UPCATET Mathematics Sample Paper-1

Duration: 80 Minutes **Maximum Marks: 320**

Instructions

- This paper contains **80** Multiple Choice Questions.
- Each correct answer carries **+4** mark. Incorrect answer: **−1** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. The value of $\lim_{x\to 0} \frac{1-\cos(2x)\cdot\cos(3x)}{x^2}$ is equal to:

(A) $\frac{5}{2}$

(B) $\frac{13}{2}$

(C) $\frac{9}{2}$

(D) $\frac{11}{2}$

Q2. A straight line L passes through the origin and makes an angle θ with the positive x-axis. It intersects the lines $x + y = 1$ and $x + y = 3$ at points A and B respectively. If the length $AB = \sqrt{2}$, then the value of θ can be:

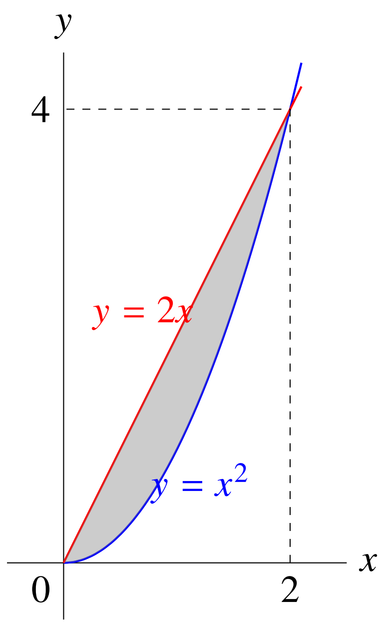
(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{12}$

Q3. Consider the region bounded by the curves $y = x^2$ and $y = 2x$. Which of the following TikZ diagrams accurately describes the shaded area enclosed between these two curves?



(A) The area bounded from below by $y = 2x$ and from above by $y = x^2$ from $x = 0$ to $x = 2$.

(B) The area bounded from below by $y = x^2$ and from above by $y = 2x$ from $x = 0$ to $x = 2$.

(C) The area bounded from $x = 0$ to $x = 4$ between the line and the x-axis.

(D) The area under the parabola from $x = 0$ to $x = 2$ only.

Q4. If the matrix $A = \begin{bmatrix} 1 & \lambda & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ is singular, then the value of λ is:

(A) 2

(B) -1

(C) -2

(D) 1

Q5. The projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $\frac{5}{\sqrt{6}}$

(B) $\frac{10}{\sqrt{6}}$

(C) $\frac{5}{3}$

(D) $\frac{10}{3}$

Q6. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$, then the value of $\cos^{-1} x + \cos^{-1} y$ is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) π

Q7. Let A and B be two independent events such that $P(A) = 0.3$ and $P(B) = 0.4$. The probability $P(A \cup B^c)$ is equal to:

(A) 0.72

(B) 0.58

(C) 0.82

(D) 0.48

Q8. The function $f(x) = 2x^3 - 9x^2 + 12x + 5$ is monotonically decreasing in the interval:

(A) $(1, 2)$

(B) $(-\infty, 1)$

(C) $(2, \infty)$

(D) $(0, 2)$

Q9. The length of the latus rectum of the parabola $y^2 - 4y - 8x + 4 = 0$ is:

(A) 4

(B) 8

(C) 2

(D) 16

Q10. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then bc^2, ca^2, ab^2 are in:

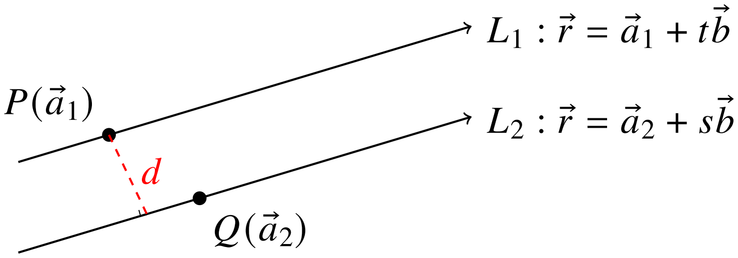
(A) Arithmetic Progression (AP)

(B) Geometric Progression (GP)

(C) Harmonic Progression (HP)

(D) None of these

Q11. The shortest distance between the parallel lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ can be calculated using vector methods. Consider the geometric representation below:



(A) $\sqrt{\frac{8}{29}}$

(B) $\sqrt{\frac{14}{29}}$

(C) $\sqrt{\frac{5}{29}}$

(D) $\sqrt{\frac{11}{29}}$

Q12. The value of $\tan(2\tan^{-1}\frac{1}{5} - \frac{\pi}{4})$ is:

(A) $-\frac{7}{17}$

(B) $-\frac{17}{7}$

(C) $\frac{7}{17}$

(D) $\frac{17}{7}$

Q13. A bag contains 5 white and 3 black balls. Two balls are drawn at random one after the other without replacement. The probability that both balls are black is:

(A) $\frac{3}{28}$

(B) $\frac{9}{64}$

(C) $\frac{1}{14}$

(D) $\frac{5}{28}$

Q14. The value of the integral $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\, dx$ is:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) 0

Q15. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is:

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{16}{25}$

(D) $\frac{2}{5}$

Q16. The number of permutations of the letters of the word 'EXAMINATION' is:

(A) 4989600

(B) 2494800

(C) 9979200

(D) 1102400

Q17. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Q18. If α and β are the roots of $x^2 - 3x + 2 = 0$, then the value of $\alpha^3 + \beta^3$ is:

(A) 7

(B) 9

(C) 15

(D) 27

Q19. The value of $\int \frac{1}{x(x^4+1)}\,dx$ is equal to:

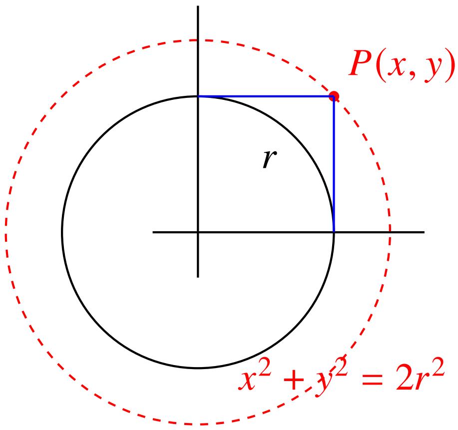
(A) $\frac{1}{4}\ln\left|\frac{x^4}{x^4+1}\right| + C$

(B) $\ln\left|\frac{x^4}{x^4+1}\right| + C$

(C) $\frac{1}{4}\ln\left|\frac{x^4+1}{x^4}\right| + C$

(D) $\frac{1}{2}\ln\left|\frac{x^2}{x^2+1}\right| + C$

Q20. The locus of the point of intersection of two perpendicular tangents to the circle $x^2 + y^2 = r^2$ is known as its director circle. Which of the diagrams accurately visualizes this locus?



(A) $x^2 + y^2 = 4r^2$

(B) $x^2 + y^2 = 2r^2$

(C) $x^2 + y^2 = \sqrt{2}r^2$

(D) $x^2 + y^2 = \frac{r^2}{2}$

Q21. The value of $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is:

(A) $a + b + c$

(B) abc

(C) 0

(D) 1

Q22. The value of $\cos 20^\circ \cos 40^\circ \cos 80^\circ$ is:

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $\frac{1}{16}$

Q23. The variance of the first n natural numbers is given by:

(A) $\frac{n^2-1}{12}$

(B) $\frac{n^2+1}{12}$

(C) $\frac{n(n+1)}{12}$

(D) $\frac{n^2-1}{6}$

Q24. If $y = \ln(\sec x + \tan x)$, then $\frac{dy}{dx}$ is equal to:

(A) $\sec x$

(B) $\tan x$

(C) $\sec x + \tan x$

(D) $\frac{1}{\sec x+\tan x}$

Q25. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$ if the value of m is:

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) -1

Q26. If $\begin{bmatrix} x+y & 2 \\ 1 & x-y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, then the values of x and y are respectively:

(A) 2, 1

(B) 1, 2

(C) 3, 0

(D) 1, 1

Q27. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, then the value of λ is:

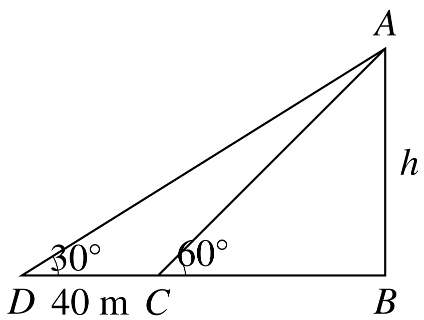
(A) -4

(B) -2

(C) 2

(D) 4

Q28. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60°. When he moves 40 meters away from the bank, the angle of elevation becomes 30°. Which geometric description using a triangle matches this height and distance configuration?



(A) The height of the tree h is $20\sqrt{3}$ meters.

(B) The height of the tree h is $40\sqrt{3}$ meters.

(C) The height of the tree h is 20 meters.

(D) The height of the tree h is 30 meters.

Q29. For two events A and B, if $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, and $P(A \cap B) = \frac{1}{4}$, then $P(A|B)$ is:

(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) $\frac{2}{3}$

(D) $\frac{3}{8}$

Q30. The maximum value of $f(x) = \sin x + \cos x$ occurs at x equal to:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Q31. The equation of the circle concentric with $x^2 + y^2 - 4x - 6y - 3 = 0$ and passing through the point $(7, 3)$ is:

(A) $x^2 + y^2 - 4x - 6y - 12 = 0$

(B) $x^2 + y^2 - 4x - 6y - 21 = 0$

(C) $x^2 + y^2 - 4x - 6y - 25 = 0$

(D) $x^2 + y^2 - 4x - 6y + 12 = 0$

Q32. The sum of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$ up to infinity is:

(A) 1.5

(B) 2

(C) 2.5

(D) ∞

Q33. The direction cosines of a line equally inclined to the three coordinate axes are:

(A) $\left(\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right)$

(B) $\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$

(C) $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$

(D) $(1, 1, 1)$

Q34. If $\tan\theta = \frac{4}{3}$, then the value of $\sin 2\theta$ is:

(A) $\frac{24}{25}$

(B) $\frac{7}{25}$

(C) $\frac{12}{25}$

(D) $\frac{16}{25}$

Q35. The arithmetic mean of 10 observations is 25. If each observation is multiplied by 2 and then increased by 5, the new mean will be:

(A) 50

(B) 55

(C) 60

(D) 45

Q36. The value of $\int e^x(\tan x + \sec^2 x)\,dx$ is equal to:

(A) $e^x \sec x + C$

(B) $e^x \tan x + C$

(C) $e^x \sec^2 x + C$

(D) $e^x(\tan x + \sec x) + C$

Q37. The condition for the line $lx + my + n = 0$ to be a tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is:

(A) $a^2l^2 + b^2m^2 = n^2$

(B) $a^2l^2 - b^2m^2 = n^2$

(C) $a^2m^2 - b^2l^2 = n^2$

(D) $a^2l^2 - b^2m^2 = n$

Q38. The value of ${}^{15}C_8 + {}^{15}C_7$ is:

(A) ${}^{15}C_9$

(B) ${}^{16}C_7$

(C) ${}^{16}C_8$

(D) ${}^{15}C_{15}$

Q39. The vector equation of the line passing through the point $(1, -2, 3)$ and parallel to the vector $2\hat{i} + 3\hat{j} - \hat{k}$ is:

(A) $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$

(B) $\vec{r} = (2\hat{i} + 3\hat{j} - \hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$

(C) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - 3\hat{j} - \hat{k})$

(D) $\vec{r} = (\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$

Q40. If $\cos^{-1}\left(\frac{x}{a}\right) + \cos^{-1}\left(\frac{y}{b}\right) = \alpha$, then $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2}$ is equal to:

(A) $\sin^2\alpha$

(B) $\cos^2\alpha$

(C) $\tan^2\alpha$

(D) 1

Q41. Three fair coins are tossed. What is the probability of getting at least two heads?

(A) $\frac{1}{4}$

(B) $\frac{3}{8}$

(C) $\frac{1}{2}$

(D) $\frac{5}{8}$

Q42. The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1} x$ is:

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) $\frac{2}{1+x^2}$

Q43. The point $(1, 2)$ lies relative to the circle $x^2 + y^2 - 4x - 2y + 1 = 0$:

(A) inside the circle

(B) outside the circle

(C) on the circle

(D) at the center of the circle

Q44. The inverse of the matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ is:

(A) $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

Q45. The angle between the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ is:

(A) $\cos^{-1}\left(\frac{1}{3}\right)$

(B) $\cos^{-1}\left(-\frac{1}{3}\right)$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Q46. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{3}$

(D) $-\frac{\pi}{3}$

Q47. For a moderately skewed distribution, the relationship between Mean, Median, and Mode is given by:

(A) Mode = 3 Median − 2 Mean

(B) Mean = 3 Median − 2 Mode

(C) Median = 3 Mode − 2 Mean

(D) Mode = 3 Mean − 2 Median

Q48. The slope of the tangent to the curve $y = x^3 - x$ at $x = 2$ is:

(A) 11

(B) 12

(C) 6

(D) 10

Q49. The coordinates of the focus of the parabola $x^2 = -16y$ are:

(A) $(4, 0)$

(B) $(-4, 0)$

(C) $(0, 4)$

(D) $(0, -4)$

Q50. If the third term of a Geometric Progression is 4, then the product of its first 5 terms is:

(A) 4^3

(B) 4^5

(C) 4^4

(D) None of these

Q51. The distance of the point $(2, 3, 4)$ from the plane $3x - 6y + 2z + 11 = 0$ is:

(A) 1 unit

(B) 2 units

(C) 3 units

(D) 0 units

Q52. The value of $\lim_{x\to 0} \frac{e^{\sin x}-1}{x}$ is:

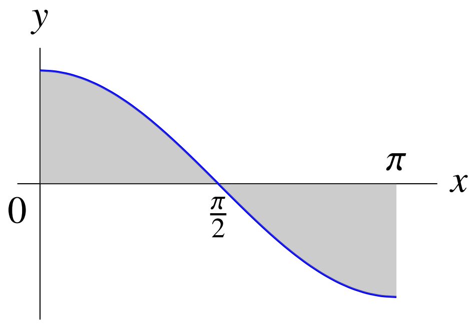
(A) 0

(B) 1

(C) e

(D) -1

Q53. The area bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ can be visualized as two segments split by the x-axis. Which of the following diagrams accurately conveys this standard geometric definite integral setup?



