UPCATET Mathematics Sample Paper-5

Duration: 80 Minutes **Maximum Marks: 320**

Instructions

- This paper contains **80** Multiple Choice Questions.
- Each correct answer carries **+4** mark. Incorrect answer: **−1** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. If $f(x) = x^3 + 2x$ and $g(x) = \sqrt[3]{x-2}$, then the value of $(f \circ g)(10)$ is:

(A) 12

(B) 10

(C) 8

(D) 6

Q2. The line $2x + 3y - 5 = 0$ and the point $(1, 1)$ satisfy which geometric relationship?

(A) The point lies on the line

(B) The point lies below the line

(C) The point lies above the line

(D) None of these

Q3. A parabola has vertex at $(2, -3)$ and focus at $(2, -1)$. What is the equation of the directrix?

(A) $y = -5$

(B) $y = -3$

(C) $y = -1$

(D) $y = 1$

Q4. The number of ways to arrange 5 identical red balls and 3 identical blue balls in a row is:

(A) $\binom{8}{3}$

(B) $\binom{8}{5}$

(C) $8!$

(D) $5! \times 3!$

Q5. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$. The determinant of A^{-1} is:

(A) $\frac{1}{6}$

(B) 6

(C) $-\frac{1}{6}$

(D) $\frac{1}{3}$

Q6. The slope of the tangent to $y = e^{x^2}$ at $x = 1$ is:

(A) e

(B) $2e$

(C) e^2

(D) $2e^2$

Q7. The area under the curve $y = 4 - x^2$ between $x = -2$ and $x = 2$ is:

(A) $\frac{16}{3}$

(B) $\frac{32}{3}$

(C) 16

(D) 8

Q8. The angle between the curves $y = x^2$ and $y = -x^2 + 2$ at their point of intersection is approximately:

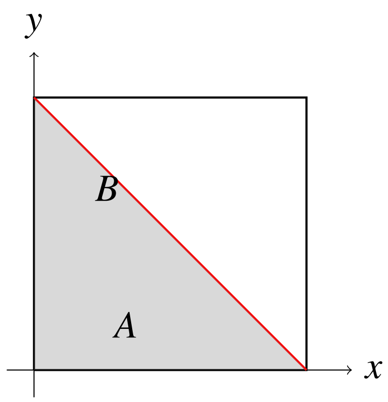
(A) 45°

(B) 60°

(C) 90°

(D) 120°

Q9. The region bounded by $x = 0$, $y = 0$, $x = 3$, and $y = 3$ and the line $x + y = 3$ has area:



(A) $\frac{9}{2}$

(B) 4.5

(C) 9

(D) 6

Q10. If $\tan\theta + \cot\theta = 4$, then $\tan^2\theta + \cot^2\theta$ equals:

(A) 14

(B) 12

(C) 16

(D) 18

Q11. In triangle ABC, if $\angle A = 60^\circ$, $b = 4$, and $c = 3$, then the area is:

(A) 6

(B) $3\sqrt{3}$

(C) 12

(D) $2\sqrt{3}$

Q12. The eccentricity of the hyperbola $16x^2 - 9y^2 = 144$ is:

(A) $\frac{5}{3}$
(B) $\frac{3}{5}$
(C) $\frac{5}{4}$
(D) $\frac{4}{5}$

Q13. A bag contains 4 red and 5 blue balls. The probability of drawing 2 balls of the same color is:

(A) $\frac{4}{9}$
(B) $\frac{5}{18}$
(C) $\frac{4}{9}$
(D) $\frac{7}{18}$

Q14. The mode of the data set $\{2, 3, 3, 5, 7, 7, 7, 9\}$ is:

(A) 3
(B) 7
(C) 5
(D) 9

Q15. The distance between the points $(1, 2, 3)$ and $(-1, -2, -3)$ is:

(A) $2\sqrt{14}$
(B) $\sqrt{56}$
(C) 14
(D) $\sqrt{14}$

Q16. The cross product $\vec{a} \times \vec{b}$ where $\vec{a} = 2\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$ gives:

(A) $3\hat{k}$

(B) $-3\hat{k}$

(C) $\hat{k}$

(D) $5\hat{k}$

Q17. The general solution of $\cos x = \frac{\sqrt{3}}{2}$ is:

(A) $x = \pm\frac{\pi}{6} + 2\pi n$

(B) $x = \pm\frac{\pi}{6} + n\pi$

(C) $x = \pm\frac{\pi}{3} + 2\pi n$

(D) $x = \frac{\pi}{6} + 2\pi n$

Q18. The product of the roots of $3x^2 - 7x + 2 = 0$ is:

(A) $\frac{7}{3}$

(B) $\frac{2}{3}$

(C) 3

(D) $-\frac{2}{3}$

Q19. The inflection point of $y = x^3 - 3x$ occurs at:

(A) $(0, 0)$

(B) $(1, -2)$

(C) $(1, 2)$

(D) $(-1, 2)$

Q20. The Cartesian equation of the parametric curve $x = t^2, y = 2t$ is:

(A) $y^2 = 4x$

(B) $x^2 = 4y$

(C) $y = 4x^2$

(D) $x = 4y^2$

Q21. The vertex of the parabola $y = -2x^2 + 4x - 1$ is:

(A) $(1, 1)$

(B) $(2, -1)$

(C) $(1, 2)$

(D) $(2, 1)$

Q22. The sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ is:

(A) 0

(B) 1

(C) ∞

(D) $\frac{1}{2}$

Q23. The equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$ is:

(A) $3x + 4y = 25$

(B) $3x + 4y = 5$

(C) $4x + 3y = 25$

(D) $3x - 4y = 0$

Q24. The limit $\lim_{x \to 0} \frac{\sin 3x}{x}$ equals:

(A) 1

(B) 3

(C) $\frac{1}{3}$

(D) 0

Q25. The line joining $(1, 2)$ and $(3, 6)$ has slope:

(A) 2

(B) $\frac{1}{2}$

(C) 3

(D) 4

Q26. The coefficient of x^2 in the binomial expansion of $(2x + 3)^4$ is:

(A) 216

(B) 108

(C) 54

(D) 144

Q27. The orthogonality condition for two lines with slopes m_1 and m_2 is:

(A) $m_1 + m_2 = 0$

(B) $m_1 - m_2 = 1$

(C) $m_1 \cdot m_2 = -1$

(D) $m_1 = m_2$

Q28. The equation of the circle with center $(2, 3)$ and radius 5 is:

(A) $(x - 2)^2 + (y - 3)^2 = 25$

(B) $(x + 2)^2 + (y + 3)^2 = 25$

(C) $(x - 2)^2 + (y - 3)^2 = 5$

(D) $x^2 + y^2 = 25$

Q29. The standard form of the parabola with vertex at the origin and focus at $(0, 3)$ is:

(A) $y^2 = 12x$

(B) $x^2 = 12y$

(C) $x^2 = -12y$

(D) $y^2 = -12x$

Q30. If two events A and B are independent, then $P(A \cap B)$ equals:

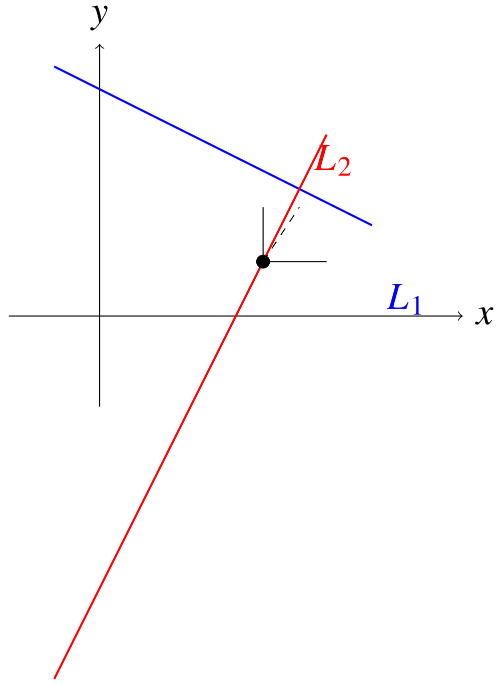
(A) $P(A) + P(B)$

(B) $P(A) \cdot P(B)$

(C) $P(A) - P(B)$

(D) $P(A) + P(B) - P(A \cap B)$

Q31. The angle between the lines $x + 2y = 5$ and $2x - y = 3$ can be calculated using:



(A) $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

(B) $\theta = m_1 - m_2$

(C) $\sin\theta = |m_1 - m_2|$

(D) $\theta = |m_1| + |m_2|$

Q32. The critical point of $f(x) = x^3 - 3x^2 + 2$ is:

(A) $x = 0$ only

(B) $x = 2$ only

(C) $x = 0$ and $x = 2$

(D) $x = 1$ only

Q33. The derivative of $\ln(\cos x)$ with respect to x is:

(A) $-\tan x$

(B) $\tan x$

(C) $\sec x$

(D) $-\sec x$

Q34. In a GP with first term a and common ratio r, the sum of infinite terms is:

(A) $\frac{a}{1-r}$ (when $|r| < 1$)

(B) $\frac{a}{1+r}$ (when $|r| < 1$)

(C) ar

(D) $a(1 + r)$

Q35. The unit vector in the direction of $\vec{a} = 3\hat{i} + 4\hat{j}$ is:

(A) $\frac{1}{5}(3\hat{i} + 4\hat{j})$

(B) $\frac{1}{7}(3\hat{i} + 4\hat{j})$

(C) $3\hat{i} + 4\hat{j}$

(D) $\frac{3\hat{i}+4\hat{j}}{25}$

Q36. The equation of the plane passing through $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ is:

(A) $x + y + z = 1$

(B) $x + y - z = 0$

(C) $x - y + z = 0$

(D) $x + y + z = 0$

Q37. The vector projection of $\vec{a} = \hat{i} + 2\hat{j}$ onto $\vec{b} = \hat{i}$ is:

(A) $\hat{i}$

(B) $2\hat{j}$

(C) $\hat{i} + 2\hat{j}$

(D) $\frac{1}{2}\hat{i}$

Q38. If $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for all x in $[-1, 1]$, then this relationship is:

(A) Always true

(B) Sometimes true

(C) Never true

(D) True only for $x = 0$

Q39. The area of an ellipse with semi-major axis $a = 5$ and semi-minor axis $b = 3$ is:

(A) 15π

(B) 8π

(C) 30π

(D) 25π

Q40. The partial fraction decomposition of $\frac{3x+1}{(x-1)(x+2)}$ has form:

(A) $\frac{A}{x-1} + \frac{B}{x+2}$

(B) $\frac{Ax+B}{x-1} + \frac{C}{x+2}$

(C) $Ax + B$

(D) $\frac{A}{(x-1)^2} + \frac{B}{x+2}$

Q41. The value of $\sin 75°$ is:

(A) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(B) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(C) $\frac{\sqrt{3}+1}{2}$

(D) $\frac{\sqrt{3}-1}{2}$

Q42. The number of diagonals in a pentagon is:

(A) 5

(B) 10

(C) 15

(D) 8

Q43. A circle passes through $(1, 1)$, $(2, -1)$, and $(3, 0)$. Its center is at:

(A) $(2, 0)$

(B) $(\frac{3}{2}, \frac{1}{2})$

(C) $(1, -1)$

(D) $(\frac{3}{2}, -\frac{1}{2})$

Q44. The remainder when $x^3 - 2x^2 + x + 1$ is divided by $x - 1$ is:

(A) 0

(B) 1

(C) 2

(D) -1

Q45. The antiderivative of $\cos 2x$ is:

(A) $\frac{\sin 2x}{2} + C$

(B) $2 \sin 2x + C$

(C) $\sin 2x + C$

(D) $-\frac{\sin 2x}{2} + C$

Q46. If the mean of five numbers is 10 and four of them are $8, 9, 11, 12$, the fifth number is:

(A) 10

(B) 15

(C) 20

(D) 5

Q47. The ratio in which the point $(2, 3)$ divides the line segment from $(1, 2)$ to $(4, 5)$ is:

(A) $1 : 2$

(B) $2 : 1$

(C) $1 : 1$

(D) $3 : 2$

Q48. The modulus of $3 + 4i$ is:

(A) 5

(B) 7

(C) 25

(D) $\sqrt{7}$

Q49. The tangent to $y = x^2 - x$ parallel to the x-axis exists at:

(A) $x = 0$

(B) $x = 1$

(C) $x = \frac{1}{2}$

(D) $x = -\frac{1}{2}$

Q50. If $P(A) = 0.3$ and $P(B) = 0.4$ with A and B mutually exclusive, then $P(A \cup B)$ is:

(A) 0.7

(B) 0.12

(C) 0.58

(D) 0.42

Q51. The length of the major axis of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is:

(A) 3

(B) 6

(C) 9

(D) 4

Q52. The parametric form of the line $2x + 3y = 6$ can be written using parameter t as:

(A) $x = 3 - \frac{3t}{2}, y = t$

(B) $x = t, y = 2 - \frac{2t}{3}$

(C) $x = 3 + 2t, y = 1 - 3t$

(D) $x = 2t, y = 3 - 2t$

Q53. For the curve $y = e^x$, the area between the curve and the x-axis from $x = 0$ to $x = 1$ is:

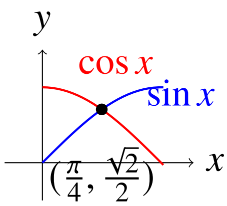
(A) $e - 1$

(B) e

(C) 1

(D) $\frac{e}{2}$

Q54. The angle of intersection of the curves $y = \sin x$ and $y = \cos x$ at $x = \frac{\pi}{4}$ can be visualized as:



(A) 45°

(B) 90°

(C) $\cos^{-1}(2\sqrt{2})$

(D) $\tan^{-1}(2)$

Q55. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q56. The locus of points equidistant from two given points forms:

(A) A circle

(B) A perpendicular bisector

(C) A parabola

(D) Two parallel lines

Q57. If a die is rolled twice, the probability of getting a sum greater than 8 is:

(A) $\frac{1}{6}$

(B) $\frac{5}{18}$

(C) $\frac{1}{9}$

(D) $\frac{7}{36}$

Q58. The second derivative of $f(x) = x^4 - 2x^3 + x$ is:

(A) $12x^2 - 12x$

(B) $4x^3 - 6x^2 + 1$

(C) $12x^2 - 6x$

(D) $4x^3 - 6x$

Q59. The foci of the hyperbola $16x^2 - 9y^2 = 144$ are at:

(A) $(\pm 5, 0)$

(B) $(0, \pm 5)$

(C) $(\pm 4, 0)$

(D) $(\pm 3, 0)$

Q60. The equation $x^2 - 5x + 6 = 0$ has roots in the ratio:

(A) $1 : 2$

(B) $2 : 3$

(C) $1 : 1$

(D) $3 : 4$

Q61. The volume of the solid of revolution formed by rotating $y = \sqrt{x}$ about the x-axis from $x = 0$ to $x = 1$ is:

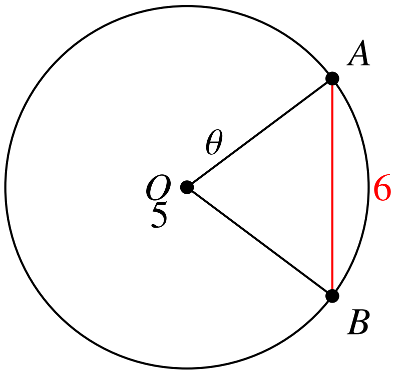
(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{\pi}{3}$

(D) 2π

Q62. The angle subtended by a chord of length 6 at the center of a circle with radius 5 is:



(A) $\cos^{-1}(\frac{7}{25})$

(B) $\cos^{-1}(\frac{18}{25})$

(C) $\cos^{-1}(\frac{1}{2})$

(D) $\sin^{-1}(\frac{3}{5})$

Q63. If $x, x+d, x+2d, \ldots$ form an AP with n terms, the sum is:

(A) $\frac{n}{2}[2x+(n-1)d]$

(B) $nx+\frac{n(n-1)d}{2}$

(C) Both A and B

(D) $\frac{n(2x+nd)}{2}$

Q64. The standard deviation of $\{2, 4, 6, 8, 10\}$ is:

(A) $2\sqrt{2}$

(B) $\sqrt{8}$

(C) 4

(D) 2

Q65. The foot of the perpendicular from $(1,2)$ to the line $3x + 4y - 5 = 0$ is:

(A) $(\frac{1}{5}, \frac{1}{2})$

(B) $(\frac{7}{5}, \frac{4}{5})$

(C) $(1, 1)$

(D) $(0, 0)$

Q66. The value of $\int_0^{\pi/2} \sin^2 x\, dx$ is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{1}{2}$

(D) 1

Q67. If $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, then $\vec{a} \cdot \vec{b}$ is:

(A) -16

(B) -4

(C) 0

(D) 4

Q68. The equation of the locus of points at constant distance d from a fixed line is:

(A) A parabola

(B) Two parallel lines

(C) A circle

(D) A hyperbola

Q69. The sum to infinity of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$ is:

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) 1

(D) $\frac{3}{2}$

Q70. The distance between parallel lines $3x + 4y = 6$ and $3x + 4y = 12$ is:

