2t = 4$, confirming the result.

Why other options are wrong:

- (A) $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ just repeats A, i.e. it assumes $A^2 = A$ (only true for idempotent matrices).
- (B) the identity $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ would require A to be its own inverse.
- (C) $\begin{pmatrix} 1 & 8 \\ 0 & 1 \end{pmatrix}$ squares the off-diagonal entry ($2^2 = 8$ wrongly) instead of doubling it ($2 \cdot 2 = 4$).

Final Answer: $A^2 = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q8**

Q9.

Solution

Concept — Scalar multiple of a determinant: Multiplying a matrix by a scalar k multiplies *every* row by k. Since pulling a common factor out of one row multiplies the determinant by that factor, pulling k out of all n rows of an $n \times n$ matrix multiplies the determinant by k^n. Hence

$$|kA| = k^n |A|.$$

The exponent n is the size of the matrix, a point that is easy to forget.

Step 1 — Identify n and k: The matrix is 3×3, so $n = 3$, and the scalar is $k = 2$.

Step 2 — Apply the rule:

$$|2A| = 2^3 \, |A| = 8 \times 5.$$

Step 3 — Simplify:

$$|2A| = 8 \times 5 = 40.$$

Why other options are wrong:

- (A) $10 = 2^1 \times 5$ uses k^1, treating the determinant as linear in the whole matrix.
- (C) $20 = 2^2 \times 5$ uses k^2, the rule for a 2×2 matrix, not 3×3.
- (D) $80 = 2^4 \times 5$ over-counts with k^4.

Final Answer: $|2A| = 40 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q9

Q10.

Solution

Concept — Evaluating a 2×2 determinant: The determinant of a 2×2 matrix is the product of the main-diagonal entries minus the product of the anti-diagonal entries:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

Setting this equal to a given value turns the determinant condition into an ordinary algebraic equation in the unknown.

Step 1 — Expand the determinant:

$$\begin{vmatrix} x & 3 \\ 2 & x \end{vmatrix} = x \cdot x - 3 \cdot 2 = x^2 - 6.$$

Step 2 — Form the equation: The condition is $x^2 - 6 = 10$, so

$$x^2 = 16.$$

Step 3 — Solve:

$$x = \pm\sqrt{16} = \pm 4.$$

Both ± 4 work; among the options the available value is $x = 4$.

Step 4 — Verify: With $x = 4$, $\begin{vmatrix} 4 & 3 \\ 2 & 4 \end{vmatrix} = 16 - 6 = 10$, exactly as required.

Why other options are wrong:

- (B) $x = 2$ gives $4 - 6 = -2 \neq 10$.
- (C) $x = \sqrt{10}$ gives $10 - 6 = 4 \neq 10$; it ignores the -6 shift.
- (D) $x = 16$ is the value of x^2, not of x.

Final Answer: $x = 4 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept — Consistency of two linear equations: For the pair $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, compare the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}, \frac{c_1}{c_2}$. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ there is a unique solution; if all three ratios are equal the lines coincide (infinitely many solutions); and if the first two ratios are equal but differ from the third, the lines are parallel and distinct, so there is no solution (inconsistent).

Step 1 — Read the coefficients: The equations are $x + y = 3$ and $2x + 2y = 7$, so $a_1 = 1, b_1 = 1, c_1 = 3$ and $a_2 = 2, b_2 = 2, c_2 = 7$.

Step 2 — Form the ratios:

$$\frac{a_1}{a_2} = \frac{1}{2}, \qquad \frac{b_1}{b_2} = \frac{1}{2}, \qquad \frac{c_1}{c_2} = \frac{3}{7}.$$

Step 3 — Compare: The coefficient ratios are equal, $\frac{1}{2} = \frac{1}{2}$, but the constant ratio differs, $\frac{1}{2} \neq \frac{3}{7}$. This is the parallel-and-distinct case.

Step 4 — Confirm directly: Doubling the first equation gives $2x + 2y = 6$, which contradicts $2x + 2y = 7$. No (x, y) can make $2x + 2y$ equal both 6 and 7, so the system has no solution.

Why other options are wrong:

- (A) a unique solution needs $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, which fails here.
- (B) infinitely many solutions would need $\frac{3}{7} = \frac{1}{2}$, which is false.
- (D) "dependent" (coincident lines) also requires all three ratios equal, again failing.

Final Answer: inconsistent (no solution) $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Permutations: ${}^nP_r$ counts the number of ordered arrangements of r objects chosen from n distinct objects. The formula is ${}^nP_r = \frac{n!}{(n-r)!}$, which equals the product of the r consecutive integers starting from n and counting down, namely $n(n-1)\cdots(n-r+1)$.

Step 1 — Substitute $n = 7$, $r = 2$:

$$^{7}P_2 = \frac{7!}{(7-2)!} = \frac{7!}{5!}.$$

Step 2 — Cancel the common factorial: Since $7! = 7 \times 6 \times 5!$,

$$\frac{7!}{5!} = 7 \times 6.$$

Step 3 — Multiply:

$$7 \times 6 = 42.$$

Why other options are wrong:

- (A) $14 = 7 \times 2$ multiplies n by r instead of taking falling factors.
- (B) $21 = \binom{7}{2} = \frac{7\times 6}{2}$ is the *combination*, which ignores order; permutations are twice as many.
- (C) $49 = 7^2$ treats the two picks as independent with repetition.

