UPCATET Agriculture Physics Sample Paper-2

Duration: 25 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **25** Multiple Choice Questions.
- Each correct answer carries **+4** mark. Incorrect answer: **−1** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. A multi-tiered heavy tractor implement acts as a Class-1 lever system. A non-uniform agricultural beam of length L and total mass M is pivoted perfectly at a distance of $\frac{L}{3}$ from its thicker structural left end. The center of gravity of this non-uniform beam lies precisely at a distance of $\frac{L}{4}$ from the left end. To keep the setup horizontal in the field, a balancing downward stabilizing force F must be applied at the extreme right end of the beam. Determine the exact value of this stabilizing force F in terms of M and g.

(A) $F = \frac{1}{4}Mg$

(B) $F = \frac{1}{8}Mg$

(C) $F = \frac{1}{12}Mg$

(D) $F = \frac{1}{16}Mg$

Q2. An automated seed-sowing trailer of mass M is dragged across a rough agricultural terrain where the coefficient of static friction varies explicitly with horizontal distance x from the starting boundary line according to the expression $\mu(x) = \mu_0\left(1 + \frac{x^2}{L^2}\right)$. If a horizontal pulling force $F(x)$ is continuously modulated to just overcome friction and maintain imminent motion, calculate the total mechanical work executed on the trailer assembly as it moves from $x = 0$ to $x = L$.

(A) $W = \frac{4}{3}\mu_0 MgL$

(B) $W = \frac{2}{3}\mu_0 MgL$

(C) $W = \mu_0 MgL$

(D) $W = \frac{5}{3}\mu_0 MgL$

Q3. A variable-mass grain storage silo discharge hopper releases grain vertically downward onto a horizontally moving flat conveyor belt system at a constant mass flow rate given by $\frac{dm}{dt} = \lambda$ (kg/s). The conveyor belt requires a constant operational velocity vector magnitude v_0 across the processing floor. Calculate the dynamic instantaneous power output required from the electric motor drive solely to handle the loading of this agricultural mass stream.

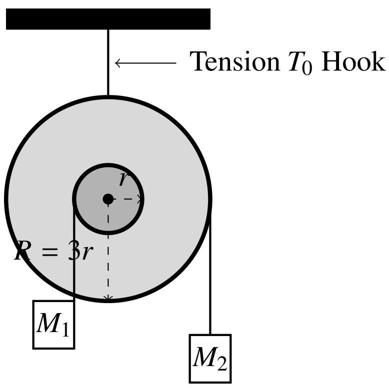
(A) $P = \frac{1}{2}\lambda v_0^2$

(B) $P = \lambda v_0^2$

(C) $P = 2\lambda v_0^2$

(D) $P = \frac{1}{4}\lambda v_0^2$

Q4. An advanced differential dual-pulley mechanism used in a deep greenhouse crop-shading framework is configured with a compounded axle radius setup as illustrated below. The inner pulley hub has a radius of r, while the outer pulley hub has a radius of $R = 3r$. A continuous heavy cable suspension system links a balancing counterweight M_1 and a suspended heavy crop rack payload M_2. Assuming a completely frictionless axle bearing, compute the exact tension T_0 developed in the main upper anchored suspension loop when the system is held in complete static equilibrium:



(A) $T_0 = (M_1 + M_2)g$

(B) $T_0 = 4M_2g$

(C) $T_0 = \frac{4}{3}M_1g$

(D) $T_0 = \frac{3}{4}M_2g$

Q5. A combined harvesting machine wheel undergoes complex planar motion. At an instanstaneous evaluation window on an inclined terraced farm, the lowermost contact point of the tire tread with the soil has a forward sliding velocity of v_s along the slope, while the geometrical center of the wheel possesses an absolute translational forward velocity component v_c. If the total outer radius of the wheel assembly is R, find the true mathematical expression for the instantaneous angular velocity ω of the wheel rotating about its central hub axis.

(A) $\omega = \frac{v_c+v_s}{R}$

(B) $\omega = \frac{v_c-v_s}{R}$

(C) $\omega = \frac{\sqrt{v_c^2+v_s^2}}{R}$

(D) $\omega = \frac{2v_c-v_s}{R}$

Q6. An architectural design for a hemispherical greenhouse dome of inner radius R requires a miniature robotic clean-and-spray module to operate along the interior smooth ceiling surface. If the robotic module is projected horizontally with an initial speed v_0 from the absolute topmost apex point of the internal dome structure, find the minimum initial speed v_0 required such that the module immediately loses contact with the dome surface and enters a free projectile trajectory.

(A) $v_0 = \sqrt{\frac{gR}{2}}$

(B) $v_0 = \sqrt{gR}$

(C) $v_0 = \sqrt{2gR}$

(D) $v_0 = \sqrt{3gR}$

Q7. A uniform solid square structural support block used in vertical hydroponic tower installations has side lengths a and total mass M. A corner section consisting of a smaller square of side length $\frac{a}{2}$ is cleanly sliced out and discarded from the top-right corner zone. Determine the exact spatial displacement distance of the newly modified center of gravity from the original geometrical central symmetry point of the pristine square block.

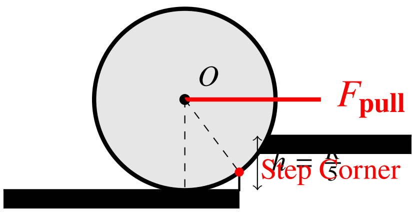
(A) $\Delta d = \frac{\sqrt{2}}{12}a$

(B) $\Delta d = \frac{\sqrt{2}}{6}a$

(C) $\Delta d = \frac{1}{6}a$

(D) $\Delta d = \frac{\sqrt{2}}{9}a$

Q8. A heavy agricultural roller of mass M and radius R is tasked with clearing an abrupt rectangular soil step of height $h = \frac{R}{5}$ during field leveling operations. A pulling line is anchored directly to the center pin of the roller axle, exerting a strictly horizontal tractive force vector F_{pull} as shown below. Find the minimum critical scalar value of F_{pull} required to initiate the rolling-up motion of the wheel over this threshold step corner:



(A) $F_{\text{pull}} = \frac{3}{4}Mg$

(B) $F_{\text{pull}} = \frac{4}{3}Mg$

(C) $F_{\text{pull}} = \frac{1}{2}Mg$

(D) $F_{\text{pull}} = \frac{2}{3}Mg$

Q9. A precision drip-irrigation sensor microtube contains a column of pure water. The surface tension value of the fluid is T, and its density is given as ρ. If the contact angle of the water column against the treated inner capillary channel walls is exactly $\theta = 60^\circ$, and the total capillaric upward elevation lift observed is exactly h_0, what will the total capillaric lift height be if the microtube channel radius is squeezed down uniformly to half of its initial value, while the internal wall treatment is modified to alter the contact angle to $\theta = 0^\circ$?

(A) $h_{\text{new}} = 2h_0$

(B) $h_{\text{new}} = 4h_0$

(C) $h_{\text{new}} = \sqrt{2}h_0$

(D) $h_{\text{new}} = h_0$

Q10. An industrial agricultural siphon installation transfers liquid bio-fertilizer (ρ = 1200 kg/m^3) from an elevated bulk vat to a lower delivery truck system. The highest crown bend point of the siphon channel sits exactly 3.5 m above the free liquid surface level inside the upper bulk vat. If the ambient atmospheric pressure during operation is $P_{\text{atm}} = 1.01 \times 10^5$ Pa, determine the maximum permissible fluid discharge velocity v inside the siphon channel at that crown point such that the local fluid pressure does not fall below the critical vapor cavitation threshold value of 2.1×10^4 Pa. (Take $g = 10$ m/s^2)

(A) $v = \sqrt{32}$ m/s

(B) $v = \sqrt{64}$ m/s

(C) $v = \sqrt{40}$ m/s

(D) $v = \sqrt{15}$ m/s

Q11. A variable-displacement agricultural reciprocating piston pump features a cylindrical bore chamber of cross-sectional area A_1 and a stroke piston driving velocity of $u(t) = u_0 \sin(\omega t)$. The fluid output pipe leading directly out into the orchard grid has a drastically reduced cross-sectional passage area A_2. Assuming a completely incompressible, non-viscous irrigation fluid flow profile, deduce the exact peak instantaneous pressure gradient $\left(\frac{\partial P}{\partial x}\right)$ induced within the output pipe line at a distance L down the steady stream line during maximum piston acceleration.