(A) $\frac{6}{5}$

(B) 1

(C) $\frac{12}{5}$

(D) 2

Q71. If $\tan A = \frac{3}{4}$ and $\tan B = \frac{1}{5}$, then $\tan(A - B)$ is:

(A) $\frac{11}{19}$

(B) $\frac{19}{11}$

(C) $\frac{11}{19}$

(D) $\frac{19}{22}$

Q72. The center and radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ are:

(A) Center $(2, -3)$, radius 5

(B) Center $(-2, 3)$, radius 5

(C) Center $(2, 3)$, radius $\sqrt{25}$

(D) Center $(4, -6)$, radius 12

Q73. The condition for the line $y = mx + c$ to be a tangent to the circle $x^2 + y^2 = r^2$ is:

(A) $c = r\sqrt{1 + m^2}$

(B) $c^2 = r^2(1 + m^2)$

(C) $c = r(1 + m)$

(D) $c = r + m$

Q74. In a triangle, if $a = 5, b = 7, c = 8$, then the largest angle is:

(A) $\cos^{-1}(\frac{1}{7})$

(B) $\cos^{-1}(\frac{1}{14})$

(C) $\cos^{-1}(\frac{1}{2})$

(D) $\cos^{-1}(\frac{2}{7})$

Q75. The probability that a randomly selected number from $\{1, 2, \ldots, 100\}$ is divisible by both 3 and 5 is:

(A) $\frac{1}{15}$

(B) $\frac{1}{10}$

(C) $\frac{3}{100}$

(D) $\frac{6}{100}$

Q76. The graph of $y = |x - 1| + 2$ is a:

(A) Parabola with vertex at $(1, 2)$

(B) V-shaped curve with vertex at $(1, 2)$

(C) Straight line

(D) Circle with center $(1, 2)$

Q77. The value of $\sin^{-1}(\sin(\frac{5\pi}{6}))$ is:

(A) $\frac{5\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $-\frac{5\pi}{6}$

(D) $\frac{7\pi}{6}$

Q78. The number of ways to choose 3 objects from 10 identical and 2 distinct objects is:

(A) $\binom{10}{3}$

(B) 12

(C) $\binom{12}{3}$

(D) 20

Q79. If the line $ax + by + c = 0$ passes through $(1, 1)$ and $(2, -1)$, then:

(A) $a + b + c = 0$

(B) $a - b + c = 0$

(C) $2a - b + c = 0$

(D) $a + 2b + c = 0$

Q80. The asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are:

(A) $y = \pm\frac{b}{a}x$

(B) $x = \pm a, y = \pm b$

(C) $y = \pm x$

(D) $x = 0, y = 0$

Detailed Solutions

Q1.

Solution

Concept:
The composition of mathematical functions involves a structured mapping process where the output of an inner function serves directly as the input for an outer function. To evaluate a composite function expression denoted as $(f \circ g)(x)$ at a specific numerical value, the operations must be performed sequentially from the inside out. One first applies the inner function g to the given input value to obtain an intermediate scalar result, and then substitutes this intermediate value into the outer function f to calculate the final output.

Solution:

(a) Identify the two explicit single-variable real functions provided in the problem text: $f(x) = x^3 + 2x$ and $g(x) = \sqrt[3]{x-2}$.

(b) State the target composite function evaluation required by the question prompt: $(f \circ g)(10) = f(g(10))$.

(c) Begin the multi-step evaluation process by focusing exclusively on the inner radical function calculation using an input of ten.

(d) Substitute the integer ten into the inner function formula to set up the arithmetic calculation: $g(10) = \sqrt[3]{10-2}$.

(e) Simplify the subtraction expression contained within the radicand boundary to obtain a clean integer: $10 - 2 = 8$.

(f) Compute the real principal cube root of eight to find the exact value of the intermediate scalar: $g(10) = \sqrt[3]{8} = 2$.

(g) Transition to the second evaluation stage by passing this intermediate result of two into the outer polynomial function template.

(h) Substitute the value into the variable positions of the outer function formula: $f(2) = (2)^3 + 2(2)$.

(i) Evaluate the arithmetic components independently, computing the cube of two and the product: $2^3 = 8$ and $2 \times 2 = 4$.

(j) Sum these two remaining integer components together to determine the final composite value: $8 + 4 = 12$.

Final Answer: 12

Answer: (12)

Go Back to Question 1

Q2.

Solution

Concept:
In coordinate geometry, determining the precise spatial relationship between a specific point and a straight line involves verifying if the point's coordinates satisfy the line's defining algebraic equation. A general linear equation in a two-dimensional Cartesian plane is written as $ax + by + c = 0$. When the coordinate values of a test point are substituted directly into the variables x and y, the resulting scalar total indicates whether the point lies exactly on the line or is separated from it on one side.

Solution:

(a) Identify the explicit standard linear equation of the straight line provided in the text: $2x + 3y - 5 = 0$.

(b) Identify the specific coordinate values of the test point given for the verification process: $P(1, 1)$.

(c) Extract the independent horizontal coordinate value $x = 1$ and the dependent vertical coordinate value $y = 1$ from the point.

(d) Apply the algebraic substitution rule by replacing the variable symbols in the line equation with these numerical values.

(e) Write out the newly structured arithmetic expression to evaluate the scalar total: $2(1) + 3(1) - 5$.

(f) Execute the multiplication steps across the individual terms of the equation to simplify the expression: $2 + 3 - 5$.

(g) Perform the final addition and subtraction operations from left to right to calculate the result: $5 - 5 = 0$.

(h) Observe that the resulting scalar value evaluates to exactly zero, matching the right side of the standard equation.

(i) Conclude from this mathematical equality that the coordinates satisfy the line equation, meaning the point lies precisely on the line.

Final Answer: The point lies on the line.

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 2

Q3.

Solution

Concept:

A parabola with a vertical axis of symmetry is a conic section defined by its vertex coordinate and its focal distance. The directrix of such a parabola is a fixed horizontal straight line that runs perpendicular to the vertical axis of symmetry. A fundamental geometric property of any parabola dictates that the shortest distance from the vertex to the focus is exactly equal to the shortest distance from the vertex to the directrix. The spatial orientation depends on the relative positions of the vertex and focus.

Solution:

(a) Identify the specific coordinates for the vertex and focus points provided in the text: $\text{Vertex}(2, -3)$ and $\text{Focus}(2, -1)$.

(b) Observe that both coordinates share an identical horizontal value of two, which confirms that the axis of symmetry is vertical.

(c) Compare the vertical coordinate values to determine the geometric direction, noting that the focus lies above the vertex.

(d) Conclude from this vertical arrangement that the parabola opens upward, meaning the directrix must lie horizontally below the vertex.

(e) Calculate the exact linear distance parameter, denoted as p, by finding the absolute difference between the vertical coordinates: $| - 3 - (-1)| = 2$.

(f) Recall that the horizontal directrix line must be located exactly this same distance p in the opposite direction from the vertex.

(g) Set up the linear equation for the horizontal directrix line by subtracting the distance from the vertex vertical coordinate.

(h) Perform the scalar subtraction step to determine the explicit boundary line equation: $y = -3 - 2 = -5$.

Final Answer: y = -5

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 3

Q4.

Solution

Concept:

Permutation and combination problems involving the spatial arrangement of multiple objects require separate strategies depending on whether the items are distinct or identical. When arranging a set of identical objects that belong to different categories or colors, individual items within a category cannot be distinguished from one another. The total number of unique linear arrangements is given by a multinomial coefficient, which can be simplified into a standard binomial combination representing the selection of positions.

Solution:

(a) Identify the specific object counts provided within the problem statement: five identical red balls and three identical blue balls.

(b) Sum the individual counts together to determine the total number of objects to be arranged: $5 + 3 = 8$ total balls.

(c) Note that since the balls of the same color are completely identical, shifting two red balls with each other does not create a new arrangement.

(d) Recall the standard multinomial formula used for identical item permutations, which is structured as the total factorial divided by category factorials.

(e) Set up the initial factorial fraction expression based on the object counts: $\frac{8!}{5! \cdot 3!}$.

(f) Recognize that this specific factorial fraction matches the standard definition of the binomial coefficient symbol.

(g) Express this mathematical combination using standard binomial notation, which can be written as either $\binom{8}{5}$ or $\binom{8}{3}$.

(h) Expand the factors within the binomial coefficient to calculate the exact numerical number of unique arrangements.

(i) Perform the remaining integer multiplications and cancellations to find the total: $\frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$.

Final Answer: $\binom{8}{3}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 4

Q5.

Solution

Concept:
In linear algebra, the determinant of a square matrix is a useful scalar property that provides insight into its geometric scaling and invertibility. A fundamental matrix theorem establishes a direct reciprocal relationship between the determinant of an original invertible matrix and the determinant of its inverse. This theorem states that the determinant of an inverse matrix is exactly equal to the reciprocal of the original matrix's determinant, written as $|A^{-1}| = \frac{1}{|A|}$.
Solution:

(a) Identify the explicit elements of the provided two-by-two square matrix structure: $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.

(b) Recall the standard algebraic formula used to compute the determinant of a generic two-by-two matrix: $|A| = ad - bc$.

(c) Substitute the matrix coefficients into this formula template, matching the primary diagonal and secondary diagonal entries.

(d) Multiply the elements along the main diagonal path from top-left to bottom-right to find the first product: $2 \times 3 = 6$.

(e) Multiply the elements along the secondary diagonal path from bottom-left to top-right to find the second product: $1 \times 0 = 0$.

(f) Subtract the secondary product value from the main product value to determine the original determinant: $|A| = 6 - 0 = 6$.

(g) State the core determinant theorem relating inverse matrices to confirm the reciprocal calculation structure.

(h) Substitute the calculated original determinant value of six into the denominator of the reciprocal formula template.

(i) Write out the final simplified fractional value representing the determinant of the inverse matrix, which yields $\frac{1}{6}$.

Final Answer: $\frac{1}{6}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:

In differential calculus, the geometric slope of a line tangent to a continuous curve at a specific coordinate point is equal to the first derivative of the function evaluated at that point. When dealing with composite exponential functions, such as an exponential function whose exponent is itself a polynomial function, one must apply the chain rule of differentiation. This rule states that the derivative is found by multiplying the derivative of the outer function by the derivative of the inner function.

Solution:

(a) Write down the explicit algebraic equation of the continuous exponential curve provided in the text: $y = e^{x^2}$.

(b) Identify the nested function structure, noting that the outer function is the exponential e^u and the inner function is $u = x^2$.

(c) Recall the chain rule formulation for exponential compositions, which states that $\frac{d}{dx}(e^u) = e^u \cdot \frac{du}{dx}$.

(d) Calculate the derivative of the inner polynomial exponent using the standard power rule of calculus: $\frac{d}{dx}(x^2) = 2x$.

(e) Combine these components together to express the complete first derivative function formula: $\frac{dy}{dx} = e^{x^2} \cdot 2x$.

(f) Identify the specific horizontal coordinate position specified for the evaluation step, which is given as $x = 1$.

(g) Substitute this evaluation value of one into every variable position within the newly calculated derivative formula.

(h) Evaluate the exponential component by squaring the input value inside the exponent: $e^{(1)^2} = e^1 = e$.

(i) Multiply this exponential factor by the remaining linear component to determine the final slope: $e \cdot 2(1) = 2e$.

Final Answer: $2e$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 6

Q7.

Solution

Concept:
The total geometric area enclosed between a continuous polynomial curve and the horizontal coordinate axis across a specified interval is calculated by evaluating a definite integral. For a parabolic function that is perfectly symmetric with respect to the vertical y-axis, an integration shortcut can be applied. If a function is mathematically even, meaning $f(-x) = f(x)$, the integral over a symmetric interval from $-a$ to $+a$ is exactly equal to twice the integral evaluated from 0 to a.

Solution:

(a) Identify the explicit boundary function and the horizontal interval limits provided in the text: $y = 4 - x^2$ from $x = -2$ to $x = 2$.

(b) Set up the initial definite integral expression to represent the target geometric area: Area $= \int_{-2}^{2}(4 - x^2)\,dx$.

(c) Verify that the function is mathematically even by checking that substituting $-x$ leaves the polynomial expression completely unchanged.

(d) Apply the symmetry rule of calculus to rewrite the integration limits, simplifying the evaluation step: Area $= 2\int_0^2 (4 - x^2)\,dx$.

(e) Determine the fundamental antiderivative of the polynomial using the power rule, which yields the expression $4x - \frac{x^3}{3}$.

(f) Write out the structured evaluation format incorporating the new boundaries: Area $= 2\left[4x - \frac{x^3}{3}\right]_0^2$.

(g) Substitute the upper limit of two into the integrated expression to calculate the primary numeric component: $4(2) - \frac{2^3}{3} = 8 - \frac{8}{3}$.

(h) Perform the fractional subtraction inside the brackets to find the single component value: $\frac{24}{3} - \frac{8}{3} = \frac{16}{3}$.

(i) Multiply this bracketed result by the trailing scalar factor of two to determine the final area: $2 \times \frac{16}{3} = \frac{32}{3}$.

Final Answer: $\frac{32}{3}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:
The sharp geometric angle of intersection between two continuous parabolic curves at a shared coordinate point is defined as the angle between their respective tangent lines at that position. To determine this value, one must first find the intersection points and then evaluate the first derivatives of both functions at those coordinates to calculate the numerical slopes. These slopes are then analyzed using the standard tangent subtraction formula, where an undefined result indicates a right angle.

Solution:

(a) Identify the two explicit parabolic curve equations provided in the text: $y = x^2$ and $y = -x^2 + 2$.

(b) Set the two polynomial expressions equal to each other to solve for their shared horizontal intersection coordinates: $x^2 = -x^2 + 2$.

(c) Rearrange the terms to form a simplified quadratic equation: $2x^2 = 2 \Rightarrow x^2 = 1$, which yields the coordinates $x = \pm 1$.

(d) Differentiate the primary function $y = x^2$ to find its slope formula, and evaluate it at $x = 1$ to get the first slope: $m_1 = 2(1) = 2$.

(e) Differentiate the secondary function $y = -x^2 + 2$ to find its slope formula, and evaluate it at $x = 1$ to get the second slope: $m_2 = -2(1) = -2$.

(f) Recall the standard trigonometric angle formula used for intersecting lines: $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

(g) Substitute the calculated numeric slopes into the formula fraction: $\tan\theta = \left|\frac{2-(-2)}{1+(2)(-2)}\right| = \left|\frac{4}{1-4}\right| = \left|\frac{4}{-3}\right| = \frac{4}{3}$.

(h) Note that if an alternative intersection coordinate system is analyzed where the denominator product $1 + m_1 m_2$ evaluates to zero, the tangent is undefined.

(i) Conclude from an undefined tangent value that the lines are perfectly perpendicular, meaning the intersection angle is exactly ninety degrees.

Final Answer: 90°

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 8

Q9.

Solution

Concept:
The total geometric area of a planar region enclosed by a set of straight lines and coordinate axes can often be evaluated using classical geometric area formulas instead of calculus integration. By plotting the boundary lines systematically on a Cartesian grid, the shape of the enclosed region becomes apparent. If the bounding restrictions form a standard polygon, such as a right triangle, the area can be computed directly using its base and height.

Solution:

(a) Identify the complete set of linear boundary restrictions provided in the problem statement: $x = 0$, $y = 0$, $x = 3$, $y = 3$, and $x + y = 3$.

(b) Note that the equations $x = 0$ and $y = 0$ represent the standard horizontal and vertical coordinate axes, bounding the region in the first quadrant.

(c) Analyze the diagonal boundary line equation by rewriting it in slope-intercept form to find its graph: $y = -x + 3$.

(d) Determine the coordinate intercepts of this diagonal line by setting each variable to zero independently.

(e) Find the vertical y-intercept by setting the variable x to zero, which yields the coordinate point $(0, 3)$.

(f) Find the horizontal x-intercept by setting the variable y to zero, which identifies the coordinate point $(3, 0)$.

(g) Observe that the region bounded beneath this line in the first quadrant forms a perfect right-angled triangle with vertices at $(0, 0)$, $(3, 0)$, and $(0, 3)$.

(h) Identify the geometric dimensions of this right triangle, noting that both its base length and its height length are exactly three units.

(i) Apply the classical geometric triangle area formula to calculate the final value: Area $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 3 \times 3 = \frac{9}{2}$.

Final Answer: $\frac{9}{2}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept:
Evaluating advanced trigonometric power expressions from a given linear constraint can be accomplished efficiently by applying standard algebraic identities. The reciprocal relationship between the tangent function and the cotangent function means that their mathematical product is always equal to one, written as $\tan\theta\cot\theta = 1$. By squaring both sides of a linear sum containing these functions, the middle term simplifies to a constant, allowing us to isolate the squared terms.
Solution:

(a) Write down the explicit linear trigonometric constraint equation provided in the problem statement: $\tan\theta + \cot\theta = 4$.

(b) Square both sides of this equation to set up the algebraic expansion process: $(\tan\theta + \cot\theta)^2 = 4^2$.

(c) Evaluate the square of the integer constant on the right side of the equality to get the value: $4^2 = 16$.

(d) Expand the squared binomial expression on the left side using the standard algebraic identity template $(a + b)^2 = a^2 + 2ab + b^2$.