(A) 0

(B) 1

(C) 2

(D) 4

Q54. The equation of the straight line making equal intercepts on the coordinate axes and passing through the point $(2, 4)$ is:

(A) $x + y = 6$

(B) $x - y = 2$

(C) $2x + y = 8$

(D) $x + 2y = 10$

Q55. The value of the determinant $\begin{vmatrix} \log a & \log b \\ \log b & \log a \end{vmatrix}$ when $a = b$ is:

(A) 1

(B) 0

(C) $\log a$

(D) $2\log a$

Q56. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{4}$

Q57. In a triangle ABC, if $a = 2, b = 3, c = 4$, then $\cos A$ is equal to:

(A) $\frac{7}{8}$

(B) $\frac{5}{8}$

(C) $\frac{11}{16}$

(D) $\frac{13}{16}$

Q58. A box contains 2 brilliant and 3 average students. If 2 students are picked at random, the probability that exactly one is brilliant is:

(A) $\frac{3}{5}$

(B) $\frac{2}{5}$

(C) $\frac{1}{5}$

(D) $\frac{4}{5}$

Q59. The point of local minima for the function $f(x) = x^2 - 4x + 3$ is:

(A) $x = 1$

(B) $x = 2$

(C) $x = 3$

(D) $x = 4$

Q60. The distance between the parallel lines $3x + 4y - 5 = 0$ and $3x + 4y + 10 = 0$ is:

(A) 1

(B) 2

(C) 3

(D) 5

Q61. The sum of the coefficients in the expansion of $(1 - x)^{10}$ is:

(A) 1024

(B) 0

(C) 1

(D) 512

Q62. The equation of the line passing through $(1, 1, 1)$ and perpendicular to the plane $x + 2y + 3z = 5$ is:

(A) $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$

(B) $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$

(C) $\frac{x+1}{1} = \frac{y+1}{2} = \frac{z+1}{3}$

(D) $\frac{x-1}{5} = \frac{y-1}{5} = \frac{z-1}{5}$

Q63. The minimum value of $3 \sin x + 4 \cos x$ is:

(A) -5

(B) 5

(C) -7

(D) 0

Q64. The mean deviation from the mean for the data $3, 5, 6, 7, 9$ is:

(A) 1.6

(B) 2.0

(C) 1.2

(D) 2.4

Q65. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ are respectively:

(A) 2, 3

(B) 2, not defined

(C) 1, 2

(D) not defined, 2

Q66. The equation of the normal to the curve $y = \sin x$ at $(0, 0)$ is:

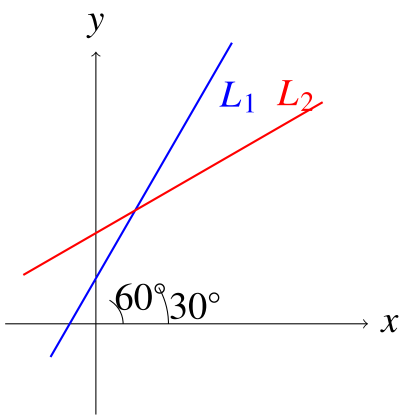
(A) $x + y = 0$

(B) $x - y = 0$

(C) $y = 0$

(D) $x = 0$

Q67. The angle between the lines $y = \sqrt{3}x + 1$ and $\sqrt{3}y = x + 2$ can be analyzed by computing their respective inclination angles. Consider the geometric scheme below:



(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q68. If A is a square matrix of order 3 such that $|A| = 5$, then the value of $|\text{adj } A|$ is:

(A) 5

(B) 25

(C) 125

(D) 1

Q69. The value of $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2$ is equal to:

(A) a^2b^2

(B) $2a^2b^2$

(C) ab

(D) 0

Q70. The value of $\sin 50° - \sin 70° + \sin 10°$ is equal to:

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Q71. The probability of throwing a sum of 7 with two fair dice is:

(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) $\frac{5}{36}$

(D) $\frac{1}{9}$

Q72. The value of $\int_{-1}^{1} |x| \, dx$ is:

(A) 0

(B) 1

(C) 2

(D) $\frac{1}{2}$

Q73. The intercept made by the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ on the x-axis is:

(A) 4

(B) 6

(C) 8

(D) 10

Q74. The value of k for which the system of equations $x + y + z = 2$, $2x + 3y + 2z = 5$, $2x + 3y + (k^2 - 1)z = k + 3$ has infinitely many solutions is:

(A) $\sqrt{3}$

(B) $-\sqrt{3}$

(C) $\pm\sqrt{3}$

(D) 0

Q75. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{b} = \vec{0}$, then which of the following must hold true?

(A) $\vec{a}$ is parallel to $\vec{b}$

(B) $\vec{a}$ is perpendicular to $\vec{b}$

(C) At least one of $\vec{a}$ or $\vec{b}$ is a null vector

(D) $\vec{a}$ and $\vec{b}$ are unit vectors

Q76. The domain of definition of the function $f(x) = \frac{1}{\sqrt{x^2-3x+2}}$ is:

(A) $(-\infty, 1) \cup (2, \infty)$

(B) $(-\infty, 1] \cup [2, \infty)$

(C) $(1, 2)$

(D) $[1, 2]$

Q77. The equation of the line passing through the intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and parallel to the x-axis is:

(A) $y = 3$

(B) $y = 2$

(C) $y = 1$

(D) $y = 0$

Q78. The number of subsets of a set containing 5 elements is:

(A) 5

(B) 10

(C) 25

(D) 32

Q79. The value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ is:

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{6}$

(D) $\frac{11\pi}{6}$

Q80. If a variable takes the values $0, 1, 2, \ldots, n$ with frequencies proportional to binomial coefficients ${}^nC_0, {}^nC_1, \ldots, {}^nC_n$, then the mean of the distribution is:

(A) $\frac{n}{2}$

(B) n

(C) $\frac{n-1}{2}$

(D) $\frac{n+1}{2}$

Detailed Solutions

Q1.

Solution

Concept:
Evaluating trigonometric limits of the indeterminate form $0/0$. We can use algebraic identities to decompose the product of cosines or apply standard expansion series and L'Hopital's rule to evaluate the limit systematically.

Solution:

(a) Consider the given expression: $\lim_{x\to 0} \frac{1-\cos(2x)\cos(3x)}{x^2}$.

(b) We rewrite the numerator by adding and subtracting $\cos(2x)$: $1-\cos(2x)\cos(3x) = 1-\cos(2x)+\cos(2x)-\cos(2x)\cos(3x) = (1-\cos 2x)+\cos 2x(1-\cos 3x)$.

(c) Split the limit into two separate parts over the denominator x^2: $\lim_{x\to 0} \frac{1-\cos 2x}{x^2} + \lim_{x\to 0} \frac{\cos 2x(1-\cos 3x)}{x^2}$.

(d) Use the standard trigonometric limit property where $\lim_{\theta\to 0} \frac{1-\cos\theta}{\theta^2} = \frac{1}{2}$.

(e) For the first term, multiply and divide by 4: $\lim_{x\to 0} 4 \cdot \frac{1-\cos 2x}{(2x)^2} = 4 \cdot \frac{1}{2} = 2$.

(f) For the second term, since $\lim_{x\to 0} \cos 2x = 1$, multiply and divide by 9: $\lim_{x\to 0} 9 \cdot \frac{1-\cos 3x}{(3x)^2} = 9 \cdot \frac{1}{2} = \frac{9}{2}$.

(g) Combining both results yields: $2 + \frac{9}{2} = \frac{13}{2}$.

Final Answer: The value of the limit is thirteen halves.

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept:
Distance between intersections of a variable line with parallel straight lines. Using the parametric form of a straight line equation allows easy calculation of coordinates and lengths along a specific directional inclination.

Solution:

(a) Let the straight line passing through the origin be represented in parametric form: $x = r\cos\theta$ and $y = r\sin\theta$, where r is the distance from the origin.

(b) The line intersects $x + y = 1$ at point A. Substituting the parametric coordinates gives $r_1\cos\theta + r_1\sin\theta = 1$, which simplifies to $r_1 = \frac{1}{\cos\theta+\sin\theta}$.

(c) Similarly, the line intersects $x + y = 3$ at point B. Substituting parametric coordinates gives $r_2\cos\theta + r_2\sin\theta = 3$, which simplifies to $r_2 = \frac{3}{\cos\theta+\sin\theta}$.

(d) The distance between A and B along the line is given by $AB = |r_2 - r_1| = \frac{2}{|\cos\theta+\sin\theta|}$.

(e) We are given that $AB = \sqrt{2}$. Setting the expressions equal: $\frac{2}{|\cos\theta+\sin\theta|} = \sqrt{2} \implies |\cos\theta + \sin\theta| = \sqrt{2}$.

(f) Squaring both sides gives $(\cos\theta + \sin\theta)^2 = 2 \implies 1 + \sin 2\theta = 2 \implies \sin 2\theta = 1$.

(g) Solving for θ within standard bounds gives $2\theta = \frac{\pi}{2} \implies \theta = \frac{\pi}{4}$.

Final Answer: The value of the angle is pi over four.

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept:
Geometric analysis of areas enclosed by intersecting curves. To properly characterize bounded regions, we determine boundary intersections and track which function establishes the upper bound versus the lower bound.

Solution:

(a) We begin by finding the points of intersection between the parabola $y = x^2$ and the straight line $y = 2x$.

(b) Equating the two expressions for y gives: $x^2 = 2x \implies x(x - 2) = 0$. Thus, the curves intersect at $x = 0$ and $x = 2$.

(c) The corresponding y-coordinates are found by substitution, yielding the intersection vertices $(0, 0)$ and $(2, 4)$.

(d) Within the interval $x \in (0, 2)$, choose a test point such as $x = 1$. Here, the line gives $y = 2$ and the parabola gives $y = 1$.

(e) Since $2 > 1$, the straight line $y = 2x$ forms the upper boundary, while the parabola $y = x^2$ forms the lower boundary.

(f) Consequently, the enclosed shaded region must be bounded from below by the parabola and from above by the line between $x = 0$ and $x = 2$.

Final Answer: The area bounded from below by the parabola and from above by the line from x=0 to x=2.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 3

Q4.

Solution

Concept:
Properties of matrix determinants and singularity conditions. A matrix is defined as singular if and only if its determinant value equals zero, indicating that the matrix is not invertible.
Solution:

(a) Given matrix $A = \begin{bmatrix} 1 & \lambda & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix}$. For A to be singular, we must set its determinant $|A| = 0$.

(b) Expand the determinant along the first row: $|A| = 1 \cdot \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} - \lambda \cdot \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} + 0 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$.

(c) Evaluate the individual 2×2 determinants: $\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = (1)(2) - (1)(0) = 2$.

(d) Evaluate the second sub-matrix: $\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix} = (0)(2) - (1)(1) = -1$.

(e) Substitute these values back into the expanded equation: $|A| = 1(2) - \lambda(-1) = 2 + \lambda$.

(f) Since the matrix is singular, set the expression to zero: $2 + \lambda = 0 \implies \lambda = -2$.

Final Answer: The value of lambda is minus two.

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 4

Q5.

Solution

Concept:
Vector projections using dot product operations. The scalar projection of vector $\vec{a}$ onto vector $\vec{b}$ measures the length of the component of $\vec{a}$ directed along the vector path of $\vec{b}$.
Solution:

(a) The formula for the scalar projection of vector $\vec{a}$ on vector $\vec{b}$ is given by the expression: Projection $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

(b) Given vectors are $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

(c) Compute the dot product $\vec{a} \cdot \vec{b}$ by multiplying corresponding components: $\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10$.

(d) Next, calculate the magnitude of vector $\vec{b}$: $|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$.

(e) Substitute the calculated dot product and magnitude back into the projection formula: Projection $= \frac{10}{\sqrt{6}}$.

Final Answer: The projection value is ten over the square root of six.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 5

Q6.

Solution

Concept:
Complementary relationships in inverse trigonometric functions. Standard identities relate inverse sine and inverse cosine functions for valid values across identical domains.
Solution:

(a) Recall the fundamental inverse trigonometric identity: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for all $x \in [-1, 1]$.

(b) Similarly, the identity holds true for variable y: $\sin^{-1} y + \cos^{-1} y = \frac{\pi}{2}$.

(c) Add both complete linear equations together to group terms: $(\sin^{-1} x + \sin^{-1} y) + (\cos^{-1} x + \cos^{-1} y) = \frac{\pi}{2} + \frac{\pi}{2} = \pi$.

(d) We are given the value for the first grouped expression: $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$.

(e) Substitute this value into the combined equation: $\frac{2\pi}{3} + (\cos^{-1} x + \cos^{-1} y) = \pi$.

(f) Isolate the desired term by subtracting $\frac{2\pi}{3}$ from both sides: $\cos^{-1} x + \cos^{-1} y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$.

Final Answer: The value of the expression is pi over three.

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:
Probability rules for independent events and set operations. The probability of unions can be calculated via additivity formulas and the product rule governing independent outcomes.
Solution:

(a) We want to determine the probability $P(A \cup B^c)$. Using the general addition rule: $P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$.

(b) Given that events A and B are independent, it follows logically that events A and B^c are also independent.

(c) Therefore, the intersection probability factors into a product: $P(A \cap B^c) = P(A) \cdot P(B^c)$.

(d) Calculate the probability of the complement of B: $P(B^c) = 1 - P(B) = 1 - 0.4 = 0.6$.

(e) Substitute these values back into our main formulation: $P(A \cup B^c) = P(A) + P(B^c) - P(A) \cdot P(B^c)$.

(f) Perform the arithmetic substitution using the given value $P(A) = 0.3$: $P(A \cup B^c) = 0.3 + 0.6 - (0.3)(0.6) = 0.9 - 0.18 = 0.72$.

Final Answer: The probability is zero point seven two.

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept:
Determining intervals of monotonicity for polynomial functions using calculus. A function decreases on intervals where its first derivative remains strictly negative.

Solution:

(a) Given polynomial function: $f(x) = 2x^3 - 9x^2 + 12x + 5$.

(b) Find the first derivative with respect to x: $f'(x) = \frac{d}{dx}(2x^3 - 9x^2 + 12x + 5) = 6x^2 - 18x + 12$.

(c) For the function to be monotonically decreasing, we set $f'(x) < 0$: $6x^2 - 18x + 12 < 0$.

(d) Factor out the common constant coefficient 6: $6(x^2 - 3x + 2) < 0 \implies x^2 - 3x + 2 < 0$.

(e) Factor the quadratic polynomial expression into linear components: $(x - 1)(x - 2) < 0$.

(f) Analyze the signs using a number line or wavy curve method. The product is negative when x lies strictly between the roots.

(g) Thus, the inequality holds for $x \in (1, 2)$.

Final Answer: The function is decreasing in the interval open interval one to two.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 8

Q9.

Solution

Concept:
Conic section reduction and standard form matching for parabolas. Converting a general second-degree equation into vertex form exposes its structural parameters, including the latus rectum length.
Solution:

(a) Given equation of the parabola: $y^2 - 4y - 8x + 4 = 0$.

(b) Group the y terms together and move the remaining terms to the right side: $y^2 - 4y = 8x - 4$.

(c) Complete the square on the left side by adding $(4/2)^2 = 4$ to both sides: $y^2 - 4y + 4 = 8x - 4 + 4 \implies (y - 2)^2 = 8x$.

(d) Compare this with the standard equation of a shifted parabola: $(y - k)^2 = 4a(x - h)$.

(e) Matching coefficients shows that the horizontal parameter is represented by $4a = 8$.

(f) The length of the latus rectum of a standard parabola is defined precisely by the value of $4a$.

(g) Therefore, the total length of the latus rectum is exactly 8.

Final Answer: The length of the latus rectum is eight.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 9

Q10.

Solution

Concept:
Theory of quadratic equations and algebraic progressions. Using Vieta's formulas links root values directly to polynomial coefficients, allowing algebraic simplification of progressional series.
Solution:

(a) Let α and β be the roots of $ax^2 + bx + c = 0$. By Vieta's formulas: $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.

(b) The given condition states: $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$.

(c) Expand the numerator: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(-\frac{b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = \frac{b^2-2ac}{a^2}$.

(d) Substitute these values into the condition: $-\frac{b}{a} = \frac{\frac{b^2-2ac}{a^2}}{\frac{c^2}{a^2}} = \frac{b^2-2ac}{c^2}$.

(e) Cross-multiplying yields: $-bc^2 = a(b^2 - 2ac) \implies -bc^2 = ab^2 - 2a^2c$.

(f) Rearranging the terms gives: $2a^2c = ab^2 + bc^2$.

(g) Divide the entire equation by abc: $2\left(\frac{a}{b}\right) = \frac{b}{c} + \frac{c}{a}$. This reveals that $\frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ form an AP, which equivalently implies that bc^2, ca^2, ab^2 are in Arithmetic Progression (AP).

Final Answer: The quantities are in Arithmetic Progression.

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept:

To find the shortest distance between two parallel lines, we determine a common parallel direction vector and find a displacement vector between any two known points lying on each respective line. The perpendicular distance is then established by computing the magnitude of the cross product of the displacement vector and the unit direction vector.