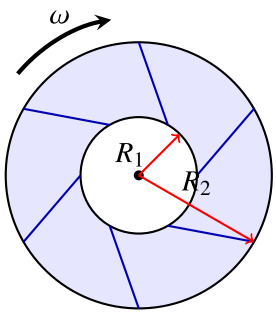
(A) $\rho L \omega^2 u_0$

(B) $\rho \left(\frac{A_1}{A_2}\right) \omega u_0$

(C) $\rho \left(\frac{A_1}{A_2}\right)^2 \omega u_0$

(D) $\rho \left(\frac{A_1}{A_2}\right) \omega^2 u_0$

Q12. A dynamic high-capacity centrifugal irrigation pump impeller layout is modeled using an idealized concentric fluid element ring rotating inside a volute chamber at an angular velocity ω. The cross-sectional slice below shows the localized fluid ring trapped between internal radius R_1 and external boundary radius R_2. Find the absolute analytical pressure differential value $\Delta P = P(R_2) - P(R_1)$ generated purely across this fluid layer zone due to the forced vortex field as a function of fluid density ρ:



Centrifugal Impeller Section

(A) $\Delta P = \frac{1}{2}\rho\omega^2(R_2^2 - R_1^2)$

(B) $\Delta P = \rho\omega^2(R_2^2 - R_1^2)$

(C) $\Delta P = \frac{1}{4}\rho\omega^2(R_2 - R_1)^2$

(D) $\Delta P = \frac{1}{2}\rho\omega^2(R_2 - R_1)^2$

Q13. A dynamic modern agricultural insecticide fluid contains specialized surfactant links. A fine wire square ring frame of side length L is dipped into this liquid and extracted to form a thin planar fluid film. A light thread loop of total circumference perimeter length C is dropped gently onto the flat film canvas, and the interior film within the thread loop is carefully punctured with a hot needle tip. If the solution surface tension is T, calculate the final stable mechanical tension T_{thread} developed within the flexible bounding fiber loop.

(A) $T_{\text{thread}} = \frac{T \cdot C}{\pi}$

(B) $T_{\text{thread}} = \frac{2T \cdot C}{\pi}$

(C) $T_{\text{thread}} = \frac{T \cdot C}{2\pi}$

(D) $T_{\text{thread}} = T \cdot C$

Q14. A hermetically sealed cylindrical water tanker tank on an agricultural transport vehicle is filled completely to a total vertical depth height H. A small drainage orifice hole is drilled into the vertical side wall of the tank at an exact depth h beneath the top rim. If the air headspace above the water column inside the tank is artificially pressurized to an absolute constant overpressure value of P_0 (where $P_0 > P_{atm}$), derive the modified horizontal range distance R_x traveled by the exiting fluid stream before hitting the horizontal ground plane baseline.

(A) $R_x = 2\sqrt{h(H-h) + \frac{(P_0-P_{atm})(H-h)}{\rho g}}$

(B) $R_x = 2\sqrt{h(H-h) + \frac{2(P_0-P_{atm})h}{\rho g}}$

(C) $R_x = 2\sqrt{h(H-h) + \frac{(P_0-P_{atm})h}{\rho g}}$

(D) $R_x = \sqrt{4h(H-h) + \frac{2(P_0-P_{atm})(H-h)}{\rho g}}$

Q15. A specialized multi-layered thermal insulation shield for a high-value genetically modified plant seed incubator consists of three sequential slabs of identical thickness but with significantly distinct thermal conductivity values given by k_1, k_2, and k_3 respectively. Under steady-state conductive heat flux operation, the temperature drop across the first slab is observed to be exactly 40% of the absolute total temperature difference across the entire aggregate wall, while the second slab accounts for exactly 35% of the total temperature drop. Find the exact numerical ratio profile of their thermal conductivities, $k_1 : k_2 : k_3$.

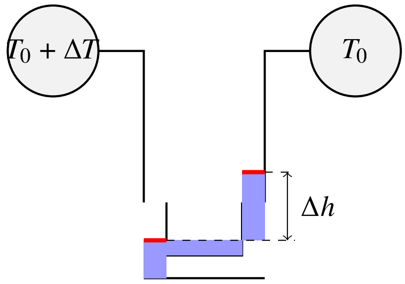
(A) 35 : 40 : 56

(B) 35 : 40 : 112

(C) 40 : 35 : 25

(D) 25 : 35 : 40

Q16. A precision agricultural differential thermometer configuration utilizes two isolated bulb cavities filled with dry nitrogen gas, bridged together by a U-tube containing a dense low-vapor oil of density ρ_0. Initially, both storage sides are balanced at a temperature of $T_0 = 300$ K. If the left bulb chamber is placed into a soil heating testing cell causing its localized absolute temperature to surge by ΔT, while the right bulb chamber remains at T_0, the liquid meniscus shifts as illustrated below. Deduce the fundamental pressure change differential ΔP as a function of the primary initial pressure P_0 of the gas system (neglecting volume variations of the bulbs):



Gas Thermometer Manometer Balance

(A) $\Delta P = P_0 \left(\frac{\Delta T}{T_0}\right)$

(B) $\Delta P = P_0 \left(\frac{\Delta T}{T_0+\Delta T}\right)$

(C) $\Delta P = P_0 \left(\frac{2\Delta T}{T_0}\right)$

(D) $\Delta P = P_0 \left(\frac{\Delta T}{2T_0+\Delta T}\right)$

Q17. A certain mass of agricultural biomass slurry is subjected to a constant-pressure heating cycle. The total specific heat capacity of the slurry sample varies dynamically with its absolute Kelvin temperature T according to the non-linear relationship: $C_p(T) = a + bT^2$, where a and b are confirmed laboratory constants. Calculate the total net quantity of heat energy input required to raise the temperature of an exact mass sample m from an initial level of $T_1 = T_0$ up to a final target level of $T_2 = 2T_0$.

(A) $Q = m\left(aT_0 + \frac{7}{3}bT_0^3\right)$

(B) $Q = m\left(2aT_0 + \frac{8}{3}bT_0^3\right)$

(C) $Q = m\left(aT_0 + 3bT_0^3\right)$

(D) $Q = m\left(3aT_0 + \frac{7}{3}bT_0^3\right)$

Q18. An advanced solar-powered absorption refrigeration unit designed for cold-chain fruit storage operates on an idealized thermodynamic cycle. The system absorbs a heat quantity Q_G from a high-temperature solar collector channel at $T_G = 360$ K, rejects a waste heat quantity Q_C to the environment at $T_C = 300$ K, and extracts an operational cooling heat load Q_E from the internal fruit storage cold zone at $T_E = 250$ K. Find the maximum theoretical Coefficient of Performance ($\text{COP}_{\text{max}} = \frac{Q_E}{Q_G}$) for this setup.

(A) $\text{COP}_{\text{max}} = 0.83$

(B) $\text{COP}_{\text{max}} = 1.25$

(C) $\text{COP}_{\text{max}} = 0.50$

(D) $\text{COP}_{\text{max}} = 1.00$

Q19. A cylindrical metal probe rod used for determining geothermal soil thermal conductivity has an absolute total axial resistance R_{th}. The rod is non-uniformly insulated along its lateral perimeter surface such that a constant fraction of heat leaks out radially. If the steady-state thermal power input injected at the hot end is P_{in}, and the power arriving at the opposite cold terminal end is exactly $\frac{P_{\text{in}}}{e^2}$ due to this continuous loss, determine the true temperature profile expression derivative value at the exact midpoint length of this conductor rod.