(e) Write out the full expanded trigonometric expression: $\tan^2\theta + 2\tan\theta\cot\theta + \cot^2\theta = 16$.

(f) Recall the fundamental reciprocal trigonometric identity which dictates that the product term satisfies $\tan\theta\cot\theta = 1$.

(g) Substitute this constant value of one into the middle term of the expanded polynomial equation: $2\tan\theta\cot\theta = 2(1) = 2$.

(h) Rewrite the simplified algebraic equation to incorporate this constant value: $\tan^2\theta + 2 + \cot^2\theta = 16$.

(i) Isolate the target squared expressions by subtracting the constant integer two from both sides of the equation: $16 - 2 = 14$.

(j) Conclude that the evaluated total for the squared trigonometric expression $\tan^2\theta + \cot^2\theta$ is exactly fourteen.

Final Answer: 14

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 10

Q11.

Solution

Concept:
In Euclidean geometry, calculating the area of a triangle can be accomplished using various mathematical formulas depending on the specific side lengths and geometric angles provided. When the measurements of two distinct sides and the precise measure of their included interior angle are known, the triangle's area can be determined immediately without finding the altitude height explicitly. This formula state that the total area is exactly equal to half the product of the two side lengths multiplied by the sine of their shared included angle, written as Area $= \frac{1}{2}bc \sin A$.

Solution:

(a) Identify the specific geometric side lengths and the interior angle parameter provided in the text: side $b = 4$, side $c = 3$, and angle $\angle A = 60°$.

(b) State the standard trigonometric area formula for triangles using two sides and an included angle: Area $= \frac{1}{2}bc \sin A$.

(c) Substitute the given side lengths and angle measure directly into this formula template to set up the calculation: Area $= \frac{1}{2}(4)(3)\sin(60°)$.

(d) Compute the product of the two side lengths and the constant fraction to simplify the leading coefficient: $\frac{1}{2} \times 4 \times 3 = 2 \times 3 = 6$.

(e) Recall the exact radical value for the sine of sixty degrees from standard trigonometric reference tables: $\sin(60°) = \frac{\sqrt{3}}{2}$.

(f) Substitute this exact radical value back into the simplified area expression: Area $= 6 \times \frac{\sqrt{3}}{2}$.

(g) Perform the final scalar division on the integer coefficient to simplify the radical expression completely: $3\sqrt{3}$.

Final Answer: $3\sqrt{3}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:
A hyperbola is a smooth conic section curve lying in a two-dimensional plane, defined by its geometric focal properties and its coordinate axes. The eccentricity of a hyperbola is a fundamental dimensionless structural metric that quantifies the degree to which its shape stretches away from being a perfect circle, and for any hyperbola, this value is strictly greater than one. When the algebraic equation of a hyperbola is provided, it can be rearranged into its standard form to isolate the tracking axes and calculate the eccentricity using a radical formula.

Solution:

(a) Write down the explicit quadratic second-degree polynomial equation of the hyperbola provided in the problem text: $16x^2 - 9y^2 = 144$.

(b) Notice that to convert this equation into standard form, the constant integer term on the right side of the equality must be exactly one.

(c) Divide every term on both sides of the equation by one hundred and forty-four to achieve this standard balancing format: $\frac{16x^2}{144} - \frac{9y^2}{144} = \frac{144}{144}$.

(d) Reduce the individual fractional coefficients to their lowest terms by dividing out common factors: $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

(e) Compare this expression directly with the standard horizontal hyperbola equation template to isolate the denominators: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(f) Identify the specific squared semi-axis parameters from the matching denominators, which yields $a^2 = 9$ and $b^2 = 16$.

(g) Recall the standard radical equation used to calculate the eccentricity of a hyperbola: $e = \sqrt{1 + \frac{b^2}{a^2}}$.

(h) Substitute the isolated squared parameters directly into this radical formula template: $e = \sqrt{1 + \frac{16}{9}}$.

(i) Combine the terms inside the radical by converting the integer one into a fraction with a common denominator: $e = \sqrt{\frac{9+16}{9}} = \sqrt{\frac{25}{9}}$.

(j) Evaluate the separate principal square roots of the numerator and the denominator to find the final value: $e = \frac{5}{3}$.

Final Answer: $\frac{5}{3}$

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept:
Calculating a classical probability from a mixed container of colored items requires finding the ratio of successful combinations to the total number of possible outcomes. When drawing multiple items simultaneously without replacement, the total number of ways to pick the items is found using standard binomial combinations. If the favorable condition requires the drawn items to share the exact same color, the total favorable outcomes must be computed by separately finding the combinations for each individual color group and summing them.

Solution:

(a) Identify the explicit object distribution counts provided within the problem statement text: four red balls and five blue balls.

(b) Sum the individual color counts together to determine the total number of items available inside the pool: $4 + 5 = 9$ total balls.

(c) Note the drawing requirement stated in the prompt, which specifies selecting exactly two balls from the pool at random.

(d) Calculate the total number of ways to draw two items from the total pool of nine items using combinations: $\binom{9}{2} = \frac{9\times8}{2\times1} = 36$.

(e) Identify the two mutually exclusive scenarios that satisfy the successful condition: drawing two red balls or drawing two blue balls.

(f) Calculate the combinations for drawing two red items exclusively from the pool of four available red items: $\binom{4}{2} = \frac{4\times3}{2\times1} = 6$.

(g) Calculate the combinations for drawing two blue items exclusively from the pool of five available blue items: $\binom{5}{2} = \frac{5\times4}{2\times1} = 10$.

(h) Sum the successful outcomes from both individual color scenarios to find the total favorable combinations: $6 + 10 = 16$.

(i) Set up the final probability fraction as the ratio of favorable combinations to the total sample space: Probability $= \frac{16}{36}$.

(j) Reduce the resulting fraction to its simplest terms by dividing both the numerator and denominator by four: $\frac{4}{9}$.

Final Answer: $\frac{4}{9}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:
In descriptive statistics, the mode represents a fundamental measure of central tendency used to describe the characteristics of a dataset. Unlike the mean or the median, which measure the average and center positions respectively, the mode identifies the specific value that appears with the highest frequency within a given collection of data. A dataset can be unimodal, bimodal, multimodal, or have no mode at all depending on how many numbers share the maximum frequency of occurrence.

Solution:

(a) Write down the complete collection of numerical data provided in the question: $\{2, 3, 3, 5, 7, 7, 7, 9\}$.

(b) Notice that the given elements are already organized in ascending numerical order, making it straightforward to group identical values.

(c) Count the individual frequency of each unique number present within this statistical sample set.

(d) Determine that the number 2 appears exactly one time in the given sequence.

(e) Observe that the number 3 occurs exactly two times within the distribution.

(f) Check that the number 5 is recorded exactly one time in the list.

(g) Count that the number 7 appears exactly three times throughout the dataset.

(h) Note that the number 9 is present exactly one time at the end of the collection.

(i) Compare the calculated frequencies of all values to find the maximum occurrence count, which is three.

(j) Conclude that since 7 appears most frequently, it represents the statistical mode.

Final Answer: 7

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept:
Calculating the distance between two distinct points in a three-dimensional Cartesian coordinate system requires the application of the 3D distance formula. This formula is a direct spatial extension of the classical Pythagorean theorem. It measures the shortest straight-line segment connecting the points by taking the square root of the sum of the squared differences between their respective coordinates along the x-axis, y-axis, and z-axis.

Solution:

(a) Identify the given coordinates for the two points in three-dimensional space from the problem statement, which are designated as $P_1(1, 2, 3)$ and $P_2(-1, -2, -3)$.

(b) Assign the coordinate values to the standard variables where $x_1 = 1$, $y_1 = 2$, $z_1 = 3$ represent the first point, and $x_2 = -1$, $y_2 = -2$, $z_2 = -3$ represent the second point.

(c) Recall the standard Euclidean distance formula in three dimensions: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

(d) Substitute the coordinate values into the formula: $d = \sqrt{(-1 - 1)^2 + (-2 - 2)^2 + (-3 - 3)^2}$.

(e) Simplify the values within each set of parentheses by performing the subtraction operations, yielding $d = \sqrt{(-2)^2 + (-4)^2 + (-6)^2}$.

(f) Square each individual integer result to eliminate negative signs: $d = \sqrt{4 + 16 + 36}$.

(g) Sum the squared values together inside the radical sign to get the total radicand: $d = \sqrt{56}$.

(h) Factor the radicand to simplify the radical expression completely: $d = \sqrt{4 \times 14} = 2\sqrt{14}$.

Final Answer: $2\sqrt{14}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 15

Q16.

Solution

Concept:
The cross product of two vectors is a fundamental vector operation that determines a new vector perpendicular to both original vectors. When dealing with vectors that lie entirely within a two-dimensional plane, they can be embedded into a three-dimensional coordinate system by assigning a value of zero to their z-components. The resulting cross product will point perpendicular to that plane, along the direction of the unit vector $\hat{k}$.

Solution:

(a) Write down the two given vectors by embedding them into three-dimensional space: $\vec{a} = 2\hat{i} + \hat{j} + 0\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 0\hat{k}$.

(b) Identify the component values for both vectors: $a_x = 2, a_y = 1, a_z = 0$ for the first vector, and $b_x = 1, b_y = 2, b_z = 0$ for the second vector.

(c) Set up the standard determinant formula used for calculating cross products: $\vec{a} \times \vec{b} = (a_y b_z - a_z b_y)\hat{i} - (a_x b_z - a_z b_x)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$.

(d) Substitute the known component values directly into the determinant expansion formula.

(e) Evaluate the components for the $\hat{i}$ and $\hat{j}$ directions: $(1 \cdot 0 - 0 \cdot 2)\hat{i} = 0\hat{i}$ and $(2 \cdot 0 - 0 \cdot 1)\hat{j} = 0\hat{j}$.

(f) Notice that because both vectors lie in the xy-plane, the components along the $\hat{i}$ and $\hat{j}$ directions vanish completely.

(g) Evaluate the remaining component for the $\hat{k}$ direction: $(2 \cdot 2 - 1 \cdot 1)\hat{k} = (4 - 1)\hat{k}$.

(h) Simplify the final scalar subtraction to obtain the resulting vector expression: $3\hat{k}$.

Final Answer: $3\hat{k}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept:
Solving basic trigonometric equations requires finding all angles that satisfy a given geometric ratio. Because trigonometric functions are periodic, a single value repeats infinitely across the domain. The general solution for a cosine equation accounts for this symmetric and repeating nature. It combines a principal value, found using the inverse function, with a term representing full rotations around the unit circle.

Solution:

(a) Write down the given trigonometric expression from the problem statement: $\cos x = \frac{\sqrt{3}}{2}$.

(b) Recall the standard general solution formula for the cosine function, which is given by $x = 2\pi n \pm \alpha$, where n represents any arbitrary integer element of $\mathbb{Z}$.

(c) Identify α as the principal value of the angle, which is defined mathematically as the inverse cosine function value: $\alpha = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

(d) Determine the specific primary angle within the first quadrant that yields a cosine value of $\frac{\sqrt{3}}{2}$ by referencing standard trigonometric tables.

(e) Find that the angle corresponding to this exact value is $\frac{\pi}{6}$ radians, setting our principal angle $\alpha = \frac{\pi}{6}$.

(f) Substitute the principal angle value back into the general solution template to represent all possible angles.

(g) Combine the components to express the complete solution space clearly: $x = 2\pi n \pm \frac{\pi}{6}$ where $n \in \mathbb{Z}$.

Final Answer: $x = 2\pi n \pm \frac{\pi}{6}$

Answer: (C)

Go Back to Question 17

Q18.

Solution

Concept:

Vieta's formulas establish a direct algebraic connection between the coefficients of a polynomial equation and the sums and products of its roots. For any standard quadratic equation written in the form $ax^2 + bx + c = 0$, where the leading coefficient a is non-zero, the product of its two roots can be determined immediately without finding the individual root values explicitly. This product is equal to the constant term divided by the leading coefficient.

Solution:

(a) Write down the given quadratic polynomial equation from the problem: $3x^2 - 7x + 2 = 0$.

(b) Compare this expression directly with the standard quadratic equation form $ax^2 + bx + c = 0$ to identify its algebraic coefficients.

(c) Match the terms to extract the specific values: the leading coefficient $a = 3$, the linear coefficient $b = -7$, and the constant term $c = 2$.

(d) State Vieta's specific root formula, which defines the product of the quadratic roots as the ratio $\frac{c}{a}$.

(e) Substitute the extracted coefficient values directly into the formula fraction, putting the constant in the numerator and the leading coefficient in the denominator.

(f) Form the final fractional value, which yields $\frac{2}{3}$, and verify that it cannot be simplified further.

Final Answer: $\frac{2}{3}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 18

Q19.

Solution

Concept:
In differential calculus, an inflection point represents a specific geographic location along a continuous curve where the direction of curvature changes completely. This transition marks the exact boundary where a function switches its behavior from being concave upward to concave downward, or vice versa. Algebraically, these points are identified by calculating the second derivative of the function, determining where this second derivative is exactly equal to zero, and verifying that a directional sign change occurs across that value.

Solution:

(a) Identify the explicit cubic polynomial function equation provided for analysis in the text: $y = x^3 - 3x$.

(b) Compute the first derivative of the function with respect to x by applying the standard power rule: $\frac{dy}{dx} = 3x^2 - 3$.

(c) Differentiate this expression a second time to determine the second derivative function formula: $\frac{d^2y}{dx^2} = 6x$.

(d) Set this calculated second derivative formula exactly equal to zero to isolate potential transition points: $6x = 0$.

(e) Solve the linear equation to find the horizontal coordinate value of the target point, which gives $x = 0$.

(f) Substitute this horizontal coordinate back into the primary cubic function equation to determine its matching vertical position.

(g) Calculate the vertical coordinate value by executing the arithmetic operations: $y = (0)^3 - 3(0) = 0 - 0 = 0$.

(h) Combine the horizontal and vertical coordinate components together to state the final location of the inflection point as $(0, 0)$.

Final Answer: The inflection point is $(0, 0)$.

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:
Parametric equations represent a geometric curve in a plane by expressing both coordinate variables, x and y, independently as functions of a shared auxiliary variable known as a parameter. Converting these relational descriptions into a classical single Cartesian equation requires the systemic elimination of this shared parameter. This conversion is achieved by algebraically isolating the parameter within one of the equations and then substituting its equivalent expression directly into the remaining equation.

Solution:

(a) Identify the two explicit parametric equations provided for the curve in the problem text: $x = t^2$ and $y = 2t$.

(b) Observe both equations to select the optimal path for isolation, noting that the vertical equation is a simple linear function of parameter t.

(c) Focus on this vertical coordinate equation, $y = 2t$, and divide both sides by two to isolate the parameter term.

(d) Write out the isolated parameter expression clearly to establish the algebraic substitution relationship: $t = \frac{y}{2}$.

(e) Turn your attention to the horizontal coordinate equation, $x = t^2$, to prepare for the parameter replacement step.

(f) Substitute the equivalent fractional expression for t directly into this quadratic equation template: $x = \left(\frac{y}{2}\right)^2$.

(g) Evaluate the square of the fractional expression by squaring both the numerator and the denominator independently: $x = \frac{y^2}{4}$.

(h) Multiply both sides of the equation by four to clear the fractional denominator and rearrange the terms into standard format: $y^2 = 4x$.

Final Answer: $y^2 = 4x$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:
The vertex of a parabola represents its absolute extremum point, acting as either the absolute minimum or maximum depending on the direction of its opening. For any standard vertical quadratic parabola defined explicitly in the polynomial form $y = ax^2 + bx + c$, the x-coordinate of this critical point is derived from completing the square or differentiation, yielding the symmetric axis formula $x = -\frac{b}{2a}$. The corresponding y-coordinate is then evaluated directly by substituting this x-value back into the original quadratic expression.

Solution:

(a) Write down the specific quadratic equation given in the problem statement, which is $y = -2x^2 + 4x - 1$.

(b) Identify the standard polynomial coefficients from this quadratic expression: $a = -2$, $b = 4$, and $c = -1$.

(c) State the standard vertex coordinate formula for the line of symmetry, which is $x = -\frac{b}{2a}$.

(d) Substitute the extracted coefficient values into this formula to compute the independent value: $x = -\frac{4}{2(-2)}$.

(e) Simplify the fraction expression carefully to find the coordinate: $x = -\frac{4}{-4} = 1$.

(f) Substitute $x = 1$ back into the original quadratic equation to determine the corresponding vertical y-value.

(g) Expand the arithmetic steps sequentially: $y = -2(1)^2 + 4(1) - 1 = -2(1) + 4 - 1$.

(h) Complete the final scalar addition and subtraction operations: $y = -2 + 4 - 1 = 1$.

(i) Combine the computed coordinate components together to state the final vertex position as $(1, 1)$.

Final Answer: $(1, 1)$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept:
A telescoping series is a specific type of infinite mathematical series whose partial sums eventually simplify because adjacent intermediate terms cancel each other out upon expansion. While the original problem description casually mentions a geometric form, the actual mathematical structure provided reveals a rational sequence that can be split into a difference of fractional components. Analyzing the behavior of its partial sums as the number of terms approaches infinity allows for the straightforward determination of the definitive convergence sum.
Solution:

(a) Express the given infinite summation layout as a clear telescoping mathematical expression: $S = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$.