Solution:

(a) Identify a point on the first line L_1 as $P = (1, 2, 3)$ with its position vector $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$.

(b) Identify a point on the second line L_2 as $Q = (2, 4, 5)$ with its position vector $\vec{a}_2 = 2\hat{i}+4\hat{j}+5\hat{k}$.

(c) Both given lines share the same directional coefficients, providing the parallel vector $\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

(d) Compute the displacement vector connecting the two points: $\vec{a}_2 - \vec{a}_1 = (2 - 1)\hat{i} + (4 - 2)\hat{j} + (5 - 3)\hat{k} = \hat{i} + 2\hat{j} + 2\hat{k}$.

(e) Determine the cross product of this displacement vector and the parallel direction vector:
$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix} = \hat{i}(8 - 6) - \hat{j}(4 - 4) + \hat{k}(3 - 4) = 2\hat{i} - \hat{k}.$$

(f) Calculate the magnitude of this cross product vector: $\left|(\vec{a}_2 - \vec{a}_1) \times \vec{b}\right| = \sqrt{2^2 + 0^2 + (-1)^2} = \sqrt{5}$.

(g) Compute the magnitude of the parallel vector $\vec{b}$: $|\vec{b}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$.

(h) The shortest distance formula yields: $d = \frac{\left|(\vec{a}_2-\vec{a}_1)\times\vec{b}\right|}{|\vec{b}|} = \frac{\sqrt{5}}{\sqrt{29}} = \sqrt{\frac{5}{29}}$.

Final Answer: The shortest distance between the lines is $\sqrt{\frac{5}{29}}$.

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept:
Evaluating trigonometric values involving differences of inverse operations. By applying the double angle formula for inverse tangents, the compound expression can be reduced step-by-step using standard subtraction identities.

Solution:

(a) Let the internal double angle part of the expression be represented as $\alpha = 2\tan^{-1}\left(\frac{1}{5}\right)$.

(b) Apply the standard duplication identity $\tan(2\theta) = \frac{2\tan\theta}{1-\tan^2\theta}$ to simplify this term:
$\tan\alpha = \tan\left(2\tan^{-1}\frac{1}{5}\right) = \frac{2\cdot(1/5)}{1-(1/5)^2} = \frac{2/5}{1-1/25} = \frac{2/5}{24/25} = \frac{2}{5}\cdot\frac{25}{24} = \frac{5}{12}$.

(c) The original full expression can now be written in terms of this simplified value as $\tan\left(\alpha - \frac{\pi}{4}\right)$.

(d) Recall the standard trigonometric difference formula: $\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}$.

(e) Substitute $A = \alpha$ and $B = \frac{\pi}{4}$ into this relation, noting that $\tan\left(\frac{\pi}{4}\right) = 1$:
$\tan\left(\alpha - \frac{\pi}{4}\right) = \frac{\tan\alpha - 1}{1+\tan\alpha\cdot 1}$.

(f) Insert the previously calculated fraction value $\tan\alpha = \frac{5}{12}$ into this expression:
$\tan\left(\alpha - \frac{\pi}{4}\right) = \frac{(5/12)-1}{1+(5/12)} = \frac{-7/12}{17/12}$.

(g) Simplify the compound fraction by canceling out the common denominators: $\frac{-7}{17} = -\frac{7}{17}$.

Final Answer: The value of the expression is $-\frac{7}{17}$.

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept:
Determining the probability of consecutive matching events occurring without replacement. The joint probability of both independent stages is computed by multiplying the conditional probability of the second selection by the initial probability of the first selection.

Solution:

(a) Count the total number of items present in the container: 5 white + 3 black = 8 total balls.

(b) Let the event of drawing a black ball on the first turn be denoted as B_1.

(c) Calculate the probability of selecting a black ball initially: $P(B_1) = \frac{\text{Number of black balls}}{\text{Total number of balls}} = \frac{3}{8}$.

(d) Since the selection is made without replacement, adjust the item counts remaining inside the container.

(e) The remaining number of black balls becomes 2, and the remaining total number of balls drops to 7.

(f) Let the event of drawing a black ball on the second turn be denoted as B_2.

(g) Calculate the conditional probability of selecting a second black ball: $P(B_2|B_1) = \frac{2}{7}$.

(h) Compute the combined joint probability for both outcomes happening in sequence: $P(B_1 \cap B_2) = P(B_1) \cdot P(B_2|B_1) = \frac{3}{8} \cdot \frac{2}{7} = \frac{6}{56}$.

(i) Reduce the resulting fraction to its lowest terms by dividing by two: $\frac{6}{56} = \frac{3}{28}$.

Final Answer: The probability that both balls are black is $\frac{3}{28}$.

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:
Evaluating a definite integral using reflective properties over symmetric limits. Applying King's property replaces the independent variable with the sum of the limits minus the variable, creating an additive system that cancels complex trigonometric components.

Solution:

(a) Let the given definite integral expression be denoted as $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$.

(b) Apply the integral identity property $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$.

(c) Substitute x with $\left(\frac{\pi}{2} - x\right)$ throughout the integrand expression:
$I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}}\,dx$.

(d) Use standard co-function relationships, $\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$, to rewrite it:
$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx$.

(e) Add these two matching integral representations together to eliminate the asymmetric numerators:
$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx + \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$.

(f) Combine them under a common denominator: $2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$.

(g) Simplify the integrand to unity: $2I = \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$.

(h) Solve for the final integral value by dividing by two: $I = \frac{\pi}{4}$.

Final Answer: The value of the integral is $\frac{\pi}{4}$.

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept:
Determining the eccentricity of a conic section from its standard algebraic formula. Normalizing the given quadratic equation allows identification of the semi-major and semi-minor axes, which can then be substituted into the eccentricity ratio formula.

Solution:

(a) Consider the given equation of the ellipse: $9x^2 + 25y^2 = 225$.

(b) Convert this equation into standard form by dividing every term by the constant 225: $\frac{9x^2}{225} + \frac{25y^2}{225} = \frac{225}{225} \implies \frac{x^2}{25} + \frac{y^2}{9} = 1$.

(c) Compare this normalized structure to the standard horizontal ellipse equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(d) Extract the squared values of the semi-axes from the denominators: $a^2 = 25$ and $b^2 = 9$.

(e) Since $a^2 > b^2$, the major axis lies horizontally along the x-axis, meaning $a = 5$ and $b = 3$.

(f) Recall the standard mathematical formula for the eccentricity of a horizontal ellipse: $e = \sqrt{1 - \frac{b^2}{a^2}}$.

(g) Substitute the values of a^2 and b^2 directly into the radical expression: $e = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}}$.

(h) Evaluate the square root to determine the final decimal or fractional ratio: $e = \frac{4}{5}$.

Final Answer: The eccentricity of the ellipse is $\frac{4}{5}$.

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept:
Calculating distinct arrangements of characters containing repeated items. The total number of unique permutations is found by dividing the factorial of the total letter count by the product of the factorials of each repeated letter count.

Solution:

(a) Write down the characters of the given word and count the total number of letters: 'EXAMINATION'.

(b) The total count of letters in this word is $n = 11$.

(c) Analyze the composition of the word to identify any repeated characters:

- The letter 'E' appears exactly 1 time.
- The letter 'X' appears exactly 1 time.
- The letter 'A' appears exactly 2 times.
- The letter 'M' appears exactly 1 time.
- The letter 'I' appears exactly 2 times.
- The letter 'N' appears exactly 2 times.
- The letter 'O' appears exactly 1 time.
- The letter 'T' appears exactly 1 time.

(d) Apply the multinomial permutation counting formula: $N = \frac{n!}{p!\cdot q!\cdot r!\ldots}$.

(e) Set up the calculation using the identified counts: $N = \frac{11!}{2!\cdot 2!\cdot 2!}$.

(f) Expand and calculate the numerical product values systematically:
$11! = 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 39,916,800$.

(g) Divide by the product of the repeated terms $(2 \cdot 2 \cdot 2 = 8)$: $N = \frac{39,916,800}{8} = 4,989,600$.

Final Answer: The number of permutations of the letters is 4989600.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 16

Q17.

Solution

Concept:
Finding the geometric angle between two planes by computing the angle between their normal vectors. The angle is determined by evaluating the dot product of the normal vectors and dividing by the product of their magnitudes.

Solution:

(a) Extract the coefficients from the first plane $2x - y + z = 6$ to find its normal vector: $\vec{n}_1 = 2\hat{i} - \hat{j} + \hat{k}$.

(b) Extract the coefficients from the second plane $x + y + 2z = 3$ to find its normal vector: $\vec{n}_2 = \hat{i} + \hat{j} + 2\hat{k}$.

(c) Recall the vector formula for the angle between two planes: $\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$.

(d) Compute the dot product of the two normal vectors:
$\vec{n}_1 \cdot \vec{n}_2 = (2)(1) + (-1)(1) + (1)(2) = 2 - 1 + 2 = 3$.

(e) Calculate the magnitude of the first normal vector: $|\vec{n}_1| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$.

(f) Calculate the magnitude of the second normal vector: $|\vec{n}_2| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1 + 1 + 4} = \sqrt{6}$.

(g) Substitute these values into the cosine equation: $\cos\theta = \frac{3}{\sqrt{6}\cdot\sqrt{6}} = \frac{3}{6} = \frac{1}{2}$.

(h) Determine the angle corresponding to this value: $\cos\theta = \frac{1}{2} \implies \theta = \frac{\pi}{3}$ or 60°.

Final Answer: The angle between the planes is $\frac{\pi}{3}$.

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 17

Q18.

Solution

Concept:
Evaluating symmetric cubic functions of quadratic roots using Vieta's relations. The sum of the cubes of the roots can be expressed in terms of the sum and product of the roots using algebraic factorizations.

Solution:

(a) Consider the given quadratic equation: $x^2 - 3x + 2 = 0$.

(b) According to Vieta's formulas for a standard quadratic equation $ax^2 + bx + c = 0$:

- The sum of the roots is $\alpha + \beta = -\frac{b}{a} = -\frac{-3}{1} = 3$.
- The product of the roots is $\alpha\beta = \frac{c}{a} = \frac{2}{1} = 2$.

(c) Recall the algebraic identity for the sum of two cubes: $\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$.

(d) Rewrite the second part of the identity using known quantities: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

(e) Combine these expressions to create a formula depending only on the sum and product: $\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$.

(f) Substitute the numerical values $\alpha + \beta = 3$ and $\alpha\beta = 2$ into this equation: $\alpha^3 + \beta^3 = 3 \cdot [3^2 - 3(2)]$.

(g) Simplify the expression step-by-step: $\alpha^3 + \beta^3 = 3 \cdot [9 - 6] = 3 \cdot 3 = 9$.

(h) Alternatively, factoring the original equation gives $(x - 1)(x - 2) = 0$, so the roots are 1 and 2. Computing their cubes yields $1^3 + 2^3 = 1 + 8 = 9$.

Final Answer: The value of $\alpha^3 + \beta^3$ is 9.

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept:
Evaluating algebraic integrals using substitution and partial fractions. Multiplying the numerator and denominator by an appropriate variable power sets up a substitution that reduces the degree of the expression.

Solution:

(a) Let the given indefinite integral be denoted as $I = \int \frac{1}{x(x^4+1)}\,dx$.

(b) Multiply both the top numerator and bottom denominator by x^3 to structure it for substitution: $I = \int \frac{x^3}{x^4(x^4+1)}\,dx$.

(c) Let $t = x^4$ be our substitution variable. Differentiating both sides gives $dt = 4x^3\,dx \implies x^3\,dx = \frac{dt}{4}$.

(d) Substitute these variables back into the integral expression: $I = \int \frac{1}{t(t+1)} \cdot \frac{dt}{4} = \frac{1}{4}\int \frac{1}{t(t+1)}\,dt$.

(e) Use a simple partial fraction decomposition to split the integrand: $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$.

(f) Rewrite and integrate the separate terms: $I = \frac{1}{4}\int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt = \frac{1}{4}(\ln|t| - \ln|t+1|) + C$.

(g) Apply logarithmic subtraction properties to merge the logs: $I = \frac{1}{4}\ln\left|\frac{t}{t+1}\right| + C$.

(h) Substitute $t = x^4$ back into the expression to return to the original variable: $I = \frac{1}{4}\ln\left|\frac{x^4}{x^4+1}\right| + C$.

Final Answer: The value of the integral is $\frac{1}{4}\ln\left|\frac{x^4}{x^4+1}\right| + C$.

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:

Determining the equation of the director circle for a given circle. The director circle represents the geometric locus of points where pairs of perpendicular tangents intersect, creating a concentric circle with a radius scaled by $\sqrt{2}$.

Solution:

(a) Let the standard equation of a circle centered at the origin be given by $x^2 + y^2 = r^2$, where r is its radius.

(b) Consider a point $P(x_1, y_1)$ from which two perpendicular tangents are drawn to this circle.

(c) The general equation for any tangent line with a slope m to this circle is written as $y = mx \pm r\sqrt{1+m^2}$.

(d) Rearrange this tangent equation so that the point P satisfies it: $(y_1 - mx_1)^2 = r^2(1+m^2)$.

(e) Expand the squared expression to form a quadratic equation in terms of the slope m: $m^2(x_1^2 - r^2) - 2mx_1y_1 + (y_1^2 - r^2) = 0$.

(f) Let m_1 and m_2 be the roots of this quadratic equation, representing the slopes of the two tangents.

(g) Since the tangents are perpendicular, their slopes must satisfy the property $m_1 \cdot m_2 = -1$.

(h) Use the product of roots formula from quadratic theory: $\frac{y_1^2 - r^2}{x_1^2 - r^2} = -1 \implies y_1^2 - r^2 = -x_1^2 + r^2$.

(i) Group the variable coordinates together: $x_1^2 + y_1^2 = 2r^2$. Replacing with general coordinates yields $x^2 + y^2 = 2r^2$.

Final Answer: The equation of the director circle is $x^2 + y^2 = 2r^2$.

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept:
Evaluating determinants using matrix column operations. When linear combinations of columns reveal identical proportional values across all rows, the determinant can be factored to show linear dependency, reducing the final matrix evaluation to zero.

Solution:

(a) Consider the given third-order determinant expression containing variables a, b, and c:
$$\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}.$$

(b) Observe the structure of the elements in the second column C_2 and the third column C_3.

(c) Apply the linear column operation replacing the third column with the sum of the second and third columns: $C_3 \to C_3 + C_2$.

(d) Perform the addition for each row entry to obtain the new configuration of the third column:
$$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}.$$

(e) Notice that the algebraic expression $(a+b+c)$ is now common to every single element in the third column.

(f) Factor out this common term $(a+b+c)$ from the third column outside of the determinant operator:
$$\Delta = (a+b+c) \begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}.$$

(g) Examine the columns of the remaining determinant matrix. The first column C_1 and the third column C_3 are completely identical.

(h) Recall the standard property of determinants stating that if any two rows or columns are identical, the value of the determinant is zero.

(i) Substitute this zero value back into the product relation to find the total result: $\Delta = (a+b+c) \cdot 0 = 0$.

Final Answer: The value of the determinant is 0.

Answer: (C)

Go Back to Question 21

Q22.

Solution

Concept:
Evaluating products of cosine functions with doubling angles using trigonometric identities. Multiplying and dividing by a corresponding sine term establishes a chain reaction of double-angle simplifications that condenses the multi-term expression.

Solution:

(a) Let the given trigonometric product expression be denoted by $P = \cos 20^\circ \cos 40^\circ \cos 80^\circ$.

(b) Multiply and divide the entire expression by $2\sin 20^\circ$ to set up the first identity step:
$P = \frac{1}{2\sin 20^\circ} \cdot (2\sin 20^\circ \cos 20^\circ)\cos 40^\circ \cos 80^\circ$.