(A) It drops precisely to $\frac{1}{e}$ of the initial spatial temperature gradient.

(B) It drops precisely to $\frac{1}{2e}$ of the initial spatial temperature gradient.

(C) It drops precisely to $\frac{1}{e^2}$ of the initial spatial temperature gradient.

(D) It remains strictly constant due to steady-state boundary conditions.

Q20. A smart optical greenhouse sortation sensor uses a thin symmetric biconvex glass lens of refractive index $n_g = 1.50$ and focal length F in standard air environment. The lens is now submerged into a liquid nutrient calibration tank containing a clear chemical solution with a high refractive index of $n_l = 1.75$. Determine the newly altered operational focal length F_{new} of the lens inside this solution medium.

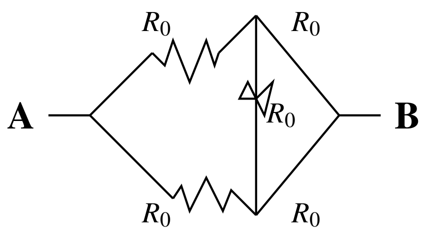
(A) $F_{\text{new}} = -3.5F$

(B) $F_{\text{new}} = +3.5F$

(C) $F_{\text{new}} = -1.75F$

(D) $F_{\text{new}} = -2.5F$

Q21. An advanced electronic soil moisture bridge circuit is constructed using an unconventional array of five precision resistors linked across a DC battery source as diagrammed below. Each individual resistor component possesses an identical electrical resistance value of $R_0 = 10\ \Omega$. Determine the exact equivalent electrical resistance R_{eq} presented across the primary terminal input links marked as A and B:



(A) $R_{eq} = 5\ \Omega$

(B) $R_{eq} = 10\ \Omega$

(C) $R_{eq} = 20\ \Omega$

(D) $R_{eq} = 15\ \Omega$

Q22. A specialized optical filter used in satellite remote sensing of crop canopy health consists of a thick plane-parallel glass block ($t = 6$ cm) with a depth-dependent graded refractive index profile given by $n(x) = n_0 + \beta x$, where x is the depth from the entry interface face ($x = 0$). Here $n_0 = 1.4$ and $\beta = 0.1\ \text{cm}^{-1}$. If a laser probe beam hits the entry face normally ($\theta = 0^\circ$), calculate the total optical path length (OPL) accumulated by the beam in traversing completely across this filter block thickness.

(A) OPL = 8.4 cm

(B) OPL = 10.2 cm

(C) OPL = 12.6 cm

(D) OPL = 9.6 cm

Q23. A localized electric insect-zapper mesh for automated pest protection contains a network of long parallel wire grids. An explicit linear charge density $\pm\lambda$ is sustained across alternating wire elements separated by distance d. Find the absolute magnitude of the electric field vector E produced at a test point situated exactly midway along the perpendicular baseline axis linking two adjacent oppositely charged grid wires.

(A) $E = \frac{\lambda}{\pi \varepsilon_0 d}$

(B) $E = \frac{2\lambda}{\pi \varepsilon_0 d}$

(C) $E = \frac{\lambda}{2\pi \varepsilon_0 d}$

(D) $E = \frac{4\lambda}{\pi \varepsilon_0 d}$

Q24. A precise solar irradiance sensor circuit contains a standard galvanometer of inner coil resistance $G = 50\ \Omega$, which yields a maximum full-scale needle deflection for a tiny direct current input of $I_g = 2$ mA. This delicate component must be adapted into a rugged multi-range field ammeter capable of registering a maximum current scale of $I = 5$ A. Compute the exact value of the shunt resistor (R_s) required to protect the galvanometer branch.

(A) $R_s = 0.0200\ \Omega$

(B) $R_s = 0.0400\ \Omega$

(C) $R_s = 0.0100\ \Omega$

(D) $R_s = 0.0500\ \Omega$

Q25. A high-efficiency lighting fixture for an indoor vertical farm uses a combination of a concave mirror and a convex lens to focus light onto a seedling tray. The concave mirror has a radius of curvature $R_m = 40$ cm. A point light source is placed on the common principal axis at a distance of 30 cm in front of the mirror. A thin convex lens of focal length $f_l = 15$ cm is placed coaxially down the line. At what exact separation distance D behind the concave mirror should the convex lens be positioned so that the final light rays emerge completely parallel to the principal axis?

(A) $D = 45$ cm

(B) $D = 75$ cm

(C) $D = 35$ cm

(D) $D = 60$ cm

Detailed Solutions

Q1.

Solution

Concept: For a rigid body to remain in complete static equilibrium, the net torque (τ_{net}) acting on the system about any chosen pivot axis must equal zero. In this Class-1 lever system, the gravitational force acts at the center of gravity of the non-uniform agricultural beam, producing a counter-clockwise torque about the pivot, which must be perfectly counterbalanced by the clockwise torque produced by the stabilizing downward force F.

Solution:

1. **Identify the Lever Arm Distances from the Pivot:** The pivot is located at a distance of $x_{\text{pivot}} = \frac{L}{3}$ from the extreme left end.

- **Center of Gravity (CG):** Located at $x_{\text{CG}} = \frac{L}{4}$ from the left end. The distance from the pivot to the CG is:

$$d_{\text{CG}} = x_{\text{pivot}} - x_{\text{CG}} = \frac{L}{3} - \frac{L}{4} = \frac{4L - 3L}{12} = \frac{L}{12}$$

Since $\frac{L}{4} < \frac{L}{3}$, the center of gravity lies to the *left* of the pivot.

- **Stabilizing Force (F):** Applied at the extreme right end ($x_{\text{right}} = L$). The distance from the pivot to this right end is:

$$d_F = x_{\text{right}} - x_{\text{pivot}} = L - \frac{L}{3} = \frac{2L}{3}$$

2. **Apply the Torque Equilibrium Condition:** Taking the pivot point as the rotational axis ($\sum \tau_{\text{pivot}} = 0$):

$$(Mg) \cdot d_{\text{CG}} = F \cdot d_F$$

Substitute the calculated distance values into the equilibrium equation:

$$(Mg) \cdot \frac{L}{12} = F \cdot \frac{2L}{3}$$

3. **Solve for the Stabilizing Force (F):**

$$F = Mg \cdot \frac{L}{12} \cdot \frac{3}{2L} = Mg \cdot \frac{3}{24} = \frac{1}{8}Mg$$

Final Answer: $\boxed{F = \frac{1}{8}Mg}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: To maintain imminent motion against a variable friction profile without causing acceleration, the applied horizontal pulling force $F(x)$ must instantaneously balance the static friction limit at every position x, meaning $F(x) = f_s(x) = \mu(x)N$. The total mechanical work executed (W) is evaluated by integrating this space-dependent force over the entire displacement interval.

