UPCATET Agriculture Statistics & Mathematics Sample Paper-2

Duration: 20 Minutes | **Maximum Marks: 80**

Instructions

- This paper contains **20** Multiple Choice Questions.
- Each correct answer carries **+4** mark. Incorrect answer: **−1** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. An agronomist collects crop yield data from 15 experimental micro-plots. Due to a calibration error in the digital weighing scale, every reading recorded is found to be exactly 12% lower than the actual yield value. If the calculated coefficient of variation (CV) of the flawed yield dataset was found to be 24%, what will be the true, corrected coefficient of variation (CV_{true}) of the actual crop yields?

(A) $CV_{\text{true}} = 12.0\%$

(B) $CV_{\text{true}} = 21.12\%$

(C) $CV_{\text{true}} = 24.0\%$

(D) $CV_{\text{true}} = 27.27\%$

Q2. A statistical analysis of the seed germination counts across 50 nursery trays yields a symmetric distribution where the mean is 85 seeds and the standard deviation is 6. If two highly anomalous trays containing extremely low germination counts of 12 and 15 seeds are completely omitted from the sample, how will the mean (μ) and standard deviation (σ) of the revised dataset behave?

(A) μ decreases, σ increases

(B) μ increases, σ decreases

(C) μ increases, σ increases

(D) μ remains constant, σ decreases

Q3. An agro-meteorological station records the daily rainfall over a multi-week monsoon period. The modal value of the rainfall distribution is calculated to be 45.2 mm, while the median rainfall stands at 38.6 mm. Assuming Karl Pearson's empirical relationship for moderately skewed biological frequency curves holds perfectly true, calculate the estimated arithmetic mean of this rainfall dataset.

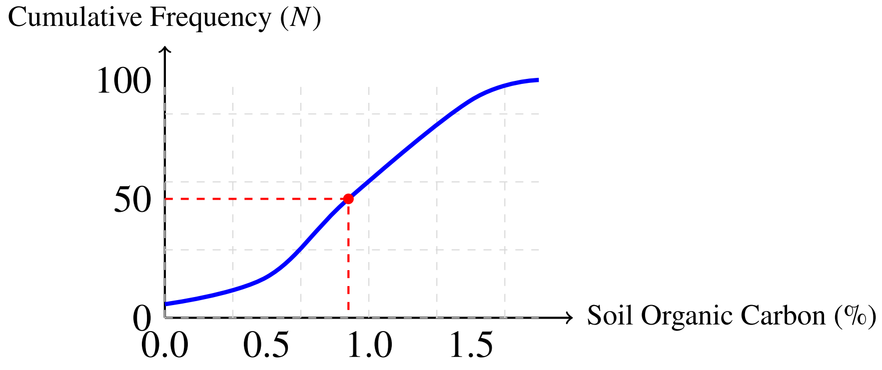
(A) 32.0 mm

(B) 35.3 mm

(C) 41.9 mm

(D) 51.8 mm

Q4. An agricultural research firm analyzes the frequency distribution of soil organic carbon percentage values across an intensive farming zone. The cumulative frequency behavior is mapped using an ogive layout shown below. Identify the approximate mathematical value of the statistical median for this soil quality dataset:



(A) 0.50%

(B) 0.90%

(C) 1.25%

(D) 1.50%

Q5. The annual yield of a newly introduced hybrid mango orchard increases every year such that the quantities harvested form a strict Geometric Progression (GP).

If the combined total yield of the first 3 years is exactly 7 times the yield obtained in the first year alone, find the common ratio (r) governing the growth of this orchard's productivity.

(A) $r = 1$

(B) $r = 2$

(C) $r = 3$

(D) $r = \sqrt{2}$

Q6. A precision drip irrigation system is programmed to deliver water to a linear row of citrus trees. The volume of water discharged at each subsequent emitter nozzle follows a strict Arithmetic Progression (AP). If the amount of water delivered to the 5^{th} tree is 18 liters and the amount delivered to the 11^{th} tree is 42 liters, determine the total cumulative volume of water discharged from the first nozzle up to the 20^{th} nozzle.

(A) 440 liters

(B) 800 liters

(C) 840 liters

(D) 920 liters

Q7. An agricultural economist formulates a quadratic profit function optimization scheme given by $P(x) = -2x^2 + kx - 18$, where x represents the quantity of specialized bio-fertilizer added per hectare. Find the range of values of the structural constant k for which the farm system guarantees to suffer a chronic economic loss under all conditions (i.e., $P(x) < 0$ for all real inputs of x).

