# UPCATET Agriculture Statistics & Mathematics Sample Paper-7

**Duration: 20 Minutes** **Maximum Marks: 80**

## Instructions

- This paper contains **20** Multiple Choice Questions.
- Each correct answer carries **+4** mark. Incorrect answer: **−1** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

**Q1.** An agronomist collects the root-zone soil salinity index ($X$) from a problematic field plot. The recorded dataset is highly positively skewed, containing an extreme high-value outlier due to localized fertilizer accumulation. If the statistician applies a strict non-linear monotonic transformation $Y = \ln(X)$ to stabilize the variance, which of the following statements correctly identifies the shifts in the central tendency metrics?

(A) The Mean remains completely unaffected by the localized extreme outlier in both distributions.

(B) The Median of the transformed dataset $Y$ is exactly equal to the natural logarithm of the original Median of $X$.

(C) The Mode of $Y$ shifts further towards positive infinity relative to its new Mean.

(D) The Standard Deviation of $Y$ becomes strictly greater than the original variance of $X$.

**Q2.** A high-throughput phenotyping system measures the canopy heights of a genetically modified rice variety. During data cleaning, it is discovered that the highest 5% of the plants were incorrectly recorded as being 15 cm taller than their actual values, while the lowest 5% were recorded as 15 cm shorter. After correcting this symmetrical calibration error, how do the sample Mean and sample Standard Deviation ($\sigma$) change?

(A) Both the Mean and the Standard Deviation remain completely unchanged.

(B) The Mean changes, but the Standard Deviation decreases.

(C) The Mean remains unchanged, but the Standard Deviation decreases.

(D) The Mean increases, and the Standard Deviation remains unchanged.

**Q3.** A precision agricultural drone maps the nitrogen density of a field. The spatial data yields a perfect symmetric distribution with a Mean of $\mu$. However, due to a severe sensor clipping fault, all field data values falling exactly above the upper quartile ($Q_3$) are completely deleted from the dataset. For this truncated, negatively skewed remaining dataset, what is the exact mathematical relationship between the new Mean ($\mu_{\text{new}}$), Median ($M_{\text{new}}$), and Mode ($Mo_{\text{new}}$)?

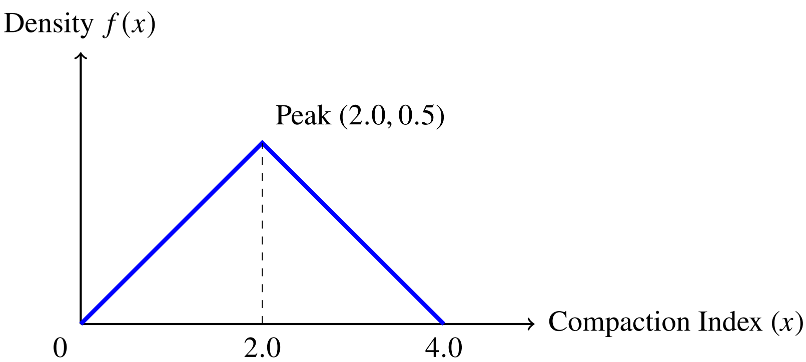
(A) $\mu_{\text{new}} = M_{\text{new}} = Mo_{\text{new}}$

(B) $\mu_{\text{new}} > M_{\text{new}} > Mo_{\text{new}}$

(C) $\mu_{\text{new}} < M_{\text{new}} < Mo_{\text{new}}$

(D) $M_{\text{new}} < \mu_{\text{new}} < Mo_{\text{new}}$

**Q4.** A digital soil-compaction sensor outputs a continuous probability density profile across a commercial orchard, as illustrated in the geometric graph below. Calculate the precise value of the Median compaction index ($m$) that splits the entire operational distribution area into two exactly equal halves:




(A) $m = 1.414$

(B) $m = 2.000$

(C) $m = 1.732$

(D) $m = 2.500$

**Q5.** An experimental farm measures the daily water-table depletion depth across two distinct soil profiles, A and B. The coefficient of variation ($CV_A$) of profile A is found to be 40%, with a mean depletion of 12 mm. Profile B exhibits a mean depletion of 18 mm with a coefficient of variation ($CV_B$) of 25%. What is the absolute ratio of the standard deviation of profile A ($\sigma_A$) to the standard deviation of profile B ($\sigma_B$)?

(A) $\frac{\sigma_A}{\sigma_B} = \frac{16}{15}$

(B) $\frac{\sigma_A}{\sigma_B} = \frac{15}{16}$

(C) $\frac{\sigma_A}{\sigma_B} = \frac{4}{3}$

(D) $\frac{\sigma_A}{\sigma_B} = \frac{1}{1}$

**Q6.** A mathematical modeling trial establishes that the distribution of seed germination times follows a structured frequency curve where the Mean is 8.5 days and the Mode is 10.0 days. Utilizing Karl Pearson's empirical formulation for moderately asymmetrical distributions, predict the exact theoretical valuation of the Median germination time for this batch.