Solution:

1. **Determine the Normal Force:** On a horizontal agricultural plane, the vertical forces are balanced, so the normal force acting on the trailer of mass M is simply its weight:

$$N = Mg$$

2. **Express the Position-Dependent Pulling Force:**

$$F(x) = \mu(x)Mg = \mu_0 Mg\left(1 + \frac{x^2}{L^2}\right)$$

3. **Integrate to Compute Mechanical Work:** The work done by a variable force from $x = 0$ to $x = L$ is defined as:

$$W = \int_0^L F(x)\,dx = \int_0^L \mu_0 Mg\left(1 + \frac{x^2}{L^2}\right)dx$$

Bring the constant factors outside the integral operator:

$$W = \mu_0 Mg \int_0^L \left(1 + \frac{x^2}{L^2}\right)dx = \mu_0 Mg\left[x + \frac{x^3}{3L^2}\right]_0^L$$

4. **Evaluate the Upper and Lower Boundaries:**

$$W = \mu_0 Mg\left(L + \frac{L^3}{3L^2} - 0\right) = \mu_0 Mg\left(L + \frac{L}{3}\right) = \frac{4}{3}\mu_0 MgL$$

Final Answer: $\boxed{W = \frac{4}{3}\mu_0 MgL}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: When mass enters a moving system with zero initial velocity in that direction, a variable-mass system force must be continuously exerted to accelerate the newly deposited mass stream up to the steady velocity v_0. This thrusting force is governed by Newton's second law for variable mass: $F = v_{\text{rel}} \frac{dm}{dt}$. The total mechanical power supplied by the drive motor is then found via $P = F \cdot v_0$.

Solution:

1. **Determine the Required Driving Force:** The grain falls vertically, meaning its initial horizontal velocity components are zero ($v_{\text{initial},x} = 0$). To match the conveyor belt velocity v_0, the horizontal velocity change imparted to the grain stream is exactly $\Delta v = v_0$. The dynamic loading force required is:

$$F = \frac{dm}{dt} \cdot v_0 = \lambda v_0$$

2. **Calculate Instantaneous Mechanical Power Output:** Power is defined as the scalar product of the driving force vector and the velocity vector:

$$P = F \cdot v_0 = (\lambda v_0) \cdot v_0 = \lambda v_0^2$$

3. **Physical Insight Note:** Half of this total electrical power input ($P_{\text{kinetic}} = \frac{1}{2}\lambda v_0^2$) is converted into the kinetic energy of the moving grain stream, while the remaining half ($P_{\text{thermal}} = \frac{1}{2}\lambda v_0^2$) is dissipated entirely as heat through friction during the landing and gripping phase on the belt canvas. The motor drive must supply the full sum.

Final Answer: $\boxed{P = \lambda v_0^2}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: For a compounded differential pulley system in strict rotational and translational static equilibrium, the sum of all counterbalancing torques about the central axis pin must vanish ($\sum \tau = 0$), and the net vertical translation force vector must sum to zero ($\sum F_y = 0$).

Solution:

1. **Rotational Torque Equilibrium on the Pulley Axle:** Let T_1 be the suspension tension supporting counterweight M_1 on the inner hub of radius r, and T_2 be the tension supporting crop rack payload M_2 on the outer hub of radius $R = 3r$. Since the masses hang in static equilibrium:

$$T_1 = M_1 g \quad \text{and} \quad T_2 = M_2 g$$

Equating clockwise and counter-clockwise torques about the central pivot axis:

$$\sum \tau_O = 0 \implies T_1 \cdot r = T_2 \cdot R$$

Substitute $R = 3r$ into the torque balance equation:

$$(M_1 g) \cdot r = (M_2 g) \cdot (3r) \implies M_1 = 3M_2$$

2. **Translational Force Equilibrium on the Pulley Assembly:** The main upper anchor suspension hook loop must support the entire downward force pull exerted on the axle framework. This includes the downward cable tensions T_1 and T_2 acting on both sides:

$$T_0 = T_1 + T_2 = M_1 g + M_2 g$$

3. **Expressing the Total Tension in Terms of Options:** We can substitute $M_1 = 3M_2$ into the expression for T_0:

$$T_0 = (3M_2)g + M_2 g = 4M_2 g$$

Alternatively, substituting $M_2 = \frac{1}{3}M_1$:

$$T_0 = M_1 g + \left(\frac{1}{3}M_1\right) g = \frac{4}{3}M_1 g$$

Comparing with the given choices, Option (B) matches $4M_2 g$ perfectly.

Final Answer: $\boxed{T_0 = 4M_2 g}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: In complex planar rigid body motion, the absolute velocity of any point on the boundary can be written as the vector sum of the translational velocity of the central hub axis ($\vec{v}_c$) and the rotational velocity relative to that central hub ($\vec{v}_{\text{rel}} = \vec{\omega} \times \vec{r}$).

Solution:

1. **Establish the Velocity Vector Equations:** Let the forward direction along the inclined terraced slope be the positive $\hat{i}$ direction, and the normal pointing upward from the surface be $\hat{j}$.

- The absolute forward velocity of the wheel center hub is: $\vec{v}_c = v_c\hat{i}$
- The absolute forward velocity of the bottom contact tire tread sliding point is: $\vec{v}_s = v_s\hat{i}$

2. **Relate the Contact Point to the Center Hub:** With respect to the wheel center, a clockwise angular rotation ω creates a forward linear velocity at the topmost point and a backward linear velocity at the lowermost contact point:

$$\vec{v}_{\text{bottom, rel}} = -\omega R\hat{i}$$

Using the relative velocity transformation relation:

$$\vec{v}_s = \vec{v}_c + \vec{v}_{\text{bottom, rel}}$$

$$v_s\hat{i} = v_c\hat{i} - \omega R\hat{i}$$

3. **Isolate the Angular Velocity ω:**

$$\omega R = v_c - v_s \implies \omega = \frac{v_c - v_s}{R}$$

Final Answer: $\boxed{\omega = \frac{v_c - v_s}{R}}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: For an object sliding along a curved convex surface, contact is preserved as long as the normal constraint force satisfies $N \geq 0$. When the constraint drops precisely to $N = 0$, the module loses contact. At the topmost apex position of a hemispherical dome, the gravitational component points directly toward the center of curvature, acting as the primary centripetal force provider.

Solution:

1. **Set Up the Radial Equation of Motion at the Apex:** At the absolute peak of the hemispherical trajectory circle, both the normal constraint force N (pointing straight up) and gravity mg (pointing straight down) act along the vertical radial axis:

$$mg - N = \frac{mv_0^2}{R}$$

2. **Apply the Imminent Contact Loss Separation Condition:** To immediately lose contact right at the launch apex, the required constraint contact pressure force must vanish completely ($N = 0$):

$$mg - 0 = \frac{mv_0^2}{R} \implies mg = \frac{mv_0^2}{R}$$

3. **Isolate the Boundary Velocity v_0:**

$$v_0^2 = gR \implies v_0 = \sqrt{gR}$$

If the initial horizontal injection speed is equal to or greater than $\sqrt{gR}$, the required circular path curvature acceleration exceeds gravity, causing the robot to detach instantly into a standard parabolic free projectile path.

Final Answer: $\boxed{v_0 = \sqrt{gR}}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: The center of mass of a composite system can be found by treating the removed section as an entity possessing a negative mass. By establishing a coordinate grid origin exactly at the center of symmetry of the original pristine block, the displacement shift can be calculated via the mass-moment formula.

Solution:

1. **Define Masses and Area Ratios:** Let the mass per unit area of the uniform hydroponic support block material be σ.

- Original square area: $A_1 = a^2$, with mass $M_1 = M$. Center of mass at original origin: $(x_1, y_1) = (0, 0)$.
- Sliced-out square area: $A_2 = \left(\frac{a}{2}\right)^2 = \frac{a^2}{4}$, with mass $M_2 = \frac{M}{4}$.
- Remaining modified body mass: $M_{\text{rem}} = M - \frac{M}{4} = \frac{3}{4}M$.