Final Answer: $^{7}P_2 = 42 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Multiplication principle for selections: When a task splits into independent stages, the total number of ways is the product of the number of ways for each stage. Choosing a committee here is two independent selections: which men, and which woman. Order does not matter within each group, so we use combinations $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$.

Step 1 — Choose the 2 men from 4:

$$\binom{4}{2} = \frac{4 \times 3}{2 \times 1} = 6 \text{ ways}.$$

Step 2 — Choose the 1 woman from 3:

$$\binom{3}{1} = 3 \text{ ways}.$$

Step 3 — Combine by the multiplication principle:

$$6 \times 3 = 18 \text{ ways.}$$

Step 4 — Sanity check: The men can be $\{12, 13, 14, 23, 24, 34\}$ (six pairs) and each pairs with any of 3 women, giving $6 \times 3 = 18$ distinct committees, as listed.

Why other options are wrong:

- (B) 12 uses $\binom{4}{1}\binom{3}{1}\cdot$ wrong split or 4×3, the wrong man-count.
- (C) $35 = \binom{7}{3}$ chooses any 3 of the 7 people, ignoring the 2-men-1-woman constraint.
- (D) 24 comes from using ${}^4P_2 = 12$ (ordered men) times $\binom{3}{1}$ wrongly, or similar miscount of order.

Final Answer: 18 ways $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Circular permutations: Around a round table only the *relative* order of people matters, since rotating everyone one seat does not create a new arrangement. There are n rotations of any linear arrangement, so the $n!$ linear orders collapse into $\frac{n!}{n} = (n-1)!$ distinct circular ones. Equivalently, fixing one person's seat as a reference point removes the rotational duplication, after which the remaining $n-1$ people fill the other seats in $(n-1)!$ ways.

Step 1 — Identify n: There are $n = 6$ friends.

Step 2 — Fix one seat and arrange the rest: With one friend's seat fixed, the other 5 friends are arranged in

$$(6-1)! = 5! \text{ ways.}$$

Step 3 — Evaluate:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

Why other options are wrong:

- (A) $720 = 6!$ is the count of *linear* arrangements; it counts the 6 rotations of each seating as distinct.

- (C) $24 = 4!$ would be the answer for 5 people, i.e. one too few.
- (D) 600 does not arise from any correct factorial computation here.

Final Answer: $5! = 120 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Vieta's formula for a cubic: For $x^3 + px^2 + qx + r = 0$ with roots α, β, γ, the sum of the roots equals $-p$, the negative of the coefficient of x^2. This lets us find the total of all roots without solving the cubic, and then peel off any known root.

Step 1 — Write the cubic in full form: The equation $x^3 - 7x + 6 = 0$ has no x^2 term, so it is $x^3 + 0 \cdot x^2 - 7x + 6 = 0$ with $p = 0$.

Step 2 — Sum of all three roots:

$$\alpha + \beta + \gamma = -p = 0.$$

Step 3 — Subtract the known root: Taking the given root $\gamma = 1$, the sum of the remaining two is

$$\alpha + \beta = (\alpha + \beta + \gamma) - \gamma = 0 - 1 = -1.$$

Step 4 — Verify by factoring: Since $x = 1$ is a root, $x^3 - 7x + 6 = (x-1)(x^2 + x - 6) = (x-1)(x+3)(x-2)$. The other roots are -3 and 2, whose sum is $-3+2 = -1$, confirming the answer.

Why other options are wrong:

- (A) 7 misreads $-q = 7$ (the sum of pairwise products) as the sum of roots.
- (B) 6 is the constant term r, related to the product, not the sum.
- (D) 1 forgets to subtract the known root $\gamma = 1$ from the total of 0.

Final Answer: sum of the other two roots $= -1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Transforming roots: To build a quadratic whose roots are obtained from the old ones by a fixed shift, the cleanest route is to recompute the sum and product of the *new* roots from the old sum and product (Vieta), then assemble $x^2 - (\text{sum})x + (\text{product}) = 0$. (Equivalently, replacing x by $x - 1$ in the original equation shifts every root up by 1.)

Step 1 — Old sum and product: From $x^2 - 5x + 6 = 0$,

$$\alpha + \beta = 5, \qquad \alpha\beta = 6.$$

Step 2 — New sum: The new roots are $\alpha + 1$ and $\beta + 1$, so

$$(\alpha + 1) + (\beta + 1) = (\alpha + \beta) + 2 = 5 + 2 = 7.$$

Step 3 — New product:

$$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1 = 6 + 5 + 1 = 12.$$

Step 4 — Assemble the equation:

$$x^2 - (\text{sum})x + (\text{product}) = x^2 - 7x + 12 = 0.$$

Step 5 — Verify with explicit roots: $x^2 - 5x + 6 = (x - 2)(x - 3)$, so $\alpha = 2, \beta = 3$. The new roots are 3 and 4, and $x^2 - 7x + 12 = (x - 3)(x - 4)$, exactly matching.

Why other options are wrong:

- (A) $x^2 - 5x + 6 = 0$ is the original equation (no shift applied).
- (B) $x^2 + 7x + 12 = 0$ has roots $-3, -4$; it shifts the roots *down* by reversing the sign of the sum.
- (C) $x^2 - 5x + 12 = 0$ keeps the old sum 5 while changing only the product, which is inconsistent.