(c) Apply the standard sine double-angle formula $2\sin\theta\cos\theta = \sin(2\theta)$ to the grouped terms:
$P = \frac{1}{2\sin 20^\circ} \cdot \sin 40^\circ \cos 40^\circ \cos 80^\circ$.

(d) Introduce a factor of 2 in both the numerator and denominator to repeat the duplication sequence:
$P = \frac{1}{4\sin 20^\circ} \cdot (2\sin 40^\circ \cos 40^\circ)\cos 80^\circ$.

(e) Simplify the combined terms using the double-angle identity again for 40°:
$P = \frac{1}{4\sin 20^\circ} \cdot \sin 80^\circ \cos 80^\circ$.

(f) Introduce another factor of 2 to combine the remaining terms into a final single sine value:
$P = \frac{1}{8\sin 20^\circ} \cdot (2\sin 80^\circ \cos 80^\circ) = \frac{\sin 160^\circ}{8\sin 20^\circ}$.

(g) Express $\sin 160^\circ$ using its supplementary angle relation: $\sin 160^\circ = \sin(180^\circ - 20^\circ) = \sin 20^\circ$.

(h) Substitute this back to cancel the sine terms from the numerator and denominator: $P = \frac{\sin 20^\circ}{8\sin 20^\circ} = \frac{1}{8}$.

Final Answer: The value of the product is $\frac{1}{8}$.

Answer: (C)

Go Back to Question 22

Q23.

Solution

Concept:
Calculating statistical variance for consecutive integer series using summation formulas. Variance measures the spread of the data and is computed as the mean of the squares of the numbers minus the square of their mean value.

Solution:

(a) Recall the general definition formula for statistical variance: $\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$.

(b) Write down the formula for the sum of the first n natural numbers: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

(c) Compute the mean value of these numbers: $\mu = \frac{\sum x_i}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$.

(d) Square this mean expression for the second part of the variance formula: $\mu^2 = \frac{(n+1)^2}{4}$.

(e) Write down the formula for the sum of the squares of the first n natural numbers: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.

(f) Calculate the mean of the squared values: $\frac{\sum x_i^2}{n} = \frac{n(n+1)(2n+1)}{6n} = \frac{(n+1)(2n+1)}{6}$.

(g) Substitute both components back into the main variance relation: $\sigma^2 = \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$.

(h) Factor out the common algebraic fraction $\frac{n+1}{2}$ to simplify the remaining terms: $\sigma^2 = \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right] = \frac{n+1}{2}\left[\frac{2(2n+1)-3(n+1)}{6}\right]$.

(i) Expand and combine the numerators inside the bracket: $\sigma^2 = \frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right] = \frac{n+1}{2}\left[\frac{n-1}{6}\right]$.

(j) Multiply the resulting linear terms together: $\sigma^2 = \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$.

Final Answer: The variance of the first n natural numbers is $\frac{n^2-1}{12}$.

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept:
Differentiating logarithmic trigonometric compositions using the chain rule. The derivative of a composite function requires differentiating the outer logarithmic operator followed by multiplying by the derivative of the inner function.

Solution:

(a) Consider the given explicit function relationship: $y = \ln(\sec x + \tan x)$.

(b) Apply the chain rule for differentiation, treating $u = \sec x + \tan x$ as the inner function core.

(c) Differentiate the outer logarithmic function with respect to u, yielding: $\frac{d}{du}(\ln u) = \frac{1}{u} = \frac{1}{\sec x + \tan x}$.

(d) Find the derivative of the inner functional part u with respect to the variable x: $\frac{d}{dx}(\sec x + \tan x) = \frac{d}{dx}(\sec x) + \frac{d}{dx}(\tan x)$.

(e) Substitute the standard derivatives of these individual trigonometric functions: $\frac{du}{dx} = \sec x \tan x + \sec^2 x$.

(f) Multiply the outer and inner derivatives together to form the complete derivative expression: $\frac{dy}{dx} = \frac{1}{\sec x + \tan x} \cdot (\sec x \tan x + \sec^2 x)$.

(g) Factor out the common trigonometric term $\sec x$ from the numerator group: $\frac{dy}{dx} = \frac{\sec x (\tan x + \sec x)}{\sec x + \tan x}$.

(h) Notice that the factor $(\sec x + \tan x)$ appears identically in both the numerator and denominator.

(i) Cancel this common factor to obtain the final simplified derivative function: $\frac{dy}{dx} = \sec x$.

Final Answer: The derivative $\frac{dy}{dx}$ is equal to $\sec x$.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 24

Q25.

Solution

Concept:
Applying tangency conditions to parabolas using slope coordinates. A line is tangent to a parabola if the system of equations has a single intersection point, which matches standard criteria relating coefficients and slopes.
Solution:

(a) Write down the equation of the given straight line: $y = mx + 1$.

(b) Write down the standard equation of the given parabola: $y^2 = 4x$.

(c) Compare this parabola with the general form $y^2 = 4ax$ to determine its specific focal parameters.

(d) Equating the coefficients shows that $4a = 4$, which determines the parameter value: $a = 1$.

(e) Recall the standard mathematical condition for a straight line $y = mx + c$ to be tangent to the parabola $y^2 = 4ax$: $c = \frac{a}{m}$.

(f) Identify the constant intercept value from the given line equation: $c = 1$.

(g) Substitute the known parameters $a = 1$ and $c = 1$ into the tangency condition relation: $1 = \frac{1}{m}$.

(h) Solve this basic algebraic equation for the unknown slope parameter: $m = 1$.

(i) Alternatively, substitute $y = mx + 1$ into $y^2 = 4x$ to get $(mx + 1)^2 = 4x \implies m^2x^2 + (2m - 4)x + 1 = 0$. Setting the discriminant to zero yields $(2m - 4)^2 - 4m^2 = 0 \implies -16m + 16 = 0 \implies m = 1$.

Final Answer: The value of m is 1.

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:
Solving system configurations by matching matrix element coordinates. Two matrices are equal if and only if their corresponding entries at every specific row and column position match identically, creating simultaneous equations.
Solution:

(a) Write down the given matrix equation containing the variable entries:
$$\begin{bmatrix} x+y & 2 \\ 1 & x-y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}.$$

(b) Equate the corresponding elements from the first row and first column position (index 1,1): $x + y = 3$.

(c) Equate the corresponding elements from the second row and second column position (index 2,2): $x - y = 1$.

(d) Set up a simultaneous linear system using these two derived algebraic relations:

- Equation 1: $x + y = 3$
- Equation 2: $x - y = 1$

(e) Add Equation 1 and Equation 2 together to eliminate the variable y: $(x + y) + (x - y) = 3 + 1 \implies 2x = 4$.

(f) Solve for the primary variable by dividing by two: $x = 2$.

(g) Substitute this calculated value $x = 2$ back into Equation 1 to find the remaining variable: $2 + y = 3$.

(h) Solve for the second variable: $y = 3 - 2 = 1$.

(i) Verify the values in Equation 2: $2 - 1 = 1$, which matches the right-hand side perfectly.

Final Answer: The values of x and y are 2, 1 respectively.

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept:
Determining vector coplanarity using scalar triple products. Three vectors are coplanar if they lie in the exact same geometric plane, meaning the volume of the parallelopiped formed by them is zero, which corresponds to a zero determinant.

Solution:

(a) Write down the components of the three given vectors:

- $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$
- $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$
- $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$

(b) Recall the mathematical definition condition for coplanarity using the scalar triple product: $[\vec{a}\ \vec{b}\ \vec{c}] = 0$.

(c) Set up the corresponding scalar matrix determinant using the vector coefficients:
$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0.$$

(d) Expand this determinant along the first row to create a linear equation in terms of λ:
$$2 \cdot \begin{vmatrix} 2 & -3 \\ \lambda & 5 \end{vmatrix} - (-1) \cdot \begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 2 \\ 3 & \lambda \end{vmatrix} = 0.$$

(e) Evaluate each of the individual second-order sub-determinants:

- First part: $2 \cdot (10 - (-3\lambda)) = 2(10 + 3\lambda) = 20 + 6\lambda$
- Second part: $+1 \cdot (5 - (-9)) = 1 \cdot (5 + 9) = 14$
- Third part: $+1 \cdot (\lambda - 6) = \lambda - 6$

(f) Combine all expanded terms into a single equation line: $(20 + 6\lambda) + 14 + (\lambda - 6) = 0$.

(g) Group the linear variable terms and constant numbers together: $7\lambda + 28 = 0$.

(h) Solve for the unknown parameter: $7\lambda = -28 \implies \lambda = -4$.

Final Answer: The value of λ is -4.

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept:

Solving heights and distances configurations using right-angled triangles. By expressing the horizontal base segments in terms of the vertical height via cotangent or tangent functions, simultaneous linear relations determine the altitude.

Solution:

(a) Let the height of the tree be represented by the vertical segment $AB = h$.

(b) Let the initial width of the river bank be denoted by the horizontal base segment $BC = x$.

(c) In the first right-angled triangle $\triangle ABC$, the angle of elevation at point C is $60°$:
$\tan 60° = \frac{AB}{BC} \implies \sqrt{3} = \frac{h}{x} \implies x = \frac{h}{\sqrt{3}}$.

(d) The person moves 40 meters further back to point D, making the total horizontal distance $DB = 40 + x$.

(e) In the second expanded right-angled triangle $\triangle ABD$, the angle of elevation at point D is $30°$:
$\tan 30° = \frac{AB}{DB} \implies \frac{1}{\sqrt{3}} = \frac{h}{40+x}$.

(f) Rearrange this equation to express the horizontal distance group: $40 + x = h\sqrt{3}$.

(g) Substitute the expression for x from the first step into this new relation: $40 + \frac{h}{\sqrt{3}} = h\sqrt{3}$.

(h) Move the height variables to one side to isolate the constant term: $40 = h\sqrt{3} - \frac{h}{\sqrt{3}}$.

(i) Find a common denominator to combine the fractional terms: $40 = h\left(\frac{3-1}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}$.

(j) Solve for the final tree height value: $2h = 40\sqrt{3} \implies h = 20\sqrt{3}$ meters.

Final Answer: The height of the tree h is $20\sqrt{3}$ meters.

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept:
Evaluating conditional probabilities from intersection and marginal events. Conditional probability scales the probability of a joint occurrence relative to the constrained sample space defined by the given condition event.

Solution:

(a) Write down the given mathematical probability values for the events:

- Probability of event A: $P(A) = \frac{1}{2}$
- Probability of event B: $P(B) = \frac{1}{3}$
- Probability of joint intersection event: $P(A \cap B) = \frac{1}{4}$

(b) Recall the standard definition formula for the conditional probability of event A given that event B has already occurred: $P(A|B) = \frac{P(A\cap B)}{P(B)}$.

(c) Ensure that the condition event probability $P(B)$ is non-zero, which is satisfied here since $P(B) = \frac{1}{3} > 0$.

(d) Substitute the given fractional values directly into the conditional ratio formula:
$P(A|B) = \frac{1/4}{1/3}$.

(e) Simplify the resulting compound division fraction by multiplying by the reciprocal of the denominator:
$P(A|B) = \frac{1}{4} \cdot \frac{3}{1} = \frac{3}{4}$.

(f) Note that the value of $P(A)$ is extraneous information here and is not required to complete the solution.

Final Answer: The conditional probability $P(A|B)$ is $\frac{3}{4}$.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 29

Q30.

Solution

Concept:
Finding extrema of single-variable trigonometric equations using differential calculus. Local extrema occur where the first derivative equals zero, and maximum points are confirmed when the second derivative value is negative.

Solution:

(a) Consider the given continuous real function definition: $f(x) = \sin x + \cos x$.

(b) Differentiate this function with respect to the variable x to find its first derivative: $f'(x) = \cos x - \sin x$.

(c) Set this first derivative expression equal to zero to locate all critical points: $\cos x - \sin x = 0 \implies \cos x = \sin x$.

(d) Divide both sides by $\cos x$ to express this condition as a standard tangent function: $\tan x = 1$.

(e) Solve for the primary angle value within the standard primary interval: $x = \frac{\pi}{4}$.

(f) Differentiate a second time to determine the stability or nature of this critical point: $f''(x) = -\sin x - \cos x$.

(g) Evaluate this second derivative expression at the critical coordinate $x = \frac{\pi}{4}$:
$f''\left(\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$.

(h) Since the second derivative value is strictly negative ($-\sqrt{2} < 0$), the function achieves a local maximum at this coordinate.

Final Answer: The maximum value occurs at x equal to $\frac{\pi}{4}$.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 30

Q31.

Solution

Concept:
Determining the equation of a concentric circle involves identifying the common center coordinates from the given circle's linear terms. The new constant term is then evaluated by substituting the given passing coordinates into the translated geometric formula.
Solution:

(a) Identify the general structural equation of the given reference circle: $x^2+y^2-4x-6y-3=0$.

(b) Recall that any circle concentric to this reference equation shares the exact same quadratic and linear terms, differing solely by its constant parameter.

(c) Write down the general algebraic expression for the required concentric family: $x^2+y^2-4x-6y+k=0$.

(d) Recognize that this targeted circle passes explicitly through the given point coordinates $(7,3)$.

(e) Substitute these coordinate values into the family equation to determine the value of k: $(7)^2+(3)^2-4(7)-6(3)+k=0$.

(f) Expand and evaluate each individual numerical component of the expression: $49+9-28-18+k=0$.

(g) Combine the computed constants to simplify the mathematical statement: $58-46+k=0 \implies 12+k=0$.

(h) Solve for the unknown constant parameter to find its unique value: $k=-12$.

(i) Substitute this parameter value back into the concentric structural expression: $x^2+y^2-4x-6y-12=0$.

Final Answer: The equation of the circle is $x^2+y^2-4x-6y-12=0$.

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept:

Evaluating the summation of an infinite geometric progression. When the absolute value of the common ratio between consecutive terms is strictly less than one, the infinite progression converges cleanly to a finite scalar value.

Solution:

(a) Write down the explicit terms of the given infinite numeric progression:
$S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

(b) Examine the structural pattern of the sequence to confirm it matches a geometric progression.

(c) Identify the first term of this sequence, denoted by the algebraic variable a: $a = 1$.

(d) Compute the common ratio, denoted by the variable r, by dividing the second term by the first:
$r = \frac{1/2}{1} = \frac{1}{2}$.

(e) Verify that the magnitude of this common ratio satisfies the strict mathematical convergence criteria:
$|r| = \left|\frac{1}{2}\right| < 1$.

(f) Recall the standard algebraic formula for calculating the sum of an infinite geometric series:
$S = \frac{a}{1-r}$.

(g) Substitute the identified values for a and r into this convergence formula:
$S = \frac{1}{1-1/2}$.

(h) Simplify the denominator expression by performing the fractional subtraction:
$1 - \frac{1}{2} = \frac{1}{2}$.

(i) Calculate the final scalar quotient by evaluating the reciprocal: $S = \frac{1}{1/2} = 2$.

Final Answer: The sum of the series up to infinity is 2.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 32

Q33.

Solution

Concept:
Determining the direction cosines of a line that forms identical inclination angles with the three standard three-dimensional coordinate axes by utilizing the fundamental trigonometric sum-of-squares vector identity.

Solution:

(a) Let the direction cosines of the inclined line be denoted by the standard symbols l, m, and n.

(b) Express these directional components using their cosine angle definitions: $l = \cos\alpha$, $m = \cos\beta$, and $n = \cos\gamma$.

(c) Apply the given geometric constraint that the line is equally inclined to the three axes: $\alpha = \beta = \gamma$.

(d) Deduce from this angular equality that all three direction cosines must possess equal values: $l = m = n$.

(e) Recall the fundamental geometric vector identity relating direction cosines: $l^2 + m^2 + n^2 = 1$.