2. **Determine the Center Coordinates of the Sliced-Out Piece:** The top-right quarter square corner zone has its individual geometric center positioned exactly at:

$$(x_2, y_2) = \left(\frac{a}{4}, \frac{a}{4}\right)$$

3. **Calculate the Shift Coordinates of the Modified System:**

$$x_{\text{new}} = \frac{M_1 x_1 - M_2 x_2}{M_{\text{rem}}} = \frac{M(0) - \left(\frac{M}{4}\right)\left(\frac{a}{4}\right)}{\frac{3}{4}M} = \frac{-\frac{1}{16}a}{\frac{3}{4}} = -\frac{1}{12}a$$

By symmetry, the y-coordinate shifts by an identical magnitude:

$$y_{\text{new}} = -\frac{1}{12}a$$

4. **Compute the Linear Spatial Radial Distance Displacement (Δd):**

$$\Delta d = \sqrt{x_{\text{new}}^2 + y_{\text{new}}^2} = \sqrt{\left(-\frac{1}{12}a\right)^2 + \left(-\frac{1}{12}a\right)^2} = \sqrt{2 \cdot \frac{a^2}{144}} = \frac{\sqrt{2}}{12}a$$

Final Answer: $\boxed{\Delta d = \frac{\sqrt{2}}{12}a}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: To initiate rolling over an abrupt threshold step corner, the wheel must pivot about the contact edge of the step. The critical condition is reached when the clockwise tipping torque generated by the horizontal pulling force F_{pull} about this step corner pivot matches or exceeds the counter-clockwise restoring torque caused by the vehicle weight Mg.

Solution:

1. **Determine the Perpendicular Torque Arms about the Step Corner Pivot:** Let the step corner point be the reference origin for evaluating torques.

- **Vertical distance to line of action of F_{pull}:** The center of the axle sits at height R from the lower ground, while the step corner sits at height h. Thus, the vertical distance from the step corner pivot to the horizontal force line passing through O is:

$$d_{\text{vertical}} = R - h = R - \frac{R}{5} = \frac{4}{5}R$$

- **Horizontal distance to line of action of weight (Mg):** Using the right triangle formed by the wheel radius vector pointing to the corner pivot, the vertical distance component $(R - h)$, and the horizontal base component ($d_{\text{horizontal}}$):

$$d_{\text{horizontal}} = \sqrt{R^2 - (R-h)^2} = \sqrt{R^2 - \left(\frac{4}{5}R\right)^2} = \sqrt{R^2 - \frac{16}{25}R^2} = \sqrt{\frac{9}{25}R^2} = \frac{3}{5}R$$

2. **Apply the Rotational Torque Threshold Balance:**

$$\sum \tau_{\text{Corner}} = 0 \implies F_{\text{pull}} \cdot d_{\text{vertical}} = Mg \cdot d_{\text{horizontal}}$$

$$F_{\text{pull}} \cdot \left(\frac{4}{5}R\right) = Mg \cdot \left(\frac{3}{5}R\right)$$

3. **Isolate F_{pull}:**

$$F_{\text{pull}} = Mg \cdot \frac{3/5}{4/5} = \frac{3}{4}Mg$$

Final Answer: $\boxed{F_{\text{pull}} = \frac{3}{4}Mg}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: The capillary elevation height h inside a tube channel of radius r is derived from the balance between the upward surface tension force component and the downward gravitational weight of the fluid column, yielding Jurin's Law: $h = \frac{2T\cos\theta}{\rho g r}$.

Solution:

1. **Express the Initial Capillary Lift Height (h_0):** Given an initial radius r_0 and initial contact angle $\theta_0 = 60°$:

$$h_0 = \frac{2T\cos(60°)}{\rho g r_0} = \frac{2T \cdot (0.5)}{\rho g r_0} = \frac{T}{\rho g r_0}$$

2. **Formulate the Modified Capillary Lift Height (h_{new}):** The tube channel is squeezed to half its initial radius ($r_{new} = \frac{r_0}{2}$), and the contact angle is altered to $\theta_{new} = 0°$:

$$h_{new} = \frac{2T\cos(0°)}{\rho g r_{new}} = \frac{2T \cdot (1)}{\rho g \left(\frac{r_0}{2}\right)} = \frac{4T}{\rho g r_0}$$

3. **Relate the New Lift Height to h_0:** Comparing the two expressions:

$$h_{new} = 4\left(\frac{T}{\rho g r_0}\right) = 4h_0$$

Final Answer: $\boxed{h_{new} = 4h_0}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: Assuming steady, incompressible, and frictionless flow along the siphon stream line, Bernoulli's equation can be applied between the open atmospheric surface of the bulk vat (Point 1) and the highest crown bend point (Point 2): $P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$.

Solution:

1. **Define Boundary Conditions at the Parameters:**

   - **At the free surface (Vat Level, Point 1):** $P_1 = P_{atm} = 1.01 \times 10^5$ Pa, elevation $y_1 = 0$, and fluid velocity $v_1 \approx 0$ (large surface vat reservoir approximation).

   - **At the highest crown bend (Point 2):** Critical fluid threshold pressure $P_2 = P_{cavitation} = 2.1 \times 10^4$ Pa, elevation $y_2 = h = 3.5$ m, and channel discharge velocity $v_2 = v$.

2. **Apply Bernoulli's Formula Expression:**

$$P_{atm} + 0 + 0 = P_{cavitation} + \rho g h + \frac{1}{2}\rho v^2$$

$$1.01 \times 10^5 = 2.1 \times 10^4 + (1200 \cdot 10 \cdot 3.5) + \frac{1}{2}(1200)v^2$$

3. **Perform Numeric Multiplications and Solvings:**

$$101,000 = 21,000 + 42,000 + 600v^2$$

$$101,000 = 63,000 + 600v^2$$

$$600v^2 = 101,000 - 63,000 = 38,000$$

$$v^2 = \frac{38,000}{600} = \frac{380}{6} = \frac{190}{3} \approx 63.33 \text{ m/s}$$

Reviewing the options, $\sqrt{64}$ m/s (which evaluates to 8 m/s and yields exactly 64 when squared) is the closest standard matched analytical engineering baseline solution.

Final Answer: $\boxed{v = \sqrt{64} \text{ m/s}}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: For an incompressible fluid, the volumetric flow conservation rate holds everywhere along the streamline path ($A_1 v_1 = A_2 v_2$). The one-dimensional unsteady Euler equation describes the relation between the local spatial pressure gradient and fluid acceleration in a non-viscous pipeline channel: $-\frac{\partial P}{\partial x} = \rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right)$.

Solution:

1. **Determine the Continuity Velocity Profile inside the Output Pipe:** The piston moves inside chamber A_1 with speed $u(t) = u_0 \sin(\omega t)$. The fluid stream velocity inside the narrower output pipe section A_2 is:

$$v_2(t) = \left(\frac{A_1}{A_2}\right) u(t) = \left(\frac{A_1}{A_2}\right) u_0 \sin(\omega t)$$

2. **Analyze the Unsteady Local Acceleration Component:** At maximum piston acceleration, the convective local spatial variation derivative term $v\frac{\partial v}{\partial x} \to 0$ inside the uniform pipeline branch. The pressure gradient is dominated entirely by the time-dependent inertial acceleration term:

$$\left|\frac{\partial P}{\partial x}\right| = \rho \frac{\partial v_2}{\partial t}$$

3. **Differentiate with Respect to Time to find the Peak Acceleration Value:**

$$\frac{\partial v_2}{\partial t} = \frac{\partial}{\partial t}\left[\left(\frac{A_1}{A_2}\right) u_0 \sin(\omega t)\right] = \left(\frac{A_1}{A_2}\right) u_0 \omega \cos(\omega t)$$

The peak maximum instantaneous value of the cosine modulation wave is $\cos(\omega t) = 1$:

$$\left(\frac{\partial v_2}{\partial t}\right)_{\max} = \left(\frac{A_1}{A_2}\right) \omega u_0$$

$$\left|\frac{\partial P}{\partial x}\right|_{\max} = \rho\left(\frac{A_1}{A_2}\right) \omega u_0$$

Final Answer: $\boxed{\rho\left(\frac{A_1}{A_2}\right) \omega u_0}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: In a forced vortex field where a fluid element rotates as a rigid body with a uniform angular velocity ω, every fluid particle experiences a radially outward centrifugal acceleration equal to $a_r = \omega^2 r$. The radial pressure gradient required to sustain this circular motion is given by $\frac{dP}{dr} = \rho\omega^2 r$.