Final Answer: $x^2 - 7x + 12 = 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — Limits of rational functions at infinity: As $x \to \infty$ the behaviour of a rational function is governed by the highest powers of x. If the numerator and denominator have the *same* degree, all lower-order terms become negligible and the limit equals the ratio of the leading coefficients. The standard technique is to divide every term by the highest power of x present.

Step 1 — Note the degrees: Both numerator $3x^2 + 2x + 1$ and denominator $5x^2 - x + 4$ have degree 2, so the limit is finite and nonzero.

Step 2 — Divide numerator and denominator by x^2:

$$\frac{3x^2 + 2x + 1}{5x^2 - x + 4} = \frac{3 + \dfrac{2}{x} + \dfrac{1}{x^2}}{5 - \dfrac{1}{x} + \dfrac{4}{x^2}}.$$

Step 3 — Take the limit term by term: As $x \to \infty$, each of $\frac{2}{x}, \frac{1}{x^2}, \frac{1}{x}, \frac{4}{x^2} \to 0$, so

$$\lim_{x \to \infty} \frac{3 + \frac{2}{x} + \frac{1}{x^2}}{5 - \frac{1}{x} + \frac{4}{x^2}} = \frac{3 + 0 + 0}{5 - 0 + 0} = \frac{3}{5}.$$

Why other options are wrong:

- (A) 0 would occur only if the numerator's degree were lower than the denominator's.
- (C) ∞ would occur only if the numerator's degree were higher.
- (D) 1 ignores the leading coefficients 3 and 5, wrongly assuming they cancel.

Final Answer: the limit is $\frac{3}{5} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q17

Q18.

Solution

Concept — Implicit differentiation: When y is defined implicitly by an equation in x and y, we differentiate both sides with respect to x, treating y as a function of x. By the chain rule, $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$. We then solve the resulting linear equation for $\frac{dy}{dx}$.

Step 1 — Differentiate each term: The equation $x^2 + y^2 = 25$ gives

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25),$$

that is

$$2x + 2y\frac{dy}{dx} = 0.$$

Step 2 — Isolate the derivative:

$$2y\frac{dy}{dx} = -2x \;\Rightarrow\; \frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}.$$

Step 3 — Geometric check: The curve is the circle of radius 5. The radius to a point (x, y) has slope $\frac{y}{x}$, and the tangent is perpendicular to it, so the tangent slope is $-\frac{x}{y}$, matching our result.

Why other options are wrong:

- (B) $\frac{x}{y}$ drops the negative sign that comes from moving $2x$ across.
- (C) $-\frac{y}{x}$ inverts the ratio (this would be the perpendicular's slope flipped).
- (D) $\frac{y}{x}$ both inverts the ratio and loses the sign.

Final Answer: $\frac{dy}{dx} = -\frac{x}{y} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q18

Q19.

Solution

Concept — Optimisation by calculus: For a fixed perimeter 20, if one side is x then the adjacent side is $10 - x$ (since the two pairs of sides sum to 20). The area $A(x) = x(10 - x)$ is a downward-opening parabola, so it attains a maximum at its vertex. We locate the vertex by setting the derivative $A'(x) = 0$ and confirm it is a maximum because $A''(x) < 0$.

Step 1 — Express the area:

$$A(x) = x(10 - x) = 10x - x^2.$$

Step 2 — Differentiate and find the critical point:

$$A'(x) = 10 - 2x = 0 \;\Rightarrow\; x = 5.$$

Step 3 — Confirm a maximum: $A''(x) = -2 < 0$, so $x = 5$ gives a maximum, not a minimum.

Step 4 — Evaluate the maximum area:

$$A(5) = 5(10 - 5) = 5 \times 5 = 25 \text{ sq. units}.$$

This is the 5×5 square, and it matches the marked vertex $(5, 25)$ on the graph.

Why other options are wrong:

- (A) 20 comes from a non-optimal rectangle, e.g. sides 2 and 8 giving 16, or $x = 2$ misread.
- (B) 50 doubles the true maximum (using $x(10 - x)$ at the wrong scaling).
- (D) $100 = 10 \times 10$ ignores that the two sides must sum to 10, not each be 10.

Final Answer: maximum area $= 25$ sq. units $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept — Velocity as a derivative: The instantaneous velocity of a particle is the rate of change of its position with respect to time, $v(t) = \frac{ds}{dt}$. We differentiate $s(t)$ term by term using the power rule $\frac{d}{dt}(t^n) = nt^{n-1}$, then substitute the required instant.

Step 1 — Differentiate the position function: For $s(t) = t^3 - 2t^2 + 5$,

$$\frac{ds}{dt} = 3t^2 - 4t + 0 = 3t^2 - 4t.$$

The constant 5 differentiates to 0.

Step 2 — Substitute $t = 2$:

$$v(2) = 3(2)^2 - 4(2) = 3 \times 4 - 8 = 12 - 8.$$

Step 3 — Simplify:

$$v(2) = 4.$$

Why other options are wrong:

- (A) 8 keeps only $3t^2 = 12$ minus a mis-evaluated $4t = 4$, or uses $2t^2$ at $t = 2$.
- (B) 12 keeps only the $3t^2$ term and forgets to subtract $4t$.
- (C) 5 is the constant in $s(t)$, i.e. the initial position, not the velocity.