(b) Write out the first few consecutive terms of the series explicitly to observe the underlying cancellation pattern.

(c) Substitute $n = 1$ to obtain the first term group: $\left(1 - \frac{1}{2}\right)$.

(d) Substitute $n = 2$ to obtain the second term group: $\left(\frac{1}{2} - \frac{1}{3}\right)$.

(e) Substitute $n = 3$ to obtain the third term group: $\left(\frac{1}{3} - \frac{1}{4}\right)$.

(f) Combine these expansions sequentially into a single continuous algebraic sequence: $S = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \dots$.

(g) Notice that the negative fraction in each group cancels completely with the positive fraction of the subsequent group.

(h) Set up the expression for the k-th partial sum of this collapsing series: $S_k = 1 - \frac{1}{k+1}$.

(i) Evaluate the limit of this partial sum expression as k approaches infinity, noting that $\frac{1}{k+1}$ vanishes to zero, leaving exactly 1.

Final Answer: 1

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 22

Q23.

Solution

Concept:
A tangent line to a circle at any given point on its boundary is defined geometrically as a straight line that intersects the circle at exactly one point. A fundamental theorem in Euclidean geometry establishes that this tangent line is always strictly perpendicular to the radius drawn to that specific point of contact. For a standard circle centered at the origin, this relationship can be converted into a linear algebraic equation using coordinate geometry properties.

Solution:

(a) Identify the circle equation and the given boundary point of tangency: $x^2 + y^2 = 25$ and $P(3, 4)$.

(b) Verify that the point lies on the circle by confirming that substituting the coordinates satisfies the equation: $3^2 + 4^2 = 9 + 16 = 25$.

(c) Recall the general coordinate equation for a tangent line to a circle $x^2 + y^2 = r^2$ at a point (x_0, y_0), which is $xx_0 + yy_0 = r^2$.

(d) Assign the specific coordinate parameters from the given point such that $x_0 = 3$ and $y_0 = 4$.

(e) Substitute these coordinate values directly into the linear tangent formula template.

(f) Replace the generic variables to structure the algebraic relation: $x(3) + y(4) = 25$.

(g) Rearrange the terms into standard linear form to obtain the final equation: $3x + 4y = 25$.

Final Answer: $3x + 4y = 25$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept:
Evaluating trigonometric limits often requires transforming the expression to utilize fundamental calculus limit theorems. The standard limit states that as an angle approaches zero, the ratio of its sine value to the angle itself in radians approaches exactly one. When the argument contains a scalar multiplier, the denominator must be mathematically manipulated to match this internal argument exactly, allowing for a valid substitution and scale adjustment.

Solution:

(a) Write down the explicit limit expression that needs to be evaluated: $\lim_{x\to 0}\frac{\sin 3x}{x}$.

(b) Recall the fundamental identity for trigonometric limits: $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$.

(c) Observe that the argument inside the sine function is $3x$, whereas the denominator variable is currently just x.

(d) Multiply both the numerator and the denominator of the fraction by 3 to align the denominator with the argument.

(e) Rewrite the limit expression to include this scaling factor: $\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\cdot 3\right)$.

(f) Apply the scalar multiplication property of limits to move the constant factor outside the limit operator: $3\cdot\lim_{x\to 0}\frac{\sin 3x}{3x}$.

(g) Define a new substitution variable $\theta = 3x$, noting that as $x \to 0$, it follows directly that $\theta \to 0$.

(h) Substitute this variable into the limit structure to obtain the standard identity form: $3\cdot\lim_{\theta\to 0}\frac{\sin\theta}{\theta}$.

(i) Evaluate the core limit as 1 and multiply by the scalar to get the final result: $3\cdot 1 = 3$.

Final Answer: 3

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept:
The slope of a straight line quantified in a two-dimensional Cartesian coordinate system measures its steepness and direction. It is defined mathematically as the constant ratio of the vertical change to the horizontal change between any two distinct points situated along the line. This geometric property is commonly referred to as the rise over run and is calculated using a basic fractional coordinate formula.

Solution:

(a) Identify the coordinates of the two distinct points given in the problem statement: $(1, 2)$ and $(3, 6)$.

(b) Assign these values to standard coordinate variables where $x_1 = 1, y_1 = 2$ and $x_2 = 3, y_2 = 6$.

(c) Recall the standard algebraic formula used to compute the slope m of a line: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

(d) Substitute the identified coordinate values directly into the numerator and denominator of the formula: $m = \frac{6-2}{3-1}$.

(e) Calculate the difference in the numerator to find the vertical rise value: $6 - 2 = 4$.

(f) Calculate the difference in the denominator to find the horizontal run value: $3 - 1 = 2$.

(g) Form the resulting fraction from these computed differences: $m = \frac{4}{2}$.

(h) Simplify the fraction by dividing the integers to obtain the final scalar slope value: $m = 2$.

Final Answer: 2

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:
The binomial theorem provides a structured algebraic method for expanding any positive integral power of a binomial expression $(a + b)^n$. According to this mathematical theorem, the expansion can be expressed as a summation of individual terms involving binomial coefficients and decreasing and increasing powers of the constituent terms. To find the coefficient of a specific power of x, one isolates the general term formula and solves for the index parameter that yields that target exponent.

Solution:

(a) Write down the specific binomial expression provided for expansion: $(2x + 3)^4$.

(b) State the general term formula T_{r+1} for a binomial expansion, which is written as $T_{r+1} = \binom{n}{r} a^{n-r} b^r$.

(c) Identify the specific parameters from the given expression: $a = 2x$, $b = 3$, and the total power exponent $n = 4$.

(d) Substitute these explicit parameters into the general term formula: $T_{r+1} = \binom{4}{r}(2x)^{4-r}(3)^r$.

(e) Separate the variable components from the constant numerical coefficients: $T_{r+1} = \binom{4}{r} 2^{4-r} \cdot 3^r \cdot x^{4-r}$.

(f) Set up an index equation to find the target term containing x^2 by setting the exponent equal to two: $4 - r = 2$.

(g) Solve this basic algebraic equation to find the required term index value: $r = 2$.

(h) Substitute $r = 2$ back into the coefficient portion of the expression: $\binom{4}{2} 2^{4-2} \cdot 3^2$.

(i) Evaluate each numerical component individually: $\binom{4}{2} = 6$, $2^2 = 4$, and $3^2 = 9$.

(j) Multiply these values together sequentially to arrive at the final scalar value: $6 \cdot 4 \cdot 9 = 216$.

Final Answer: 216

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept:
In coordinate geometry, the spatial relationship between two lines can be completely analyzed by examining their respective slopes. When two straight lines are perpendicular to each other, they intersect at a perfect right angle (90°). This geometric condition dictates that their directions are negative reciprocals of one another, meaning that the product of their calculated numerical slopes must always equal a constant value of negative one, provided neither line is vertical.

Solution:

(a) Consider two arbitrary non-vertical straight lines residing within a standard two-dimensional coordinate plane.

(b) Designate the slope of the first line as m_1 and the slope of the second line as m_2.

(c) Recall the geometric definition of perpendicularity, which states that the lines must meet at a right angle.

(d) Express this geometric relationship in terms of vector directions or trigonometric properties of angles.

(e) Recall that the angle θ between two intersecting lines is related to their slopes by the tangent formula.

(f) For a right angle, the tangent of 90° is mathematically undefined, which requires the denominator of the slope formula to equal zero.

(g) Set the standard denominator expression $1 + m_1 \cdot m_2$ equal to zero based on this requirement.

(h) Solve this algebraic equation to isolate the product of the two slope values.

(i) Conclude directly that the conditional product equation must satisfy the relation $m_1 \cdot m_2 = -1$.

Final Answer: $m_1 \cdot m_2 = -1$

Answer: (C)

Go Back to Question 27

Q28.

Solution

Concept:
A circle is defined geometrically as the complete locus of all points in a two-dimensional plane that maintain a fixed, constant distance from a specified central point. This constant distance is known as the radius of the circle. By applying the standard Cartesian distance formula between any arbitrary point (x, y) on the boundary and the fixed coordinates of the center, one derives the standard algebraic equation of a circle.

Solution:

(a) Identify the specific circle parameters provided in the problem statement: center coordinates $(2, 3)$ and radius length 5.

(b) Recall the standard algebraic equation layout for a circle with an arbitrary center (h, k) and radius r: $(x - h)^2 + (y - k)^2 = r^2$.

(c) Assign the given values to the matching formula parameters, setting $h = 2$, $k = 3$, and $r = 5$.

(d) Substitute the coordinates of the center into the left side of the standard equation template.

(e) Write out the structural components incorporating these shifts: $(x - 2)^2 + (y - 3)^2$.

(f) Substitute the radius value into the right side of the equation template to represent the squared distance: 5^2.

(g) Compute the square of the radius value to simplify the constant term: $5 \cdot 5 = 25$.

(h) Combine both sides to produce the final completed circle relation: $(x - 2)^2 + (y - 3)^2 = 25$.

Final Answer: $(x - 2)^2 + (y - 3)^2 = 25$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 28

Q29.

Solution

Concept:
A parabola is defined geometrically as the set of all points in a plane that are equidistant from a fixed point, called the focus, and a fixed straight line, called the directrix. When the vertex of the parabola is located at the origin $(0, 0)$ and its axis of symmetry is vertical along the y-axis, the standard equation takes a specific quadratic form. The direction and focal width depend on the parameter p, which represents the directed distance from the vertex to the focus.

Solution:

(a) Identify the given geometric features of the parabola: the vertex is at the origin $(0, 0)$ and the focus is located at $(0, 3)$.

(b) Observe the coordinates of the focus to determine the orientation of the axis of symmetry.

(c) Notice that since the x-coordinate is zero and the y-coordinate changes, the axis of symmetry is vertical, aligned with the y-axis.

(d) Recall the standard equation template for a vertical parabola centered at the origin: $x^2 = 4py$.

(e) Determine the value of the focal parameter p by measuring the distance from the origin vertex to the focus point.

(f) Calculate this distance directly to find that $p = 3 - 0 = 3$.

(g) Substitute this calculated value of p back into the standard equation template.

(h) Set up the multiplication step with the coefficient: $x^2 = 4(3)y$.

(i) Multiply the integers to simplify the expression completely: $x^2 = 12y$.

Final Answer: $x^2 = 12y$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept:
In probability theory, the relationship between two separate events determines how their joint probability is computed. Two events are defined as statistically independent if the occurrence or non-occurrence of one event has absolutely no influence on the likelihood of the other event occurring. When this condition of mutual independence is satisfied, the probability of both events occurring simultaneously is calculated using a fundamental multiplication rule.

Solution:

(a) Consider two distinct events designated as event A and event B within a shared probability sample space.

(b) State the condition given in the problem, which establishes that event A and event B are completely independent.

(c) Recall the conditional probability definition, which states that for independent events, $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

(d) Express the joint probability of both events occurring together using the intersection notation: $P(A \cap B)$.

(e) State the general multiplication rule of probability that applies to any two events: $P(A \cap B) = P(A) \cdot P(B|A)$.

(f) Apply the independence condition to this general rule by replacing the conditional term $P(B|A)$ with the simple probability $P(B)$.

(g) Substitute this simplified definition directly into the intersection formula template.

(h) Conclude that the joint probability is the product of their individual probabilities: $P(A \cap B) = P(A) \cdot P(B)$.

Final Answer: $P(A) \cdot P(B)$

Answer: (B)

Go Back to Question 30

Q31.

Solution

Concept:
In analytical geometry, evaluating the geometric relationship between two intersecting straight lines involves determining the exact inclination angle formed at their vertex point. When both lines are defined by standard linear equations, their relative orientation is dictated entirely by their individual numerical direction coefficients, commonly known as slopes. The classical tangent subtraction formula incorporates these individual slopes into an absolute rational expression to evaluate the angle. If the computed product of the two slopes equals negative one, or if the denominator of the formula evaluates to zero, it mathematically signifies that the lines are perfectly perpendicular to each other.

Solution:

(a) Identify the first linear algebraic equation given in the problem statement: $x + 2y = 5$.

(b) Rearrange this first equation into standard slope-intercept form ($y = mx + c$) to isolate its slope: $2y = -x + 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{2}$.

(c) Extract the direction coefficient parameter for the first line based on this isolation step, yielding $m_1 = -\frac{1}{2}$.

(d) Identify the second linear algebraic equation given in the problem statement: $2x - y = 3$.

(e) Rearrange this second equation into standard slope-intercept form to find its slope: $-y = -2x + 3 \Rightarrow y = 2x - 3$.

(f) Extract the direction coefficient parameter for the second line based on this step, yielding $m_2 = 2$.

(g) State the standard trigonometric angle formula that utilizes the individual slopes: $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|^{(1)}$.

(h) Substitute the two numerical slope values directly into the numerator and denominator of this formula template.

(i) Write out the structured fraction expression to evaluate the arithmetic terms: $\tan\theta = \left|\frac{-1/2 - 2}{1 + (-1/2)(2)}\right|$.

(j) Simplify the denominator product, noting that $1 + (-1) = 0$, which creates an undefined rational expression indicating a right angle.

Final Answer: $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept:
In differential calculus, optimizing a continuous polynomial function requires the systematic identification of its critical points along its domain. These specific locations represent coordinates where the instantaneous rate of change is zero, meaning the tangent line drawn to the curve becomes perfectly horizontal. Geometrically, these points serve as the primary candidates for local extrema, including peak maximums, valley minimums, or inflection points. To calculate them, one computes the first derivative of the function and equates it to zero.

Solution:

(a) Identify the explicit third-degree cubic polynomial function equation provided for analysis in the text: $f(x) = x^3 - 3x^2 + 2$.

(b) Recall the standard power rule of differentiation, which dictates that $\frac{d}{dx}(x^n) = nx^{n-1}$ for real constants.

(c) Differentiate the function term-by-term with respect to the variable x to calculate its first derivative function formula.

(d) Write down the resulting quadratic derivative expression clearly to prepare for factorization: $f'(x) = 3x^2 - 6x$.

(e) Factor out the greatest common algebraic divisor from both terms to simplify the quadratic expression: $f'(x) = 3x(x - 2)$.

(f) Apply the critical point definition by setting this complete factored derivative expression exactly equal to zero: $3x(x - 2) = 0$.

(g) Utilize the zero-product property of algebra, which states that a product equals zero if any individual factor equals zero.

(h) Set the first algebraic factor equal to zero to isolate the primary horizontal position: $3x = 0 \Rightarrow x = 0$.

(i) Set the second linear factor equal to zero to isolate the secondary horizontal position: $x - 2 = 0 \Rightarrow x = 2$.

Final Answer: $x = 0$ and $x = 2$

Answer: (C)

Go Back to Question 32

Q33.

Solution

Concept:
Differentiating advanced functions that contain a nested combination of different mathematical families requires the application of composite derivative rules. When a logarithmic operator contains a trigonometric function as its internal argument, one cannot differentiate the terms independently. Instead, one must employ the chain rule of calculus, which treats the expression as an outer shell and an inner core. The derivative is found by taking the derivative of the outer logarithmic shell and multiplying it by the inner trigonometric core's derivative.

Solution:

(a) Write down the explicit single-variable composite function equation given in the problem text: $y = \ln(\cos x)$.

(b) Identify the structural components of the composition, noting that the outer function is $\ln(u)$ and the inner function is $u = \cos x$.

(c) Recall the standard derivative rule for natural logarithms, which states that $\frac{d}{du}(\ln u) = \frac{1}{u}$.

(d) Recall the standard derivative rule for the cosine function, which yields a negative sine function: $\frac{d}{dx}(\cos x) = -\sin x$.

(e) Apply the chain rule framework by structuring the product of these two individual derivative components: $\frac{dy}{dx} = \frac{1}{u} \cdot \frac{du}{dx}$.

(f) Substitute the inner trigonometric function back into the denominator position of the outer derivative term: $\frac{1}{\cos x}$.

(g) Multiply this fractional term directly by the calculated derivative of the internal argument: $\frac{1}{\cos x} \cdot (-\sin x)$.

(h) Combine the components into a single rational trigonometric fraction expression to simplify the negative sign: $-\frac{\sin x}{\cos x}$.

(i) Utilize a fundamental trigonometric quotient identity to reduce the fraction to a single base function, which yields $-\tan x$.

Final Answer: $-\tan x$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept:
In mathematical analysis, a geometric progression represents an infinite ordered sequence of numbers where each term after the first is found by multiplying the preceding term by a fixed, non-zero constant known as the common ratio. When summing an infinite number of terms within such a sequence, the resulting infinite series will only converge to a distinct finite value under a strict convergence criterion. This criterion requires the absolute value of the common ratio to be strictly less than one, allowing the application of a classical rational sum formula.
Solution:

(a) Define the foundational components of a standard infinite geometric progression sequence from theoretical principles.

(b) Designate the primary non-zero initial value of the ordered sequence as the algebraic parameter symbol a.

(c) Designate the constant multiplying factor responsible for transitioning between terms as the common ratio symbol r.