(f) Substitute the equality relation into this identity to form a single variable equation: $l^2 + l^2 + l^2 = 1$.

(g) Combine the identical terms together to simplify the algebraic expression: $3l^2 = 1$.

(h) Isolate the squared variable by dividing through by the scalar coefficient: $l^2 = \frac{1}{3}$.

(i) Solve for the linear component by evaluating the square root, yielding both signs: $l = \pm\frac{1}{\sqrt{3}}$.

(j) Conclude that all three coordinate direction cosines share this identical value set: $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$.

Final Answer: The direction cosines of the line are $\left(\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}\right)$.

Answer: (C)

Go Back to Question 33

Q34.

Solution

Concept:

Evaluating double-angle trigonometric functions using single-angle tangent values. Converting double-angle sine expressions into rational tangent formulations allows direct algebraic evaluation without requiring explicit determination of the angle.

Solution:

(a) Identify the given primary single-angle trigonometric value from the problem statement: $\tan\theta = \frac{4}{3}$.

(b) Recall the standard double-angle identity that expresses $\sin 2\theta$ entirely in terms of $\tan\theta$: $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$.

(c) Substitute the given fractional value of $\tan\theta$ into the numerator of the identity: Numerator $= 2 \cdot \left(\frac{4}{3}\right) = \frac{8}{3}$.

(d) Substitute the given fractional value of $\tan\theta$ into the denominator of the identity: Denominator $= 1 + \left(\frac{4}{3}\right)^2 = 1 + \frac{16}{9}$.

(e) Simplify the denominator expression by converting to a common fractional base: Denominator $= \frac{9+16}{9} = \frac{25}{9}$.

(f) Reconstruct the full compound fraction using the evaluated components: $\sin 2\theta = \frac{8/3}{25/9}$.

(g) Invert the denominator fraction and multiply to complete the scalar calculation: $\sin 2\theta = \frac{8}{3} \cdot \frac{9}{25} = \frac{8\cdot 3}{25} = \frac{24}{25}$.

Final Answer: The value of $\sin 2\theta$ is $\frac{24}{25}$.

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept:
Analyzing linear transformations on statistical arithmetic means. The arithmetic mean behaves linearly; if every data value is scaled and shifted by constants, the resulting mean undergoes the exact same mathematical transformation.
Solution:

(a) Identify the initial parameters given for the sample data: number of observations $n = 10$, initial mean $\bar{X} = 25$.

(b) Express the definition of the initial arithmetic mean mathematically: $\bar{X} = \frac{\sum x_i}{10} = 25$, which implies $\sum x_i = 250$.

(c) Define the transformation applied to each individual observation to create a new dataset: $y_i = 2x_i + 5$.

(d) Write down the algebraic formula for calculating the new transformed arithmetic mean $\bar{Y}$:

$$\bar{Y} = \frac{\sum y_i}{10} = \frac{\sum(2x_i+5)}{10}.$$

(e) Distribute the summation operator across the linear terms inside the expression: $\bar{Y} = \frac{2\sum x_i+\sum 5}{10} = \frac{2\sum x_i+10\cdot 5}{10}$.

(f) Separate the fraction to express it in terms of the initial mean function: $\bar{Y} = 2\left(\frac{\sum x_i}{10}\right) + \frac{50}{10} = 2\bar{X} + 5$.

(g) Substitute the known initial mean value $\bar{X} = 25$ into this transformation rule: $\bar{Y} = 2(25) + 5$.

(h) Perform the scalar multiplication and addition to find the final mean: $\bar{Y} = 50 + 5 = 55$.

Final Answer: The new mean will be 55.

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept:
Evaluating exponential calculus integrals using the product derivative rule structure. An integral matching the configuration of an exponential multiplied by a function plus its derivative simplifies directly to a product term.

Solution:

(a) Identify the structure of the given calculus integration expression:
$I = \int e^x(\tan x + \sec^2 x)\, dx$.

(b) Recall the standard integration theorem for exponential products:
$\int e^x[f(x) + f'(x)]\, dx = e^x f(x) + C$.

(c) Assign the primary internal trigonometric function to match the theorem layout: let $f(x) = \tan x$.

(d) Differentiate this assigned function with respect to x to find its corresponding derivative:
$f'(x) = \frac{d}{dx}(\tan x) = \sec^2 x$.

(e) Observe that the integrand contains exactly the combination $f(x) + f'(x) = \tan x + \sec^2 x$.

(f) Match the components perfectly to the statement of the integration theorem.

(g) Conclude that the integral evaluates directly to the product of the exponential and the primary function.

(h) Write down the final expression including the mandatory constant of integration C:
$I = e^x \tan x + C$.

Final Answer: The value of the integral is $e^x \tan x + C$.

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept:
Deriving the condition of tangency for a hyperbola using linear substitution. Converting a line into slope-intercept form and equating it to the hyperbola's auxiliary equation defines constraints between the coefficients.

Solution:

(a) Write down the given explicit equation of the straight line: $lx + my + n = 0$.

(b) Rearrange this linear equation into standard slope-intercept form $y = Mx + C$:
$my = -lx - n \implies y = \left(-\frac{l}{m}\right)x + \left(-\frac{n}{m}\right)$.

(c) Identify the slope M and intercept C from this configuration: $M = -\frac{l}{m}$ and $C = -\frac{n}{m}$.

(d) Write down the standard equation of the hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(e) Recall the standard condition for a line $y = Mx + C$ to be tangent to this hyperbola: $C^2 = a^2M^2 - b^2$.

(f) Substitute the expressions for M and C into this algebraic condition:
$\left(-\frac{n}{m}\right)^2 = a^2\left(-\frac{l}{m}\right)^2 - b^2$.

(g) Expand the squared terms to clear the negative signs: $\frac{n^2}{m^2} = a^2 \cdot \frac{l^2}{m^2} - b^2$.

(h) Multiply the entire equation by m^2 to eliminate all denominators: $n^2 = a^2l^2 - b^2m^2$.

(i) Rearrange the final terms to match the required format: $a^2l^2 - b^2m^2 = n^2$.

Final Answer: The condition for tangency is $a^2l^2 - b^2m^2 = n^2$.

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept:
Simplifying binomial coefficient combinations using Pascal's identity. When adding two combinations with identical upper indices and consecutive lower indices, the indices merge into a single higher-order term.

Solution:

(a) Write down the given expression containing the binomial coefficients: ${}^{15}C_8 + {}^{15}C_7$.

(b) Recall the fundamental combinatorial relation known as Pascal's Identity:
${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$.

(c) Compare the given problem parameters with the variables in the identity:

- Set the upper index parameter: $n = 15$
- Set the primary lower index parameter: $r = 8$
- Verify the secondary lower index parameter: $r - 1 = 8 - 1 = 7$

(d) Confirm that the parameters perfectly match the structural conditions required by the identity.

(e) Increment the upper index value by one as specified by the formula rule: $n + 1 = 15 + 1 = 16$.

(f) Select the larger of the two original lower indices to serve as the new lower index: $r = 8$.

(g) Assemble the final unified binomial coefficient term from these results: ${}^{16}C_8$.

Final Answer: The value of the expression is ${}^{16}C_8$.

Answer: (C)

Go Back to Question 38

Q39.

Solution

Concept:
Formulating the vector parameter equation of a line in three-dimensional space. The path is constructed by taking the position vector of a known fixed point and adding a scaled scalar multiple of the parallel direction vector.

Solution:

(a) Identify the coordinates of the fixed point through which the line passes: $(1, -2, 3)$.

(b) Construct the position vector $\vec{a}$ corresponding to these spatial coordinates: $\vec{a} = 1\hat{i} - 2\hat{j} + 3\hat{k}$.

(c) Identify the direction vector $\vec{b}$ to which the required line must run parallel: $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$.

(d) Recall the standard structural vector formula for a straight line in three dimensions: $\vec{r} = \vec{a} + \lambda\vec{b}$.

(e) Recognize that the variable λ represents an arbitrary scalar parameter tracking points along the line.

(f) Substitute the constructed position vector $\vec{a}$ into the first part of the formula layout.

(g) Substitute the given parallel direction vector $\vec{b}$ into the parameterized part of the layout.

(h) Combine these terms to write out the full parameter vector representation:
$\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

Final Answer: The vector equation of the line is $\vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept:
Simplifying inverse trigonometric combinations using cosine addition formulas. Isolating the angle and applying cosine expansion establishes an algebraic relation that removes inverse operators through squaring techniques.

Solution:

(a) Let $\cos^{-1}\left(\frac{x}{a}\right) = A$ and $\cos^{-1}\left(\frac{y}{b}\right) = B$, which implies $\cos A = \frac{x}{a}$ and $\cos B = \frac{y}{b}$.

(b) Express the corresponding sine values using identities: $\sin A = \sqrt{1 - \frac{x^2}{a^2}}$ and $\sin B = \sqrt{1 - \frac{y^2}{b^2}}$.

(c) Write down the given expression using these substitutions: $A + B = \alpha$.

(d) Apply the cosine function to both sides of this equation: $\cos(A + B) = \cos\alpha$.

(e) Expand the left-hand side using the standard cosine addition formula:
$\cos A\cos B - \sin A\sin B = \cos\alpha$.

(f) Substitute the algebraic variable relations back into this expanded formula:
$\left(\frac{x}{a}\right)\left(\frac{y}{b}\right) - \sqrt{1 - \frac{x^2}{a^2}}\sqrt{1 - \frac{y^2}{b^2}} = \cos\alpha$.

(g) Isolate the radical product term on one side of the equation:
$\frac{xy}{ab} - \cos\alpha = \sqrt{1 - \frac{x^2}{a^2}}\sqrt{1 - \frac{y^2}{b^2}}$.

(h) Square both sides to eliminate the radicals: $\left(\frac{xy}{ab} - \cos\alpha\right)^2 = \left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)$.

(i) Expand both sides and cancel the common term $\frac{x^2y^2}{a^2b^2}$ to arrive at the final form:
$\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2} = 1 - \cos^2\alpha = \sin^2\alpha$.

Final Answer: The value of the expression is $\sin^2\alpha$.

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept:
Calculating sample space probabilities for multiple independent Bernoulli trials involves enumerating the exhaustive outcomes. The likelihood of a specific compound target outcome is determined by dividing the favorable event count by the total sample space size.

Solution:

(a) Identify that throwing three fair coins creates a binary experiment with two outcomes per coin.

(b) Determine the total cardinality of the sample space by calculating the exponential power: $2^3 = 8$.

(c) List out all eight exhaustive elements present within this sample space: $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$.

(d) Define the target condition which requires obtaining at least two heads.

(e) Recognize that at least two heads means getting exactly two heads or exactly three heads.

(f) Identify all outcomes from the sample space that satisfy this specific criteria: $E = \{HHH, HHT, HTH, THH\}$.

(g) Count the number of favorable elements contained within this target event set: $n(E) = 4$.

(h) Formulate the classical probability equation by taking the ratio of favorable to total outcomes: $P(E) = \frac{n(E)}{n(S)} = \frac{4}{8}$.

(i) Reduce the resulting fractional ratio to its lowest terms by dividing out the common factor: $\frac{1}{2}$.

Final Answer: The probability of getting at least two heads is $\frac{1}{2}$.

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 41

Q42.

Solution

Concept:
Evaluating the derivative of one inverse trigonometric function with respect to another. Utilizing parametric substitution allows complex compound functions to be mapped onto a simpler tracking variable, revealing hidden linear relationships.

Solution:

(a) Define the primary compound function as a dependent tracking variable: $u = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

(b) Define the base reference function as a secondary tracking variable: $v = \tan^{-1} x$.

(c) State the core objective which requires finding the rate of change: $\frac{du}{dv}$.

(d) Recall the standard trigonometric substitution designed for handling these specific rational structures: let $x = \tan\theta$.

(e) Substitute this expression into the primary compound function formula: $u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$.

(f) Recognize the inner rational structure as the standard identity for double-angle sine: $\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$.

(g) Simplify the expression by evaluating the inverse composition: $u = \sin^{-1}(\sin 2\theta) = 2\theta$.

(h) Substitute the definition of x back into the base reference function equation: $v = \tan^{-1}(\tan\theta) = \theta$.

(i) Express the variable u directly as a linear function of v by substituting the relation: $u = 2v$.

(j) Differentiate this linear equation directly with respect to v to obtain the final value: $\frac{du}{dv} = 2$.

Final Answer: The derivative with respect to $\tan^{-1} x$ is 2.

Answer: (B)

Go Back to Question 42

Q43.

Solution

Concept:
Determining the relative position of a point with respect to a circle by evaluating its power. Substituting coordinate points into the standard quadratic form reveals whether it lies inside, on, or outside the curve.

Solution:

(a) Write down the standard structural equation representing the reference circle: $x^2 + y^2 - 4x - 2y + 1 = 0$.

(b) Identify the specific coordinates of the point to be tested: $(x_1, y_1) = (1, 2)$.

(c) Recall the geometric notation for evaluating a point's power relative to a circle: $S_1 = x_1^2 + y_1^2 - 4x_1 - 2y_1 + 1$.

(d) Substitute the coordinate values into this power expression: $S_1 = (1)^2+(2)^2-4(1)-2(2)+1$.

(e) Expand and evaluate each arithmetic term: $S_1 = 1 + 4 - 4 - 4 + 1$.

(f) Combine the positive and negative components to compute the net value: $S_1 = 6 - 8 = -2$.

(g) Analyze the sign of the computed result: since $S_1 = -2$, it is strictly less than zero ($S_1 < 0$).

(h) Recall the geometric position classification rules: if $S_1 > 0$ it lies outside, if $S_1 = 0$ it lies on, and if $S_1 < 0$ it lies inside.

(i) Conclude that the point resides inside the boundaries of the circle based on its negative power value.

Final Answer: The point lies inside the circle.

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept:
Computing the inverse of a square matrix of order two. The operational method requires evaluating the scalar determinant, transposing the main diagonal entries, and reversing the algebraic signs of the off-diagonal entries.

Solution:

(a) Write down the given matrix elements: $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

(b) Compute the determinant of the matrix by taking the difference of the diagonal products: $|A| = (2)(3) - (5)(1)$.

(c) Evaluate the arithmetic products: $|A| = 6 - 5 = 1$.

(d) Confirm that the matrix is non-singular since its determinant is non-zero ($|A| \neq 0$).

(e) Recall the general formula for the adjoint of a two-by-two matrix: $\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

(f) Swap the positions of the primary diagonal elements (2 and 3): the new entries are 3 and 2.

(g) Reverse the signs of the secondary diagonal elements (5 and 1): the new entries are -5 and -1.

(h) Assemble these modified entries to form the adjoint matrix: $\text{adj } A = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$.

(i) Use the inverse matrix relation: $A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{1}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$.

Final Answer: The inverse of the matrix is $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$.

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept:
Determining the angle between two three-dimensional vectors by utilizing the geometric definition of the vector dot product. The calculation relies on finding the ratio of their inner scalar product to the product of their magnitudes.
Solution:

(a) Write down the component expressions for both given vectors: $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$.

(b) Compute the inner scalar dot product of the vectors by multiplying corresponding directional parts:
$\vec{a} \cdot \vec{b} = (1)(1) + (1)(-1) + (-1)(1)$.

(c) Simplify the arithmetic sum to find the scalar product: $\vec{a} \cdot \vec{b} = 1 - 1 - 1 = -1$.

(d) Calculate the magnitude of the first vector using the square root of the sum of squares:
$|\vec{a}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$.

(e) Calculate the magnitude of the second vector following the same method:
$|\vec{b}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{1 + 1 + 1} = \sqrt{3}$.

(f) Recall the standard cosine formula relating vector angles: $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$.

(g) Substitute the computed scalar values into this formula: $\cos\theta = \frac{-1}{\sqrt{3} \cdot \sqrt{3}}$.