Final Answer: velocity $= 4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Solving a separable/direct ODE: When $\frac{dy}{dx}$ is given purely as a function of x, the general solution is obtained by integrating that function: $y = \int f(x)\,dx$. Because indefinite integration is determined only up to a constant, the *general* solution must include an arbitrary constant C, which represents the whole family of solution curves.

Step 1 — Set up the integral:

$$\frac{dy}{dx} = e^x \Rightarrow y = \int e^x\,dx.$$

Step 2 — Integrate: Since e^x is its own antiderivative,

$$y = e^x + C.$$

Step 3 — Verify by differentiating back: $\frac{d}{dx}(e^x + C) = e^x$, which reproduces the original differential equation, confirming the solution.

Why other options are wrong:

- (B) $y = e^x$ omits the arbitrary constant, so it is only one particular solution, not the general one.
- (C) $y = xe^x + C$ differentiates to $e^x + xe^x \neq e^x$.
- (D) $y = \frac{e^x}{x} + C$ differentiates to $\frac{e^x(x-1)}{x^2} \neq e^x$.

Final Answer: $y = e^x + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — Power rule with a linear inner function: For a linear inner function, $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$. This comes from the substitution $u = ax + b$, $du = a\,dx$, so $dx = \frac{du}{a}$ and $\int u^n \frac{du}{a} = \frac{u^{n+1}}{a(n+1)}$. The crucial extra factor is the $\frac{1}{a}$ that compensates for the chain rule.

Step 1 — Identify the parameters: Here $a = 3$, $b = 2$, $n = 4$, so $n + 1 = 5$.

Step 2 — Apply the formula:

$$\int (3x+2)^4\,dx = \frac{(3x+2)^5}{3\cdot 5} + C = \frac{(3x+2)^5}{15} + C.$$

Step 3 — Check by differentiating:

$$\frac{d}{dx}\left[\frac{(3x+2)^5}{15}\right] = \frac{5(3x+2)^4\cdot 3}{15} = (3x+2)^4,$$

recovering the integrand, so the answer is correct.

Why other options are wrong:

- (A) $\frac{(3x+2)^5}{5}$ forgets the $\frac{1}{a} = \frac{1}{3}$ factor from the inner derivative.
- (C) $\frac{(3x+2)^5}{3}$ divides only by a and forgets $n+1=5$.
- (D) $3(3x+2)^3$ differentiates the integrand instead of integrating it.

Final Answer: $\dfrac{(3x+2)^5}{15} + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — The "King" reflection property: The identity $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ follows from the substitution $x \mapsto a-x$, which reverses the interval $[0,a]$ onto itself. For trigonometric integrals on $[0,\frac{\pi}{2}]$ this swaps $\sin$ and $\cos$, since $\sin(\frac{\pi}{2}-x) = \cos x$ and $\cos(\frac{\pi}{2}-x) = \sin x$. Adding the original integral to its reflected copy often collapses a hard integrand into a constant.

Step 1 — Name the integral:

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx.$$

Step 2 — Reflect with $x \to \frac{\pi}{2} - x$: Using $\sin(\frac{\pi}{2}-x) = \cos x$ and $\cos(\frac{\pi}{2}-x) = \sin x$,

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx.$$

Step 3 — Add the two expressions for I:

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2}.$$

Step 4 — Solve for I:

$$I = \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi}{4}.$$

Why other options are wrong:

- (A) $\frac{\pi}{2}$ is the value of $2I$, not I; it forgets the final halving.

- (B) 1 mistakes the constant integrand's coefficient for the answer.
- (D) 0 ignores that the integrand is positive throughout $[0, \frac{\pi}{2}]$.

Final Answer: $I = \dfrac{\pi}{4} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q23

Q24.

Solution

Concept — Area between two curves: The area enclosed between an upper curve $y = g(x)$ and a lower curve $y = h(x)$ over $[p, q]$ is $\displaystyle\int_p^q \big(g(x) - h(x)\big)\, dx$. Here the line $y = 4$ lies above the parabola $y = x^2$ between their intersection points, so the height of a vertical strip is $4 - x^2$ and the limits are the x-values where the curves meet.

Step 1 — Find the intersection points: Setting $x^2 = 4$ gives

$$x = \pm 2,$$

so the region runs from $x = -2$ to $x = 2$.

Step 2 — Set up the area integral:

$$A = \int_{-2}^{2} (4 - x^2)\, dx.$$

Step 3 — Use symmetry and integrate: The integrand is even, so

$$A = 2\int_0^2 (4 - x^2)\, dx = 2\left[4x - \frac{x^3}{3}\right]_0^2 = 2\left(8 - \frac{8}{3}\right) = 2 \cdot \frac{16}{3} = \frac{32}{3}.$$

Step 4 — Confirm without symmetry: Directly,

$$\left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = 16 - \frac{16}{3} = \frac{32}{3},$$

the same value.

Why other options are wrong:

- (A) $\frac{16}{3}$ integrates over only half the interval (0 to 2) and forgets the factor 2.
- (B) 8 comes from $\int_{-2}^{2} 4\, dx$ alone, dropping the $-x^2$ term.
- (C) 16 doubles that same error or uses the wrong limits.