(A) 9.5 days

(B) 9.0 days

(C) 8.0 days

(D) 7.5 days

**Q7.** The yield data of a high-density apple plantation consisting of $N$ trees yields a calculated standard deviation value of $\sigma$. If every single tree's yield value is simultaneously scaled up by multiplying it by a constant value $k$ ($k > 0$) and then systematically reduced by subtracting a base operational processing tare weight $C$, what will be the exact mathematical expression for the updated population variance ($\sigma^2_{\text{new}}$)?

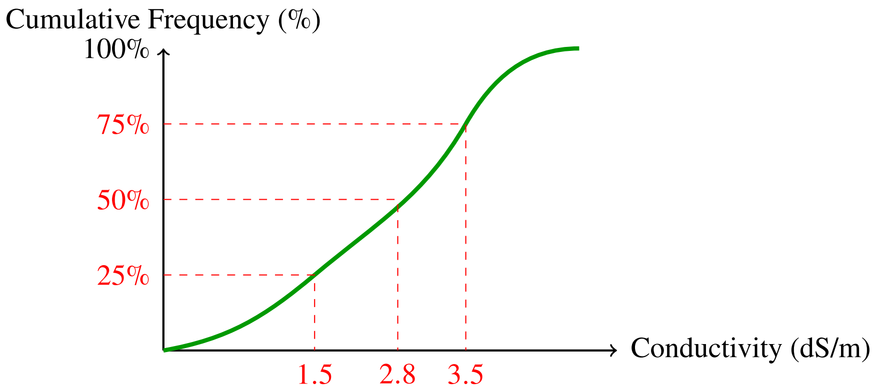
(A) $\sigma^2_{\text{new}} = k\sigma^2 - C$

(B) $\sigma^2_{\text{new}} = k^2\sigma^2$

(C) $\sigma_{\text{new}}^2 = k^2\sigma^2 - C^2$

(D) $\sigma_{\text{new}}^2 = (k - C)^2\sigma^2$

**Q8.** A sub-surface hydrology station measures the electrical conductivity of irrigation water across several wells. The cumulative frequency ogive below illustrates the distribution. Based on this historical curve, estimate the approximate Interquartile Range (IQR = $Q_3 - Q_1$) of the water quality samples:




(A) IQR = 1.3 dS/m

(B) IQR = 2.0 dS/m

(C) IQR = 1.5 dS/m

(D) IQR = 2.3 dS/m

**Q9.** An intensive hydroponics system scales its daily nutrient delivery pump cycle according to an Arithmetic Progression (AP). If the sum of the first 4 cycles is exactly 24 liters and the sum of the next 4 consecutive cycles (from the $5^{\text{th}}$ to the $8^{\text{th}}$ cycle) is exactly 56 liters, determine the precise volume of nutrient solution discharged during the very first cycle ($a_1$).

(A) 1.5 liters

(B) 3.0 liters

(C) 4.5 liters

(D) 2.0 liters

**Q10.** The vertical growth rate profile of a highly invasive weed colony expands over time such that its biomass increases daily in a strict Geometric Progression (GP).

If the third day's total biomass is 18 grams and the sixth day's total biomass escalates to 486 grams, compute the definitive total accumulated biomass of the weed colony at the end of the fourth day.

(A) 36 grams

(B) 54 grams

(C) 72 grams

(D) 108 grams

**Q11.** An optimization equation modeling the critical balance between nitrogenous fertilizer inputs ($x$) and crop profit yields results in a quadratic equation $ax^2 + bx + c = 0$, where the roots are $\alpha$ and $\beta$. If the roots are highly asymmetric such that $\alpha = 3\beta$, find the correct algebraic constraint condition among the system coefficients $a$, $b$, and $c$.

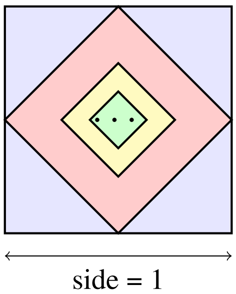
(A) $3b^2 = 16ac$

(B) $16b^2 = 3ac$

(C) $9b^2 = 4ac$

(D) $4b^2 = 9ac$

**Q12.** A complex solar-powered irrigation channel is partitioned into multiple sub-zones whose areas decrease exponentially. The layout creates an infinite geometric cascade, as mapped out in the coordinate block diagram below. Find the total limiting sum of the infinite series of areas if the first boundary square has side length 1 unit, and each subsequent inner square is formed by connecting the midpoints of the preceding outer square:




(A) Total Area = 4.0 sq units

(B) Total Area = 2.0 sq units

(C) Total Area = 1.5 sq units

(D) Total Area = 3.0 sq units

**Q13.** If the arithmetic mean ($AM$) of the layout dimensions of a rectangular greenhouse foundation plot is 10 meters and its geometric mean ($GM$) is 8 meters, formulate the underlying quadratic equation whose roots directly represent the true length and width dimensions of this greenhouse structure.