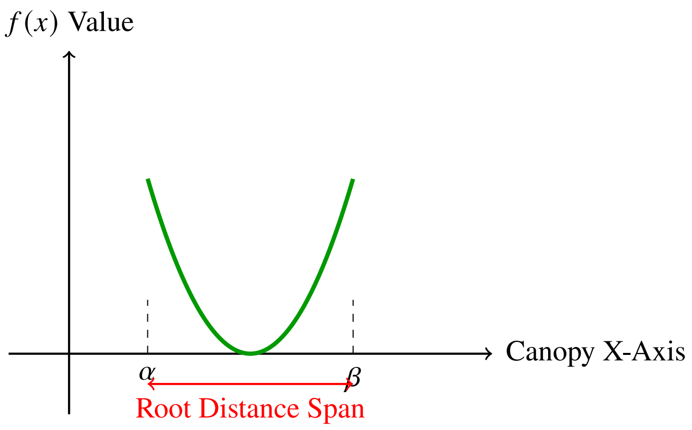
(A) $-6 < k < 6$

(B) $-12 < k < 12$

(C) $k > 12$ or $k < -12$

(D) $k = 0$ only

Q8. A team of bio-engineers measures the vegetative canopy coverage radius expansion across four sequential generation lines. The dynamic boundary area behaves according to a parabolic function as mapped out below. If the roots of this geometric boundary equation α and β fulfill the design condition $(\alpha - \beta)^2 = 16$, find the value of coefficient m in the underlying function $f(x) = x^2 - mx + 12$:



(A) $m = \pm 4$

(B) $m = \pm 8$

(C) $m = \pm 12$

(D) $m = \pm 16$

Q9. A large agricultural rainwater harvesting unit is constructed in the shape of an inverted right circular cone with its vertex pointing downwards. The semi-vertical angle of the inner conical cavity is exactly 30°. If water is being pumped into this reservoir at a uniform volumetric rate of 3 m^3/hr, what is the instantaneous rate of increase in the depth of water when the water depth reaches exactly 2 meters?

(A) $\frac{3}{4\pi}$ m/hr

(B) $\frac{9}{4\pi}$ m/hr

(C) $\frac{1}{4\pi}$ m/hr

(D) $\frac{3}{\pi}$ m/hr

Q10. A cylindrical metal grain silo has a fixed total surface area (including both flat circular base ends and the curved lateral wall structure) equal to S m^2. In order to maximize the total storage capacity volume (V) of grain inside this structure, what must be the precise mathematical relationship between the height (h) of the silo cylinder and its base radius (r)?

(A) $h = r$

(B) $h = 2r$

(C) $h = \frac{r}{2}$

(D) $h = 4r$

Q11. An irregular agricultural field is surveyed and modeled as a combination of a central rectangular layout flanked by two identical semi-circular grazing patches on its shorter parallel boundaries. If the total perimeter of this entire combined field system is structurally restricted to exactly 400 meters, find the maximum possible area that this field can enclose.

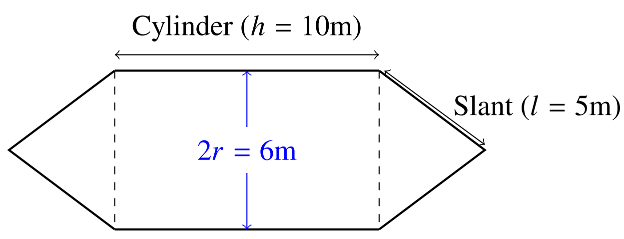
(A) $\frac{20000}{\pi}$ m^2

(B) $\frac{40000}{\pi}$ m^2

(C) 10000 m^2

(D) 40000 m^2

Q12. A custom-engineered agricultural pesticide tank consists of a central hollow cylinder terminated at both ends by matching right circular solid cones, as illustrated in the cross-sectional diagram below. If the radius (r) of both the cylinder and the cones is 3 meters, the height of the cylindrical body is 10 meters, and the slant height (l) of each cone is 5 meters, calculate the total surface area of this tank structure requiring an anti-corrosive chemical liner:



(A) 60π m^2

(B) 75π m^2

(C) 90π m^2

(D) 120π m^2

Q13. A laser-guided automated tractor operates along a precise path where its steering guidance system depends on the calculation of a variable expression. Simplify the trigonometric value given by the expression: $\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ$.

(A) $\frac{1}{8}$

(B) $\frac{1}{16}$

(C) $\frac{1}{32}$

(D) $\frac{\sqrt{3}}{16}$

Q14. The biochemical degradation of a systemic pesticide in soil layers over time is governed by an exponential decay function containing a logarithmic component. Find the exact numerical solution for x in the following logarithmic equation: $\log_2(x+1) + \log_2(x-1) = 3$.

(A) $x = 3$

(B) $x = \pm 3$

(C) $x = \sqrt{7}$

(D) $x = 4$

Q15. A dynamic solar tracking solar panel mounted on a green-house roof tracks sunlight angles. The angular elevation factor is governed by the expression $\tan\theta + \cot\theta = 4$. Under this operational condition, find the exact derived value of the parameter represented by $\tan^4\theta + \cot^4\theta$.