Final Answer: area $= \dfrac{32}{3}$ sq. units $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — Fundamental theorem of calculus: A definite integral is evaluated by finding an antiderivative and taking its difference at the limits: $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F' = f$. For $f(x) = x$ the antiderivative is $\dfrac{x^2}{2}$ (by the power rule), so $\int_a^b x\,dx = \left[\dfrac{x^2}{2}\right]_a^b$.

Step 1 — Write the antiderivative with limits:

$$\int_1^2 x\,dx = \left[\frac{x^2}{2}\right]_1^2.$$

Step 2 — Substitute the limits:

$$= \frac{2^2}{2} - \frac{1^2}{2} = \frac{4}{2} - \frac{1}{2} = 2 - \frac{1}{2}.$$

Step 3 — Simplify:

$$= \frac{3}{2}.$$

Step 4 — Geometric check: The region under $y = x$ from $x = 1$ to $x = 2$ is a trapezium with parallel sides 1 and 2 and width 1, of area $\frac{1}{2}(1+2)(1) = \frac{3}{2}$, confirming the result.

Why other options are wrong:

- (B) 2 keeps only $\frac{2^2}{2}$ and forgets to subtract the lower-limit value.
- (C) $\frac{1}{2}$ keeps only the lower-limit term $\frac{1^2}{2}$.
- (D) 3 uses $2^2 - 1^2 = 3$ but omits the factor $\frac{1}{2}$ from the antiderivative.

Final Answer: $\int_1^2 x\,dx = \dfrac{3}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Foot of perpendicular onto the x-axis: The x-axis is the line $y = 0$. The perpendicular from a point to the x-axis is vertical, so it keeps the same x-coordinate and meets the axis where $y = 0$. Hence the foot of the perpendicular from (x_0, y_0) is $(x_0, 0)$, and the dropped distance is $|y_0|$.

Step 1 — Identify the point: The given point is $(x_0, y_0) = (2, 3)$.

Step 2 — Drop the vertical: Keep the x-coordinate and set $y = 0$:

$$(2, 3) \longrightarrow (2, 0).$$

Step 3 — Note the distance: The length of this perpendicular is $|y_0| = 3$, the point's distance from the axis, which is consistent.

Why other options are wrong:

- (A) $(0, 3)$ is the foot of the perpendicular onto the y-axis, not the x-axis.
- (C) $(3, 2)$ simply swaps the coordinates.
- (D) $(0, 0)$ is the origin, which would require both coordinates to vanish.

Final Answer: foot $= (2, 0) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — A point lies on a curve iff it satisfies the equation: A circle (or any curve) passes through a given point exactly when substituting that point's coordinates makes the equation true. For the general circle $x^2+y^2+2gx+2fy+c = 0$, putting $(0, 0)$ kills every term except the constant, so "passes through the origin" is equivalent to $c = 0$.

Step 1 — Substitute $(x, y) = (0, 0)$:

$$0^2 + 0^2 + 4(0) - 6(0) + c = 0.$$

Step 2 — Simplify: All but the constant vanish, leaving

$$c = 0.$$

Step 3 — Interpret: The condition for any circle of this form to pass through the origin is simply that its constant term be zero, which is what we found.

Why other options are wrong:

- (A) 4 is the coefficient $2g$ from the x-term, not the constant.
- (B) -6 is the coefficient $2f$ from the y-term.
- (D) $13 = 2^2 + 3^2$ relates to the centre $(-2, 3)$ and radius, not to the passing-through-origin condition.

Final Answer: $c = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Latus rectum of an ellipse: For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$ (major axis along x), each latus rectum is the chord through a focus perpendicular to the major axis. Its length is $\frac{2b^2}{a}$. The key is to identify a (the semi-major axis, from the larger denominator) and b^2 correctly.

Step 1 — Identify a and b: Comparing $\frac{x^2}{25} + \frac{y^2}{9} = 1$ with the standard form, the larger denominator is 25, so $a^2 = 25 \Rightarrow a = 5$, and $b^2 = 9 \Rightarrow b = 3$. Since $a > b$, the major axis is horizontal, matching the wide ellipse in the figure.

Step 2 — Apply the formula:

$$\text{length} = \frac{2b^2}{a} = \frac{2 \times 9}{5}.$$

Step 3 — Simplify:

$$\frac{2 \times 9}{5} = \frac{18}{5}.$$

Why other options are wrong:

- (A) $\frac{9}{5} = \frac{b^2}{a}$ drops the factor 2.
- (B) $\frac{25}{3} = \frac{2a^2}{2b}$ wrongly swaps the roles of a and b.
- (C) $6 = 2b$ uses the minor-axis length rather than $\frac{2b^2}{a}$.

Final Answer: latus rectum $= \frac{18}{5} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Slope as the tangent of the inclination: The slope m of a line measures its steepness as rise over run. If a line makes an angle θ (its inclination) with the positive x-axis, then $m = \tan\theta$, because over a horizontal run the vertical rise is exactly $\tan\theta$ times the run.

Step 1 — Substitute the given angle: Here $\theta = 45°$, so

$$m = \tan 45°.$$

Step 2 — Evaluate: The tangent of $45°$ is 1 (the legs of a $45°$ right triangle are equal), so

$$m = 1.$$

Step 3 — Interpret: A slope of 1 means the line rises one unit for every one unit it moves right, i.e. the line $y = x + c$, which indeed sits at $45°$.