(A) $x^2 - 20x + 64 = 0$

(B) $x^2 - 10x + 8 = 0$

(C) $x^2 - 20x + 16 = 0$

(D) $x^2 - 8x + 10 = 0$

**Q14.** A surveyor divides a large agricultural field into a perfect rhombus shape. The shorter diagonal pathway measures 120 meters. If the total planimetric area of this agricultural field is exactly 9600 square meters, calculate the absolute perimeter distance required to completely encircle this field layout with protective barbed fencing wire.

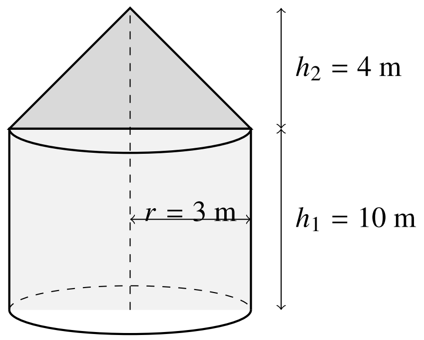
(A) 360 meters

(B) 400 meters

(C) 480 meters

(D) 520 meters

**Q15.** An industrial grain storage facility utilizes a complex composite bin composed of a right circular cylinder topped by a perfect right circular cone, as detailed in the technical schematic below. The uniform radius of both the cylinder and cone is $r = 3$ meters. The height of the cylindrical body is $h_1 = 10$ meters, and the vertical height of the conical roof cap is $h_2 = 4$ meters. Calculate the absolute total storage volume capability of this facility:




(A) Total Volume = $96\pi$ m$^3$

(B) Total Volume = $102\pi$ m$^3$

(C) Total Volume = $114\pi$ m$^3$

(D) Total Volume = $126\pi$ m$^3$

**Q16.** A cylindrical metallic water tanker has an internal radius of 2 meters and a total length of 7 meters. It completely discharges its entire volume of liquid liquid into an open, empty rectangular reservoir tank whose base dimensions have a length of 11 meters and a width of 4 meters. Determine the exact vertical rise in the water level within the rectangular reservoir tank (Take $\pi = \frac{22}{7}$).

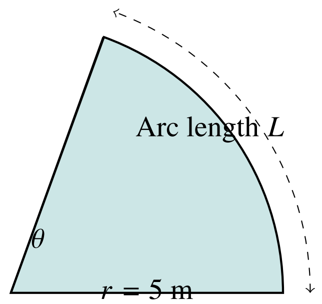
(A) 1.5 meters

(B) 2.0 meters

(C) 2.5 meters

(D) 3.0 meters

**Q17.** A dynamic micro-irrigation system sprays water over a specialized sector plot layout. The physical dimensions and boundaries of this sector zone are traced on the coordinate chart below. If the perimeter of this sector track is exactly 25 meters and its radius length is 5 meters, compute the absolute surface area available for crop seeding within this sector zone:



(A) Area = 37.5 $m^2$

(B) Area = 50.0 $m^2$

(C) Area = 75.0 $m^2$

(D) Area = 30.0 $m^2$

**Q18.** A forestry officer uses a precision clinometer instrument to evaluate the structural posture of an old timber tree standing on flat ground. From an initial observation point $A$, the angle of elevation to the top of the tree canopy is recorded as exactly $30°$. The officer then walks a distance of 40 meters directly towards the base of the tree to point $B$, where the new angle of elevation increases to exactly $60°$. Calculate the absolute structural height of this tree.

(A) $20\sqrt{3}$ meters

(B) $40\sqrt{3}$ meters

(C) 20 meters

(D) 30 meters

**Q19.** An environmental chemical decay equation modeling pesticide breakdown pathways is governed by a logarithmic relation. Simplify the following expression to find the effective concentration constant value ($x$):

$$\log_{10}(x) = 2\log_{10}(5) + \frac{1}{2}\log_{10}(64) - 3\log_{10}(2)$$

(A) $x = 10$

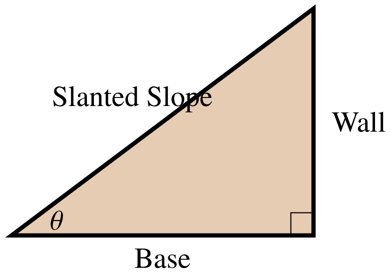
(B) $x = 25$

(C) $x = 50$

(D) $x = 100$

**Q20.** A mechanical engineers builds a hillside retaining wall structure for a terraced vineyard step. The cross-sectional right-angled triangle structural system is diagrammed below. If the structural layout enforces the trigonometric relation

$\tan(\theta) + \sec(\theta) = 3$, calculate the exact operating value of $\sin(\theta)$ representing the safe tilt gradient slope:




(A) $\sin(\theta) = \frac{4}{5}$

(B) $\sin(\theta) = \frac{3}{5}$

(C) $\sin(\theta) = \frac{1}{3}$

(D) $\sin(\theta) = \frac{2}{3}$

## Detailed Solutions

## Q1.

### Solution

**Concept:** A strictly monotonic transformation preserves the order of data points. For any continuous variable $X$ undergoing a strictly increasing monotonic transformation $Y = g(X)$, the quantiles (including the median) scale exactly with the function: $\text{Median}(Y) = g(\text{Median}(X))$. Non-linear transformations like logarithms compress extreme values, shifting the mean and mode non-linearly.