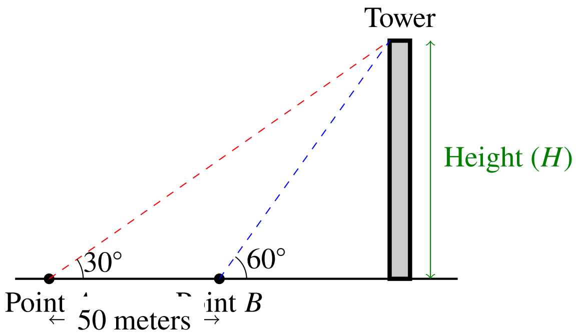
(A) 14

(B) 194

(C) 196

(D) 254

Q16. An agricultural surveyor utilizes an inclinometer to evaluate the height of an ultra-tall forest canopy observation tower situated across a steep irrigation drainage trench. Based on the geometric angular configuration and benchmark dimensions provided in the schematic below, calculate the absolute true vertical height (H) of the tower:



(A) 25 meters

(B) $25\sqrt{3}$ meters

(C) 50 meters

(D) $50\sqrt{3}$ meters

Q17. A rigorous agricultural census registers the number of standard livestock units managed per smallholder farm across a sub-district. The final compilation shows that the values of the Mean and Median of this discrete dataset are 22 and 24 respectively. Which of the following options defines the most accurate characterization of this livestock data distribution profile?

(A) The distribution is perfectly symmetric.

(B) The distribution is positively skewed with Mode = 28.

(C) The distribution is negatively skewed with Mode = 28.

(D) The distribution is negatively skewed with Mode = 20.

Q18. The total structural capacity of an underground cylindrical concrete slurry pit must be expanded by exactly 44% to meet environmental runoff regulations. If the depth (height) of this slurry tank must remain entirely constant due to groundwater table restrictions, by what percentage value must the inner base radius of the cylinder be increased?

(A) 20%

(B) 22%

(C) 44%

(D) 12%

Q19. The progression of three successive node points along a high-tech experimental plant stem is found to follow the mathematical relation where $\log a$, $\log b$, and $\log c$ form a continuous Arithmetic Progression (AP). Which statement below correctly describes the fundamental baseline relationship existing between the raw spatial coordinates a, b, and c?

(A) They form an Arithmetic Progression ($2b = a + c$).

(B) They form a Geometric Progression ($b^2 = ac$).

(C) They form a Harmonic Progression ($b = \frac{2ac}{a+c}$).

(D) They satisfy the condition $b = a^2c^2$.

Q20. During a deep-bed grain drying procedure, the operational efficiency factor E depends on an angle parameter according to the transcendental relationship $E = \sin^6\theta + \cos^6\theta$. If an operator alters the machine settings such that the value of $\sin\theta \cdot \cos\theta = \frac{1}{3}$, calculate the resulting value of the efficiency parameter E.

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{7}{9}$

(D) $\frac{25}{27}$

Detailed Solutions

Q1.

Solution

Concept: The Coefficient of Variation (CV) is a dimensionless measure of relative variability defined as the ratio of the standard deviation (σ) to the arithmetic mean (μ), expressed as a percentage:

$$CV = \left(\frac{\sigma}{\mu}\right) \times 100$$

When every observation in a dataset is scaled by a constant factor c (i.e., decreased by 12%, which means multiplying by 0.88), both the mean and the standard deviation are scaled by that exact same absolute factor.

Solution:

Let the true mean be μ_{true} and the true standard deviation be σ_{true}.

(a) Because of the calibration error, every reading is 12% lower than actual, so each flawed data point is $y_i = 0.88x_i$.

(b) By the linear properties of central tendency and dispersion, the flawed dataset metrics are:

$$\mu_{\text{flawed}} = 0.88\mu_{\text{true}} \quad \text{and} \quad \sigma_{\text{flawed}} = 0.88\sigma_{\text{true}}$$

(c) Substitute these into the formula for the flawed coefficient of variation ($CV_{\text{flawed}} = 24\%$):

$$CV_{\text{flawed}} = \left(\frac{\sigma_{\text{flawed}}}{\mu_{\text{flawed}}}\right) \times 100 = \left(\frac{0.88\sigma_{\text{true}}}{0.88\mu_{\text{true}}}\right) \times 100 = \left(\frac{\sigma_{\text{true}}}{\mu_{\text{true}}}\right) \times 100 = CV_{\text{true}}$$

(d) Since the scaling factor 0.88 cancels out completely, the relative variation remains invariant. Therefore, $CV_{\text{true}} = 24.0\%$.