Why other options are wrong:

- (B) $\frac{1}{\sqrt{2}}$ is $\sin 45°$ (or $\cos 45°$), not the tangent.
- (C) $\sqrt{3}$ is $\tan 60°$, the slope at a steeper $60°$.
- (D) 0 is $\tan 0°$, the slope of a horizontal line.

Final Answer: slope $= 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — Intersection of a line and a plane: A line given in symmetric form $\frac{x}{1} = \frac{y}{2} = \frac{z}{3} = t$ has the parametric point $(t, 2t, 3t)$. To find where it pierces a plane, substitute these coordinates into the plane's equation, solve the resulting single equation for the parameter t, then plug t back to get the point.

Step 1 — Parametrise the line: Every point on the line is

$$(x, y, z) = (t,\ 2t,\ 3t).$$

Step 2 — Substitute into the plane $x + y + z = 12$:

$$t + 2t + 3t = 12 \;\Rightarrow\; 6t = 12.$$

Step 3 — Solve for t:

$$t = 2.$$

Step 4 — Recover the point and verify:

$$(t, 2t, 3t) = (2, 4, 6), \qquad 2 + 4 + 6 = 12. \checkmark$$

Why other options are wrong:

- (A) $(1, 2, 3)$ uses $t = 1$, giving $1 + 2 + 3 = 6 \neq 12$.
- (B) $(4, 4, 4)$ has $4+4+4 = 12$ but does not satisfy the ratios $x : y : z = 1 : 2 : 3$, so it is not on the line.
- (D) $(3, 4, 5)$ neither lies on the line nor sums to 12.

Final Answer: intersection $= (2, 4, 6) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q30

Q31.

Solution

Concept — Direction cosines: The direction cosines of a vector are the cosines of the angles it makes with the x-, y- and z-axes. They equal the components of the corresponding *unit* vector, namely $\left(\frac{a_1}{|\vec{a}|}, \frac{a_2}{|\vec{a}|}, \frac{a_3}{|\vec{a}|}\right)$. A built-in check is that they satisfy $l^2 + m^2 + n^2 = 1$.

Step 1 — Compute the magnitude:

$$|\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

Step 2 — Divide each component by the magnitude:

$$(l, m, n) = \left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right).$$

Step 3 — Verify the identity:

$$\left(\tfrac{2}{7}\right)^2+\left(\tfrac{3}{7}\right)^2+\left(\tfrac{6}{7}\right)^2=\frac{4+9+36}{49}=\frac{49}{49}=1,\ \checkmark$$

so the triple is a valid set of direction cosines.

Why other options are wrong:

- (A) $(2,3,6)$ are the raw components (direction *ratios*), not normalised cosines; they fail $l^2+m^2+n^2=1$.
- (C) $\left(\frac{2}{11},\frac{3}{11},\frac{6}{11}\right)$ uses the wrong magnitude $11=2+3+6$ instead of $\sqrt{49}=7$.
- (D) $\left(\frac{1}{2},\frac{1}{3},\frac{1}{6}\right)$ inverts the components rather than dividing by $|\vec{a}|$.

Final Answer: $\left(\frac{2}{7},\frac{3}{7},\frac{6}{7}\right)\Rightarrow\boxed{\text{B}}$

Answer: (B) Go Back to Q31

Q32.

Solution

Concept — Condition for parallel vectors: Two nonzero vectors are parallel exactly when one is a scalar multiple of the other, equivalently when their cross product is the zero vector. In component form this means the components are proportional: $\frac{a_1}{b_1}=\frac{a_2}{b_2}$. Setting up this proportion turns the parallelism condition into a simple equation for the unknown.

Step 1 — Write the proportionality: For $\vec{a}=2\hat{i}+\lambda\hat{j}$ and $\vec{b}=6\hat{i}+9\hat{j}$,

$$\frac{2}{6}=\frac{\lambda}{9}.$$

Step 2 — Cross-multiply and solve:

$$6\lambda=2\times 9=18\ \Rightarrow\ \lambda=\frac{18}{6}=3.$$

Step 3 — Verify via the cross product: With $\lambda=3$, $\vec{a}=(2,3)$ and $\vec{b}=(6,9)$, the 2D cross component is $a_1b_2-a_2b_1=2\cdot 9-3\cdot 6=18-18=0$, so $\vec{a}\times\vec{b}=\vec{0}$ as required. Indeed $\vec{b}=3\vec{a}$.

Why other options are wrong:

- (A) 6 and (B) 9 just copy components of $\vec{b}$ rather than solving the proportion.
- (C) $\frac{1}{3}$ is the ratio $\frac{2}{6}$ itself, not the value of λ.

Final Answer: $\lambda = 3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q32

Q33.

Solution

Concept — Vector triple product (BAC–CAB rule): The expansion $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ is a standard identity. A handy mnemonic is "BAC minus CAB": the result is $\vec{b}$ times $(\vec{a} \cdot \vec{c})$ minus $\vec{c}$ times $(\vec{a} \cdot \vec{b})$. Geometrically the answer must lie in the plane spanned by $\vec{b}$ and $\vec{c}$, since $\vec{b} \times \vec{c}$ is perpendicular to that plane and crossing again with $\vec{a}$ brings the vector back into it.

Step 1 — Recall the structure: The outer vector $\vec{a}$ dots with each of the inner vectors, and the inner vectors $\vec{b}, \vec{c}$ appear as the directions of the result.