(h) Simplify the denominator product: $\cos\theta = -\frac{1}{3}$.

(i) Isolate the angle parameter by applying the inverse function: $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$.

Final Answer: The angle between the vectors is $\cos^{-1}\left(-\frac{1}{3}\right)$.

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept:
Evaluating the principal value of inverse trigonometric functions. The principal value range of the inverse cosine function is strictly bounded within the closed interval $[0, \pi]$, requiring a quadrant adjustment for negative inputs.

Solution:

(a) let the target inverse expression be assigned to an angle variable: $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$.

(b) Rewrite this equation in its base trigonometric form: $\cos\theta = -\frac{1}{2}$.

(c) Recall the core mathematical constraint restricting the principal branch of this function: $\theta \in [0, \pi]$.

(d) Identify the corresponding reference angle in the first quadrant where the function is positive: $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

(e) Recognize that the negative sign dictates that the angle must lie within the second quadrant.

(f) Apply the standard trigonometric angle transformation rule for the second quadrant: $\cos\theta = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right)$.

(g) Evaluate the fractional subtraction inside the argument: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

(h) Equate the expressions to find the final principal angle: $\cos\theta = \cos\left(\frac{2\pi}{3}\right) \implies \theta = \frac{2\pi}{3}$.

(i) Verify that this result satisfies the principal domain boundaries: $\frac{2\pi}{3} \in [0, \pi]$.

Final Answer: The principal value is $\frac{2\pi}{3}$.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 46

Q47.

Solution

Concept:
Identifying the fundamental empirical relationship connecting central tendency metrics. For moderately asymmetrical distributions, Karl Pearson's theorem describes a fixed linear constraint balancing mean, median, and mode.

Solution:

(a) Recall that in perfectly symmetrical statistical distributions, the mean, median, and mode align identically.

(b) Recognize that as a distribution becomes moderately skewed, these three values deviate in a predictable manner.

(c) State Karl Pearson's empirical formula established for skewed tracking:
$\text{Mean} - \text{Mode} = 3(\text{Mean} - \text{Median})$.

(d) Expand the right-hand side of this linear relation by distributing the scalar factor:
$\text{Mean} - \text{Mode} = 3\,\text{Mean} - 3\,\text{Median}$.

(e) Rearrange the algebraic terms to isolate the mode variable on one side.

(f) Shift the mode term to the right and move the other components to the left:
$3\,\text{Median} - 3\,\text{Mean} + \text{Mean} = \text{Mode}$.

(g) Combine the like terms representing the mean values together: $-3\,\text{Mean} + \text{Mean} = -2\,\text{Mean}$.

(h) Write out the simplified linear combination expression: $\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$.

(i) Compare this arrangement with the provided choices to select the matching structural identity.

Final Answer: The relationship is $\text{Mode} = 3\,\text{Median} - 2\,\text{Mean}$.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 47

Q48.

Solution

Concept:
Calculating the geometric slope of a curve's tangent line via differentiation. Evaluating the first derivative of the function at a specific coordinate point provides the exact numerical rate of change representing the slope.

Solution:

(a) Write down the explicit algebraic formula representing the curve: $y = x^3 - x$.

(b) Recall that the geometric slope m of the tangent line is given by the first derivative: $m = \frac{dy}{dx}$.

(c) Differentiate the function with respect to x by applying the power rule to each term individually:
$\frac{d}{dx}(x^3) = 3x^2$ and $\frac{d}{dx}(x) = 1$.

(d) Combine these derivative terms to form the complete slope function expression: $\frac{dy}{dx} = 3x^2 - 1$.

(e) Identify the specific evaluation point along the x-axis from the problem statement: $x = 2$.

(f) Substitute this coordinate value into the newly derived slope function expression:
$m = \left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 - 1$.

(g) Square the coordinate value before applying the scalar multiplier: $3(4) - 1$.

(h) Complete the arithmetic multiplication and subtraction steps: $12 - 1 = 11$.

Final Answer: The slope of the tangent to the curve is 11.

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept:
Determining the focus coordinates of a parabola by matching it to a standard form. Analyzing the quadratic variable and the sign of the linear coefficient reveals the directional orientation of the conic curve.

Solution:

(a) Write down the given specific quadratic equation of the parabola: $x^2 = -16y$.

(b) Recall the standard structural equation for parabolas symmetric about the vertical y-axis: $x^2 = -4ay$.

(c) Recognize that this particular structural equation describes a parabola opening vertically downwards.

(d) Compare the coefficients of the linear term from both equations to find the parameter a: $-4a = -16$.

(e) Solve this linear relationship for the parameter a by dividing through: $a = \frac{-16}{-4} = 4$.

(f) Recall the general formula for the focus coordinates of a vertically downward-opening parabola.

(g) State that the focus lies along the axis of symmetry on the negative y-axis, defined by: $(0, -a)$.

(h) Substitute the calculated parameter value $a = 4$ into this coordinate template.

(i) Conclude that the exact focus coordinates are given by the ordered pair: $(0, -4)$.

Final Answer: The coordinates of the focus are $(0, -4)$.

Answer: (D)

Go Back to Question 49

Q50.

Solution

Concept:
Analyzing geometric progressions using exponential variable indices. Expressing sequential terms using a base term and common ratio allows large product strings to collapse into a power of the central term.

Solution:

(a) Let the first term of the progression be denoted by a and its common ratio be denoted by r.

(b) Express the general formula for any arbitrary n-th term of a geometric sequence: $t_n = ar^{n-1}$.

(c) Identify the value of the third term given in the problem statement: $t_3 = 4$.

(d) Substitute $n = 3$ into the general formula to create an algebraic statement: $t_3 = ar^{3-1} = ar^2 = 4$.

(e) Write out the product of the first five consecutive terms of this sequence: $P = t_1 \cdot t_2 \cdot t_3 \cdot t_4 \cdot t_5$.

(f) Replace each term with its definition using the parameters a and r:
$P = (a) \cdot (ar) \cdot (ar^2) \cdot (ar^3) \cdot (ar^4)$.

(g) Group the base terms and sum the exponential indices of the common ratios: $P = a^5 \cdot r^{1+2+3+4}$.

(h) Evaluate the sum in the exponent: $1 + 2 + 3 + 4 = 10$, which simplifies the product to: $P = a^5 r^{10}$.

(i) Rewrite this expression as a perfect power of a simpler base: $P = (ar^2)^5$.

(j) Substitute the known value $ar^2 = 4$ into this equation to get the final answer: $P = 4^5$.

Final Answer: The product of its first 5 terms is 4^5.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 50

Q51.

Solution

Concept:
Calculating the perpendicular distance of a given point from a three-dimensional plane surface. The calculation requires substituting the coordinate point values into the normalized linear algebraic equation representing the target plane.

Solution:

(a) Write down the standard scalar linear equation of the given plane: $3x - 6y + 2z + 11 = 0$.

(b) Identify the specific structural components from the plane equation: $A = 3$, $B = -6$, $C = 2$, and $D = 11$.

(c) Identify the coordinates of the target point given in the space layout: $(x_1, y_1, z_1) = (2, 3, 4)$.

(d) State the standard mathematical distance formula: $d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$.

(e) Substitute the coordinate values and linear constants into the formula numerator: $|3(2) - 6(3) + 2(4) + 11|$.

(f) Expand and simplify the arithmetic products inside the modulus: $|6 - 18 + 8 + 11| = |7| = 7$.

(g) Substitute the plane direction components into the square root denominator formula: $\sqrt{3^2 + (-6)^2 + 2^2}$.

(h) Evaluate the sum of the squared coefficients underneath the radical: $\sqrt{9 + 36 + 4} = \sqrt{49} = 7$.

(i) Formulate the final numerical ratio by dividing the calculated values: $d = \frac{7}{7} = 1$.

Final Answer: The distance of the point from the plane is 1 unit.

Answer: (A)

Go Back to Question 51

Q52.

Solution

Concept:
Evaluating a trigonometric limits problem involving an exponential composite form. The standard solution methodology leverages basic limit properties to split the complex structure into two foundational limit identities.

Solution:

(a) Write down the given limit expression for evaluation: $\lim_{x\to 0} \frac{e^{\sin x}-1}{x}$.

(b) Modify the rational algebraic expression by multiplying and dividing the function by the sine term.

(c) Express the adjusted expression in a product format: $\lim_{x\to 0} \left(\frac{e^{\sin x}-1}{\sin x} \cdot \frac{\sin x}{x}\right)$.

(d) Recall the product rule of limits to separate the individual operations: $\left(\lim_{x\to 0} \frac{e^{\sin x}-1}{\sin x}\right) \cdot \left(\lim_{x\to 0} \frac{\sin x}{x}\right)$.

(e) Analyze the secondary limit term using the fundamental tracking property: $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

(f) Establish a variable substitution for the primary component expression by defining a new parameter: let $u = \sin x$.

(g) Observe that as the independent tracking variable x approaches 0, the new parameter u also approaches 0.

(h) Rewrite the primary tracking component expression using the substituted parameter: $\lim_{u\to 0} \frac{e^{u}-1}{u}$.

(i) Recognize that this expression matches the standard natural exponential derivative limit identity, which evaluates to 1.

(j) Multiply both evaluated tracking terms together to arrive at the final result: $1 \cdot 1 = 1$.

Final Answer: The value of the limit is 1.

Answer: (B)

Go Back to Question 52

Q53.

Solution

Concept:
Calculating the area bounded by a trigonometric function across an interval that crosses the axis. The total area requires integrating the absolute value, splitting the calculation where the curve changes polarity.

Solution:

(a) Identify the bounding interval along the horizontal x-axis from the geometric layout: $x = 0$ to $x = \pi$.

(b) Express the area using the standard definite integral formulation: $A = \int_0^{\pi} |\cos x|\, dx$.

(c) Identify the root where the trigonometric curve intersects the axis within this interval: $x = \frac{\pi}{2}$.

(d) Split the integration interval into two distinct sub-intervals based on this geometric crossing point.

(e) Analyze the algebraic sign of the function in the first interval: $\cos x \geq 0$ for $x \in [0, \frac{\pi}{2}]$.

(f) Analyze the algebraic sign of the function in the second interval: $\cos x \leq 0$ for $x \in [\frac{\pi}{2}, \pi]$.

(g) Expand the absolute value terms to set up the separate integrations: $\int_0^{\pi/2} \cos x\, dx + \int_{\pi/2}^{\pi} (-\cos x)\, dx$.

(h) Compute the antiderivative and evaluate the first definite integral part: $[\sin x]_0^{\pi/2} = \sin(\frac{\pi}{2}) - \sin(0) = 1$.

(i) Compute the antiderivative and evaluate the second definite integral part: $[-\sin x]_{\pi/2}^{\pi} = (-\sin \pi) - (-\sin \frac{\pi}{2}) = 1$.

(j) Combine the calculated numerical results to obtain the total area value: $1 + 1 = 2$.

Final Answer: The area bounded by the curve is 2.

Answer: (C)

Go Back to Question 53

Q54.

Solution

Concept:
Determining the linear equation of a straight line using the coordinate axis intercept formulation. A line making identical intercepts on both axes can be modeled with a simplified slope component.
Solution:

(a) Recall the standard algebraic intercept equation representing a straight line: $\frac{x}{a} + \frac{y}{b} = 1$.

(b) Apply the specific condition given in the problem statement that intercepts are equal: let $a = b$.

(c) Substitute this equality condition into the base relation formula: $\frac{x}{a} + \frac{y}{a} = 1$.

(d) Simplify the expression by multiplying both sides by the common denominator: $x + y = a$.

(e) Identify the specific coordinate point that must satisfy this linear condition: $(x, y) = (2, 4)$.

(f) Substitute these coordinate values directly into the simplified linear equation: $2 + 4 = a$.

(g) Evaluate the arithmetic sum to find the numerical value of the intercept parameter: $a = 6$.

(h) Substitute the calculated intercept value back into the simplified linear equation formula: $x + y = 6$.

(i) Verify that this matches the initial criteria by confirming equal horizontal and vertical intercepts.

Final Answer: The equation of the straight line is $x + y = 6$.

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept:
Evaluating the scalar value of a logarithmic determinant under a specific boundary condition. The calculation involves applying logarithmic transformations alongside standard matrix row operations.

Solution:

(a) Write down the explicit structural layout of the given determinant: $\Delta = \begin{vmatrix} \log a & \log b \\ \log b & \log a \end{vmatrix}$.

(b) State the specific logarithmic equality condition provided in the problem context: $a = b$.

(c) Apply this variable substitution condition directly to every individual component inside the determinant.

(d) Replace all occurrences of the term b with the term a: $\Delta = \begin{vmatrix} \log a & \log a \\ \log a & \log a \end{vmatrix}$.

(e) Analyze the structural columns and rows of the modified matrix layout.

(f) Observe that the first horizontal row is now completely identical to the second horizontal row.

(g) Observe alternatively that the first vertical column is completely identical to the second vertical column.

(h) Recall the core mathematical property of determinants stating that if any two rows or columns are identical, the matrix is singular.

(i) Conclude directly from this structural property that the scalar value evaluates to exactly zero.

Final Answer: The value of the determinant is 0.

Answer: (B)

Go Back to Question 55

Q56.

Solution

Concept:

Determining the angle between unit vectors using the vector magnitude identity. Squaring the sum vector allows the expression to be expanded into an equation containing a scalar dot product.

Solution:

(a) Identify that both vector quantities are defined as unit vectors: $|\vec{a}| = 1$ and $|\vec{b}| = 1$.

(b) State the additional constraint given for their sum vector component: $|\vec{a} + \vec{b}| = 1$.

(c) Square both sides of the sum magnitude equation to remove the radical property: $|\vec{a} + \vec{b}|^2 = 1^2 = 1$.

(d) Expand the vector magnitude expression using the standard scalar dot product expansion rule.

(e) Write out the expanded scalar equation components: $|\vec{a}|^2 + |\vec{b}|^2 + 2(\vec{a} \cdot \vec{b}) = 1$.

(f) Substitute the known individual unit magnitude values into this expanded algebraic formula: $1^2 + 1^2 + 2(\vec{a} \cdot \vec{b}) = 1$.

(g) Simplify the arithmetic constants and group the remaining scalar terms: $1 + 1 + 2(\vec{a} \cdot \vec{b}) = 1 \implies 2 + 2(\vec{a} \cdot \vec{b}) = 1$.

(h) Isolate the dot product by subtracting and dividing out the constants: $2(\vec{a} \cdot \vec{b}) = -1 \implies \vec{a} \cdot \vec{b} = -\frac{1}{2}$.

(i) Express the dot product using the geometric angle formula: $|\vec{a}||\vec{b}| \cos\theta = -\frac{1}{2} \implies (1)(1)\cos\theta = -\frac{1}{2}$.

(j) Evaluate the inverse cosine function to find the final principal angle: $\cos\theta = -\frac{1}{2} \implies \theta = \frac{2\pi}{3}$.

Final Answer: The angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$.

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 56

Q57.

Solution

Concept:

Calculating the internal angles of a triangle when given all three side lengths. The structural calculation employs the Law of Cosines, which relates the sides of a triangle to the cosine of one of its angles.

Solution:

(a) Identify the specific side lengths of the triangle from the problem text: $a = 2$, $b = 3$, and $c = 4$.

(b) State the standard Law of Cosines formula corresponding to the target internal angle A: $a^2 = b^2 + c^2 - 2bc\cos A$.

(c) Rearrange the algebraic formula to explicitly isolate the cosine tracking variable: $\cos A = \frac{b^2+c^2-a^2}{2bc}$.

(d) Substitute the numerical side lengths into the numerator expression: $\cos A = \frac{3^2+4^2-2^2}{2(3)(4)}$.