Final Answer: $\boxed{CV_{\text{true}} = 24.0\%}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: The arithmetic mean (μ) acts as the center of mass of a dataset, while the standard deviation (σ) measures the average distance of data points from that mean. Removing extreme outliers far below the mean alters both parameters predictably.

Solution:

Let's analyze the mathematical impact of dropping the two highly anomalous trays (12 and 15 seeds):

(a) The original baseline mean is $\mu = 85$. The two omitted values (12 and 15) are substantially lower than the average seed count.

(b) Removing numbers that are lower than the current mean reduces the deficit from the sum total, causing the average of the remaining values to increase (μ increases).

(c) The standard deviation measures overall spread. Since the data points 12 and 15 lie many standard deviations away from the mean ($85 - 12 = 73$, which is $> 12\sigma$), they contribute heavily to the sum of squared deviations. Removing these highly distant anomalous points significantly compacts the dataset distribution, causing the standard deviation to drop (σ decreases).

Final Answer: $\boxed{\mu \text{ increases, } \sigma \text{ decreases}}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: For unimodal distributions that are moderately asymmetrical or skewed, Karl Pearson established an empirical mathematical approximation linking the three central metrics:

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

Solution:
Let's calculate the estimated arithmetic mean from the given weather parameters:

(a) Identify the given metrics: Mode = 45.2 mm and Median = 38.6 mm.

(b) Set up the empirical formula and substitute the parameters, letting Mean = $\bar{x}$:

$$\bar{x} - 45.2 = 3(\bar{x} - 38.6)$$

(c) Expand the right side of the equation:

$$\bar{x} - 45.2 = 3\bar{x} - 115.8$$

(d) Rearrange terms to collect the variable $\bar{x}$ on one side:

$$115.8 - 45.2 = 3\bar{x} - \bar{x}$$

$$70.6 = 2\bar{x}$$

(e) Divide by 2 to find the final mean value:

$$\bar{x} = \frac{70.6}{2} = 35.3 \text{ mm}$$

Final Answer: $\boxed{35.3 \text{ mm}}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: An ogive represents the graphical curve of a cumulative frequency distribution. The statistical median can be determined from this graph by locating the position corresponding to half of the total cumulative frequency ($N/2$) on the vertical axis and tracking it to its horizontal coordinate.

Solution:

Let's trace the coordinate path on the provided soil layout:

(a) Identify the total cumulative sample size from the maximum value on the vertical Y-axis: $N = 100$.

(b) Compute the median rank position on the frequency axis:

$$\text{Median Position} = \frac{N}{2} = \frac{100}{2} = 50$$

(c) Locate 50 on the vertical axis and follow the horizontal dashed line to its intersection point with the blue cumulative curve.

(d) Move vertically downward from that intersection point to the horizontal axis to read the corresponding soil carbon concentration. The point lands between 0.5 and 1.0, precisely at 0.90%.

Final Answer: 0.90%

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: A Geometric Progression (GP) is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed, non-zero number called the common ratio (r). The sum of the first n terms of a GP is given by:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Solution:
Let's construct the algebraic problem using the yield criteria:

(a) Let the harvest yield in year 1 be a. The total yield over the first three years is S_3.

(b) According to the problem statement, $S_3 = 7 \times a$.

(c) Write out the structural expansion for the sum of the first three terms ($S_3 = a + ar + ar^2$):

$$a + ar + ar^2 = 7a$$

(d) Since a represents a non-zero harvest quantity, divide both sides of the equation by a:

$$1 + r + r^2 = 7$$

(e) Rearrange this into a standard quadratic equation form:

$$r^2 + r - 6 = 0$$

(f) Factor the quadratic equation to find the roots:

$$(r + 3)(r - 2) = 0 \implies r = -3 \quad \text{or} \quad r = 2$$

(g) Since orchard productivity cannot alternate negatively from year to year, we select the valid physical growth ratio, $r = 2$.

Final Answer: $\boxed{r = 2}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: In an Arithmetic Progression (AP), any specific term n is defined by $a_n = a + (n-1)d$, where a is the initial term and d is the common difference. The total sum of the first n terms is computed via:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Solution:
Let's find the progression characteristics to compute the total volume:

(a) Write down the linear equations for the 5th and 11th tree emitter nozzles:

$$a_5 = a + 4d = 18$$

$$a_{11} = a + 10d = 42$$

(b) Subtract the first linear equation from the second to isolate the common difference d:

$$(a + 10d) - (a + 4d) = 42 - 18 \implies 6d = 24 \implies d = 4 \text{ liters}$$