Step 2 — Place the signs by BAC–CAB: The first (positive) term keeps $\vec{b}$ with coefficient $(\vec{a} \cdot \vec{c})$; the second (negative) term keeps $\vec{c}$ with coefficient $(\vec{a} \cdot \vec{b})$:

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\, \vec{b} - (\vec{a} \cdot \vec{b})\, \vec{c}.$$

Step 3 — Quick numeric check: Take $\vec{a} = \vec{b} = \hat{i}$, $\vec{c} = \hat{j}$. Then $\vec{b} \times \vec{c} = \hat{i} \times \hat{j} = \hat{k}$ and $\vec{a} \times \hat{k} = \hat{i} \times \hat{k} = -\hat{j}$. The formula gives $(\hat{i} \cdot \hat{j})\hat{i} - (\hat{i} \cdot \hat{i})\hat{j} = 0 - \hat{j} = -\hat{j}$, matching.

Why other options are wrong:

- (B) $(\vec{a} \cdot \vec{b})\vec{c} - (\vec{a} \cdot \vec{c})\vec{b}$ reverses the two terms (it is the expansion of $(\vec{a} \times \vec{b}) \times \vec{c}$ up to sign).
- (C) pairs the wrong dot products with the wrong vectors.
- (D) $\vec{0}$ holds only in degenerate cases (e.g. $\vec{b} \parallel \vec{c}$), not in general.

Final Answer: $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q33

Q34.

Solution

Concept — Effect of a constant shift on the mean: The mean is $\bar{x} = \frac{1}{n}\sum x_i$. If every observation is increased by a constant k, the new sum is $\sum(x_i + k) = \sum x_i + nk$, so the new mean is $\frac{\sum x_i + nk}{n} = \bar{x} + k$. In words, adding a constant to all data shifts the mean by exactly that constant (the spread is unchanged).

Step 1 — Identify the shift: The original mean is $\bar{x} = 15$ and the constant added is $k = 4$.

Step 2 — Apply the rule:

$$\bar{x}_{\text{new}} = \bar{x} + k = 15 + 4 = 19.$$

Step 3 — Derive it explicitly: New sum $= \sum x_i + 4n = 15n + 4n = 19n$, so new mean $= \frac{19n}{n} = 19$, independent of n.

Why other options are wrong:

- (A) 15 ignores the shift entirely.
- (C) $60 = 15 \times 4$ multiplies by the constant instead of adding it.
- (D) $11 = 15 - 4$ subtracts instead of adding.

Final Answer: new mean $= 19 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q34

Q35.

Solution

Concept — Multiplication rule with conditional probability: For two events drawn in sequence, $P(A \cap B) = P(A)\,P(B \mid A)$. "Without replacement" means the first ball is not returned, so both the count of red balls and the total shrink by one for the second draw. We therefore multiply the first-draw probability by the *conditional* second-draw probability.

Step 1 — First draw red: There are 4 red out of $4 + 6 = 10$ balls, so

$$P(\text{1st red}) = \frac{4}{10}.$$

Step 2 — Second draw red, given the first was red: One red is gone, leaving 3

red out of 9 total:

$$P(\text{2nd red} \mid \text{1st red}) = \frac{3}{9}.$$

Step 3 — Multiply:

$$P(\text{both red}) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}.$$

Step 4 — Cross-check by combinations: The number of ways to pick 2 red from 4 over 2 from 10 is $\dfrac{\binom{4}{2}}{\binom{10}{2}} = \dfrac{6}{45} = \dfrac{2}{15}$, the same value.

Why other options are wrong:

- (A) $\frac{4}{25} = \left(\frac{2}{5}\right)^2$ assumes *with* replacement (second draw still $\frac{4}{10}$).
- (B) $\frac{2}{5} = \frac{4}{10}$ is only the first-draw probability.
- (D) $\frac{1}{5}$ comes from miscounting the second draw as $\frac{3}{6}$ or similar.

Final Answer: $P = \dfrac{2}{15} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q35

Q36.

Solution

Concept — Complement rule for "at least one": Computing "at least one head" directly would require summing the cases of exactly 1, 2 and 3 heads. It is far quicker to use the complement: $P(X \geq 1) = 1 - P(X = 0)$, since the only excluded outcome is "no heads at all". Each toss of a fair coin is independent with $P(\text{tail}) = \frac{1}{2}$.

Step 1 — List the sample space size: Three tosses give $2^3 = 8$ equally likely outcomes.

Step 2 — Probability of no heads: "No heads" means all three tosses are tails, with probability

$$P(X = 0) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Step 3 — Apply the complement rule:

$$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{1}{8} = \frac{7}{8}.$$

Step 4 — Cross-check by counting: Of the 8 outcomes only TTT has no head, so 7 of the 8 contain at least one head, giving $\frac{7}{8}$ directly.

Why other options are wrong:

- (A) $\frac{1}{8}$ is $P(X=0)$, the complement we subtract, not the answer.
- (B) $\frac{3}{8} = \binom{3}{2}\left(\frac{1}{2}\right)^3$ is $P(\text{exactly } 2 \text{ heads})$.
- (C) $\frac{1}{2}$ is $P(\text{exactly } 1 \text{ head}) = \binom{3}{1}\left(\frac{1}{2}\right)^3 = \frac{3}{8}$ mis-simplified, or a guess.