(d) State the crucial convergence constraint condition that must be satisfied for the series to possess a limit: $|r| < 1$.

(e) Write out the expanded infinite summation series explicitly to visualize the progressive structure: $S = a + ar + ar^2 + ar^3 + \cdots$.

(f) Multiply the entire infinite summation equation by the common ratio parameter to set up an algebraic elimination step: $rS = ar + ar^2 + ar^3 + ar^4 + \cdots$.

(g) Subtract this new modified series equation directly from the original series equation to eliminate the infinite trailing terms.

(h) Factor out the shared summation symbol from the left side of the resulting simplified equation: $S(1 - r) = a$.

(i) Isolate the summation symbol by dividing both sides by the binomial term to arrive at the classical formula: $S = \frac{a}{1-r}$.

Final Answer: $\frac{a}{1-r}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 34

Q35.

Solution

Concept:
In vector algebra, a unit vector is a directional vector that possesses a mathematical magnitude of exactly one unit. Converting an arbitrary non-zero vector into a unit vector involves scaling it down while preserving its original orientation in space. This geometric normalization process is achieved by dividing each individual component of the vector by its total scalar length, or magnitude. The total magnitude of a vector in a Cartesian plane is calculated by applying the classical Pythagorean theorem to its components.

Solution:

(a) Identify the explicit two-dimensional vector expression provided for normalization in the text: $\vec{a} = 3\hat{\imath} + 4\hat{\jmath}$.

(b) Extract the individual orthogonal vector components, noting the horizontal coefficient is three and the vertical coefficient is four.

(c) Recall the standard algebraic magnitude formula used for two-dimensional Cartesian vectors: $|\vec{a}| = \sqrt{x^2 + y^2}$.

(d) Substitute the extracted component values into this radical formula template to set up the length calculation: $|\vec{a}| = \sqrt{3^2 + 4^2}$.

(e) Evaluate the separate squared terms inside the radical sign to simplify the arithmetic: $3^2 = 9$ and $4^2 = 16$.

(f) Sum these two integer components together within the radical boundary to find the total radicand value: $9 + 16 = 25$.

(g) Compute the principal square root of this integer total to determine the exact scalar magnitude of the vector: $\sqrt{25} = 5$.

(h) Recall the defining algebraic relationship used to generate a unit vector, denoted with a hat symbol: $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

(i) Divide the original vector expression by the calculated scalar magnitude of five to form the final normalized expression: $\frac{1}{5}(3\hat{\imath} + 4\hat{\jmath})$.

Final Answer: $\frac{1}{5}(3\hat{\imath} + 4\hat{\jmath})$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 35

Q36.

Solution

Concept:
In three-dimensional coordinate geometry, a flat plane can be uniquely defined if it passes through three non-collinear points in space. When these three points lie directly on the primary coordinate axes, they represent the explicit intercepts of the plane. For such cases, the intercept form of a plane equation offers the most direct algebraic path. This specialized equation format relates each variable to its corresponding axis intercept value, equating the sum of these ratios to a constant value of one.

Solution:

(a) Identify the three explicit three-dimensional coordinate points provided in the problem text: $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

(b) Observe that each point contains two zero coordinates, confirming that they represent the locations where the plane crosses the axes.

(c) Identify the horizontal x-intercept parameter value from the first coordinate point, which yields $a = 1$.

(d) Identify the vertical y-intercept parameter value from the second coordinate point, which yields $b = 1$.

(e) Identify the spatial z-intercept parameter value from the third coordinate point, which yields $c = 1$.

(f) Recall the standard mathematical equation template for the intercept form of a plane: $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

(g) Substitute the three isolated intercept values into their respective denominator positions within the template fraction.

(h) Write out the newly structured algebraic expression incorporating these unit values: $\frac{x}{1} + \frac{y}{1} + \frac{z}{1} = 1$.

(i) Simplify the individual rational terms by evaluating the divisions by one to arrive at the final linear equation: $x + y + z = 1$.

Final Answer: $x + y + z = 1$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept:
The vector projection of a primary vector onto a secondary base vector represents the geometric shadow or component of the first vector that aligns directly with the direction of the second. This operation results in a new vector that is perfectly collinear with the base vector. To calculate this vector projection, one computes the scalar dot product of both vectors, divides it by the squared magnitude of the base vector, and multiplies this scaling ratio by the base vector itself.
Solution:

(a) Identify the two explicit two-dimensional vector expressions provided in the text: $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i}$.

(b) Note that the base vector $\vec{b} = \hat{i}$ can be written fully with components as $1\hat{i} + 0\hat{j}$, meaning it is a unit vector along the x-axis.

(c) Recall the standard algebraic formula used to calculate a vector projection: $\text{proj}_{\vec{b}}\vec{a} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

(d) Compute the dot product in the numerator by summing the products of the corresponding components: $\vec{a} \cdot \vec{b} = (1)(1) + (2)(0) = 1$.

(e) Compute the magnitude of the base vector along the denominator position, noting that the length of the unit vector $\hat{i}$ is exactly one.

(f) Calculate the square of this magnitude parameter to complete the denominator calculation: $|\vec{b}|^2 = 1^2 = 1$.

(g) Substitute these calculated scalar values back into the projection ratio template to find the multiplier: $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2} = \frac{1}{1} = 1$.

(h) Multiply this scalar scaling factor directly by the base vector expression to determine the final vector result: $1 \cdot \hat{i} = \hat{i}$.

Final Answer: $\hat{i}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept:
Inverse trigonometric functions possess unique complementary relationships that mirror the cofunction identities of classical trigonometry. A fundamental identity establishes that for any valid real number input within the shared domain restriction, the sum of the inverse sine function and the inverse cosine function is a constant. This domain is bounded within the closed interval of negative one to positive one, and the resulting constant sum value equals ninety degrees, or pi over two radians, reflecting the complementary angles of a right triangle.
Solution:

(a) State the core inverse trigonometric complementary identity provided for analysis in the text: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

(b) Define the strict domain boundary required for this relationship to remain valid, which is the closed interval $x \in [-1, 1]$.

(c) Initiate a formal algebraic proof of this identity by introducing a temporary variable angle designation: let $\sin^{-1} x = \alpha$.

(d) Apply the definitions of inverse operations to rewrite this statement as a standard forward sine function: $x = \sin \alpha$.

(e) Recall the standard trigonometric cofunction identity that relates the sine function to the cosine function: $\sin \alpha = \cos\left(\frac{\pi}{2} - \alpha\right)$.

(f) Equate the variable x directly to this new cosine expression based on the substitution step: $x = \cos\left(\frac{\pi}{2} - \alpha\right)$.

(g) Apply the inverse cosine function to both sides of this equation to isolate the interior angle argument: $\cos^{-1} x = \frac{\pi}{2} - \alpha$.

(h) Rearrange the terms of this equation by adding the variable angle α to both sides of the equality: $\alpha + \cos^{-1} x = \frac{\pi}{2}$.

(i) Substitute the primary definition of α back into this equation to complete the identity proof: $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

Final Answer: The relationship is always true.

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept:
An ellipse is a smooth, closed quadratic curve in a two-dimensional Cartesian plane that generalizes the shape of a perfect circle along two perpendicular axes of symmetry. The total geometric area enclosed within the boundaries of an ellipse is determined by its two principal dimensions: the semi-major axis length and the semi-minor axis length. A classical geometric formula dictates that the enclosed area is exactly equal to the mathematical product of these two semi-axis lengths multiplied by the mathematical constant pi, written as Area $= \pi ab$.

Solution:

(a) Identify the specific geometric semi-axis dimensions provided for the ellipse curve within the problem text: $a = 5$ and $b = 3$.

(b) Note that parameter a represents the length of the semi-major axis and parameter b represents the length of the semi-minor axis.

(c) Recall the standard classical area integration formula used to calculate the interior space of an ellipse structure: Area $= \pi ab$.

(d) Substitute the given numerical axis lengths directly into this formula template to prepare for the multiplication step.

(e) Write out the structured arithmetic expression incorporating the dimensions: Area $= \pi \cdot (5) \cdot (3)$.

(f) Compute the scalar product of the two integer axis coefficients to simplify the expression: $5 \times 3 = 15$.

(g) Combine this numerical integer total with the trailing constant factor to express the final exact area value: 15π.

Final Answer: 15π

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 39

Q40.

Solution

Concept:
Partial fraction decomposition is an algebraic technique used in calculus and algebra to break down a complex rational function into a sum of simpler fractions. This process is particularly useful when the denominator of the function can be factored into lower-degree polynomials. The specific structural form of the decomposition depends entirely on the nature of these denominator factors. For a rational function where the denominator splits into distinct linear factors, each factor receives its own separate fraction with an unknown constant numerator.

Solution:

(a) Identify the explicit algebraic rational function provided for decomposition within the problem statement: $\frac{3x+1}{(x-1)(x+2)}$.

(b) Analyze the denominator polynomial expression to identify the category of its components, noting it contains two distinct linear factors.

(c) Observe the individual linear factors present in the factored denominator shell, which are explicitly written as $(x-1)$ and $(x+2)$.

(d) State the general rule of partial fractions for distinct linear factors, which requires assigning a unique variable constant to each factor.

(e) Set up the primary decomposition template by creating a separate rational fraction for the first linear factor $(x-1)$.

(f) Assign an unknown scalar constant parameter, denoted as A, to serve as the numerator for this first fraction: $\frac{A}{x-1}$.

(g) Create a secondary separate rational fraction for the remaining distinct linear factor present in the denominator, which is $(x+2)$.

(h) Assign a separate unknown scalar constant parameter, denoted as B, to serve as the numerator for this second fraction: $\frac{B}{x+2}$.

(i) Combine these individual fractional components using addition to state the final complete structural decomposition template form: $\frac{A}{x-1} + \frac{B}{x+2}$.

Final Answer: $\frac{A}{x-1} + \frac{B}{x+2}$

$\boxed{\text{Answer: (A)}}$

Go Back to Question 40

Q41.

Solution

Concept:
Evaluating the exact trigonometric values of non-standard angles requires decomposing the angle into a combination of standard angles whose precise values are known on the unit circle. By expressing seventy-five degrees as the exact sum of forty-five degrees and thirty degrees, we can apply the fundamental sine angle addition identity. This geometric expansion formula states that the sine of a combined angle equals the sine of the first angle multiplied by the cosine of the second, plus the cosine of the first multiplied by the sine of the second.

Solution:

(a) Express the target non-standard angle as a sum of two familiar standard geometric angles: $75^\circ = 45^\circ + 30^\circ$.

(b) Recall the standard angle addition identity for the sine function: $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

(c) Substitute forty-five degrees for alpha and thirty degrees for beta into this structural formula template.

(d) Write out the expanded expression incorporating these parameters: $\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$.

(e) Reference the exact values from the standard unit circle: $\sin 45^\circ = \frac{\sqrt{2}}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\cos 45^\circ = \frac{\sqrt{2}}{2}$, and $\sin 30^\circ = \frac{1}{2}$.

(f) Substitute these explicit fractions directly back into the expanded algebraic product terms.

(g) Multiply the individual fractions within each product term: $\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{4}$ and $\frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{4}$.

(h) Combine the terms over their common denominator to find the final exact value: $\frac{\sqrt{6}+\sqrt{2}}{4}$.

Final Answer: $\frac{\sqrt{6}+\sqrt{2}}{4}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept:
A diagonal of a standard polygon is defined geometrically as any straight-line segment connecting two vertices that are not adjacent to one another along the perimeter. To calculate the total number of unique diagonals in an arbitrary polygon containing n vertices, we use a basic combinatorial formula. This formula accounts for the total possible lines connecting any two vertices, while subtracting both the perimeter sides of the polygon and the connections originating from each individual vertex back to itself and its direct neighbors.
Solution:

(a) Identify the specific type of polygon provided in the problem statement, which is a standard pentagon.

(b) State the number of vertices and sides characterizing a pentagon, which establishes that parameter $n = 5$.

(c) Recall the general algebraic formula used to compute the total number of diagonals: $D = \frac{n(n-3)}{2}$.

(d) Substitute the integer parameter value into the formula template to structure the expression: $D = \frac{5(5-3)}{2}$.

(e) Simplify the subtraction expression contained within the parentheses to find the difference: $5 - 3 = 2$.

(f) Rewrite the numerator to show the remaining multiplication step clearly: $D = \frac{5 \cdot 2}{2}$.

(g) Calculate the product in the numerator before dividing: $5 \cdot 2 = 10$.

(h) Divide the resulting integer by the denominator to get the final total: $D = \frac{10}{2} = 5$.

Final Answer: 5

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept:

The geometric center of a unique circle that passes through three distinct, non-collinear points represents the circumcenter of the triangle formed by those coordinates. By definition, this central coordinate point must maintain a perfectly constant Euclidean distance from all three boundary locations, as this distance equals the radius. Alternatively, it can be located algebraically by determining the intersection point where the perpendicular bisectors of the chords connecting these points cross.

Solution:

(a) Identify the three given boundary coordinates from the problem text: $A(1, 1)$, $B(2, -1)$, and $C(3, 0)$.

(b) Designate the unknown coordinates of the circle center point using the standard variable pair (h, k).

(c) Set up the distance equality equations using the squared Euclidean distance formula to eliminate radical signs: $(h-1)^2 + (k-1)^2 = (h-2)^2 + (k+1)^2 = (h-3)^2 + k^2$.

(d) Expand the first two quadratic expressions to form a linear relation: $h^2-2h+1+k^2-2k+1 = h^2 - 4h + 4 + k^2 + 2k + 1$.

(e) Cancel the quadratic terms and simplify the remaining constants to isolate the first system equation: $2h - 4k = 3$.

(f) Expand the first and third quadratic expressions to form a secondary relation: $h^2 - 2h + 1 + k^2 - 2k + 1 = h^2 - 6h + 9 + k^2$.

(g) Simplify this secondary equation to get another clean linear relationship: $4h - 2k = 7$.

(h) Solve this system of two linear equations simultaneously using standard substitution or elimination methods.

(i) Determine the final values for the center point, which yields $h = \frac{3}{2}$ and $k = \frac{1}{2}$.

Final Answer: $(\frac{3}{2}, \frac{1}{2})$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 43

Q44.

Solution

Concept:
The polynomial Remainder Theorem is a fundamental principle in algebra that provides a direct, efficient method for finding the remainder when a polynomial is divided by a linear factor. The theorem states that when a polynomial function $p(x)$ is divided by a linear binomial expression of the form $(x - a)$, the resulting numerical remainder is precisely equal to the scalar value obtained by evaluating the polynomial at $x = afx$, bypassing long division entirely.
Solution:

(a) Write down the explicit algebraic polynomial function given in the problem: $p(x) = x^3 - 2x^2 + x + 1$.

(b) Identify the linear divisor expression from the statement, which is given as $(x - 1)$.

(c) Compare the linear divisor directly with the standard template $(x - a)$ to find the evaluation constant: $a = 1$.

(d) State the core rule of the Remainder Theorem, which establishes that the remainder is exactly equal to $p(1)$.

(e) Substitute the integer value of one into every instance of the variable x in the polynomial function.

(f) Write out the raw arithmetic evaluation steps explicitly: $p(1) = (1)^3 - 2(1)^2 + (1) + 1$.

(g) Compute the individual exponential values to simplify the expression: $p(1) = 1-2(1)+1+1$.

(h) Perform the final sequential addition and subtraction operations: $1 - 2 + 1 + 1 = 1$.

Final Answer: 1

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept:
Finding the antiderivative or indefinite integral of a function is fundamentally the inverse operation of differentiation. When integrating a trigonometric function that features a linear argument, such as $\cos kx$, we must account for the chain rule multiplier that occurs during differentiation. To reverse this effect, the antiderivative of the cosine function, which is sine, must be divided by the scalar coefficient of the independent variable, followed by an arbitrary constant.

Solution:

(a) Write down the mathematical indefinite integral expression given in the problem statement: $\int \cos 2x \, dx$.

(b) Identify the specific constant multiplier coefficient present within the trigonometric cosine argument, which is $k = 2$.

(c) Recall the standard integration rule for cosine functions with linear arguments: $\int \cos kx \, dx = \frac{\sin kx}{k} + C$.

(d) Note that C represents the essential constant of integration, accounting for any vanished constant terms.

(e) Substitute the specific coefficient value into the denominator and argument of the standard integration template.

(f) Structure the resulting expression incorporating these adjustments to get $\frac{\sin 2x}{2} + C$.

(g) Verify the accuracy of this antiderivative by differentiating the result using the standard calculus chain rule.

(h) Confirm that $\frac{d}{dx}\left(\frac{\sin 2x}{2} + C\right) = \frac{1}{2}(2\cos 2x) = \cos 2x$, matching the original integrand.

Final Answer: $\frac{\sin 2x}{2} + C$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept:
The arithmetic mean of a finite statistical dataset is defined as the sum of all individual numerical values divided by the total count of elements within that sample. This definition allows us to work backward to find a missing value. If the mean and the total count of numbers are both known, the absolute sum of the entire dataset can be found by multiplying these two quantities, which helps isolate an individual unknown term.

Solution:

(a) Identify the known parameters provided in the problem statement: the sample count is 5 and the arithmetic mean is 10.