(c) Substitute $d = 4$ back into the a_5 equation to find the initial volume a:

$$a + 4(4) = 18 \implies a + 16 = 18 \implies a = 2 \text{ liters}$$

(d) Calculate the total volume delivered across the first 20 nozzles ($n = 20$):

$$S_{20} = \frac{20}{2}[2(2) + (20-1)4] = 10 \cdot [4 + 19 \cdot 4] = 10 \cdot [4 + 76] = 10 \cdot 80 = 800 \text{ liters}$$

Final Answer: $\boxed{800 \text{ liters}}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: For a standard quadratic expression $f(x) = ax^2 + bx + c$ to remain strictly negative under all real input conditions ($f(x) < 0$ for all $x \in \mathbb{R}$), two geometric requirements must be met simultaneously: the parabola must open downward ($a < 0$) and it must never intersect or touch the horizontal axis ($D = b^2 - 4ac < 0$).

Solution:

Let's evaluate these conditions for the profit function $P(x) = -2x^2 + kx - 18$:

(a) Identify the quadratic coefficients: $a = -2$, $b = k$, and $c = -18$.

(b) Check the leading term condition: $a = -2$, which is less than 0, confirming that the parabola opens downward.

(c) Set up the discriminant inequality ($D < 0$) to ensure there are no real roots:

$$D = b^2 - 4ac = k^2 - 4(-2)(-18) < 0$$

$$k^2 - 4(36) < 0 \implies k^2 - 144 < 0$$

(d) Solve the quadratic inequality for the structural constant k:

$$k^2 < 144 \implies |k| < 12 \implies -12 < k < 12$$

Final Answer: $\boxed{-12 < k < 12}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: For any standard quadratic equation $ax^2 + bx + c = 0$ with real roots α and β, the roots relate directly to the coefficients via Vieta's formulas: $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$. The squared distance between the roots can be expressed as:

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$$

Solution:
Let's analyze the canopy boundary equation $x^2 - mx + 12 = 0$:

(a) Identify the relevant coefficient parameters from the function:

$$\alpha + \beta = m \quad \text{and} \quad \alpha\beta = 12$$

(b) Substitute these expressions into the squared difference identity:

$$(\alpha - \beta)^2 = (m)^2 - 4(12) = m^2 - 48$$

(c) We are given that the design condition is $(\alpha - \beta)^2 = 16$. Set the two expressions equal to each other:

$$m^2 - 48 = 16$$

(d) Isolate and solve for the coefficient parameter m:

$$m^2 = 16 + 48 = 64 \implies m = \pm\sqrt{64} = \pm 8$$

Final Answer: $\boxed{m = \pm 8}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: This problem involves related rates from calculus. The volume of a right circular cone is $V = \frac{1}{3}\pi r^2 h$. Using the semi-vertical angle, we can express the radius r in terms of the depth h, reducing the volume formula to a single variable.

Solution:

Let's find the rate of depth increase step-by-step:

(a) From the geometry of the cone with semi-vertical angle $\alpha = 30°$:

$$\tan 30° = \frac{r}{h} \implies \frac{1}{\sqrt{3}} = \frac{r}{h} \implies r = \frac{h}{\sqrt{3}}$$

(b) Substitute this expression for r into the volume formula:

$$V = \frac{1}{3}\pi \left(\frac{h}{\sqrt{3}}\right)^2 h = \frac{1}{3}\pi \left(\frac{h^2}{3}\right) h = \frac{\pi h^3}{9}$$

(c) Differentiate both sides with respect to time (t) using the chain rule:

$$\frac{dV}{dt} = \frac{\pi}{9} \cdot 3h^2 \cdot \frac{dh}{dt} = \frac{\pi h^2}{3}\frac{dh}{dt}$$

(d) Substitute the given rate of water flow ($\frac{dV}{dt} = 3 \text{ m}^3/\text{hr}$) and the target water depth ($h = 2$ m):

$$3 = \frac{\pi (2)^2}{3}\frac{dh}{dt} \implies 3 = \frac{4\pi}{3}\frac{dh}{dt}$$

(e) Solve for the instantaneous rate of depth increase ($\frac{dh}{dt}$):

$$\frac{dh}{dt} = 3 \cdot \frac{3}{4\pi} = \frac{9}{4\pi} \text{ m/hr}$$

Final Answer: $\boxed{\frac{9}{4\pi} \text{ m/hr}}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: To find the maximum storage capacity under a fixed material constraint, we use optimization. Express the volume function in terms of a single variable using the surface area constraint, then find where its first derivative equals zero.