(e) Evaluate the exponential terms in the upper part of the fraction: $3^2 = 9$, $4^2 = 16$, and $2^2 = 4$.

(f) Combine the calculated tracking constants in the numerator: $9 + 16 - 4 = 21$.

(g) Evaluate the arithmetic product in the lower part of the fraction: $2 \cdot 3 \cdot 4 = 24$.

(h) Formulate the complete fractional ratio based on these calculations: $\cos A = \frac{21}{24}$.

(i) Simplify the resulting fraction by dividing out the greatest common divisor, which is 3: $\cos A = \frac{7}{8}$.

Final Answer: The value of $\cos A$ is $\frac{7}{8}$.

Answer: (A)

Go Back to Question 57

Q58.

Solution

Concept:
Calculating hyper-geometric sampling probabilities for disjoint student subgroups. The combination rule determines the total configuration counts for both the specific event scenario and the entire selection pool.

Solution:

(a) Count the total number of students present within the pool: 2 brilliant + 3 average = 5 total students.

(b) Identify the size of the random sample group being drawn from the box: $r = 2$.

(c) Compute the total number of possible combinations for the sample size: $n(S) = \binom{5}{2} = \frac{5\times4}{2\times1} = 10$.

(d) Define the target outcome requirement, which specifies selecting exactly one brilliant student.

(e) Recognize that choosing exactly one brilliant student implies selecting exactly one average student to complete the group.

(f) Calculate the combinations for selecting one student from the brilliant pool: $\binom{2}{1} = 2$.

(g) Calculate the combinations for selecting one student from the average pool: $\binom{3}{1} = 3$.

(h) Multiply these disjoint counts to find the total favorable sample combinations: $n(E) = \binom{2}{1} \times \binom{3}{1} = 2 \times 3 = 6$.

(i) Apply the classical probability formula by taking the ratio of these counts: $P(E) = \frac{n(E)}{n(S)} = \frac{6}{10}$.

(j) Reduce the resulting fractional ratio to its lowest terms by dividing out the common factor: $\frac{3}{5}$.

Final Answer: The probability that exactly one is brilliant is $\frac{3}{5}$.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 58

Q59.

Solution

Concept:
Locating the local minima point of a real-valued polynomial function. The optimization procedure entails finding critical points through the first derivative and verifying minimum conditions using the second derivative.
Solution:

(a) Write down the explicit algebraic statement of the quadratic function: $f(x) = x^2 - 4x + 3$.

(b) Differentiate the polynomial function with respect to x to find the first derivative: $f'(x) = 2x - 4$.

(c) Set the expression for the first derivative equal to zero to identify the critical tracking points: $2x - 4 = 0$.

(d) Solve the linear equation for the variable x: $2x = 4 \implies x = 2$.

(e) Differentiate the first derivative function a second time to determine the curvature: $f''(x) = \frac{d}{dx}(2x - 4) = 2$.

(f) Analyze the sign of the calculated second derivative value: observe that $f''(x) = 2$, which is strictly positive ($f''(x) > 0$).

(g) Recall the second derivative test criteria: a positive second derivative at a critical point confirms a local minimum.

(h) Conclude that the function reaches its local minimum state at the coordinate position $x = 2$.

Final Answer: The point of local minima is $x = 2$.

Answer: (B)

Go Back to Question 59

Q60.

Solution

Concept:
Calculating the perpendicular spatial distance separating two parallel straight lines. The calculation uses a normalized ratio comparing the difference between the constant intercept parameters of the lines.

Solution:

(a) Write down the equations for both parallel lines: $3x + 4y - 5 = 0$ and $3x + 4y + 10 = 0$.

(b) Confirm that the lines are parallel by verifying their identical directional coefficients: $A = 3$ and $B = 4$.

(c) Identify the specific constant intercept parameters from both line equations: $C_1 = -5$ and $C_2 = 10$.

(d) State the standard parallel lines distance formula: $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$.

(e) Substitute the constant intercept parameters into the numerator expression: $|-5 - 10|$.

(f) Evaluate the arithmetic subtraction and absolute value inside the numerator: $|-15| = 15$.

(g) Substitute the shared direction coefficients into the denominator formula: $\sqrt{3^2 + 4^2}$.

(h) Evaluate the sum of the squared coefficients underneath the radical: $\sqrt{9 + 16} = \sqrt{25} = 5$.

(i) Formulate the final numerical distance ratio based on these calculations: $d = \frac{15}{5}$.

(j) Reduce the fraction to find the final integer distance value: 3.

Final Answer: The distance between the parallel lines is 3.

Answer: (C)

Go Back to Question 60

Q61.

Solution

Concept:
The total sum of all combinatorial expansion coefficients within a given algebraic expression is determined efficiently by setting all the independent variables to unity. This method simplifies the variable binomial expansion direct evaluation.
Solution:

(a) Identify the given mathematical binomial function provided for the coefficient calculation: $(1-x)^{10}$.

(b) Understand that any regular binomial expansion takes the algebraic form of $P(x) = c_0 + c_1x + c_2x^2 + \cdots + c_nx^n$.

(c) Note that to obtain the pure aggregate sum value of these constant terms $(c_0 + c_1 + \cdots + c_n)$, the variable component x must be removed mathematically.

(d) Set the independent real tracking variable x equal to 1 inside the polynomial structure.

(e) Substitute this numerical value back directly into the base binomial expression formula: $(1-1)^{10}$.

(f) Perform the elementary arithmetic subtraction inside the structural parenthesis bounds: $1-1=0$.

(g) Raise the resulting single integer core base value to the designated exponent power: $0^{10} = 0$.

(h) Conclude that the aggregate sum value of all coefficients within this particular algebraic expansion evaluates to exactly zero.

Final Answer: The sum of the coefficients is 0.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 61

Q62.

Solution

Concept:

Formulating the unique three-dimensional symmetric vector equation of a straight line. The line direction values must match the perpendicular layout coordinates of the given boundary plane normal vector.

Solution:

(a) Write down the linear coordinate equation of the boundary plane surface layout: $x+2y+3z = 5$.

(b) Extract the direction coefficients representing the orthogonal normal vector of this plane: $\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$.

(c) Identify the specific direction ratios from the normal vector coordinates: $a = 1$, $b = 2$, and $c = 3$.

(d) Recognize that a line oriented perpendicular to a plane surface moves exactly parallel to that plane's normal vector.

(e) Assign the extracted direction ratios as the parallel direction numbers for the target line configuration.

(f) Identify the given specific point coordinates through which the straight line passes: $(x_1, y_1, z_1) = (1, 1, 1)$.

(g) Recall the standard symmetric linear coordinate equation format for a space line: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$.

(h) Substitute the known spatial coordinates and parallel direction numbers directly into the standard format.

(i) Arrive at the final structural representation of the line: $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$.

Final Answer: The equation of the line is $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{3}$.

Answer: (A)

Go Back to Question 62

Q63.

Solution

Concept:
Determining the absolute lowest real value constraint of a general linear trigonometric combination. The solution exploits the mathematical bound identities linked to composite wave amplitudes.
Solution:

(a) Identify the explicit real-valued trigonometric wave expression provided for minimization: $f(x) = 3\sin x + 4\cos x$.

(b) Recall the standard structural form representing linear sine and cosine combinations: $a\sin x + b\cos x$.

(c) Map the specific numerical constants from the given expression: $a = 3$ and $b = 4$.

(d) State the core trigonometric range inequality theorem which bounds this expression: $-\sqrt{a^2+b^2} \le a\sin x + b\cos x \le \sqrt{a^2+b^2}$.

(e) Observe that the absolute minimum potential value possible is given by the radical formula: $-\sqrt{a^2+b^2}$.

(f) Substitute the identified scalar numbers into the squared radical component formula: $-\sqrt{3^2+4^2}$.

(g) Compute the squares of the individual parameters underneath the radical sign: $3^2 = 9$ and $4^2 = 16$.

(h) Evaluate the arithmetic sum inside the radical boundaries: $-\sqrt{9+16} = -\sqrt{25}$.

(i) Simplify the square root value to find the final integer minimal constraint: -5.

Final Answer: The minimum value is -5.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 63

Q64.

Solution

Concept:
Calculating the average statistical variation of an explicit numeric dataset. The process requires computing the central arithmetic mean value, finding absolute distance differences, and averaging them.

Solution:

(a) List out the explicit numbers containing the specific statistical data observations: $3, 5, 6, 7, 9$.

(b) Count the total number of observations present in the dataset layout: $n = 5$.

(c) Calculate the total arithmetic sum of these elements: $\sum x_i = 3 + 5 + 6 + 7 + 9 = 30$.

(d) Compute the central arithmetic mean value by dividing the total sum: $\bar{x} = \frac{30}{5} = 6$.

(e) Calculate the absolute differences between each observation value and the central mean value: $|x_i - \bar{x}|$.

(f) Evaluate the individual absolute deviation values: $|3 - 6| = 3$, $|5 - 6| = 1$, $|6 - 6| = 0$, $|7 - 6| = 1$, and $|9 - 6| = 3$.

(g) Aggregate all the computed absolute deviation parameters together: $\sum |x_i - \bar{x}| = 3 + 1 + 0 + 1 + 3 = 8$.

(h) State the standard statistical formulation for mean deviation: $\text{MD} = \frac{\sum |x_i - \bar{x}|}{n}$.

(i) Substitute the aggregated sum and count values to calculate the final decimal value: $\text{MD} = \frac{8}{5} = 1.6$.

Final Answer: The mean deviation from the mean is 1.6.

Answer: (A)

Go Back to Question 64

Q65.

Solution

Concept:
Analyzing the order and degree properties characterizing a standard differential equation. The degree becomes undefined if the highest-order derivative component is embedded inside a transcendental function expansion.

Solution:

(a) Write down the explicit algebraic layout of the differential equation: $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$.

(b) Recall that the order is defined as the absolute highest derivative step present anywhere in the expression.

(c) Scan the equation and identify the highest derivative term layout: the second derivative term $\frac{d^2y}{dx^2}$.

(d) Conclude directly from this structural term that the order of the differential equation is exactly 2.

(e) Recall that the degree represents the exponent power of the highest order derivative when expressed as a polynomial.

(f) Check if the differential equation can be written as a valid polynomial function containing all derivative terms.

(g) Notice the presence of a non-linear transcendental function component containing a derivative: $\sin\left(\frac{dy}{dx}\right)$.

(h) Realize that because a derivative exists inside a sine function, the equation cannot be expressed as a regular polynomial in terms of its derivatives.

(i) Determine from this restriction that the degree property cannot be defined mathematically.

Final Answer: The order is 2 and the degree is not defined.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 65

Q66.

Solution

Concept:

Determining the linear equation representing a perpendicular normal line targeting a curve. The method calculates the negative reciprocal value of the tangent slope evaluated at the given intersection coordinate.

Solution:

(a) Identify the specific geometric function equation provided for analysis: $y = \sin x$.

(b) Identify the specific origin point coordinate given along the curve path: $(x_1, y_1) = (0, 0)$.

(c) Differentiate the function with respect to x to form the general tangent slope function: $\frac{dy}{dx} = \cos x$.

(d) Evaluate this derivative at the given coordinate point to find the tangent slope value: $m_t = \cos(0) = 1$.

(e) Recall the perpendicular geometric slope condition relating tangent and normal lines: $m_n \cdot m_t = -1$.

(f) Calculate the specific normal slope value by taking the negative reciprocal: $m_n = \frac{-1}{1} = -1$.

(g) State the standard point-slope linear formula representing a straight line path: $y - y_1 = m_n(x - x_1)$.

(h) Substitute the origin point coordinates and calculated normal slope into the formula: $y - 0 = -1(x - 0)$.

(i) Simplify the remaining linear terms into standard algebraic format: $y = -x \implies x + y = 0$.

Final Answer: The equation of the normal is $x + y = 0$.

Answer: (A)

Go Back to Question 66

Q67.

Solution

Concept:
Calculating the angle formed between two intersecting straight lines in a two-dimensional layout. The solution determines the individual slope inclination angles relative to the horizontal base coordinate line.

Solution:

(a) Write down the linear function equation representing the first line: $L_1 : y = \sqrt{3}x + 1$.

(b) Extract the slope value for this line from its slope-intercept form: $m_1 = \tan\theta_1 = \sqrt{3}$.

(c) Find the primary inclination angle corresponding to this slope value: $\theta_1 = 60°$.

(d) Write down the linear function equation representing the second line: $L_2 : \sqrt{3}y = x + 2$.

(e) Rearrange this second line equation to isolate the dependent variable: $y = \frac{1}{\sqrt{3}}x + \frac{2}{\sqrt{3}}$.

(f) Extract the secondary slope value from this rearranged linear layout: $m_2 = \tan\theta_2 = \frac{1}{\sqrt{3}}$.

(g) Find the secondary inclination angle corresponding to this slope value: $\theta_2 = 30°$.

(h) State the geometric formula for the acute intersection angle between two lines: $\theta = |\theta_1 - \theta_2|$.

(i) Substitute the calculated inclination angles to find the final difference value: $\theta = |60° - 30°| = 30°$.

Final Answer: The angle between the lines is 30°.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 67

Q68.

Solution

Concept:
Evaluating the numerical determinant value of an adjugate matrix. The transformation relies on a fundamental determinant matrix theorem that raises the base scalar determinant to an exponential power.

Solution:

(a) Identify the given matrix properties from the problem context statement: order $n = 3$.

(b) Identify the given scalar determinant value of the primary square matrix: $|A| = 5$.

(c) Recall the core mathematical determinant property defining adjugate matrix scaling: $|\text{adj } A| = |A|^{n-1}$.

(d) Understand that this exponential relation holds true for any invertible square matrix system of order n.

(e) Substitute the specific order dimension variable value into the exponent power expression: $n - 1 = 3 - 1 = 2$.

(f) Rewrite the adjugate identity expression using the evaluated square exponent power: $|\text{adj } A| = |A|^2$.

(g) Substitute the known base scalar determinant value directly into the exponent expression: 5^2.

(h) Evaluate the final arithmetic exponential product calculation to obtain the integer result: $5 \times 5 = 25$.

Final Answer: The value of $|\text{adj } A|$ is 25.

Answer: (B)

Go Back to Question 68

Q69.

Solution

Concept:
Simplifying a combined vector identity involving cross product and dot product squared magnitudes. The calculation uses geometric vector rules to merge separate parts into Lagrange's identity.
Solution:

(a) Write down the complete scalar vector expression given for algebraic evaluation: $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2$.

(b) Recall the geometric magnitude formula defining a standard vector cross product: $|\vec{a} \times \vec{b}| = ab \sin \theta$.

(c) Square this cross product definition component to form the first part: $(\vec{a} \times \vec{b})^2 = a^2 b^2 \sin^2 \theta$.

(d) Recall the geometric scalar formula defining a standard vector dot product: $\vec{a} \cdot \vec{b} = ab \cos \theta$.

(e) Square this dot product definition component to form the second part: $(\vec{a} \cdot \vec{b})^2 = a^2 b^2 \cos^2 \theta$.

(f) Substitute both squared trigonometric definitions back into the initial complete expression formula.

(g) Write out the combined trigonometric algebraic expression: $a^2 b^2 \sin^2 \theta + a^2 b^2 \cos^2 \theta$.

(h) Factor out the common structural scalar magnitude term from both parts: $a^2 b^2 (\sin^2 \theta + \cos^2 \theta)$.

(i) Apply the fundamental Pythagorean trigonometric identity to simplify the parenthesis: $\sin^2 \theta + \cos^2 \theta = 1$.

(j) Multiply out the remaining components to establish the final identity expression: $a^2 b^2 \cdot 1 = a^2 b^2$.

Final Answer: The value of the expression is $a^2 b^2$.

Answer: (A)

Go Back to Question 69

Q70.