**Solution:**

Let's analyze the properties of the logarithmic transformation $Y = \ln(X)$:

(a) Evaluate the Mean: The mean is highly sensitive to outliers. Compressing the right tail shifts the mean significantly, making option (A) false.

(b) Evaluate the Median: Since $Y = \ln(X)$ is a strictly increasing monotonic function, the positional order of the data remains unchanged. The middle value of $Y$ corresponds directly to the log-transformed middle value of $X$:

$$M_Y = \ln(M_X)$$

This matches option (B).

(c) Evaluate the Mode and Standard Deviation: The mode does not shift toward infinity relative to the mean, and the variance decreases due to tail compression, making options (C) and (D) false.

**Final Answer:** The Median of the transformed dataset $Y$ is exactly equal to the natural logarithm of the original Median of $X$.

**Answer: (B)**

**Go Back to Question 1**

# Q2.

## Solution

**Concept:** The sample Mean ($\mu$) represents the balance point of a distribution, while the Standard Deviation ($\sigma$) measures the dispersion of data points relative to that mean. Symmetrical adjustments to extreme values on opposite sides of the distribution cancel out when summing values, but changing how far data points sit from the center alters the overall spread.

**Solution:**

Let's analyze the data corrections systematically:

(a) Evaluate the Mean: The highest 5% of plants were recorded as 15 cm too tall, so correcting them subtracts $0.05N \times 15$ from the sum. The lowest 5% were recorded as 15 cm too short, so correcting them adds $0.05N \times 15$ back to the sum. The net change to $\sum X$ is exactly zero, leaving the sample Mean completely unchanged.

(b) Evaluate the Standard Deviation: The extreme values in both tails are brought closer to the center of the distribution (the highest values are decreased, and the lowest values are increased). This reduces the total squared deviations from the mean ($\sum(X - \mu)^2$), causing the sample Standard Deviation to decrease.

**Final Answer:** The Mean remains unchanged, but the Standard Deviation decreases.

**Answer: (C)**

Go Back to Question 2

# Q3.

## Solution

**Concept:** Truncating a symmetric distribution by removing all data points above the upper quartile ($Q_3$) eliminates the upper right tail. This creates a highly negatively skewed (left-skewed) dataset. In a negatively skewed distribution, the measures of central tendency follow a fixed inequality order:

$$\text{Mean} < \text{Median} < \text{Mode}$$

**Solution:**
Let's analyze the structural shift caused by the sensor clipping fault:

(a) Identify the shape change: Dropping the values above $Q_3$ pulls the mean heavily toward the left because it is highly sensitive to the missing upper values.

(b) Determine the median position: The median is positional and shifts left less aggressively than the mean, sitting at the new $50^{\text{th}}$ percentile mark of the remaining data.

(c) Determine the mode position: The mode corresponds to the peak frequency of the distribution, which remains at the original center peak of the distribution profile, unaffected by the truncation of the upper tail.

(d) Arrange the terms in order: $\mu_{\text{new}} < M_{\text{new}} < Mo_{\text{new}}$.

**Final Answer:** $\boxed{\mu_{\text{new}} < M_{\text{new}} < Mo_{\text{new}}}$

**Answer: (C)**

Go Back to Question 3

# Q4.

## Solution

**Concept:** The median ($m$) of a continuous probability density function splits the total area under the curve into two equal halves of 0.5 each. For a perfectly symmetric triangular distribution, the vertical line passing through the peak acts as the axis of symmetry, dividing the area into two equal parts.

**Solution:**

Let's analyze the geometric profile of the compaction index:

(a) Note the boundaries of the triangular distribution: it starts at $x = 0$ and ends at $x = 4.0$.

(b) Identify the peak point: The vertex of the triangle is located exactly at $x = 2.0$, where the density reaches its maximum of 0.5.

(c) Check for symmetry: The distance from the start to the peak ($2.0 - 0 = 2.0$) is exactly equal to the distance from the peak to the end ($4.0 - 2.0 = 2.0$).

(d) Determine the median: Because the triangle is perfectly symmetrical, the vertical line at $x = 2.0$ divides the total area under the curve into two equal halves. Thus, $m = 2.000$.