Solution:

Let's optimize the cylinder configuration step-by-step:

(a) State the formulas for total surface area (S) and volume (V):

$$S = 2\pi r^2 + 2\pi rh \quad \text{and} \quad V = \pi r^2 h$$

(b) Rearrange the surface area formula to express the height h in terms of r:

$$2\pi rh = S - 2\pi r^2 \implies h = \frac{S - 2\pi r^2}{2\pi r} = \frac{S}{2\pi r} - r$$

(c) Substitute this expression for h into the volume formula:

$$V = \pi r^2 \left(\frac{S}{2\pi r} - r\right) = \frac{Sr}{2} - \pi r^3$$

(d) Differentiate V with respect to r and set the derivative to zero to find the critical point:

$$\frac{dV}{dr} = \frac{S}{2} - 3\pi r^2 = 0 \implies S = 6\pi r^2$$

(e) Substitute this optimal value for S back into the surface area constraint equation to find the relationship between h and r:

$$6\pi r^2 = 2\pi r^2 + 2\pi rh \implies 4\pi r^2 = 2\pi rh$$

(f) Simplify the equation by dividing both sides by $2\pi r$:

$$2r = h \implies h = 2r$$

Final Answer: $\boxed{h = 2r}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: This problem requires maximizing the area of a composite geometric shape under a fixed perimeter constraint. Combining two matching semi-circles at opposite ends forms one complete circle of radius r.

Solution:

Let's define the dimensions and optimize the total area:

(a) Let the rectangular region have length x and width $2r$. The two semi-circular ends have radius r, which together form a full circle with circumference $2\pi r$ and area πr^2.

(b) The perimeter consists of the two long straight edges of the rectangle plus the circular boundaries:

$$P = 2x + 2\pi r = 400 \implies x + \pi r = 200 \implies x = 200 - \pi r$$

(c) Write out the total area equation (A) as the sum of the rectangular and circular components:

$$A = \text{Area}_{\text{rect}} + \text{Area}_{\text{circle}} = x(2r) + \pi r^2$$

(d) Substitute the expression for x into the area equation:

$$A = (200 - \pi r)(2r) + \pi r^2 = 400r - 2\pi r^2 + \pi r^2 = 400r - \pi r^2$$

(e) Differentiate A with respect to r and set it to zero to maximize the area:

$$\frac{dA}{dr} = 400 - 2\pi r = 0 \implies 2\pi r = 400 \implies r = \frac{200}{\pi}$$

(f) Substitute this optimal value of r back into the area equation:

$$A_{\max} = 400\left(\frac{200}{\pi}\right) - \pi\left(\frac{200}{\pi}\right)^2 = \frac{80000}{\pi} - \frac{40000}{\pi} = \frac{40000}{\pi}\ \text{m}^2$$

Final Answer: $\boxed{\frac{40000}{\pi}\ \text{m}^2}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: The total surface area of this composite tank structure is the sum of the exposed surface areas of its components. Because the cones cap the ends of the cylinder, the flat circular bases are interior walls and are not counted; only the lateral curved surface areas require coating.

Solution:

Let's calculate the outer lateral surface area of each component:

(a) Identify the given dimensions: radius $r = 3$ m, cylinder height $h = 10$ m, and cone slant height $l = 5$ m.

(b) Calculate the curved surface area (CSA) of the central cylinder:

$$CSA_{\text{cylinder}} = 2\pi rh = 2\pi \cdot (3) \cdot (10) = 60\pi \text{ m}^2$$

(c) Calculate the lateral curved surface area of one solid end cone:

$$CSA_{\text{cone}} = \pi rl = \pi \cdot (3) \cdot (5) = 15\pi \text{ m}^2$$

(d) Sum the lateral surface areas of the cylinder and both end cones to find the total area:

$$\text{Area}_{\text{total}} = CSA_{\text{cylinder}} + 2 \cdot (CSA_{\text{cone}})$$

$$\text{Area}_{\text{total}} = 60\pi + 2 \cdot (15\pi) = 60\pi + 30\pi = 90\pi \text{ m}^2$$

Final Answer: $\boxed{90\pi \text{ m}^2}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: To simplify product products of cosines with arguments that double at each step ($\theta, 2\theta, 4\theta\ldots$), we use the sine double-angle identity: $\sin 2\theta = 2\sin\theta\cos\theta \implies \cos\theta = \frac{\sin 2\theta}{2\sin\theta}$.