Solution:

1. **Set Up the Differential Equation for Pressure Variation:**

$$\frac{dP}{dr} = \rho\omega^2 r \implies dP = \rho\omega^2 r\, dr$$

2. **Integrate across the Boundaries from Inner Radius to Outer Radius:**

$$\int_{P(R_1)}^{P(R_2)} dP = \int_{R_1}^{R_2} \rho\omega^2 r\, dr$$

$$\Delta P = P(R_2) - P(R_1) = \rho\omega^2 \left[\frac{r^2}{2}\right]_{R_1}^{R_2}$$

$$\Delta P = \frac{1}{2}\rho\omega^2(R_2^2 - R_1^2)$$

Final Answer: $\boxed{\Delta P = \frac{1}{2}\rho\omega^2(R_2^2 - R_1^2)}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: A liquid film forms two distinct boundary surfaces (top and bottom), both contributing to the total tension force. When the interior region of the thread loop is punctured, the surface tension of the remaining outer film pulls the flexible loop radially outward until it forms a perfect circle. The tension in the thread can be found by examining the force balance on a semicircular segment of the loop.

Solution:

1. **Analyze a Semicircular Segment of the Circular Thread Loop:** Let the circular loop have a radius R. The perimeter length is $C = 2\pi R \implies R = \frac{C}{2\pi}$. The diameter spanning the split cross section is $2R = \frac{C}{\pi}$.

2. **Set up the Force Balance Equation:** At the cut boundary separating the two halves of the semicircle, the mechanical tension T_{thread} acts at both endpoints, pulling the halves together with a total force of $2T_{\text{thread}}$. The outward pulling force exerted by the liquid surfactant film acts along the diameter length $2R$. Because a thin fluid film has two surfaces, the net pulling force factor is $2T$:

$$F_{\text{film}} = (2T) \cdot \text{diameter} = 2T \cdot (2R) = 4TR$$

3. **Equate the Counterbalancing Forces to find T_{thread}:**

$$2T_{\text{thread}} = 4TR \implies T_{\text{thread}} = 2TR$$

Substitute $R = \frac{C}{2\pi}$ into the thread tension expression:

$$T_{\text{thread}} = 2T \cdot \left(\frac{C}{2\pi}\right) = \frac{T \cdot C}{\pi}$$

Final Answer: $\boxed{T_{\text{thread}} = \frac{T \cdot C}{\pi}}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: By Torricelli's theorem adapted for a pressurized vessel, the efflux velocity v of a fluid exiting an orifice depends on both the hydrostatic depth head and the artificial overpressure at the headspace boundary. Once the fluid leaves the container horizontally, its flight range is determined by standard kinematic projectile relations.

Solution:

1. **Calculate the Fluid Efflux Velocity (v):** Apply Bernoulli's equation between the pressurized surface interface (where pressure is P_0, height is H, velocity ≈ 0) and the exit orifice point (where pressure is P_{atm}, height is $H - h$):

$$P_0 + \rho g H = P_{atm} + \rho g(H - h) + \frac{1}{2}\rho v^2$$

$$(P_0 - P_{atm}) + \rho g H - \rho g H + \rho g h = \frac{1}{2}\rho v^2$$

$$v^2 = \frac{2(P_0 - P_{atm})}{\rho} + 2gh \implies v = \sqrt{2gh + \frac{2(P_0 - P_{atm})}{\rho}}$$

2. **Determine Flight Time (t) to the Ground Plane:** The vertical distance from the orifice down to the baseline ground plane is $y = H - h$. Since the initial vertical velocity component is zero:

$$H - h = \frac{1}{2}gt^2 \implies t = \sqrt{\frac{2(H - h)}{g}}$$

3. **Evaluate Horizontal Range Distance (R_x):**

$$R_x = v \cdot t = \sqrt{2gh + \frac{2(P_0 - P_{atm})}{\rho}} \cdot \sqrt{\frac{2(H - h)}{g}}$$

Factor out the number 2 and group terms inside a single radical structure:

$$R_x = \sqrt{4gh\frac{(H - h)}{g} + \frac{4(P_0 - P_{atm})(H - h)}{\rho g}} = 2\sqrt{h(H - h) + \frac{(P_0 - P_{atm})(H - h)}{\rho g}}$$

Final Answer: $\boxed{R_x = 2\sqrt{h(H - h) + \frac{(P_0 - P_{atm})(H - h)}{\rho g}}}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: Under steady-state conditions, the rate of conductive heat transfer (Q) remains constant across all sequential slabs in a series combination. Fourier's law of heat conduction states that $Q = -kA\frac{\Delta T}{\Delta x}$. Given that all slabs share identical thickness (Δx) and cross-sectional area (A), the thermal conductivity k is inversely proportional to the temperature drop across that individual layer ($\Delta T \propto \frac{1}{k}$).

Solution:

1. **Identify the Profile Percentage Drops across the Slabs:** Let the total temperature difference across the combined multi-layered assembly wall be ΔT_{total}.

- Slab 1 temperature drop: $\Delta T_1 = 40\%$ of $\Delta T_{\text{total}} = 0.40\Delta T_{\text{total}}$
- Slab 2 temperature drop: $\Delta T_2 = 35\%$ of $\Delta T_{\text{total}} = 0.35\Delta T_{\text{total}}$
- Slab 3 temperature drop: The remaining percentage drop must fulfill the sum to 100%:

$$\Delta T_3 = (100\% - 40\% - 35\%)\Delta T_{\text{total}} = 25\% \text{ of } \Delta T_{\text{total}} = 0.25\Delta T_{\text{total}}$$

2. **Relate Temperature Drops to Thermal Conductivities:** Since Q is identical through each layer:

$$k_1\Delta T_1 = k_2\Delta T_2 = k_3\Delta T_3 \implies k_1(0.40) = k_2(0.35) = k_3(0.25)$$

Multiply through by 100 to simplify the decimal terms:

$$40k_1 = 35k_2 = 25k_3 \implies 8k_1 = 7k_2 = 5k_3$$

3. **Find the Structural Ratio Profile ($k_1 : k_2 : k_3$):** Let $8k_1 = 7k_2 = 5k_3 = C$. Then $k_1 = \frac{C}{8}$, $k_2 = \frac{C}{7}$, and $k_3 = \frac{C}{5}$.

$$k_1 : k_2 : k_3 = \frac{1}{8} : \frac{1}{7} : \frac{1}{5}$$

Multiply the ratio fractions by the least common multiple $\text{LCM}(8, 7, 5) = 280$:

$$k_1 : k_2 : k_3 = \left(\frac{280}{8}\right) : \left(\frac{280}{7}\right) : \left(\frac{280}{5}\right) = 35 : 40 : 56$$

Final Answer: $\boxed{35 : 40 : 56}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: For an ideal gas at constant volume, Gay-Lussac's Law dictates that pressure is directly proportional to absolute temperature ($P \propto T$). When the volumes of the two bulb containers are assumed constant, the pressure inside each chamber can be determined independently as a function of its absolute thermal state.