Solution

Concept:
Evaluating a composite combination of single-variable sine wave terms. The solution simplifies the expression by grouping terms and using standard trigonometric sum-to-product transformation formulas.

Solution:

(a) Write down the explicit trigonometric sequence expression provided for evaluation: $\sin 50° - \sin 70° + \sin 10°$.

(b) Rearrange the sequence order to group the primary positive sine terms together: $(\sin 50° + \sin 10°) - \sin 70°$.

(c) Recall the standard trigonometric identity for a sum of two sine functions: $\sin C + \sin D = 2\sin(\frac{C+D}{2})\cos(\frac{C-D}{2})$.

(d) Apply this identity directly to the grouped terms inside the parenthesis by setting $C = 50°$ and $D = 10°$.

(e) Calculate the sum and difference angle tracking arguments: $\frac{50°+10°}{2} = 30°$ and $\frac{50°-10°}{2} = 20°$.

(f) Write down the transformed expression tracking components: $2\sin 30° \cos 20° - \sin 70°$.

(g) Substitute the exact standard unit value known for the sine component: $\sin 30° = \frac{1}{2}$.

(h) Simplify the multiplied numerical terms in the front section: $2 \cdot (\frac{1}{2}) \cdot \cos 20° = \cos 20°$.

(i) Write out the updated intermediate difference expression formula: $\cos 20° - \sin 70°$.

(j) Apply the standard complementary co-function transformation rule to the sine term: $\sin 70° = \cos(90° - 70°) = \cos 20°$.

(k) Perform the final subtraction with the identical transformed values to arrive at zero: $\cos 20° - \cos 20° = 0$.

Final Answer: The value of the expression is 0.

Answer: (B)

Go Back to Question 70

Q71.

Solution

Concept:
The total number of combined elemental sample space events resulting from throwing two identical standard independent cubical dice simultaneously equals thirty-six. Finding the specific probability of matching a desired numerical score requires identifying all compliant coordinate subsets.
Solution:

(a) Identify the comprehensive outcome group structure when rolling a pair of independent six-sided dice: $S = \{(i, j) : 1 \le i, j \le 6\}$.

(b) Calculate the absolute grand total of discrete event combinations available within this configuration space: $n(S) = 6 \times 6 = 36$.

(c) Identify the specific arithmetic requirement given for the target event, which mandates that the two face values sum to seven.

(d) Establish the precise target conditional equation modeling this required system behavior: $i + j = 7$.

(e) Enumerate systematically every possible integer pair matching this sum constraint: $(1, 6)$, $(2, 5)$, $(3, 4)$, $(4, 3)$, $(5, 2)$, and $(6, 1)$.

(f) Count the individual matching outcomes to establish the size of the successful subset group: $n(E) = 6$.

(g) Recall the classical probability formulation definition: $P(E) = \frac{n(E)}{n(S)}$.

(h) Substitute the calculated integer numbers directly into the fraction template layout: $P(E) = \frac{6}{36}$.

(i) Reduce the fraction components to their lowest possible rational terms: $\frac{1}{6}$.

Final Answer: The probability is $\frac{1}{6}$.

Answer: (A)

Go Back to Question 71

Q72.

Solution

Concept:
Evaluating a definite integral featuring a symmetric coordinate bound range enclosing an absolute value function. The system can be simplified by leveraging the algebraic properties of symmetric even functions.

Solution:

(a) Write down the explicit definite mathematical integral function provided for calculation: $I = \int_{-1}^{1} |x|\,dx$.

(b) Analyze the internal function properties to check for symmetry: $f(x) = |x|$.

(c) Evaluate the function using a inverted input parameter step: $f(-x) = |-x| = |x| = f(x)$.

(d) Determine from this functional behavior that the core expression layout constitutes a perfectly symmetric even function.

(e) Recall the specific calculus definition theorem simplifying symmetric even integrals: $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

(f) Transform the initial coordinate boundaries using this rule: $I = 2\int_{0}^{1} |x|\,dx$.

(g) Observe that within the newly established non-negative domain from zero to one, $|x| = x$.

(h) Rewrite the single variable integration structure explicitly without the modulus notation: $I = 2\int_{0}^{1} x\,dx$.

(i) Apply the standard power integration rule to find the anti-derivative function: $\int x\,dx = \frac{x^2}{2}$.

(j) Write out the bounded expression: $I = 2\left[\frac{x^2}{2}\right]_0^1 = \left[x^2\right]_0^1$.

(k) Evaluate the limits to calculate the final integer value: $1^2 - 0^2 = 1$.

Final Answer: The value of the integral is 1.

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 72

Q73.

Solution

Concept:
Calculating the straight line geometric distance slice isolated on the horizontal coordinate axis by a circular path boundary. This intersection distance length depends on the constant equation parameters.

Solution:

(a) Write down the general polynomial equation representing the circle boundary line: $x^2 + y^2 - 4x - 6y - 12 = 0$.

(b) Recall the standard generic coordinate model for a circle: $x^2 + y^2 + 2gx + 2fy + c = 0$.

(c) Match the corresponding algebraic coefficients from the given equation layout: $2g = -4 \implies g = -2$, and $c = -12$.

(d) Recall the standard geometric distance formula defining the intercept length left on the horizontal x-axis: Intercept $= 2\sqrt{g^2 - c}$.

(e) Substitute the extracted scalar constants directly into the radical calculation layout: $2\sqrt{(-2)^2 - (-12)}$.

(f) Compute the inner squared integer parameter value underneath the radical sign: $(-2)^2 = 4$.

(g) Simplify the combined subtraction operations inside the radical core: $4 - (-12) = 4 + 12 = 16$.

(h) Rewrite the updated mathematical radical expression template: $2\sqrt{16}$.

(i) Compute the perfect square root value of the interior integer: $\sqrt{16} = 4$.

(j) Multiply the remaining numbers together to find the final integer distance length: $2 \times 4 = 8$.

Final Answer: The intercept made on the x-axis is 8.

Answer: (C)

Go Back to Question 73

Q74.

Solution

Concept:

Determining the unknown coefficient value that produces infinite solutions for a linear matrix system. This configuration requires the core determinant and its corresponding variable component determinants to equal zero.

Solution:

(a) Write out the structured system of linear equations: $x + y + z = 2$, $2x + 3y + 2z = 5$, and $2x + 3y + (k^2 - 1)z = k + 3$.

(b) Form the primary coefficient matrix determinant to check the system consistency: $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & k^2 - 1 \end{vmatrix}$.

(c) Set the primary matrix determinant equation equal to zero to fulfill the infinite solution constraint criteria: $D = 0$.

(d) Perform elementary row operations to simplify the determinant layout: apply $R_3 \to R_3 - R_2$.

(e) Write down the newly simplified determinant matrix form: $\begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & k^2 - 3 \end{vmatrix} = 0$.

(f) Expand the determinant along the third row to isolate the variable terms: $(k^2 - 3) \cdot (3 - 2) = 0$.

(g) Simplify the remaining algebraic expression directly: $(k^2 - 3) \cdot 1 = 0 \implies k^2 - 3 = 0$.

(h) Solve the remaining quadratic equation for the variable term: $k^2 = 3 \implies k = \pm\sqrt{3}$.

(i) Verify that substituting $k = \pm\sqrt{3}$ makes the remaining augment variable plane determinants equal zero, ensuring infinite solutions.

Final Answer: The values of k are $\pm\sqrt{3}$.

Answer: (C)

Go Back to Question 74

Q75.

Solution

Concept:
Analyzing joint geometric constraints linking vector dot and cross operations. If both products return zero simultaneously, it eliminates all possible spatial angle configurations, implying a null vector presence.

Solution:

(a) Note the first mathematical condition given for the two spatial vectors: $\vec{a} \cdot \vec{b} = 0$.

(b) Recall that a zero dot product implies either vector is null, or they are orthogonal: $\vec{a} \perp \vec{b}$ ($\theta = 90°$).

(c) Note the second mathematical condition given for the same vectors: $\vec{a} \times \vec{b} = \vec{0}$.

(d) Recall that a zero cross product implies either vector is null, or they are collinear: $\vec{a} \parallel \vec{b}$ ($\theta = 0°$ or $180°$).

(e) Observe that the directional angle parameter θ cannot be perpendicular and parallel at the exact same time.

(f) Realize that no single pair of non-zero directional vectors can satisfy both spatial angle conditions simultaneously.

(g) Conclude that the simultaneous zero values must be caused by a lack of magnitude in at least one component.

(h) Determine that it is mathematically mandatory for at least one of the vectors to be a null vector.

Final Answer: At least one of $\vec{a}$ or $\vec{b}$ is a null vector.

Answer: (C)

Go Back to Question 75

Q76.

Solution

Concept:
Finding the absolute valid mathematical domain for a real function containing a radical fraction denominator. The interior quadratic expression must remain strictly greater than zero.
Solution:

(a) Identify the explicit real single-variable function expression provided for analysis: $f(x) = \frac{1}{\sqrt{x^2-3x+2}}$.

(b) Note that for the radical fraction to yield real values, the denominator core cannot equal zero or contain a negative value.

(c) State the strict mathematical inequality rule defining this constraint: $x^2 - 3x + 2 > 0$.

(d) Factor the quadratic polynomial expression into its distinct linear components: $x^2 - 2x - x + 2 > 0$.

(e) Group the factored terms to reveal the roots of the expression: $(x-1)(x-2) > 0$.

(f) Identify the specific critical boundary transition points on the real number line: $x = 1$ and $x = 2$.

(g) Apply the standard sign interval test method across the partitioned tracking segments.

(h) Observe that the product is positive when x is less than 1 or when x is greater than 2.

(i) Write out the final combined open interval set representing the function domain: $(-\infty, 1) \cup (2, \infty)$.

Final Answer: The domain is $(-\infty, 1) \cup (2, \infty)$.

Answer: (A)

Go Back to Question 76

Q77.

Solution

Concept:
Formulating a straight line equation passing through a specific coordinate intersection point. A line oriented perfectly parallel to the horizontal axis contains a constant independent of x.
Solution:

(a) Write down the linear equations tracking the two primary intersecting coordinate lines: $x - y + 1 = 0$ and $2x - 3y + 5 = 0$.

(b) Solve the system of linear equations simultaneously to find their point of intersection.

(c) Multiply the first line equation by two to enable variable elimination: $2x - 2y + 2 = 0$.

(d) Subtract this updated equation from the second line equation: $(2x-3y+5)-(2x-2y+2) = 0 \implies -y + 3 = 0$.

(e) Isolate the variable to find the vertical coordinate position of the intersection: $y = 3$.

(f) Substitute $y = 3$ back into the first line equation to find the horizontal position: $x - 3 + 1 = 0 \implies x = 2$.

(g) Identify the precise coordinate point matching this intersection: $(x_1, y_1) = (2, 3)$.

(h) Note that a line moving parallel to the x-axis has a slope of zero, meaning its equation takes the form $y = c$.

(i) Match this constant directly with the required intersection height coordinate to obtain the final equation: $y = 3$.

Final Answer: The equation of the line is $y = 3$.

Answer: (A)

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Q78.

Solution

Concept:
Determining the complete total count of distinct sub-elements or combinations that can be generated from a set. This calculation utilizes an exponential formula based on the total element count.
Solution:

(a) Identify the total count of distinct base elements contained within the given set group: $n = 5$.

(b) Recall the fundamental set theory theorem defining subset power structures.

(c) State the standard exponential equation template used to calculate the total subset count: $\text{Total Subsets} = 2^n$.

(d) Understand that this formula accounts for all possible selection counts, ranging from the empty set up to the full set.

(e) Substitute the given element count parameter directly into the exponent power slot: 2^5.

(f) Expand the exponential terms into an explicit multiplication sequence: $2 \times 2 \times 2 \times 2 \times 2$.

(g) Perform the continuous arithmetic multiplication steps to calculate the final product value: $2 \times 2 = 4$, $4 \times 2 = 8$, $8 \times 2 = 16$, and $16 \times 2 = 32$.

(h) Conclude that a set containing five elements yields exactly thirty-two unique subsets.

Final Answer: The number of subsets is 32.

Answer: (D)

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Q79.

Solution

Concept:
Evaluating the unique principal branch angle value for an inverse cosine function. The output angle must fall inside the restricted standard interval range from zero to pi.

Solution:

(a) Write down the composite inverse trigonometric expression provided for calculation: $\theta = \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$.

(b) State the principal branch value interval range restriction that bounds the inverse cosine function: $[0, \pi]$.

(c) Observe the initial internal radian angle input value given in the expression: $\frac{7\pi}{6}$.

(d) Notice that this initial angle value lies outside the mandatory principal interval boundaries since $\frac{7\pi}{6} > \pi$.

(e) Use standard trigonometric quadrant reduction identities to map the angle into the primary branch.

(f) Rewrite the angle argument using a full rotation difference step: $\cos\left(\frac{7\pi}{6}\right) = \cos\left(2\pi - \frac{7\pi}{6}\right)$.

(g) Compute the fractional subtraction operation inside the parenthesis: $2\pi - \frac{7\pi}{6} = \frac{12\pi - 7\pi}{6} = \frac{5\pi}{6}$.

(h) Substitute this equivalent reduced term back into the function layout: $\theta = \cos^{-1}\left(\cos\frac{5\pi}{6}\right)$.

(i) Verify that the new radian angle value satisfies the required principal domain restriction: $\frac{5\pi}{6} \in [0, \pi]$.

(j) Cancel the inverse operations to find the final principal radian value: $\frac{5\pi}{6}$.

Final Answer: The principal value is $\frac{5\pi}{6}$.

Answer: (B)

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Q80.

Solution

Concept:
Calculating the central mean parameter of a discrete frequency distribution. When the weights are exactly proportional to binomial coefficients, the system models a symmetric binomial distribution.
Solution:

(a) Identify the sequence of discrete integer values taken by the variable: $x = 0, 1, 2, \ldots, n$.

(b) Identify the corresponding weight frequencies assigned to each value position: $f = \binom{n}{0}, \binom{n}{1}, \binom{n}{2}, \ldots, \binom{n}{n}$.

(c) State the standard statistical formula used to calculate a weighted mean value: $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$.

(d) Evaluate the denominator sum using the standard binomial coefficient sum identity: $\sum_{r=0}^{n} \binom{n}{r} = 2^n$.

(e) Set up the numerator sum sequence using the variable and frequency products: $\sum_{r=0}^{n} r \cdot \binom{n}{r}$.

(f) Recall the standard combinatorial factor identity used to simplify the numerator terms: $r \cdot \binom{n}{r} = n \cdot \binom{n-1}{r-1}$.

(g) Factor out the constant parameter to evaluate the remaining sum expression: $n \sum_{r=1}^{n} \binom{n-1}{r-1} = n \cdot 2^{n-1}$.

(h) Substitute both evaluated sum components back into the weighted mean formula layout: $\bar{x} = \frac{n \cdot 2^{n-1}}{2^n}$.

(i) Simplify the exponential terms to calculate the final mean value: $\bar{x} = \frac{n}{2}$.

Final Answer: The mean of the distribution is $\frac{n}{2}$.

Answer: (A)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	C	5	B
6	B	7	A	8	A	9	B	10	A
11	C	12	A	13	A	14	B	15	B
16	A	17	C	18	B	19	A	20	B
21	C	22	C	23	A	24	A	25	A
26	A	27	A	28	A	29	B	30	B
31	A	32	B	33	C	34	A	35	B
36	B	37	B	38	C	39	A	40	A
41	C	42	B	43	A	44	A	45	B
46	B	47	A	48	A	49	D	50	B
51	A	52	B	53	C	54	A	55	B
56	C	57	A	58	A	59	B	60	C
61	B	62	A	63	A	64	A	65	A
66	A	67	A	68	B	69	A	70	B
71	A	72	B	73	C	74	C	75	C
76	A	77	A	78	D	79	B	80	A