**Final Answer:** $\boxed{m = 2.000}$

**Answer: (B)**

**Go Back to Question 4**

# Q5.

## Solution

**Concept:** The Coefficient of Variation (CV) relates the standard deviation ($\sigma$) of a distribution to its mean ($\mu$) via the formula:

$$\mathrm{CV} = \frac{\sigma}{\mu} \implies \sigma = \mathrm{CV} \times \mu$$

**Solution:**
Let's compute the standard deviations for both soil profiles and find their ratio:

(a) Calculate the standard deviation of profile A ($\sigma_A$) using $\mathrm{CV}_A = 40\% = 0.40$ and $\mu_A = 12$ mm:

$$\sigma_A = 0.40 \times 12 = 4.8 \text{ mm}$$

(b) Calculate the standard deviation of profile B ($\sigma_B$) using $\mathrm{CV}_B = 25\% = 0.25$ and $\mu_B = 18$ mm:

$$\sigma_B = 0.25 \times 18 = 4.5 \text{ mm}$$

(c) Compute the absolute ratio $\frac{\sigma_A}{\sigma_B}$:

$$\frac{\sigma_A}{\sigma_B} = \frac{4.8}{4.5} = \frac{48}{45} = \frac{16}{15}$$

**Final Answer:** $\boxed{\frac{\sigma_A}{\sigma_B} = \frac{16}{15}}$

**Answer: (A)**

Go Back to Question 5

## Q6.

### Solution

**Concept:** Karl Pearson's empirical relationship for moderately asymmetrical distributions states that the difference between the mean and the mode is roughly three times the difference between the mean and the median:

$$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$$

**Solution:**
Let's substitute the given parameters to solve for the median germination time:

(a) Identify the given metrics: Mean = 8.5 days and Mode = 10.0 days.

(b) Set up the empirical relationship equation:

$$8.5 - 10.0 = 3(8.5 - \text{Median})$$

(c) Simplify the left side of the expression:

$$-1.5 = 3(8.5 - \text{Median})$$

(d) Divide both sides by 3:

$$-0.5 = 8.5 - \text{Median}$$

(e) Isolate the median variable:

$$\text{Median} = 8.5 - (-0.5) = 8.5 + 0.5 = 9.0 \text{ days}$$

**Final Answer:** $\boxed{9.0 \text{ days}}$

**Answer: (B)**

Go Back to Question 6

# Q7.

## Solution

**Concept:** For any random variable $X$ with variance $\sigma^2$, applying a linear transformation of the form $Y = kX - C$ alters its variance according to the property:

$$\text{Var}(kX - C) = k^2 \cdot \text{Var}(X)$$

Subtracting or adding a constant shift factor $C$ has no effect on the spread or variance of the distribution.

**Solution:**

Let's calculate the updated population variance step-by-step:

(a) State the original variance of the apple plantation yield: $\text{Var}_{\text{old}} = \sigma^2$.

(b) Identify the transformation applied to each tree's yield: $X_{\text{new}} = kX_{\text{old}} - C$.

(c) Apply the variance transformation rule:

$$\sigma^2_{\text{new}} = \text{Var}(kX_{\text{old}} - C) = k^2 \cdot \text{Var}(X_{\text{old}}) = k^2\sigma^2$$

**Final Answer:** $\boxed{k^2\sigma^2}$

**Answer: (B)**

**Go Back to Question 7**

# Q8.

## Solution

**Concept:** The Interquartile Range (IQR) measures the statistical spread of the middle 50% of a dataset. It is calculated as the difference between the third quartile ($Q_3$, corresponding to the 75% cumulative frequency mark) and the first quartile ($Q_1$, corresponding to the 25% cumulative frequency mark):

$$\text{IQR} = Q_3 - Q_1$$

**Solution:**
Let's read the quartile values directly from the cumulative frequency ogive curve:

(a) Find $Q_1$: Locate 25% on the vertical axis, move horizontally to intersect the green curve, and read down to the horizontal axis to find $Q_1 = 1.5$ dS/m.

(b) Find $Q_3$: Locate 75% on the vertical axis, move horizontally to intersect the green curve, and read down to the horizontal axis to find $Q_3 = 3.5$ dS/m.

(c) Calculate the difference to find the Interquartile Range:

$$\text{IQR} = Q_3 - Q_1 = 3.5 - 1.5 = 2.0 \text{ dS/m}$$

**Final Answer:** 2.0 dS/m

**Answer: (B)**

**Go Back to Question 8**

## Q9.

### Solution

**Concept:** The sum of the first $n$ terms of an Arithmetic Progression (AP) is given by $S_n = \frac{n}{2}[2a_1 + (n-1)d]$, where $a_1$ is the first term and $d$ is the common difference.