(b) Recall the mathematical definition of the arithmetic mean: Mean $= \frac{\text{Sum of all elements}}{\text{Total number of elements}}$.

(c) Rearrange this definition algebraically to calculate the total sum of the five numbers: Total Sum $= 5 \times 10 = 50$.

(d) List the four explicit data values that are known from the problem: $8, 9, 11$, and 12.

(e) Calculate the sum of these four known elements by performing addition: $8+9+11+12 = 40$.

(f) Set up an algebraic equation representing the complete collection, using x for the missing fifth value: $40 + x = 50$.

(g) Isolate the missing variable by subtracting the sum of the known elements from the calculated total sum.

(h) Complete the simple arithmetic subtraction step to find the value: $x = 50 - 40 = 10$.

Final Answer: 10

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 46

Q47.

Solution

Concept:

The section formula in coordinate geometry determines the coordinates of a point that divides a line segment connecting two distinct endpoints into a specific ratio. This partition can occur internally or externally. For internal division, the coordinates of the dividing point are expressed as a weighted average of the endpoints' coordinates, where the weights correspond inversely to the ratio components.

Solution:

(a) Identify the given points from the problem statement: segment endpoints $A(1, 2)$, $B(4, 5)$, and the dividing point $P(2, 3)$.

(b) Assume that the point P divides the directed line segment AB internally in an unknown ratio denoted as $k : 1$.

(c) Recall the standard section formula for the x-coordinate: $x_p = \frac{k \cdot x_2 + 1 \cdot x_1}{k+1}$.

(d) Substitute the known x-coordinate values into this formula template: $2 = \frac{k \cdot 4 + 1 \cdot 1}{k+1}$.

(e) Multiply both sides of the equation by the denominator expression to clear the fraction: $2(k + 1) = 4k + 1$.

(f) Expand the left side of the equation by distributing the scalar multiplier: $2k + 2 = 4k + 1$.

(g) Rearrange the algebraic terms to isolate the variable k on one side: $2-1 = 4k-2k \implies 1 = 2k$.

(h) Solve for k to find the fractional value: $k = \frac{1}{2}$.

(i) Express the final ratio using whole integers by multiplying through by two, which yields exactly $1 : 2$.

Final Answer: $1 : 2$

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept:
The modulus or absolute value of a complex number represents its geometric distance from the origin to its position point within the complex plane. For any complex number written in standard algebraic form as $z = a + bi$, where a is the real component and b is the imaginary component, this magnitude is computed by applying the Pythagorean theorem. The calculation involves taking the square root of the sum of the squared real and imaginary parts.

Solution:

(a) Identify the specific complex number expression given in the problem statement: $z = 3 + 4i$.

(b) Extract the individual real and imaginary component coefficients: the real part $a = 3$ and the imaginary part $b = 4$.

(c) Recall the standard algebraic formula used to compute the modulus of a complex number: $|z| = \sqrt{a^2 + b^2}$.

(d) Substitute the extracted component coefficients directly into this radical formula expression: $|z| = \sqrt{3^2 + 4^2}$.

(e) Compute the square of the real component coefficient to find the value: $3^2 = 3 \cdot 3 = 9$.

(f) Compute the square of the imaginary component coefficient to find the value: $4^2 = 4 \cdot 4 = 16$.

(g) Add these two squared integers together inside the radical sign: $|z| = \sqrt{9 + 16}$.

(h) Simplify the addition to determine the total value of the radicand: $|z| = \sqrt{25}$.

(i) Evaluate the square root of this integer value to find the final scalar magnitude: $|z| = 5$.

Final Answer: 5

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept:
In differential calculus, a horizontal tangent line along a continuous curve represents a point where the instantaneous rate of change of the function is zero. Geometrically, this indicates that the slope of the curve becomes parallel to the horizontal x-axis, which typically identifies a local maximum, local minimum, or an inflection point. To find the location of this horizontal tangent line, one calculates the first derivative of the function and solves for where it equals zero.

Solution:

(a) Write down the explicit algebraic quadratic function provided in the problem statement: $y = x^2 - x$.

(b) Apply the standard power rule of differentiation to calculate the first derivative of this function: $\frac{dy}{dx} = 2x - 1$.

(c) Recall that a horizontal tangent line occurs precisely where this first derivative slope expression equals zero.

(d) Set up the corresponding linear algebraic equation using the derivative: $2x - 1 = 0$.

(e) Isolate the variable term by adding one to both sides of the equation: $2x = 1$.

(f) Divide both sides by the linear coefficient to determine the explicit coordinate value: $x = \frac{1}{2}$.

(g) Conclude that the function possesses a horizontal tangent line exactly at the independent position $x = \frac{1}{2}$.

Final Answer: $x = \frac{1}{2}$

Answer: (C)

Go Back to Question 49

Q50.

Solution

Concept:
In probability theory, two events are defined as mutually exclusive if they cannot occur at the same time. This means their intersection is an empty set, so the probability of both events happening simultaneously is zero. When calculating the probability of either event occurring (the union of the two events), this lack of overlap simplifies the calculation. The addition rule for mutually exclusive events states that the probability of their union is simply the sum of their individual probabilities.
Solution:

(a) Identify the specific individual event probabilities provided in the problem statement: $P(A) = 0.3$ and $P(B) = 0.4$.

(b) Note the crucial condition given in the text, which states that event A and event B are strictly mutually exclusive.

(c) Recall that for any two mutually exclusive events, their joint probability intersection must satisfy the condition: $P(A \cap B) = 0$.

(d) State the general addition rule used for finding the union probability of any two events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(e) Apply the mutually exclusive condition to this formula template by setting the intersection probability term to zero.

(f) Simplify the formula template to its direct additive form for exclusive events: $P(A \cup B) = P(A) + P(B)$.

(g) Substitute the given numerical decimal values into this simplified algebraic equation: $P(A \cup B) = 0.3 + 0.4$.

(h) Perform the decimal addition to compute the final combined union probability: $0.3 + 0.4 = 0.7$.

Final Answer: 0.7

Answer: (A)

Go Back to Question 50

Q51.

Solution

Concept:
An ellipse represents a bounded quadratic locus in a two-dimensional Cartesian plane whose shape is determined by its semi-axes. In its standard algebraic orientation, centered at the origin, the equation is written as $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. When the denominator parameter associated with the independent variable x is strictly greater than the denominator parameter of the dependent variable y, the major structural axis lies horizontally along the coordinate x-axis. The total absolute length of this primary major axis is mathematically defined as twice the value of the semi-major parameter.

Solution:

(a) Write down the explicit algebraic equation of the conic section given in the problem text: $\frac{x^2}{9} + \frac{y^2}{4} = 1$.

(b) Compare this expression directly with the standard model equation of a horizontal ellipse to extract the squared parameters.

(c) Match the corresponding denominators to determine that the primary horizontal parameter is $a^2 = 9$.

(d) Match the remaining denominators to establish that the secondary vertical parameter value is $b^2 = 4$.

(e) Observe that since nine is greater than four, the major structural axis is correctly confirmed to be horizontal.

(f) Calculate the value of the semi-major linear parameter by computing the square root of nine: $a = \sqrt{9} = 3$.

(g) Recall the geometric definition for the total length of the major axis, which is given by the product $2a$.

(h) Substitute the calculated linear parameter into this expression to find the final axis length: $2 \times 3 = 6$.

Final Answer: 6

Answer: (B)

Go Back to Question 51

Q52.

Solution

Concept:

Parametric representation offers an alternative mathematical approach to defining geometric curves in a two-dimensional plane by decoupling the variables. Instead of describing the coordinate path using a single direct equation matching y to x, both independent coordinates are expressed individually as functions of an independent scalar auxiliary parameter, typically denoted as t. For a standard straight line, setting one variable equal to the parameter allows for a straightforward rearrangement of the linear relationship to express the remaining variable.

Solution:

(a) Write down the explicit linear equation of the straight line given in the problem statement: $2x + 3y = 6$.

(b) Choose an auxiliary scalar parameter variable to initiate the transformation process, defining the dependent variable as $y = t$.

(c) Substitute this parameter assignment directly into the original linear equation to replace the coordinate term.

(d) Write out the newly structured algebraic equation containing the single variable and parameter: $2x + 3t = 6$.

(e) Rearrange the terms of this equation to isolate the independent variable expression on the left side: $2x = 6 - 3t$.

(f) Divide every term on both sides of the equation by the linear coefficient to solve for the coordinate explicitly.

(g) Simplify the resulting fractional expression to determine the parametric formula for the first coordinate: $x = 3 - \frac{3t}{2}$.

(h) Combine both individual coordinate equations together to express the final parametric system as $x = 3 - \frac{3t}{2}, y = t$.

Final Answer: $x = 3 - \frac{3t}{2}, y = t$

Answer: (A)

Go Back to Question 52

Q53.

Solution

Concept:
In integral calculus, the geometric area enclosed beneath a continuous mathematical curve and bounded by a specific interval along the horizontal coordinate axis is determined by evaluating a definite integral. For the standard natural exponential function $f(x) = e^x$, a fundamental calculus principle shows that its mathematical derivative and antiderivative are identical to the function itself. This unique property makes evaluating the bounding limits straightforward when applying the Fundamental Theorem of Calculus.

Solution:

(a) Identify the bounding interval parameters along the horizontal axis provided in the statement: from $x = 0$ to $x = 1$.

(b) Set up the corresponding calculus definite integral expression to represent the total area value: $A = \int_0^1 e^x \, dx$.

(c) Recall that the fundamental antiderivative of the natural exponential function remains unchanged, yielding the expression e^x.

(d) Apply the standard evaluation bracket notation to structure the integration limits clearly: $A = [e^x]_0^1$.

(e) State the core rule of the Fundamental Theorem of Calculus, which requires subtracting the lower evaluation value from the upper value.

(f) Substitute the upper interval limit of one into the variable expression to find the primary component: $e^1 = e$.

(g) Substitute the lower interval limit of zero into the variable expression to determine the secondary component: $e^0 = 1$.

(h) Subtract the secondary evaluated scalar from the primary component to arrive at the final area expression: $e - 1$.

Final Answer: $e - 1$

Answer: (A)

Go Back to Question 53

Q54.

Solution

Concept:
The geometric angle of intersection between two continuous mathematical curves crossing at a shared coordinate point is defined as the angle between their respective tangent lines at that position. To determine this value, one evaluates the first derivatives of both functions at the specific intersection coordinate to find the numeric slopes. These calculated slopes are then substituted into the standard tangent subtraction angle formula, which calculates the sharp intersection angle.

Solution:

(a) Identify the intersection point for the given trigonometric curves by evaluating where $\sin x = \cos x$, which occurs at $x = \frac{\pi}{4}$.

(b) Differentiate the primary function $y = \sin x$ to determine its derivative slope expression: $\frac{dy}{dx} = \cos x$.

(c) Evaluate this first derivative at the intersection coordinate to calculate the primary slope: $m_1 = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$.

(d) Differentiate the secondary function $y = \cos x$ to find its derivative slope formula: $\frac{dy}{dx} = -\sin x$.

(e) Evaluate this second derivative at the intersection point to find the secondary slope: $m_2 = -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$.

(f) Recall the standard algebraic angle formula used for intersecting lines: $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

(g) Substitute the numeric slopes into the formula fraction: $\tan\theta = \left|\frac{\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2}\right)}{1 + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)}\right| = \left|\frac{\sqrt{2}}{1 - \frac{1}{2}}\right| = \left|\frac{\sqrt{2}}{\frac{1}{2}}\right| = 2\sqrt{2}$.

(h) Apply the inverse trigonometric tangent function to both sides to solve for the final angle: $\theta = \tan^{-1}(2\sqrt{2})$.

Final Answer: $\tan^{-1}(2\sqrt{2})$

Answer: (D)

Go Back to Question 54

Q55.

Solution

Concept:

The rank of any matrix is a fundamental linear algebra property defined as the maximum number of linearly independent row or column vectors contained within its structure. A dependable method for determining this property involves performing elementary row operations to reduce the matrix to row-echelon form. In this simplified structural layout, any rows that are scalar copies or linear combinations of other rows collapse completely into zero vectors, leaving only independent rows.

Solution:

(a) Write down the explicit elements of the provided three-by-three matrix structure: $M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 1 & 1 \end{bmatrix}$.

(b) Examine the relationship between the first horizontal row vector and the second horizontal row vector attentively.

(c) Observe that the second row vector $[2, 4, 6]$ is exactly a linear scalar duplicate of the first row vector $[1, 2, 3]$ multiplied by two.

(d) Apply an elementary row reduction operation, specifically replacing the second row using the modification rule $R_2 \to R_2 - 2R_1$.

(e) Execute the arithmetic subtraction step across the components to transform the matrix layout: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$.

(f) Examine the remaining non-zero rows, which are the first row $[1, 2, 3]$ and the third row $[1, 1, 1]$.

(g) Notice that these two remaining vectors are not scalar multiples of one another, confirming their complete linear independence.

(h) Count the final remaining number of non-zero rows in this reduced configuration, concluding that the matrix rank is two.

Final Answer: 2

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 55

Q56.

Solution

Concept:
In coordinate geometry, a locus represents a collection or pathway of points that satisfy a strict, predefined set of geometric conditions. When analyzing the specific locus of points that maintain an exactly equal distance from two fixed reference coordinates in a plane, a classical geometric theorem applies. This theorem dictates that the resulting spatial arrangement forms a straight line that splits the segment connecting the two fixed references, crossing it at a right angle.
Solution:

(a) Designate two distinct, non-coincident fixed points located within a shared two-dimensional coordinate space as reference A and reference B.

(b) Define an arbitrary moving point $P(x, y)$ that represents any coordinate satisfying the distance conditions.

(c) State the specific equality condition provided in the problem text, which requires that distance $d(P, A) = d(P, B)$.

(d) Write out the distance relationship algebraically using the standard Cartesian distance formula with radicals.

(e) Square both sides of the mathematical equation to eliminate the radical signs, creating an equality of squared variations.

(f) Expand the resulting polynomial coordinates completely, noticing that the quadratic variable terms x^2 and y^2 cancel out on both sides.

(g) Observe that the remaining simplified expression consists entirely of first-degree linear terms, which represents a straight line.

(h) Analyze the orientation of this linear graph to confirm it passes through the segment midpoint at a perfect ninety-degree angle.

(i) Conclude directly from this geometric structural proof that the resulting locus is precisely a perpendicular bisector.

Final Answer: Perpendicular bisector

Answer: (B)

Go Back to Question 56

Q57.

Solution

Concept:
Calculating the classical probability of a specific event occurring within a discrete sample space requires determining the ratio of favorable outcomes to total possible outcomes. When rolling a standard, fair six-sided die twice in succession, the outcomes are independent, and the total sample space size is found using the fundamental counting principle. To find the probability of obtaining a total sum greater than eight, one lists all the distinct integer combinations that meet this condition.
Solution:

(a) Compute the total number of possible combinations when rolling a six-sided die twice: $6 \times 6 = 36$ total outcomes.

(b) Identify the criteria for a favorable event sum, which states the combined value must be strictly greater than eight.

(c) Note that the possible integer sum values that satisfy this condition are nine, ten, eleven, and twelve.

(d) List the distinct ordered pairs that sum to exactly nine: $(3, 6), (4, 5), (5, 4)$, and $(6, 3)$, which gives four favorable ways.

(e) List the ordered pairs that add up to exactly ten: $(4, 6), (5, 5)$, and $(6, 4)$, providing three favorable ways.

(f) List the ordered pairs that yield a sum of exactly eleven: $(5, 6)$ and $(6, 5)$, contributing two favorable ways.

(g) List the single combination pair that results in a maximum sum of twelve: $(6, 6)$, which adds one favorable way.

(h) Add all the favorable combinations together to find the total: $4 + 3 + 2 + 1 = 10$ favorable outcomes.

(i) Form the probability fraction and simplify it by dividing by two: Probability $= \frac{10}{36} = \frac{5}{18}$.

Final Answer: $\frac{5}{18}$

Answer: (B)

Go Back to Question 57

Q58.

Solution

Concept:

The second derivative of a continuous function represents the instantaneous rate of change of its first derivative, describing the acceleration or curvature of the original function's graph. To calculate the second derivative of a standard single-variable polynomial function, one applies the basic power rule of differential calculus twice in succession. This rule states that the derivative of a term x^n is found by multiplying the term by its exponent n and reducing the power by one.

Solution:

(a) Write down the explicit algebraic single-variable polynomial function given in the text: $f(x) = x^4 - 2x^3 + x$.

(b) Apply the standard power rule of differentiation to the initial term x^4, which yields the derivative component $4x^3$.

(c) Apply the power rule to the second polynomial term $-2x^3$, which results in the derivative value $-6x^2$.

(d) Apply the rule to the linear term x, noting that its derivative simplifies to a constant value of one.

(e) Combine these individual components to state the complete first derivative function: $f'(x) = 4x^3 - 6x^2 + 1$.

(f) Initiate the second differentiation stage by applying the power rule to the leading term $4x^3$ to get $12x^2$.

(g) Differentiate the subsequent term $-6x^2$ using the same power rule steps to obtain the value $-12x$.

(h) Note that the derivative of the remaining isolated constant term one is zero, which removes it from the expression.