Final Answer: $P(X \geq 1) = \dfrac{7}{8} \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q36**

Q37.

Solution

Concept — Amplitude of $a \sin x + b \cos x$: Any expression of the form $a \sin x + b \cos x$ can be rewritten as $R \sin(x + \varphi)$ where $R = \sqrt{a^2 + b^2}$ and $\tan \varphi = \frac{b}{a}$. Since $\sin(x + \varphi)$ ranges over $[-1, 1]$, the expression ranges over $[-R, R]$. Hence its maximum value is $\sqrt{a^2 + b^2}$ and its minimum is $-\sqrt{a^2 + b^2}$.

Step 1 — Identify the coefficients: For $3 \sin x + 4 \cos x$ we have $a = 3$ and $b = 4$.

Step 2 — Compute the amplitude:

$$R = \sqrt{a^2 + b^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Step 3 — State the maximum: The maximum value of $3 \sin x + 4 \cos x$ is $R = 5$, attained when $\sin(x + \varphi) = 1$.

Why other options are wrong:

- (B) $7 = 3 + 4$ adds the coefficients instead of taking $\sqrt{a^2 + b^2}$.
- (C) 1 is the maximum of $\sin$ alone, ignoring the amplitudes.
- (D) $25 = a^2 + b^2$ forgets to take the square root.

Final Answer: maximum $= 5 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q37**

Q38.

Solution

Concept — Double-angle identity: One standard form of the cosine double-angle formula is $\cos 2\theta = 1 - 2\sin^2\theta$. It expresses $\cos 2\theta$ purely in terms of $\sin\theta$, so once $\sin\theta$ is known the value follows by direct substitution, without needing θ itself.

Step 1 — Substitute $\sin\theta = \frac{1}{2}$:

$$\cos 2\theta = 1 - 2\left(\frac{1}{2}\right)^2.$$

Step 2 — Evaluate the square:

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}, \qquad 2\cdot\frac{1}{4} = \frac{1}{2}.$$

Step 3 — Simplify:

$$\cos 2\theta = 1 - \frac{1}{2} = \frac{1}{2}.$$

Step 4 — Cross-check: $\sin\theta = \frac{1}{2}$ corresponds to $\theta = 30^\circ$, so $2\theta = 60^\circ$ and $\cos 60^\circ = \frac{1}{2}$, matching exactly.

Why other options are wrong:

- (A) $\frac{1}{4}$ stops at $\sin^2\theta = \frac{1}{4}$ without applying the full formula.
- (B) $-\frac{1}{2}$ flips a sign, e.g. using $2\sin^2\theta - 1$.
- (D) 1 drops the $-2\sin^2\theta$ term entirely.

Final Answer: $\cos 2\theta = \frac{1}{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q38

Q39.

Solution

Concept — Principal value branch of $\cos^{-1}$: The cosine function is many-to-one, so to invert it we restrict its domain to one interval on which it is one-to-one. The standard choice is $[0,\pi]$, on which $\cos$ decreases from 1 to -1 and covers every value in $[-1,1]$ exactly once. Therefore the principal values of $\cos^{-1}x$ lie in the *closed* interval $[0,\pi]$, and the endpoints are included: $\cos^{-1}1 = 0$ and $\cos^{-1}(-1) = \pi$.

Step 1 — State the domain and range: For every $x \in [-1, 1]$, the output satisfies

$$\cos^{-1} x \in [0, \pi].$$

Step 2 — Check the endpoints: $\cos^{-1}(1) = 0$ and $\cos^{-1}(-1) = \pi$ are both attained, confirming the interval is closed at both ends.

Why other options are wrong:

- (A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is the principal range of $\sin^{-1}$, not $\cos^{-1}$.
- (C) $(0, \pi)$ wrongly excludes the endpoints 0 and π, which are actually achieved.
- (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is both the wrong interval and open.

Final Answer: range $= [0, \pi] \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q39

Q40.

Solution

Concept — Heights and distances: The line of sight to the top of the pole, the horizontal ground, and the vertical pole form a right triangle with the right angle at the foot of the pole. The angle of elevation is the angle at the observer, the pole is the side opposite it, and the ground distance is the adjacent side. Hence $\tan(\text{elevation}) = \dfrac{\text{height (opposite)}}{\text{horizontal distance (adjacent)}}$.

Step 1 — Set up the equation: With elevation $60°$, height h and base 10 m,

$$\tan 60° = \frac{h}{10}.$$

Step 2 — Insert the known value: Since $\tan 60° = \sqrt{3}$,

$$\sqrt{3} = \frac{h}{10}.$$

Step 3 — Solve for h:

$$h = 10\sqrt{3}\text{ m} \approx 17.3\text{ m}.$$

Why other options are wrong:

- (A) 10 m uses $\tan 45° = 1$, the wrong angle.

- (B) $\frac{10}{\sqrt{3}}$ m inverts the ratio, solving $\tan 60^\circ = \frac{10}{h}$.
- (C) 20 m would need $\tan 60^\circ = 2$, which is false.

Final Answer: height $= 10\sqrt{3}$ m $\Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q40**

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	A	4	D	5	A
6	C	7	B	8	D	9	B	10	A
11	C	12	D	13	A	14	B	15	C
16	D	17	B	18	A	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	C
31	B	32	D	33	A	34	B	35	C
36	D	37	A	38	C	39	B	40	D