**Solution:**

Let's set up a system of linear equations to solve for the initial discharge volume $a_1$:

(a) Write the expression for the sum of the first 4 cycles ($S_4 = 24$):

$$S_4 = \frac{4}{2}[2a_1 + 3d] = 24 \implies 2(2a_1 + 3d) = 24 \implies 2a_1 + 3d = 12 \quad \text{— (Eq. 1)}$$

(b) Use the total sum of the first 8 cycles ($S_8$) to frame the next equation. The sum of the first 8 terms is the sum of the first 4 cycles plus the next 4 cycles: $S_8 = 24 + 56 = 80$.

$$S_8 = \frac{8}{2}[2a_1 + 7d] = 80 \implies 4(2a_1 + 7d) = 80 \implies 2a_1 + 7d = 20 \quad \text{— (Eq. 2)}$$

(c) Subtract Eq. 1 from Eq. 2 to eliminate $a_1$ and solve for $d$:

$$(2a_1 + 7d) - (2a_1 + 3d) = 20 - 12 \implies 4d = 8 \implies d = 2$$

(d) Substitute $d = 2$ back into Eq. 1 to find $a_1$:

$$2a_1 + 3(2) = 12 \implies 2a_1 + 6 = 12 \implies 2a_1 = 6 \implies a_1 = 3.0 \text{ liters}$$

**Final Answer:** $\boxed{3.0 \text{ liters}}$

**Answer: (B)**

Go Back to Question 9

# Q10.

## Solution

**Concept:** Any term in a Geometric Progression (GP) can be written as $a_n = a_1 \cdot r^{n-1}$, where $a_1$ is the initial term and $r$ is the common multiplier ratio.

**Solution:**

Let's calculate the biomass accumulated on the fourth day:

(a) Write down the expressions for the third and sixth days:

$$a_3 = a_1 \cdot r^2 = 18$$

$$a_6 = a_1 \cdot r^5 = 486$$

(b) Divide the expression for $a_6$ by the expression for $a_3$ to isolate $r^3$:

$$\frac{a_1 \cdot r^5}{a_1 \cdot r^2} = \frac{486}{18} \implies r^3 = 27 \implies r = \sqrt[3]{27} = 3$$

(c) Compute the biomass for the fourth day ($a_4$) by multiplying the third day's biomass by the common ratio $r$:

$$a_4 = a_3 \cdot r = 18 \times 3 = 54 \text{ grams}$$

**Final Answer:** 54 grams

**Answer: (B)**

**Go Back to Question 10**

# Q11.

## Solution

**Concept:** For any quadratic equation $ax^2 + bx + c = 0$ with roots $\alpha$ and $\beta$, Vieta's formulas state that:

$$\alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}$$

**Solution:**
Let's use the root relation $\alpha = 3\beta$ to find the required coefficient constraint:

(a) Substitute $\alpha = 3\beta$ into the sum of roots equation:

$$3\beta + \beta = -\frac{b}{a} \implies 4\beta = -\frac{b}{a} \implies \beta = -\frac{b}{4a}$$

(b) Substitute $\alpha = 3\beta$ into the product of roots equation:

$$(3\beta)(\beta) = \frac{c}{a} \implies 3\beta^2 = \frac{c}{a}$$

(c) Substitute the expression for $\beta$ from step 1 into the product equation:

$$3\left(-\frac{b}{4a}\right)^2 = \frac{c}{a} \implies 3\left(\frac{b^2}{16a^2}\right) = \frac{c}{a}$$

(d) Simplify the equation by canceling one factor of $a$ from the denominators:

$$\frac{3b^2}{16a} = c \implies 3b^2 = 16ac$$

**Final Answer:** $\boxed{3b^2 = 16ac}$

**Answer: (A)**

Go Back to Question 11

## Q12.

### Solution

**Concept:** Connecting the midpoints of any square creates a smaller inner square whose area is exactly half ($\frac{1}{2}$) the area of the outer square. The total area of this geometric pattern forms an infinite geometric series with a common ratio of $r = \frac{1}{2}$.

**Solution:**

Let's compute the sum of this infinite geometric progression:

(a) Calculate the area of the first large square ($a_1$) with side length 1:

$$a_1 = 1^2 = 1 \text{ sq unit}$$

(b) Identify the scaling factor for subsequent squares: since connecting midpoints halves the area, the common ratio is $r = \frac{1}{2}$.

(c) Use the infinite geometric series sum formula $S_\infty = \frac{a_1}{1-r}$:

$$S_\infty = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2.0 \text{ sq units}$$

**Final Answer:** $\boxed{\text{Total Area } = 2.0 \text{ sq units}}$

**Answer: (B)**

**Go Back to Question 12**

## Q13.

### Solution

**Concept:** For two dimensions (roots) $x_1$ and $x_2$, the Arithmetic Mean is AM $= \frac{x_1+x_2}{2}$ and the Geometric Mean is GM $= \sqrt{x_1 x_2}$. Any quadratic equation can be written in terms of its roots as:

$$x^2 - (\text{Sum of Roots})x + (\text{Product of Roots}) = 0$$

**Solution:**
Let's find the components of the quadratic equation:

(a) Calculate the sum of roots using AM = 10:

$$\frac{x_1 + x_2}{2} = 10 \implies \text{Sum} = x_1 + x_2 = 20$$

(b) Calculate the product of roots using GM = 8:

$$\sqrt{x_1 x_2} = 8 \implies \text{Product} = x_1 x_2 = 8^2 = 64$$

(c) Substitute the sum and product into the quadratic template:

$$x^2 - 20x + 64 = 0$$

**Final Answer:** $\boxed{x^2 - 20x + 64 = 0}$

**Answer: (A)**

Go Back to Question 13

# Q14.

## Solution

**Concept:** The area ($A$) of a rhombus is given by half the product of its two diagonals: $A = \frac{1}{2} \cdot d_1 \cdot d_2$. The diagonals of a rhombus intersect at right angles ($90^\circ$) and bisect each other, forming four identical right-angled triangles that can be used to find the side length ($s$) via the Pythagorean theorem.

**Solution:**

Let's find the side length and total perimeter step-by-step:

(a) Use the given area ($A = 9600$) and shorter diagonal ($d_1 = 120$) to solve for the second diagonal ($d_2$):

$$9600 = \frac{1}{2} \cdot 120 \cdot d_2 \implies 9600 = 60 \cdot d_2 \implies d_2 = \frac{9600}{60} = 160 \text{ meters}$$

(b) Find the half-lengths of the two intersecting diagonals to construct a right triangle:

$$\frac{d_1}{2} = \frac{120}{2} = 60 \text{ m}, \quad \frac{d_2}{2} = \frac{160}{2} = 80 \text{ m}$$

(c) Apply the Pythagorean theorem to calculate the side length ($s$) of the rhombus:

$$s = \sqrt{60^2 + 80^2} = \sqrt{3600 + 6400} = \sqrt{10000} = 100 \text{ meters}$$

(d) Compute the total fencing perimeter ($P = 4s$):

$$P = 4 \times 100 = 400 \text{ meters}$$

**Final Answer:** $\boxed{400 \text{ meters}}$

**Answer: (B)**

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# Q15.

## Solution

**Concept:** The total volume ($V_{\text{total}}$) of this composite grain storage bin is the sum of the volume of its cylindrical base ($V_{\text{cylinder}} = \pi r^2 h_1$) and the volume of its conical roof cap ($V_{\text{cone}} = \frac{1}{3}\pi r^2 h_2$).

**Solution:**

Let's compute the individual capacities using the radius $r = 3$ meters:

(a) Calculate the volume of the cylindrical lower body ($h_1 = 10$ m):

$$V_{\text{cylinder}} = \pi \cdot (3)^2 \cdot 10 = 90\pi \text{ m}^3$$

(b) Calculate the volume of the conical roof cap ($h_2 = 4$ m):

$$V_{\text{cone}} = \frac{1}{3} \cdot \pi \cdot (3)^2 \cdot 4 = \frac{1}{3} \cdot \pi \cdot 9 \cdot 4 = 12\pi \text{ m}^3$$

(c) Combine both individual volumes to find the total capacity:

$$V_{\text{total}} = 90\pi + 12\pi = 102\pi \text{ m}^3$$

**Final Answer:** $\boxed{\text{Total Volume } = 102\pi \text{ m}^3}$

**Answer: (B)**

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## Q16.

### Solution

**Concept:** According to the principle of conservation of volume, the total liquid volume discharged from the cylindrical tanker must exactly equal the volume of water added to the rectangular reservoir tank:

$$V_{\text{cylinder}} = V_{\text{rectangular box}} \implies \pi r^2 h = \text{length} \times \text{width} \times \Delta H$$

**Solution:**
Let's solve for the vertical rise in the water level ($\Delta H$):

(a) Calculate the volume of the cylindrical tanker with $r = 2$ m and $h = 7$ m using $\pi = \frac{22}{7}$:

$$V_{\text{cylinder}} = \frac{22}{7} \times 2^2 \times 7 = 22 \times 4 = 88 \text{ m}^3$$

(b) Set up the volume equation for the rectangular reservoir with base dimensions 11 m $\times$ 4 m:

$$V_{\text{rectangular box}} = 11 \times 4 \times \Delta H = 44 \times \Delta H$$

(c) Equate the two volumes and solve for $\Delta H$:

$$88 = 44 \times \Delta H \implies \Delta H = \frac{88}{44} = 2.0 \text{ meters}$$

**Final Answer:** 2.0 meters

**Answer: (B)**

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## Q17.

### Solution

**Concept:** The perimeter ($P$) of a circular sector is the sum of its two straight boundary radii and its curved arc length ($L$): $P = 2r + L$. Once the arc length is found, the total surface area ($A$) of the sector zone can be calculated using the formula:

$$A = \frac{1}{2} \cdot L \cdot r$$

**Solution:**
Let's solve for the arc length and surface area step-by-step:

(a) Use the given perimeter ($P = 25$) and radius ($r = 5$) to isolate the arc length $L$:

$$25 = 2(5) + L \implies 25 = 10 + L \implies L = 15 \text{ meters}$$

(b) Substitute $L = 15$ and $r = 5$ into the sector area formula:

$$A = \frac{1}{2} \times 15 \times 5 = \frac{75}{2} = 37.5 \text{ m}^2$$

**Final Answer:** $\boxed{\text{Area } = 37.5 \text{ m}^2}$

**Answer: (A)**

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## Q18.