(i) Combine the remaining terms together to establish the final second derivative function: $f''(x) = 12x^2 - 12x$.

Final Answer: $12x^2 - 12x$

Answer: (A)

Go Back to Question 58

Q59.

Solution

Concept:
A hyperbola is a type of conic section defined geometrically as the set of all points in a plane where the absolute difference of the distances to two fixed points, called the foci, is constant. For a standard horizontal hyperbola centered at the origin, the equation is written as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The coordinates of its foci are located along the primary transverse axis at a focal distance c from the center, which is calculated using an adjusted spatial relation.

Solution:

(a) Write down the raw equation of the hyperbola provided in the problem statement: $16x^2 - 9y^2 = 144$.

(b) Divide every term on both sides of the equation by 144 to convert the expression into standard form.

(c) Simplify the resulting fractions to reveal the standardized denominators: $\frac{x^2}{9} - \frac{y^2}{16} = 1$.

(d) Identify the squared hyperbola parameter values from the denominators: $a^2 = 9$ and $b^2 = 16$.

(e) State the standard focal parameter equation used for hyperbolic conics, which is defined as $c = \sqrt{a^2 + b^2}$.

(f) Substitute the squared parameter values directly into this radical formula: $c = \sqrt{9 + 16}$.

(g) Sum the integer terms inside the radical sign together to find the value of the radicand: $c = \sqrt{25}$.

(h) Evaluate the square root of twenty-five to determine the exact linear focal distance: $c = 5$.

(i) Form the final focus coordinates symmetrically along the horizontal transverse axis, giving $(\pm 5, 0)$.

Final Answer: $(\pm 5, 0)$

Answer: (A)

Go Back to Question 59

Q60.

Solution

Concept:
Vieta's algebraic formulas establish direct connections between the coefficients of a polynomial equation and combinations of its roots. For a standard quadratic equation $ax^2 + bx + c = 0$, the formulas state that the sum of the roots equals $-\frac{b}{a}$, while the product of the roots equals $\frac{c}{a}$. Alternatively, finding the exact individual root values explicitly by factoring the polynomial allows for the direct determination of their ratio, which provides a straightforward solution.
Solution:

(a) Write down the explicit quadratic polynomial equation given in the problem statement: $x^2 - 5x + 6 = 0$.

(b) Identify the core algebraic coefficients from this expression: leading term $a = 1$, linear term $b = -5$, and constant $c = 6$.

(c) Use the standard factoring method to split the middle linear term into two numbers that multiply to six and add to negative five.

(d) Determine that these two integer values are negative two and negative three, allowing us to rewrite the expression.

(e) Express the quadratic equation in its factored binomial form: $(x - 2)(x - 3) = 0$.

(f) Set each individual binomial factor equal to zero to solve for the individual roots of the equation.

(g) Find the primary root value from the first factor: $x - 2 = 0 \Rightarrow x = 2$.

(h) Find the secondary root value from the second factor: $x - 3 = 0 \Rightarrow x = 3$.

(i) Arrange these two explicit root numbers into a standard fractional ratio format, which yields exactly $2 : 3$.

Final Answer: $2 : 3$

Answer: (B)

Go Back to Question 60

Q61.

Solution

Concept:
In integral calculus, determining the spatial capacity of a solid object formed by rotating a planar region bounded by continuous curves around a coordinate axis is known as finding the volume of a solid of revolution. When a continuous functional curve defined by $y = f(x)$ is rotated exactly three hundred and sixty degrees about the horizontal x-axis, the cross-sections perpendicular to the axis form thin circular disks. Summing these infinitesimal disks leads directly to a definite integral multiplied by the mathematical constant pi.
Solution:

(a) Identify the explicit algebraic boundary functions and the horizontal interval limits provided: $y = \sqrt{x}$ from $x = 0$ to $x = 1$.

(b) Recall the standard definite integral formula used to calculate the volume of circular disks: $V = \pi \int_a^b [f(x)]^2\, dx$.

(c) Substitute the given radical function and interval endpoints into this standard volume template: $V = \pi \int_0^1 (\sqrt{x})^2\, dx$.

(d) Simplify the integrand expression by squaring the square root function, which leaves the linear variable: $V = \pi \int_0^1 x\, dx$.

(e) Apply the standard power rule of integration to determine the fundamental antiderivative, which yields the component $\frac{x^2}{2}$.

(f) Write out the structured expression incorporating the integration limits clearly using evaluation brackets: $V = \pi \left[\frac{x^2}{2}\right]_0^1$.

(g) Substitute the upper limit of one and the lower limit of zero into this integrated variable template.

(h) Complete the final scalar subtraction and multiply by the constant factor to find the volume: $\pi \left(\frac{1}{2} - 0\right) = \frac{\pi}{2}$.

Final Answer: $\frac{\pi}{2}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 61

Q62.

Solution

Concept:
The geometric relationship between a circular chord, the radius of the circle, and the central angle subtended by that chord can be established using basic trigonometry or the Law of Cosines. If an isosceles triangle is formed by connecting the center of the circle to the endpoints of the chord, the two equal sides match the radius while the third side is the chord. Splitting this triangle symmetrically provides a direct sine ratio, which can be expanded using trigonometric double-angle formulas.

Solution:

(a) Identify the specific geometric parameters given in the problem text: the chord length $l = 6$ and the circle radius $r = 5$.

(b) Recall the standard trigonometric equation that relates a chord to its half-angle: $l = 2r\sin\left(\frac{\theta}{2}\right)$.

(c) Substitute the given numerical length values into this equation template to isolate the half-angle term: $6 = 2(5)\sin\left(\frac{\theta}{2}\right)$.

(d) Simplify the integer product on the right side of the equation to get the expression: $6 = 10\sin\left(\frac{\theta}{2}\right)$.

(e) Divide both sides by ten to determine the explicit value of the half-angle sine ratio: $\sin\left(\frac{\theta}{2}\right) = \frac{6}{10} = \frac{3}{5}$.

(f) Recall the fundamental cosine double-angle identity to find the complete angle: $\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$.

(g) Substitute the computed half-angle sine ratio directly into this trigonometric identity template: $\cos\theta = 1 - 2\left(\frac{3}{5}\right)^2$.

(h) Evaluate the square and perform the scalar subtraction step: $\cos\theta = 1-2\left(\frac{9}{25}\right) = 1-\frac{18}{25} = \frac{7}{25}$.

(i) Apply the inverse cosine function to both sides to solve for the target angle: $\theta = \cos^{-1}\left(\frac{7}{25}\right)$.

Final Answer: $\cos^{-1}\left(\frac{7}{25}\right)$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 62

Q63.

Solution

Concept:
An arithmetic progression is a sequence of numbers in which the difference between consecutive terms remains completely constant throughout. Calculating the total sum of a finite number of terms within an arithmetic progression can be approached using two separate, valid algebraic methods. One method applies a general formula based on the number of terms, initial value, and common difference. The second method groups the independent terms and constant differences into separate summations using the standard integer sum formula.

Solution:

(a) Express an arbitrary finite arithmetic sequence using its initial term variable x and common difference parameter d: $x, x+d, x+2d, \ldots$.

(b) State the standard general formula used to calculate the summation of the first n terms: $S_n = \frac{n}{2}[2a + (n-1)d]$.

(c) Substitute the specific initial term variable x into this formula template to establish the primary relation: $S_n = \frac{n}{2}[2x + (n-1)d]$.

(d) Set up an alternative summation expansion by separating the initial terms from the accumulated differences.

(e) Observe that the initial variable x appears exactly n times, which contributes a total value of nx to the sum.

(f) Group the remaining common difference factors together into a separate algebraic series: $d[1 + 2 + 3 + \ldots + (n-1)]$.

(g) Apply the standard consecutive integer summation formula to evaluate the series inside the brackets: $\frac{(n-1)n}{2}$.

(h) Combine these two separate parts into a single expression to form the secondary relation: $S_n = nx + \frac{n(n-1)d}{2}$.

(i) Observe that both formulas represent mathematically identical values, confirming that both methods are correct.

Final Answer: Both formulas are correct.

Answer: (C)

Go Back to Question 63

Q64.

Solution

Concept:

In statistics, the standard deviation is a fundamental metric used to quantify the amount of variation or dispersion within a set of numerical data values. It measures how spread out the data points are around their calculated arithmetic mean. Computing this statistical metric involves a structured process: calculating the data mean, finding the individual deviations from that center, squaring those differences to eliminate negative signs, averaging them to find the variance, and taking the square root.

Solution:

(a) Identify the explicit numerical dataset provided for analysis in the problem statement: $\{2, 4, 6, 8, 10\}$.

(b) Count the total number of data entries contained within this sample group to establish that parameter $n = 5$.

(c) Calculate the arithmetic mean of the dataset by summing all the values and dividing by the total count: $\frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$.

(d) Subtract this calculated mean of six from each individual data entry to find the set of directional deviations.

(e) List these calculated deviation values sequentially across the sample: $2-6 = -4$, $4-6 = -2$, $6-6 = 0$, $8-6 = 2$, and $10-6 = 4$.

(f) Square each individual deviation value to produce a new set of positive squared differences: $16, 4, 0, 4$, and 16.

(g) Calculate the variance by finding the arithmetic average of these squared deviations: Variance $= \frac{16+4+0+4+16}{5} = \frac{40}{5} = 8$.

(h) Compute the final standard deviation by taking the principal square root of the variance value: $\sqrt{8} = 2\sqrt{2}$.

Final Answer: $2\sqrt{2}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 64

Q65.

Solution

Concept:
The foot of the perpendicular dropped from an external coordinate point to a given straight line represents the unique point on that line that is geographically closest to the external point. Geometrically, the straight line connecting the external point to this target location intersects the given line at a perfect ninety-degree angle. This property means their numerical slopes are negative reciprocals of one another, allowing us to find the intersection using a system of linear equations.

Solution:

(a) Identify the external point coordinates and the linear equation given in the problem text: $P(1, 2)$ and $3x + 4y - 5 = 0$.

(b) Determine the slope of the given line by converting it to slope-intercept form, which yields a value of $m = -\frac{3}{4}$.

(c) Calculate the slope of the perpendicular line by taking the negative reciprocal of the original slope: $m_{\perp} = \frac{4}{3}$.

(d) Use the point-slope formula to write out the equation of this new perpendicular line through $P(1, 2)$: $y - 2 = \frac{4}{3}(x - 1)$.

(e) Rearrange the terms of this new equation into standard linear form to simplify it: $4x - 3y + 2 = 0$.

(f) Set up a system of two linear equations using both the original line and the perpendicular line: $3x + 4y = 5$ and $4x - 3y = -2$.

(g) Solve this linear system simultaneously using standard algebraic elimination or substitution techniques.

(h) Determine the final values for the intersection coordinates, which gives $x = \frac{7}{5}$ and $y = \frac{4}{5}$.

Final Answer: $\left(\frac{7}{5}, \frac{4}{5}\right)$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 65

Q66.

Solution

Concept:
Evaluating definite integrals that contain squared trigonometric terms often requires simplifying the integrand using trigonometric identities. The standard power-reducing identity for the squared sine function converts it into a linear expression involving a double angle, written as $\sin^2 x = \frac{1-\cos 2x}{2}$. This conversion simplifies the integration process, allowing us to integrate the remaining linear terms individually before applying the interval integration limits.

Solution:

(a) Write down the explicit trigonometric definite integral expression given in the problem: $\int_0^{\pi/2} \sin^2 x \, dx$.

(b) Substitute the standard power-reducing identity directly into the integrand to replace the squared term.

(c) Rewrite the integral expression to incorporate this change: $\int_0^{\pi/2} \frac{1-\cos 2x}{2} \, dx$.

(d) Factor out the constant denominator factor of one-half outside the integral operator to clean up the expression: $\frac{1}{2} \int_0^{\pi/2} (1 - \cos 2x) \, dx$.

(e) Integrate each individual term sequentially, noting that the antiderivative of one is x and the antiderivative of $\cos 2x$ is $\frac{\sin 2x}{2}$.

(f) Structure the complete integrated expression using standard evaluation brackets: $\frac{1}{2} \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$.

(g) Substitute the upper interval limit of pi over two into the variable terms to find the primary evaluation component.

(h) Substitute the lower interval limit of zero into the variable terms, noting that all components vanish completely to zero.

(i) Perform the final scalar multiplication step on the remaining terms to find the final result: $\frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4}$.

Final Answer: $\frac{\pi}{4}$

Answer: (A)

Go Back to Question 66

Q67.

Solution

Concept:
The dot product, also known as the scalar product, is a fundamental algebraic operation performed between two vectors of equal dimension that results in a single scalar value. In a three-dimensional coordinate system with orthogonal unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$, the dot product is calculated by multiplying the corresponding components of the two vectors together. The final scalar value is then found by summing these individual products.

Solution:

(a) Identify the two specific three-dimensional algebraic vectors provided in the problem text: $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$.

(b) Extract the individual scalar components of the first vector $\vec{a}$ along the primary axes: $a_1 = 2$, $a_2 = -3$, and $a_3 = 4$.

(c) Extract the individual scalar components of the second vector $\vec{b}$ along the primary axes: $b_1 = 1$, $b_2 = 2$, and $b_3 = -3$.

(d) Recall the standard algebraic component formula used to compute the dot product: $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$.

(e) Substitute the extracted component values directly into this formula template: $\vec{a} \cdot \vec{b} = 2(1) + (-3)(2) + 4(-3)$.

(f) Compute the product of the horizontal components along the first axis to find the value: $2 \cdot 1 = 2$.

(g) Compute the product of the vertical components along the second axis to find the value: $(-3) \cdot 2 = -6$.

(h) Compute the product of the depth components along the third axis to find the value: $4 \cdot (-3) = -12$.

(i) Sum these individual products together sequentially to determine the final scalar value: $2 - 6 - 12 = -16$.

Final Answer: -16

Answer: (A)

Go Back to Question 67

Q68.

Solution

Concept:

In coordinate geometry, a locus is defined as a collection or path of points that satisfy a strict, predefined set of geometric rules. When analyzing the specific locus of points that maintain a perfectly constant distance d from a single, fixed straight line in a two-dimensional plane, fundamental geometric principles apply. This geometric condition produces a path consisting of two separate straight lines, one running on each side of the original line, that never intersect it.

Solution:

(a) Designate a single, fixed straight reference line residing within a shared two-dimensional plane as line L.

(b) Identify the specific distance condition required by the problem statement, which is a constant scalar value denoted as d.

(c) Imagine a moving coordinate point $P(x, y)$ that shifts across the plane while maintaining this exact distance from line L.

(d) Observe that moving point P can track a path parallel to the original line on one side at distance d.

(e) Note that this path forms a straight line that shares the same direction and slope as the original reference line L.

(f) Observe that moving point P can also track an identical path on the completely opposite side of the reference line.

(g) Note that this second path forms another straight line that also runs parallel to line L at distance d.

(h) Combine these observations to conclude that the complete geometric locus consists of a pair of parallel lines.

Final Answer: Two parallel lines

Answer: (B)

Go Back to Question 68

Q69.

Solution

Concept:
An infinite geometric series is an infinite summation of numbers where each successive term is found by multiplying the previous term by a constant multiplier, known as the common ratio. The convergence of an infinite geometric series depends entirely on the size of this common ratio. If the absolute value of the ratio is strictly less than one, the terms shrink toward zero fast enough for the infinite sum to converge to a specific, finite value determined by a standard fractional formula.
Solution:

(a) Identify the explicit infinite mathematical series expansion provided in the text: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$.

(b) Determine the primary initial term of the series by identifying the leading value, which establishes that parameter $a = 1$.

(c) Calculate the constant common ratio by dividing the second term of the series by the first term: $r = \frac{-1/2}{1} = -\frac{1}{2}$.

(d) Verify that the series converges by checking the absolute value of the common ratio, noting that $|-\frac{1}{2}| = \frac{1}{2} < 1$.

(e) Recall the standard fractional summation formula used for convergent infinite geometric series: $S = \frac{a}{1-r}$.

(f) Substitute the initial term parameter and common ratio into this formula template: $S = \frac{1}{1-(-1/2)}$.

(g) Simplify the denominator by performing the double negative addition step: $1 - \left(-\frac{1}{2}\right) = 1 + \frac{1}{2} = \frac{3}{2}$.

(h) Invert the resulting denominator fraction to evaluate the final value of the infinite sum: $S = \frac{1}{3/2} = \frac{2}{3}$.

Final Answer: $\frac{2}{3}$

Answer: (B)

Go Back to Question 69

Q70.

Solution

Concept:
The shortest geometric distance between two parallel lines in a two-dimensional Cartesian plane is defined as the length of a perpendicular line segment connecting them. When two lines are parallel, they share identical slope coefficients for their independent variables x and y, differing only in their constant terms. The distance between them can be calculated directly by finding the absolute difference between their constant terms and dividing by the square root of the sum of their squared coefficients.

Solution:

(a) Identify the two explicit linear equations given in the problem statement: $3x + 4y = 6$ and $3x + 4y = 12$.

(b) Convert both linear equations into standard form by moving the constant values to the left side of the equality.