Solution:

Let's evaluate the product step-by-step:

$$P = \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 60^\circ \cdot \cos 80^\circ$$

(a) Substitute the known constant numerical value $\cos 60^\circ = \frac{1}{2}$ into the expression:

$$P = \frac{1}{2} \cdot [\cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ]$$

(b) Multiply and divide the bracketed terms by $2\sin 20^\circ$:

$$P = \frac{1}{2 \cdot 2\sin 20^\circ} \cdot [2\sin 20^\circ \cos 20^\circ \cdot \cos 40^\circ \cdot \cos 80^\circ]$$

(c) Apply the double-angle identity ($2\sin 20^\circ \cos 20^\circ = \sin 40^\circ$):

$$P = \frac{1}{4\sin 20^\circ} \cdot [\sin 40^\circ \cos 40^\circ \cdot \cos 80^\circ]$$

(d) Multiply and divide by 2 again to combine the next pair of terms:

$$P = \frac{1}{8\sin 20^\circ} \cdot [2\sin 40^\circ \cos 40^\circ \cdot \cos 80^\circ] = \frac{1}{8\sin 20^\circ} \cdot [\sin 80^\circ \cos 80^\circ]$$

(e) Repeat the process for the final terms:

$$P = \frac{1}{16\sin 20^\circ} \cdot [2\sin 80^\circ \cos 80^\circ] = \frac{\sin 160^\circ}{16\sin 20^\circ}$$

(f) Use the identity $\sin(180^\circ - \theta) = \sin\theta$ to simplify the fraction:

$$\sin 160^\circ = \sin(180^\circ - 20^\circ) = \sin 20^\circ \implies P = \frac{\sin 20^\circ}{16\sin 20^\circ} = \frac{1}{16}$$

Final Answer: $\boxed{\frac{1}{16}}$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: Logarithmic equations can be simplified using standard logarithm rules, such as the product rule: $\log_b(M) + \log_b(N) = \log_b(M \cdot N)$. When solving, any solutions that result in a negative argument inside a logarithm must be excluded.

Solution:

Let's isolate the variable x:

$$\log_2(x+1) + \log_2(x-1) = 3$$

(a) Apply the product rule to combine the logs on the left side:

$$\log_2[(x+1)(x-1)] = 3 \implies \log_2(x^2-1) = 3$$

(b) Rewrite the equation in exponential form:

$$x^2 - 1 = 2^3 \implies x^2 - 1 = 8$$

(c) Solve for x:

$$x^2 = 9 \implies x = \pm 3$$

(d) Validate the solutions against the original domain restrictions ($x+1 > 0$ and $x - 1 > 0$, meaning $x > 1$):

- For $x = 3$: the arguments are positive, so this solution is valid.
- For $x = -3$: the arguments become negative, which is undefined for real logarithms.

Final Answer: $\boxed{x = 3}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: An algebraic expression can be raised to higher powers by sequentially squaring it and applying the algebraic identity $(a+b)^2 = a^2 + 2ab + b^2$. Note that $\tan\theta \cdot \cot\theta = 1$.

Solution:

Let's find the value of $\tan^4\theta + \cot^4\theta$ from the given equation:

(a) Square both sides of the initial equation ($\tan\theta + \cot\theta = 4$):

$$(\tan\theta + \cot\theta)^2 = 4^2$$

$$\tan^2\theta + 2(\tan\theta\cot\theta) + \cot^2\theta = 16$$

(b) Substitute $\tan\theta \cdot \cot\theta = 1$ and simplify:

$$\tan^2\theta + 2(1) + \cot^2\theta = 16 \implies \tan^2\theta + \cot^2\theta = 14$$

(c) Square both sides of this new equation to reach the fourth power:

$$(\tan^2\theta + \cot^2\theta)^2 = 14^2$$

$$\tan^4\theta + 2(\tan^2\theta\cot^2\theta) + \cot^4\theta = 196$$

(d) Since $\tan^2\theta\cot^2\theta = 1$, substitute and solve:

$$\tan^4\theta + 2(1) + \cot^4\theta = 196 \implies \tan^4\theta + \cot^4\theta = 194$$

Final Answer: $\boxed{194}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: This problem uses right-triangle trigonometry involving two different angles of elevation to find an unknown height (H). We can set up a system of linear equations using the tangent ratios from both observation points.

Solution:

Let's set up and solve the equations for the height of the tower:

(a) Let the horizontal distance from Point B to the base of the tower be x. The total distance from Point A to the base is then $50 + x$.

(b) Set up the tangent equation for the right triangle from Point B ($\theta = 60°$):

$$\tan 60° = \frac{H}{x} \implies \sqrt{3} = \frac{H}{x} \implies x = \frac{H}{\sqrt{3}}$$

(c) Set up the tangent equation for the right triangle from Point A ($\theta = 30°$):

$$\tan 30° = \frac{H}{50 + x} \implies \frac{1}{\sqrt{3}} = \frac{H}{50 + x} \implies 50 + x = H\sqrt{3}$$

(d) Substitute the expression for x from step 2 into the equation from step 3:

$$50 + \frac{H}{\sqrt{3}} = H\sqrt{3}$$

(e) Multiply the entire equation by $\sqrt{3}$ to clear the fraction:

$$50\sqrt{3} + H = 3H \implies 2H = 50\sqrt{3} \implies H = 25\sqrt{3} \text{ meters}$$

Final Answer: $\boxed{25\sqrt{3} \text{ meters}}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: The skewness of a dataset is determined by the relative positions of its mean, median, and mode. In a perfectly symmetric distribution, Mean = Median = Mode. In a negatively skewed distribution, the values follow the order Mean < Median < Mode.