### Solution

**Concept:** This problem can be modeled using right-triangle trigonometry. Let $h$ be the height of the tree, and $x$ be the horizontal distance from point $B$ to the base of the tree. Set up tangent ratio equations for both observation points.

**Solution:**

Let's solve for the height of the tree using the two angular perspectives:

(a) Set up the tangent equation for the closer observation point $B$ ($60^\circ$ angle):

$$\tan(60^\circ) = \frac{h}{x} \implies \sqrt{3} = \frac{h}{x} \implies x = \frac{h}{\sqrt{3}}$$

(b) Set up the tangent equation for the initial observation point $A$ ($30^\circ$ angle), where the total distance to the base is $40 + x$:

$$\tan(30^\circ) = \frac{h}{40 + x} \implies \frac{1}{\sqrt{3}} = \frac{h}{40 + x} \implies 40 + x = h\sqrt{3}$$

(c) Substitute the expression for $x$ from step 1 into the equation from step 2:

$$40 + \frac{h}{\sqrt{3}} = h\sqrt{3}$$

(d) Multiply the entire equation by $\sqrt{3}$ to eliminate the denominator:

$$40\sqrt{3} + h = 3h \implies 2h = 40\sqrt{3} \implies h = 20\sqrt{3} \text{ meters}$$

**Final Answer:** $\boxed{20\sqrt{3} \text{ meters}}$

**Answer: (A)**

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## Q19.

### Solution

**Concept:** Apply the standard laws of logarithms to simplify the decay constant expression: the power rule $k\log_b(a) = \log_b(a^k)$, the product rule $\log_b(a) + \log_b(c) = \log_b(ac)$, and the quotient rule $\log_b(a) - \log_b(c) = \log_b(\frac{a}{c})$.

**Solution:**

Let's simplify the right side of the logarithmic equation step-by-step:

(a) Apply the power rule to rewrite each individual term:

$$2\log_{10}(5) = \log_{10}(5^2) = \log_{10}(25)$$

$$\frac{1}{2}\log_{10}(64) = \log_{10}(64^{1/2}) = \log_{10}(8)$$

$$3\log_{10}(2) = \log_{10}(2^3) = \log_{10}(8)$$

(b) Substitute these simplified terms back into the original expression:

$$\log_{10}(x) = \log_{10}(25) + \log_{10}(8) - \log_{10}(8)$$

(c) Combine the terms (the positive and negative $\log_{10}(8)$ terms cancel out):

$$\log_{10}(x) = \log_{10}(25)$$

(d) Drop the logarithms from both sides to find $x$:

$$x = 25$$

**Final Answer:** $\boxed{x = 25}$

**Answer: (B)**

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# Q20.

## Solution

**Concept:** Use the fundamental trigonometric identity $\sec^2\theta - \tan^2\theta = 1$. Factoring this expression as a difference of squares yields $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$, which allows us to find the value of $(\sec\theta - \tan\theta)$ and solve for the individual functions.

**Solution:**

Let's calculate the operating value of $\sin(\theta)$ step-by-step:

(a) Use the identity property to find the value of $(\sec\theta - \tan\theta)$ given that $\tan\theta + \sec\theta = 3$:

$$\sec\theta - \tan\theta = \frac{1}{\sec\theta + \tan\theta} = \frac{1}{3} \quad \text{— (Eq. 1)}$$

(b) Write down the original given equation:

$$\sec\theta + \tan\theta = 3 \quad \text{— (Eq. 2)}$$

(c) Subtract Eq. 1 from Eq. 2 to isolate the tangent function:

$$2\tan\theta = 3 - \frac{1}{3} = \frac{8}{3} \implies \tan\theta = \frac{4}{3}$$

(d) Add Eq. 1 and Eq. 2 together to isolate the secant function:

$$2\sec\theta = 3 + \frac{1}{3} = \frac{10}{3} \implies \sec\theta = \frac{5}{3}$$

(e) Use the relation $\sin\theta = \frac{\tan\theta}{\sec\theta}$ to find the final value:

$$\sin\theta = \frac{4/3}{5/3} = \frac{4}{5}$$

**Final Answer:** $\boxed{\sin(\theta) = \frac{4}{5}}$

**Answer: (A)**

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# Answer Key

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | B | 2 | C | 3 | C | 4 | B | 5 | A |
| 6 | B | 7 | B | 8 | B | 9 | B | 10 | B |
| 11 | A | 12 | B | 13 | A | 14 | B | 15 | B |
| 16 | B | 17 | A | 18 | A | 19 | B | 20 | A |