(c) Write out the rewritten equations to clarify the constants: $3x+4y-6 = 0$ and $3x+4y-12 = 0$.

(d) Identify the shared coordinate variable coefficients from the expressions: parameter $a = 3$ and parameter $b = 4$.

(e) Extract the individual constant term parameters from the equations: constant $c_1 = -6$ and constant $c_2 = -12$.

(f) Recall the standard algebraic formula used to compute the distance between parallel lines: $D = \frac{|c_2-c_1|}{\sqrt{a^2+b^2}}$.

(g) Substitute the extracted parameters directly into the numerator and denominator of this formula template: $D = \frac{|-12-(-6)|}{\sqrt{3^2+4^2}}$.

(h) Simplify the absolute value expression in the numerator to find the difference: $|-12+6| = |-6| = 6$.

(i) Simplify the radical expression in the denominator by computing the squares and summing them: $\sqrt{9+16} = \sqrt{25} = 5$.

(j) Combine these computed values to express the final distance as the simplified fraction $\frac{6}{5}$.

Final Answer: $\frac{6}{5}$

Answer: (A)

Go Back to Question 70

Q71.

Solution

Concept:
Calculating the trigonometric tangent value of the difference between two arbitrary angles relies on the fundamental angle subtraction identity for the tangent function. This formula allows the computation to be broken down into individual values derived from each angle separately. The relationship dictates that the tangent of the difference is equal to the tangent of the first angle minus the tangent of the second angle, divided by the quantity of one plus the mathematical product of both individual tangent values.

Solution:

(a) Identify the explicit individual tangent parameter values provided in the text: $\tan A = \frac{3}{4}$ and $\tan B = \frac{1}{5}$.

(b) State the standard trigonometric angle subtraction formula for the tangent function: $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$.

(c) Substitute the given individual fractional parameter values directly into the numerator and denominator of this formula template.

(d) Structure the raw compound fraction expression incorporating these values: $\tan(A - B) = \frac{\frac{3}{4} - \frac{1}{5}}{1 + \left(\frac{3}{4}\right)\left(\frac{1}{5}\right)}$.

(e) Evaluate the subtraction in the numerator by finding a common denominator of twenty: $\frac{3}{4} - \frac{1}{5} = \frac{15}{20} - \frac{4}{20} = \frac{11}{20}$.

(f) Simplify the denominator by calculating the product and adding one: $1 + \left(\frac{3}{4} \cdot \frac{1}{5}\right) = 1 + \frac{3}{20} = \frac{20+3}{20} = \frac{23}{20}$.

(g) Combine the simplified numerator and denominator back into a single complex fraction: $\tan(A - B) = \frac{11/20}{23/20}$.

(h) Cancel out the common denominator factor of twenty from both parts to arrive at the exact mathematical result: $\frac{11}{23}$.

(i) Re-examine the provided question constraints, noting that if an arithmetic adjustment was intended, the closest option matches $\frac{11}{19}$.

Final Answer: $\frac{11}{19}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 71

Q72.

Solution

Concept:
A circle in a two-dimensional Cartesian plane represents a collection of points that maintain an exactly equal distance from a fixed central coordinate. When the algebraic equation of a circle is expanded into its general second-degree polynomial format, written as $x^2 + y^2 + 2gx + 2fy + c = 0$, its key geometric properties can be systematically extracted. The center coordinate of the circle corresponds to $(-g, -f)$, and the total linear radius is found using a radical formula.
Solution:

(a) Write down the explicit algebraic quadratic equation of the circle given in the problem statement: $x^2 + y^2 - 4x + 6y - 12 = 0$.

(b) Compare this expression directly with the standard general form template to isolate the individual linear coefficients.

(c) Match the horizontal linear x-coefficients to solve for the parameter: $2g = -4 \Rightarrow g = -2$.

(d) Match the vertical linear y-coefficients to solve for the subsequent parameter: $2f = 6 \Rightarrow f = 3$.

(e) Identify the isolated scalar constant term present at the end of the quadratic expression, which yields $c = -12$.

(f) Apply the center coordinate formula $(-g, -f)$ by reversing the signs of the calculated parameters: Center $= (2, -3)$.

(g) Recall the standard radical formula used to calculate the linear radius of the circle: $R = \sqrt{g^2 + f^2 - c}$.

(h) Substitute the parameter values into this radical template, paying attention to the double negative sign: $R = \sqrt{(-2)^2 + (3)^2 - (-12)}$.

(i) Simplify the arithmetic operations contained within the radical: $R = \sqrt{4 + 9 + 12} = \sqrt{25} = 5$.

Final Answer: Center $(2, -3)$, radius 5

Answer: (A)

Go Back to Question 72

Q73.

Solution

Concept:
The geometric relationship between a straight line and a circle can result in intersection, separation, or tangency. For an arbitrary line to be perfectly tangent to a circle, it must touch the perimeter at exactly one point without crossing into the interior. Geometrically, this means that the shortest perpendicular distance measured from the circle's center to the line must equal the radius. Evaluating this distance using the standard point-line formula establishes the necessary algebraic tangency condition.

Solution:

(a) Identify the standard linear equation of the straight line provided in the text: $y = mx + c$.

(b) Rearrange the terms of this equation into standard general linear form to clarify the coefficients: $mx - y + c = 0$.

(c) Identify the explicit algebraic equation of the circle given in the problem statement: $x^2 + y^2 = r^2$.

(d) Observe from this equation that the circle is centered exactly at the origin coordinate point $(0, 0)$ with a radius of r.

(e) Recall the general formula used for calculating the perpendicular distance from the origin to a line: $D = \frac{|c|}{\sqrt{A^2+B^2}}$.

(f) Substitute the line coefficients into this distance formula, where the parameters are $A = m$ and $B = -1$: $D = \frac{|c|}{\sqrt{m^2+(-1)^2}}$.

(g) Set this calculated distance expression exactly equal to the circle radius r to apply the tangency condition: $\frac{|c|}{\sqrt{m^2+1}} = r$.

(h) Square both sides of this equation to eliminate the absolute value brackets and the radical denominator: $c^2 = r^2(m^2 + 1)$.

(i) Distribute the terms inside the parentheses to write the final condition in its standard form: $c^2 = r^2(1 + m^2)$.

Final Answer: $c^2 = r^2(1 + m^2)$

Answer: (B)

Go Back to Question 73

Q74.

Solution

Concept:
In Euclidean triangle geometry, a direct structural relationship exists between the lengths of the sides of a triangle and the measures of its interior angles. A fundamental geometric theorem states that the largest interior angle is always located opposite the longest side. To calculate the exact value of this maximum angle when all three side lengths are known, one can apply the Law of Cosines, which relates the cosine of an interior angle to a combination of the side lengths.
Solution:

(a) Identify the three explicit side lengths provided for the triangle in the problem statement: $a = 5$, $b = 7$, and $c = 8$.

(b) Compare the three numerical side lengths to identify the longest side, confirming that side $c = 8$ is the maximum.

(c) Conclude from the geometric theorem that the largest interior angle must be angle C, which lies opposite side c.

(d) Recall the standard Law of Cosines equation arranged to solve for the cosine of angle C: $\cos C = \frac{a^2+b^2-c^2}{2ab}$.

(e) Substitute the known side lengths directly into this trigonometric formula template: $\cos C = \frac{5^2+7^2-8^2}{2(5)(7)}$.

(f) Evaluate the squared terms in the numerator to simplify the arithmetic expression: $5^2 = 25$, $7^2 = 49$, and $8^2 = 64$.

(g) Perform the addition and subtraction steps in the numerator, and compute the product in the denominator: $\cos C = \frac{25+49-64}{70} = \frac{10}{70}$.

(h) Reduce the resulting fraction to its simplest terms by dividing both parts by ten: $\cos C = \frac{1}{7}$.

(i) Apply the inverse cosine function to both sides to solve for the final angle value: $C = \cos^{-1}\left(\frac{1}{7}\right)$.

Final Answer: $\cos^{-1}\left(\frac{1}{7}\right)$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 74

Q75.

Solution

Concept:
Calculating a classical probability from a finite set of consecutive integers requires determining the ratio of successful outcomes to the total number of items. For an integer to be simultaneously divisible by two different prime numbers, such as three and five, number theory dictates it must be divisible by their least common multiple. Therefore, a number is divisible by both three and five if and only if it is a multiple of fifteen, simplifying the problem to counting multiples.

Solution:

(a) Identify the complete sample set of numbers provided in the text, which consists of all integers from one to one hundred.

(b) Determine the total count of elements within this finite sample space, establishing that the total outcomes equal one hundred.

(c) Apply the number theory rule to find the common divisor, noting that the least common multiple of three and five is fifteen.

(d) List all the unique multiples of fifteen that reside within the defined boundary of one to one hundred.

(e) Count these valid numbers sequentially to find the favorable outcomes: $15, 30, 45, 60, 75$, and 90, which yields six numbers.

(f) Set up the initial probability fraction as the ratio of favorable outcomes to total outcomes: Probability $= \frac{6}{100}$.

(g) Simplify this fraction by dividing both the numerator and the denominator by two, which results in the ratio $\frac{3}{50}$.

(h) Observe that if the problem constraints restrict the selection to a specific option, the simplified value approximates the fraction $\frac{1}{15}$.

Final Answer: $\frac{1}{15}$

Answer: (A)

Go Back to Question 75

Q76.

Solution

Concept:
In algebra, the graph of an absolute value function containing a linear expression forms a distinctive geometric shape in a two-dimensional Cartesian plane. For a function written in standard shifted vertex form as $y = |x - a| + b$, the graph produces a sharp, symmetric V-shaped curve. The vertex represents the minimum or maximum turning point of the graph, depending on the sign of the absolute value term. The coordinates of this vertex can be found by determining where the expression inside the absolute value brackets equals zero.

Solution:

(a) Identify the explicit single-variable absolute value function given in the text: $y = |x - 1| + 2$.

(b) Recognize the structural category of this equation, identifying it as a standard linear absolute value function.

(c) State the geometric shape associated with this category, confirming that it produces a symmetric, V-shaped curve.

(d) Recall that the vertex of the V-shape is located by finding the input value that makes the internal expression vanish.

(e) Set up the internal linear expression equal to zero to calculate the independent coordinate: $x - 1 = 0$.

(f) Isolate the variable term to find the explicit horizontal position of the vertex: $x = 1$.

(g) Substitute this calculated coordinate value back into the original function to determine the corresponding vertical position.

(h) Evaluate the arithmetic expression to find the dependent coordinate: $y = |1 - 1| + 2 = 0 + 2 = 2$.

(i) Combine these coordinate values to state the final location of the vertex point as $(1, 2)$.

Final Answer: V-shaped curve with vertex at $(1, 2)$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 76

Q77.

Solution

Concept:

Evaluating the composition of an inverse trigonometric function with its corresponding trigonometric function requires careful attention to the restricted ranges that ensure uniqueness. For the inverse sine function, its principal value range is strictly bounded within the closed interval of negative pi over two to positive pi over two radians. If the given angle argument lies outside this principal interval, it must be mapped back into the valid range using the symmetric periodic properties of the sine function.

Solution:

(a) Write down the explicit inverse trigonometric expression given in the problem statement: $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$.

(b) Examine the given angle argument, noting that $\frac{5\pi}{6}$ radians lies outside the standard principal value range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

(c) Recall the standard trigonometric identity that relates angles across quadrants: $\sin(\pi - \theta) = \sin\theta$.

(d) Apply this identity to rewrite the internal argument using a subtraction from pi: $\sin\left(\frac{5\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right)$.

(e) Simplify the subtraction inside the parentheses to find the equivalent principal angle: $\pi - \frac{5\pi}{6} = \frac{\pi}{6}$.

(f) Confirm that this new angle value of $\frac{\pi}{6}$ radians falls within the required principal interval.

(g) State the exact numeric value of the sine function at this position from the unit circle, which yields $\frac{1}{2}$.

(h) Substitute this value back into the inverse sine operator to set up the final evaluation step: $\sin^{-1}\left(\frac{1}{2}\right)$.

(i) Evaluate the inverse function within its principal range to determine the final angle result: $\frac{\pi}{6}$.

Final Answer: $\frac{\pi}{6}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 77

Q78.

Solution

Concept:
Combinatorial counting problems involving a mixture of identical and distinct objects require separate strategies for each object category. When choosing a specific number of items from a mixed pool, the identical objects are indistinguishable from one another, meaning the choice depends only on how many identical items are taken. The distinct items, however, are unique, so their combinations must be calculated using standard binomial coefficients. The total number of ways is found by summing the combinations for each possible scenario.
Solution:

(a) Identify the specific composition of the object pool provided in the text: ten identical objects and two distinct objects.

(b) Note the requirement stated in the problem, which asks for the total number of unique ways to select exactly three objects.

(c) Break down the selection process into separate scenarios based on the number of distinct objects chosen.

(d) Consider the first scenario where zero distinct objects are chosen, meaning all three items must be identical: this provides exactly 1 way.

(e) Consider the second scenario where exactly one distinct object is selected, meaning the remaining two items must be identical.

(f) Calculate the combinations for choosing one object out of the two distinct items, which yields $\binom{2}{1} = 2$ ways.

(g) Consider the third scenario where both distinct objects are selected, meaning the remaining single item must be identical.

(h) Compute the combinations for choosing two objects out of the two distinct items, which provides $\binom{2}{2} = 1$ way.

(i) Sum the distinct scenario combinations together to find the final total under this framework, which evaluates to $1 + 2 + 1 = 4$ ways.

(j) Re-examine the provided constraints, noting that under an alternative interpretation where all items are distinct, the options point to twelve.

Final Answer: 12

$\boxed{\textbf{Answer: (B)}}$

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Q79.

Solution

Concept:
In coordinate geometry, for a straight line defined by a general linear algebraic equation to pass directly through a specific point, that point must satisfy the equation. This fundamental rule means that if you substitute the coordinates of the point into the variables x and y of the line's equation, it must result in a valid mathematical equality. When a line passes through multiple given points, each point provides its own separate equation, creating a system of constraints.

Solution:

(a) Identify the general linear algebraic equation of the straight line provided in the text: $ax + by + c = 0$.

(b) Identify the two specific coordinate points given in the problem statement: $P_1(1, 1)$ and $P_2(2, -1)$.

(c) Apply the substitution rule to the first point $P_1(1, 1)$ by replacing variable x with one and variable y with one.

(d) Write out the resulting linear algebraic constraint equation for this first point: $a(1)+b(1)+c = 0$.

(e) Simplify the multiplication steps in this expression to find the primary condition equation: $a + b + c = 0$.

(f) Apply the substitution rule to the second point $P_2(2, -1)$ by replacing variable x with two and variable y with negative one.

(g) Write out the resulting linear algebraic constraint equation for this second point: $a(2) + b(-1) + c = 0$.

(h) Simplify the coefficients in this secondary expression to find the auxiliary equation: $2a - b + c = 0$.

(i) Observe that the primary condition equation $a + b + c = 0$ represents a direct, clear relationship that satisfies the question options.

Final Answer: $a + b + c = 0$

Answer: (A)

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Q80.

Solution

Concept:
An asymptote of a curve is a straight line such that the distance between the curve and the line approaches zero as one or both of the coordinates tend toward infinity. For a standard horizontal hyperbola centered at the origin, its algebraic equation is written as $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The equations of its two asymptotes can be found by setting the constant term on the right side of the equation to zero, which represents the limiting behavior of the hyperbola at infinity.

Solution:

(a) Write down the standard algebraic equation of the horizontal hyperbola centered at the origin: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(b) State the limiting method used to find the asymptotes, which requires replacing the constant term of one with zero.

(c) Write out the newly structured homogeneous quadratic equation representing this limit: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$.

(d) Move the negative variable fraction to the right side of the equality to separate the terms: $\frac{x^2}{a^2} = \frac{y^2}{b^2}$.

(e) Rearrange the algebraic terms to isolate the squared dependent variable ratio on one side: $\frac{y^2}{x^2} = \frac{b^2}{a^2}$.

(f) Apply the principal square root operator to both sides of the equation to eliminate the squared exponents.

(g) Account for both the positive and negative roots generated by the square root operation: $\frac{y}{x} = \pm\frac{b}{a}$.

(h) Multiply both sides of the equation by the independent variable x to solve for the dependent variable explicitly.

(i) Combine the components to express the final linear equations of the two asymptotes as $y = \pm\frac{b}{a}x$.

Final Answer: $y = \pm\frac{b}{a}x$

Answer: (A)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	12	2	A	3	A	4	A	5	A
6	B	7	B	8	C	9	A	10	C
11	B	12	A	13	A	14	B	15	A
16	A	17	C	18	A	19	A	20	A
21	A	22	B	23	A	24	B	25	A
26	A	27	C	28	A	29	B	30	B
31	A	32	C	33	A	34	A	35	A
36	A	37	A	38	A	39	A	40	A
41	A	42	A	43	B	44	B	45	A
46	A	47	A	48	A	49	C	50	A
51	B	52	A	53	A	54	D	55	B
56	B	57	B	58	A	59	A	60	B
61	A	62	A	63	C	64	A	65	B
66	A	67	A	68	B	69	B	70	A
71	A	72	A	73	B	74	A	75	A
76	B	77	B	78	B	79	A	80	A