Solution:

Let's analyze the properties of the livestock dataset:

(a) Compare the given metrics: Mean = 22 and Median = 24.

(b) Since Mean < Median (22 < 24), the distribution has a longer tail on the left, meaning it is **negatively skewed**.

(c) Use Karl Pearson's empirical formula to estimate the mode and confirm the distribution profile:

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

$$22 - \text{Mode} = 3(22 - 24) = 3(-2) = -6$$

$$\text{Mode} = 22 + 6 = 28$$

Final Answer: $\boxed{\text{The distribution is negatively skewed with Mode} = 28}$

Answer: (C)

Go Back to Question 17

Q18.

Solution

Concept: The volume (V) of a cylinder is calculated from its radius (r) and height (h) using the formula $V = \pi r^2 h$. If the height remains constant, the volume is directly proportional to the square of the radius ($V \propto r^2$).

Solution:

Let's find the required percentage increase for the inner base radius:

(a) Let the initial capacity be $V_1 = \pi r_1^2 h$ and the expanded capacity be $V_2 = \pi r_2^2 h$.

(b) The capacity must expand by 44%, which means:

$$V_2 = 1.44 V_1$$

(c) Substitute the volume formulas into the relation, noting that π and h cancel out:

$$\pi r_2^2 h = 1.44(\pi r_1^2 h) \implies r_2^2 = 1.44 r_1^2$$

(d) Take the square root of both sides to find the relationship between the radii:

$$r_2 = \sqrt{1.44} \cdot r_1 = 1.20 r_1$$

(e) An updated radius of 1.20 times the original value represents an increase of exactly 20%.

Final Answer: $\boxed{20\%}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: If three sequential terms x, y, z form an Arithmetic Progression (AP), the difference between consecutive terms is constant: $y - x = z - y \implies 2y = x + z$. Logarithmic properties allow us to convert these linear relationships into geometric ones.

Solution:

Let's analyze the logarithmic progression properties:

(a) We are given that $\log a$, $\log b$, and $\log c$ form an AP. Apply the standard AP relationship:

$$2 \log b = \log a + \log c$$

(b) Use the power rule for logarithms ($\rightarrow n \log M = \log M^n$) on the left side:

$$\log(b^2) = \log a + \log c$$

(c) Use the product rule for logarithms ($\rightarrow \log M + \log N = \log(MN)$) on the right side:

$$\log(b^2) = \log(ac)$$

(d) Remove the logarithms from both sides to find the relationship between the spatial coordinates:

$$b^2 = ac$$

By definition, this means the values a, b, c form a **Geometric Progression (GP)**.

Final Answer: They form a Geometric Progression $(b^2 = ac)$

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: Higher-order trigonometric expressions can be simplified using basic algebraic identities. We can rewrite the sum of cubes ($a^3 + b^3 = (a+b)(a^2 - ab + b^2)$) by substituting $a = \sin^2\theta$ and $b = \cos^2\theta$.

Solution:

Let's calculate the efficiency parameter E using the given condition:

(a) Express $E = \sin^6\theta + \cos^6\theta$ as a sum of cubes:

$$E = (\sin^2\theta)^3 + (\cos^2\theta)^3$$

(b) Apply the sum of cubes identity:

$$E = (\sin^2\theta + \cos^2\theta)[(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2]$$

(c) Substitute the Pythagorean identity ($\sin^2\theta + \cos^2\theta = 1$):

$$E = 1 \cdot [(\sin^4\theta + \cos^4\theta) - \sin^2\theta\cos^2\theta]$$

(d) Rewrite the term $\sin^4\theta + \cos^4\theta$ by completing the square:

$$\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 - 2\sin^2\theta\cos^2\theta$$

(e) Substitute this back into the expression for E:

$$E = (1 - 2\sin^2\theta\cos^2\theta) - \sin^2\theta\cos^2\theta = 1 - 3\sin^2\theta\cos^2\theta = 1 - 3(\sin\theta\cos\theta)^2$$

(f) Substitute the given machine setting value ($\sin\theta\cos\theta = \frac{1}{3}$) into the simplified equation:

$$E = 1 - 3\left(\frac{1}{3}\right)^2 = 1 - 3\left(\frac{1}{9}\right) = 1 - \frac{1}{3} = \frac{2}{3}$$

Final Answer: $\boxed{\frac{2}{3}}$

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	B	4	B	5	B
6	B	7	B	8	B	9	B	10	B
11	B	12	C	13	B	14	A	15	B
16	B	17	C	18	A	19	B	20	B