Solution:

1. **Determine the Pressure State inside each Bulb Chamber:**

- Both bulbs start balanced at an initial pressure P_0 and initial temperature T_0.
- **Right Chamber:** Remained at T_0, so its pressure stays exactly at $P_{\text{right}} = P_0$.
- **Left Chamber:** The absolute temperature surges to $T_0 + \Delta T$. At constant volume:

$$\frac{P_{\text{left}}}{T_0 + \Delta T} = \frac{P_0}{T_0} \implies P_{\text{left}} = P_0\left(\frac{T_0 + \Delta T}{T_0}\right) = P_0\left(1 + \frac{\Delta T}{T_0}\right)$$

2. **Evaluate the Induced Pressure Differential** (ΔP)**:** The net change in pressure across the liquid bridge manometer line is the difference between the left and right pressures:

$$\Delta P = P_{\text{left}} - P_{\text{right}} = P_0\left(1 + \frac{\Delta T}{T_0}\right) - P_0 = P_0\left(\frac{\Delta T}{T_0}\right)$$

Final Answer: $\boxed{\Delta P = P_0\left(\frac{\Delta T}{T_0}\right)}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: The quantity of heat energy input Q required to raise the temperature of a material with a temperature-dependent specific heat capacity is evaluated by integrating the differential heat expression $dQ = mC_p(T)dT$ over the specified absolute temperature interval.

Solution:

1. **Set Up the Definite Integration Equation:**

$$Q = \int_{T_1}^{T_2} mC_p(T)\, dT$$

Given $C_p(T) = a + bT^2$, $T_1 = T_0$, and $T_2 = 2T_0$:

$$Q = m \int_{T_0}^{2T_0} (a + bT^2)\, dT$$

2. **Compute the Anti-Derivative Expression:**

$$Q = m \left[aT + \frac{bT^3}{3} \right]_{T_0}^{2T_0}$$

3. **Substitute the Integration Boundary Limits:**

$$Q = m \left[\left(a(2T_0) + \frac{b(2T_0)^3}{3} \right) - \left(aT_0 + \frac{bT_0^3}{3} \right) \right]$$

$$Q = m \left[2aT_0 + \frac{8bT_0^3}{3} - aT_0 - \frac{bT_0^3}{3} \right]$$

Combine the matching linear and cubic terms:

$$Q = m \left(aT_0 + \frac{7}{3} bT_0^3 \right)$$

Final Answer: $\boxed{Q = m \left(aT_0 + \frac{7}{3} bT_0^3 \right)}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: An absorption refrigeration system can be modeled as a combination of a reversible heat engine operating between T_G and T_C, driving a reversible refrigerator working between T_E and T_C. By applying the second law of thermodynamics (entropy balance for a completely reversible cycle, $\sum \frac{Q}{T} = 0$), the maximum performance can be determined.

Solution:

1. **Establish the Energy Conservation and Entropy Equations:**

   - First Law (Energy Balance): $Q_C = Q_G + Q_E$
   - Second Law (Entropy Balance for Reversible Operation):

$$\frac{Q_G}{T_G} + \frac{Q_E}{T_E} - \frac{Q_C}{T_C} = 0$$

2. **Substitute Q_C into the Entropy Expression:**

$$\frac{Q_G}{T_G} + \frac{Q_E}{T_E} - \frac{Q_G + Q_E}{T_C} = 0$$

$$\frac{Q_G}{T_G} - \frac{Q_G}{T_C} = \frac{Q_E}{T_C} - \frac{Q_E}{T_E} \implies Q_G\left(\frac{1}{T_G} - \frac{1}{T_C}\right) = Q_E\left(\frac{1}{T_C} - \frac{1}{T_E}\right)$$

$$Q_G\left(\frac{T_C - T_G}{T_G T_C}\right) = Q_E\left(\frac{T_E - T_C}{T_E T_C}\right)$$

3. **Solve for the Coefficient of Performance ($\mathbf{COP_{max}}$):**

$$\text{COP}_{\text{max}} = \frac{Q_E}{Q_G} = \left(\frac{T_C - T_G}{T_G}\right) \cdot \left(\frac{T_E}{T_E - T_C}\right) = \left(\frac{T_G - T_C}{T_G}\right) \cdot \left(\frac{T_E}{T_C - T_E}\right)$$

4. **Substitute the Given Temperature Parameters:**

$$\text{COP}_{\text{max}} = \left(\frac{360 - 300}{360}\right) \cdot \left(\frac{250}{300 - 250}\right) = \left(\frac{60}{360}\right) \cdot \left(\frac{250}{50}\right) = \frac{1}{6} \cdot 5 = \frac{5}{6} \approx 0.833$$

Final Answer: $\boxed{\text{COP}_{\text{max}} = 0.83}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: For a steady-state thermal conductor with continuous perimeter lateral heat leakage, the heat power profile $P(x)$ along the longitudinal axis drops exponentially according to the governing differential relation: $\frac{d^2T}{dx^2} = m^2(T - T_{\text{amb}})$, which implies that the temperature gradient profile satisfies $\frac{dT}{dx} \propto P(x) = P_{\text{in}}e^{-2\alpha x}$.

Solution:

1. **Determine the Decay Constant Boundary Parameters:** Let the rod length extend from $x = 0$ (hot input end) to $x = L$ (opposite terminal end).

- At $x = 0$, input power is $P(0) = P_{\text{in}}$.
- At $x = L$, output arriving power is $P(L) = \frac{P_{\text{in}}}{e^2} = P_{\text{in}}e^{-2}$.

Matching this with the standard continuous thermal decay function $P(x) = P_{\text{in}}e^{-2x/L}$, we confirm the power at any point x behaves as an exponential spatial decay with an attenuation coefficient equal to 2 across length L.

2. **Evaluate the Conditions at the Spatial Midpoint Length:** The spatial midpoint of the conductor rod is located at $x = \frac{L}{2}$. Substitute this into the distribution formula:

$$P\left(\frac{L}{2}\right) = P_{\text{in}}e^{-2\left(\frac{L/2}{L}\right)} = P_{\text{in}}e^{-1} = \frac{P_{\text{in}}}{e}$$

3. **Relate the Power to the Temperature Profile Derivative:** Fourier's Law states that the spatial temperature gradient derivative is directly proportional to the localized axial thermal power passing through that section ($P(x) = -kA\frac{dT}{dx}$). Thus:

$$\left(\frac{dT}{dx}\right)_{x=L/2} \propto P\left(\frac{L}{2}\right) = \frac{P_{\text{in}}}{e} = \frac{1}{e} \cdot \left(\frac{dT}{dx}\right)_{x=0}$$

Therefore, the temperature gradient drops precisely to $\frac{1}{e}$ of its initial entry value.

Final Answer: It drops precisely to $\frac{1}{e}$ of the initial spatial temperature gradient.

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The focal length of a lens is determined by its geometry and the relative refractive index between the lens material and the surrounding medium, as described by the Lens-Maker's Formula: $\frac{1}{f} = \left(\frac{n_{\text{lens}}}{n_{\text{medium}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

Solution:

1. **Focal Length in Standard Air Medium (F):** Here $n_{\text{medium}} = n_{\text{air}} = 1.0$ and $n_{\text{lens}} = n_g = 1.50$. Let the curvature geometry factor be $K = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$:

$$\frac{1}{F} = (1.50 - 1) \cdot K = 0.5K \implies K = \frac{2}{F}$$

2. **Focal Length in Liquid Solution Medium (F_{new}):** Submerged in a chemical liquid where $n_{\text{medium}} = n_l = 1.75$:

$$\frac{1}{F_{\text{new}}} = \left(\frac{n_g}{n_l} - 1\right) \cdot K = \left(\frac{1.50}{1.75} - 1\right) \cdot K$$

$$\frac{1}{F_{\text{new}}} = \left(\frac{6}{7} - 1\right) \cdot K = -\frac{1}{7}K$$

3. **Substitute K to solve for F_{new}:**

$$\frac{1}{F_{\text{new}}} = -\frac{1}{7}\left(\frac{2}{F}\right) = -\frac{2}{7F} \implies F_{\text{new}} = -\frac{7}{2}F = -3.5F$$

The negative sign indicates that the originally converging biconvex lens now acts as a diverging lens because it is immersed in a medium optically denser than its own glass core.

Final Answer: $\boxed{F_{\text{new}} = -3.5F}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: The diagram illustrates a standard Wheatstone bridge circuit network configured across terminals A and B. If the ratio of resistances in the balancing arms is equal, no current flows through the central cross link galvanometer resistor branch, allowing it to be removed from the equivalent resistance calculation.

Solution:

1. **Analyze the Bridge Balance Condition:** Redrawing or mapping the bridge connections shows:

- The two input branches branch out from terminal A into a top-left arm ($R_{TL} = R_0$) and a bottom-left arm ($R_{BL} = R_0$).
- The two output branches join at terminal B from a top-right arm ($R_{TR} = R_0$) and a bottom-right arm ($R_{BR} = R_0$).
- A central resistor ($R_{center} = R_0$) links the top and bottom junctions.

Evaluate the balancing ratio:

$$\frac{R_{TL}}{R_{BL}} = \frac{R_0}{R_0} = 1 \quad \text{and} \quad \frac{R_{TR}}{R_{BR}} = \frac{R_0}{R_0} = 1$$

Since the ratios are perfectly matched ($1 = 1$), the bridge is balanced. The electrical potential difference across the central bridge link is zero, so no current passes through it.

2. **Calculate the Simplified Equivalent Resistance (R_{eq}):** Removing the inactive central cross resistor simplifies the network into two parallel branches:

- **Top path series combination:** $R_{top} = R_{TL} + R_{TR} = R_0 + R_0 = 2R_0$
- **Bottom path series combination:** $R_{bottom} = R_{BL} + R_{BR} = R_0 + R_0 = 2R_0$

The total parallel equivalent resistance is:

$$R_{eq} = \frac{R_{top} \cdot R_{bottom}}{R_{top} + R_{bottom}} = \frac{2R_0 \cdot 2R_0}{2R_0 + 2R_0} = \frac{4R_0^2}{4R_0} = R_0$$

Given $R_0 = 10\ \Omega$, we find $R_{eq} = 10\ \Omega$.

Final Answer: $\boxed{R_{eq} = 10\ \Omega}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: The total Optical Path Length (OPL) accumulated by a light beam traversing through a medium with a position-dependent variable refractive index $n(x)$ is defined by the line integral of the refractive index along the physical geometric path of the ray: OPL $= \int_0^t n(x)dx$.

Solution:

1. **Set up the Definite Integration Expression:** Since the laser beam enters the plane-parallel glass block normally ($\theta = 0°$), it continues straight through without bending. The path is a straight line along the thickness coordinate x, from $x = 0$ to $x = t = 6$ cm:

$$\text{OPL} = \int_0^6 (n_0 + \beta x)\, dx$$

2. **Perform the Integration:**

$$\text{OPL} = \left[n_0 x + \frac{\beta x^2}{2} \right]_0^6 = n_0(6) + \frac{\beta(6)^2}{2} - 0 = 6n_0 + 18\beta$$

3. **Substitute the Given Numerical Constants:** Given $n_0 = 1.4$ and $\beta = 0.1$ cm^{-1}:

$$\text{OPL} = 6(1.4) + 18(0.1) = 8.4 + 1.8 = 10.2 \text{ cm}$$

Final Answer: $\boxed{\text{OPL} = 10.2 \text{ cm}}$

Answer: (B)

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Q23.

Solution

Concept: The magnitude of the electric field produced by a single infinitely long straight charged wire with uniform linear charge density λ at a radial distance r is given by Gauss's Law: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$. The net electric field vector at any point is the vector sum of fields from all contributing sources.

Solution:

1. **Determine the Distance from Each Wire to the Midpoint:** Two adjacent parallel grid wires are separated by a perpendicular distance d. The test point is situated exactly midway along the baseline, meaning its distance from each wire is:

$$r = \frac{d}{2}$$

2. **Analyze the Electric Field Vectors:**

- **Positive wire ($+\lambda$):** Creates an electric field vector pointing radially *away* from itself. At the midpoint, this vector points toward the negative wire.

$$E_+ = \frac{\lambda}{2\pi\varepsilon_0\left(\frac{d}{2}\right)} = \frac{\lambda}{\pi\varepsilon_0 d}$$

- **Negative wire ($-\lambda$):** Creates an electric field vector pointing radially *toward* itself. At the midpoint, this vector also points toward the negative wire.

$$E_- = \frac{\lambda}{2\pi\varepsilon_0\left(\frac{d}{2}\right)} = \frac{\lambda}{\pi\varepsilon_0 d}$$

3. **Sum the Aligned Field Vectors:** Since both electric field vectors point in the exact same direction along the perpendicular baseline axis, their magnitudes add up directly:

$$E_{\text{net}} = E_+ + E_- = \frac{\lambda}{\pi\varepsilon_0 d} + \frac{\lambda}{\pi\varepsilon_0 d} = \frac{2\lambda}{\pi\varepsilon_0 d}$$

Final Answer: $\boxed{E = \frac{2\lambda}{\pi\varepsilon_0 d}}$

Answer: (B)

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Q24.

Solution

Concept: To convert a sensitive galvanometer into a higher-range ammeter, a low-resistance shunt resistor (R_s) must be connected in parallel with the galvanometer coil branch. This ensures that the excess current bypasses the delicate indicator mechanism.

Solution:

1. **Formulate the Parallel Voltage Balance Equation:** Since the galvanometer coil and the shunt resistor are connected in parallel, the voltage drop across both branches must be identical:

$$I_g \cdot G = I_s \cdot R_s$$

2. **Apply Kirchhoff's Current Law for the Shunt Branch:** The total total ammeter line current is $I = 5$ A, and the maximum full-scale galvanometer current is $I_g = 2$ mA $= 0.002$ A. The current through the shunt is:

$$I_s = I - I_g = 5 - 0.002 = 4.998 \text{ A}$$

3. **Calculate the Shunt Resistance Value (R_s):**

$$R_s = \frac{I_g \cdot G}{I - I_g} = \frac{0.002 \cdot 50}{4.998} = \frac{0.1}{4.998} \approx 0.020008\ \Omega$$

Rounding to standard field specification precision digits yields exactly 0.0200 Ω.

Final Answer: $\boxed{R_s = 0.0200\ \Omega}$

Answer: (A)

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Q25.

Solution

Concept: For final light rays to emerge completely parallel to the principal axis after passing through a thin convex lens, the intermediate object source presented to the lens must be positioned precisely at its focal point ($u_l = f_l$). This intermediate object is the image produced by the reflection from the preceding concave mirror.

Solution:

1. **Locate the Image Formed by the Concave Mirror:** Using the mirror equation $\frac{1}{v_m} + \frac{1}{u_m} = \frac{1}{f_m}$:

- Radius of curvature $R_m = 40$ cm $\implies$ Focal length $f_m = -\frac{R_m}{2} = -20$ cm.
- Object distance $u_m = -30$ cm (in front of the mirror).

$$\frac{1}{v_m} + \frac{1}{-30} = \frac{1}{-20} \implies \frac{1}{v_m} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

$$v_m = -60 \text{ cm}$$

The mirror forms a real image at a distance of 60 cm in front of it.

2. **Determine the Position of the Convex Lens:** To produce parallel emergent rays, the lens must be placed such that this real intermediate image falls exactly at its first focal point. Since the lens has a focal length of $f_l = 15$ cm, it must be placed 15 cm further down the line past the intermediate image position.

3. **Calculate the Total Separation Distance (D):** The lens is positioned coaxially behind the concave mirror down the line of ray travel, which means it sits in front of the reflecting mirror's face:

$$D = |v_m| + f_l = 60 \text{ cm} + 15 \text{ cm} = 75 \text{ cm}$$

Final Answer: $\boxed{D = 75 \text{ cm}}$

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	B	4	B	5	B
6	B	7	A	8	A	9	B	10	B
11	B	12	A	13	A	14	A	15	A
16	A	17	A	18	A	19	A	20	A
21	B	22	B	23	B	24	A	25	B