UPCATET Mathematics Sample Paper-8

Duration: 80 Minutes **Maximum Marks: 320**

Instructions

- This paper contains **80** Multiple Choice Questions.
- Each correct answer carries **+4** mark. Incorrect answer: **−1** marks. Only **one** correct option.
- Unattempted questions carry **0** marks.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. The value of $\lim_{x\to 0} \dfrac{\sin 3x}{x}$ is:

(A) 3

(B) $\frac{1}{3}$

(C) 0

(D) 1

Q2. The distance between the points $(1, 2)$ and $(5, 5)$ is:

(A) 5

(B) 25

(C) $\sqrt{34}$

(D) 7

Q3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then A^2 equals:

(A) $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

(B) $\begin{bmatrix} 5 & 8 \\ 11 & 14 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 4 \\ 9 & 16 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & 6 \\ 9 & 12 \end{bmatrix}$

Q4. If $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$, then $\vec{a} \cdot \vec{b}$ equals:

(A) -6

(B) 6

(C) 0

(D) 10

Q5. If $\sin\theta = \frac{3}{5}$, then the value of $\cos 2\theta$ is:

(A) $\frac{24}{25}$

(B) $\frac{7}{25}$

(C) $-\frac{7}{25}$

(D) $\frac{4}{5}$

Q6. The mean of the first 10 natural numbers is:

(A) 5

(B) 5.5

(C) 10

(D) 55

Q7. If $y = \ln(\cos x)$, then $\frac{dy}{dx}$ equals:

(A) $\tan x$

(B) $\cot x$

(C) $-\tan x$

(D) $-\cot x$

Q8. The slope of the line joining the points $(2, 3)$ and $(4, 7)$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q9. The sum of the first 15 terms of the AP $3, 7, 11, \ldots$ is:

(A) 465

(B) 450

(C) 480

(D) 495

Q10. If $f(x) = x^x$, then $f'(1)$ equals:

(A) 0

(B) e

(C) 1

(D) 2

Q11. The value of $\tan 15^\circ$ is:

(A) $2 + \sqrt{3}$

(B) $2 - \sqrt{3}$

(C) $\sqrt{3} - 2$

(D) 1

Q12. A unit vector perpendicular to the plane $x + y + z = 1$ is:

(A) $\frac{\hat{i}+\hat{j}+\hat{k}}{3}$

(B) $\hat{i} + \hat{j} + \hat{k}$

(C) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

(D) $\frac{\hat{i}+\hat{j}+\hat{k}}{2}$

Q13. The value of $\int xe^x\,dx$ is:

(A) $xe^x + e^x + C$

(B) $e^x(x+1) + C$

(C) $xe^x + C$

(D) $xe^x - e^x + C$

Q14. The centre of the circle $x^2 + y^2 - 4x + 6y - 3 = 0$ is:

(A) $(2, -3)$

(B) $(-2, 3)$

(C) $(4, -6)$

(D) $(-4, 6)$

Q15. If α and β are the roots of $x^2 - 5x + 6 = 0$, then $\alpha^2 + \beta^2$ equals:

(A) 13

(B) 25

(C) 12

(D) 37

Q16. The value of $\int \frac{dx}{x\ln x}$ is:

(A) $\ln|\ln x| + C$

(B) $\ln x + C$

(C) $(\ln x)^2 + C$

(D) $\frac{1}{\ln x} + C$

Q17. If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$, then $x^2 + y^2$ equals:

(A) 0

(B) 1

(C) 2

(D) $\frac{1}{2}$

Q18. The distance between the points $(1, 2, 3)$ and $(4, 6, 9)$ is:

(A) $\sqrt{61}$

(B) 61

(C) 7

(D) $\sqrt{77}$

Q19. The local maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 3$ is:

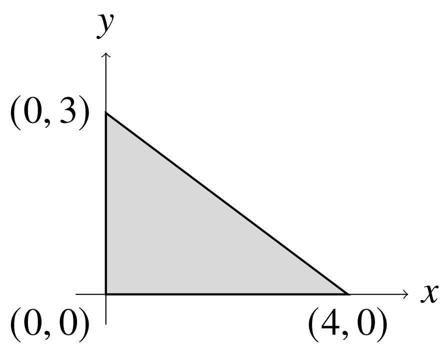
(A) 2 at $x = 1$

(B) 1 at $x = 2$

(C) 3 at $x = 0$

(D) -2 at $x = 3$

Q20. The area of the triangle with vertices $(0, 0)$, $(4, 0)$ and $(0, 3)$ is:



(A) 6

(B) 12

(C) 7

(D) 5

Q21. If ${}^nC_5 = {}^nC_9$, then n equals:

(A) 14

(B) 45

(C) 9

(D) 5

Q22. The order and degree of the differential equation $y' + y^2 = x^3$ are respectively:

(A) $1, 1$

(B) $1, 2$

(C) $2, 1$

(D) $2, 3$

Q23. If $\sin A = \frac{1}{2}$, then the value of $\sin 3A$ is:

(A) 1

(B) 0

(C) $\frac{1}{2}$

(D) $\frac{3}{2}$

Q24. The direction ratios of the line joining $(1, 2, 3)$ and $(4, 5, 6)$ are:

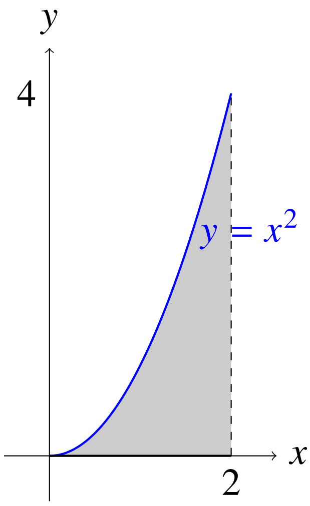
(A) $(1, 1, 1)$

(B) $(3, 3, 3)$

(C) $(4, 5, 6)$

(D) $(1, 2, 3)$

Q25. The area bounded by $y = x^2$, $x = 0$, $x = 2$ and the x-axis is:



(A) $\frac{4}{3}$

(B) 2

(C) $\frac{8}{3}$

(D) 8

Q26. The equation of the tangent to the circle $x^2 + y^2 = 25$ at $(3, 4)$ is:

(A) $3x + 4y = 25$

(B) $4x - 3y = 0$

(C) $3x - 4y = 25$

(D) $x + y = 7$

Q27. If ω is a complex cube root of unity, then $(1 + \omega)(1 + \omega^2)$ equals:

(A) 0

(B) ω

(C) ω^2

(D) 1

Q28. The value of $\lim_{x\to 0} \dfrac{\tan x - x}{x^3}$ is:

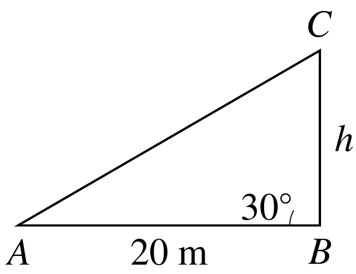
(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) 0

(D) 1

Q29. From a point 20 metres from the foot of a tower, the angle of elevation is 30°. The height of the tower is:



(A) $20\sqrt{3}$

(B) 10

(C) 40

(D) $\frac{20}{\sqrt{3}}$

Q30. The equation of the plane passing through $(1, 2, 3)$ and perpendicular to $2\hat{i} - 3\hat{j} + 4\hat{k}$ is:

(A) $2x + 3y + 4z = 8$

(B) $2x - 3y + 4z = 8$

(C) $x - 2y + 3z = 4$

(D) $2x - 3y + 4z = 4$

Q31. The order and degree of $(y'')^2 + (y')^3 + y = 0$ are respectively:

(A) 2, 3

(B) 2, 1

(C) 2, 2

(D) 3, 2

Q32. The eccentricity of the ellipse $9x^2 + 16y^2 = 144$ is:

(A) $\frac{\sqrt{7}}{4}$

(B) $\frac{\sqrt{7}}{3}$

(C) $\frac{1}{4}$

(D) $\frac{3}{4}$

Q33. If A and B are symmetric matrices, then $AB - BA$ is:

(A) symmetric

(B) skew-symmetric

(C) null

(D) diagonal

Q34. The maximum value of $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0, 4]$ is:

(A) 5

(B) 3

(C) 17

(D) 1

Q35. If $\tan\theta = 2$, then $\sin 2\theta$ equals:

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{2}{5}$

(D) $\frac{1}{5}$

Q36. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ and the plane $x + y + z = 5$ is:

(A) 90°

(B) 0°

(C) 45°

(D) 60°

Q37. The function $f(x) = |x|$ at $x = 0$ is:

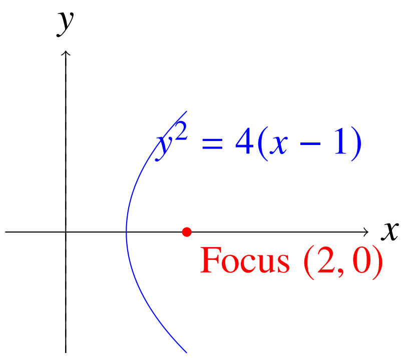
(A) continuous but not differentiable

(B) differentiable but not continuous

(C) neither continuous nor differentiable

(D) both continuous and differentiable

Q38. The locus of a point equidistant from $(2, 0)$ and the y-axis is:



(A) $y^2 = 4(x - 1)$

(B) $y^2 = 4(x + 1)$

(C) $x^2 = 4(y - 1)$

(D) $x^2 = 4(y + 1)$

Q39. The sum of the first 10 terms of the GP $1, -\frac{1}{2}, \frac{1}{4}, \ldots$ is:

(A) $\frac{341}{512}$

(B) $\frac{1023}{1024}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

Q40. The area bounded by the curve $y = \sin x$, the lines $x = 0$, $x = \pi$ and the x-axis is:

(A) 0

(B) 2

(C) 1

(D) π

Q41. The principal value of $\tan^{-1}(-1)$ is:

(A) $-\frac{\pi}{4}$

(B) $\frac{3\pi}{4}$

(C) $\frac{\pi}{4}$

(D) $\frac{5\pi}{4}$

Q42. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, then $\vec{a} \times \vec{b}$ equals:

(A) $4\hat{i} - \hat{j} - 3\hat{k}$

(B) $4\hat{i} + \hat{j} + 3\hat{k}$

(C) $2\hat{i} - 3\hat{j} + \hat{k}$

(D) $\hat{i} - \hat{j} - \hat{k}$

Q43. The value of $\int_0^{\pi/2} \sin^2 x \, dx$ is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) 1

(D) 0

Q44. The equation of the line passing through $(1, 2)$ and $(3, 4)$ is:

(A) $x + y - 3 = 0$

(B) $2x - y = 0$

(C) $x - y + 1 = 0$

(D) $x - 2y + 3 = 0$

Q45. If the system $x + 2y = 3$ and $4x + 8y = k$ has infinitely many solutions, then k equals:

(A) 3

(B) 8

(C) 12

(D) 6

Q46. The value of $\int e^x(\sec x + \sec x \tan x)\,dx$ is:

(A) $e^x \tan x + C$

(B) $e^x \sec x + C$

(C) $e^x \sec^2 x + C$

(D) $e^x(\sec x + \tan x) + C$

Q47. If $\sin x + \cos x = 1$, then $\sin 2x$ equals:

(A) 1

(B) -1

(C) 0

(D) $\frac{1}{2}$

Q48. The probability of getting exactly 2 heads in 4 tosses of a fair coin is:

(A) $\frac{3}{8}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{5}{16}$

Q49. The derivative of $\sin^{-1}(2x)$ with respect to x is:

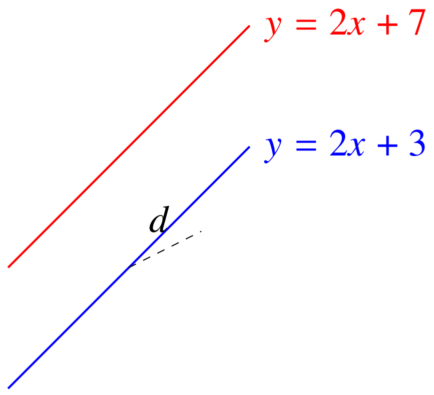
(A) $\frac{2}{\sqrt{1-4x^2}}$

(B) $\frac{1}{\sqrt{1-4x^2}}$

(C) $\frac{2}{\sqrt{1-2x^2}}$

(D) $\frac{1}{\sqrt{1-2x^2}}$

Q50. The distance between the parallel lines $y = 2x + 3$ and $y = 2x + 7$ is:



(A) $\frac{4}{\sqrt{5}}$

(B) $\frac{4}{\sqrt{13}}$

(C) $\frac{2}{\sqrt{5}}$

(D) $\frac{7}{\sqrt{5}}$

Q51. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, then A^{-1} is:

(A) $\begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$

(B) $\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$

Q52. The minimum value of $f(x) = x^2 - 4x + 5$ is:

(A) 5

(B) 1

(C) 0

(D) 4

Q53. The mean of the first 20 even natural numbers is:

(A) 20

(B) 21

(C) 22

(D) 42

Q54. The variance of the first 5 natural numbers is:

(A) 2.5

(B) 3

(C) 2

(D) 10

Q55. The value of $\int \frac{dx}{x^2+4x+5}$ is:

(A) $\ln(x^2 + 4x + 5) + C$

(B) $\frac{1}{x+2} + C$

(C) $\sin^{-1}(x + 2) + C$

(D) $\tan^{-1}(x + 2) + C$

Q56. The equation of the parabola with vertex at $(0, 0)$ and focus at $(0, 2)$ is:

(A) $y^2 = 8x$

(B) $x^2 = 4y$

(C) $x^2 = 8y$

(D) $y^2 = 4x$

Q57. The number of diagonals of a decagon is:

(A) 45

(B) 25

(C) 35

(D) 20

Q58. The shortest distance from the point $(1, 2, 3)$ to the plane $2x - y + z + 1 = 0$ is:

(A) $\frac{2\sqrt{6}}{3}$

(B) $\frac{4}{\sqrt{6}}$

(C) $\sqrt{6}$

(D) $\frac{2}{\sqrt{6}}$

Q59. If ${}^{n}P_3 = 120$, then n equals:

(A) 5

(B) 6

(C) 4

(D) 7

Q60. If $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$, then $P(A \cup B)$ equals:

(A) 0.9

(B) 0.3

(C) 0.7

(D) 0.1

Q61. The number of real roots of $x^3 - 3x + 1 = 0$ is:

(A) 1

(B) 3

(C) 2

(D) 0

Q62. The equation of the hyperbola with vertices at $(\pm 3, 0)$ and foci at $(\pm 5, 0)$ is:

(A) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(B) $\frac{x^2}{25} - \frac{y^2}{9} = 1$

(C) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(D) $\frac{x^2}{9} - \frac{y^2}{25} = 1$

Q63. If the coefficient of x^2 in $(1 + x)^n$ is 28, then n equals:

(A) 8

(B) 7

(C) 9

(D) 6

Q64. The area of the parallelogram with adjacent sides $\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = \hat{\imath} + 4\hat{\jmath}$ is:

(A) 5

(B) 11

(C) 10

(D) 14

Q65. The equation of the tangent to the parabola $y^2 = 8x$ at $(2, 4)$ is:

(A) $x + y - 2 = 0$

(B) $2x - y = 0$

(C) $x - y + 2 = 0$

(D) $x - 2y + 4 = 0$

Q66. The mode of the data $2, 3, 3, 4, 5, 5, 5, 6, 7$ is:

(A) 3

(B) 5

(C) 7

(D) 2

Q67. The derivative of $e^x \sin x$ with respect to x is:

(A) $e^x(\sin x + \cos x)$

(B) $e^x(\sin x - \cos x)$

(C) $e^x \cos x$

(D) $e^x \sin x$

Q68. The equation of the line through $(1, 2)$ perpendicular to $3x + 4y = 7$ is:

(A) $4x - 3y + 2 = 0$

(B) $3x - 4y + 5 = 0$

(C) $4x + 3y - 10 = 0$

(D) $3x + 4y - 11 = 0$

Q69. If the determinant $\begin{vmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & x \end{vmatrix} = 0$, then x equals:

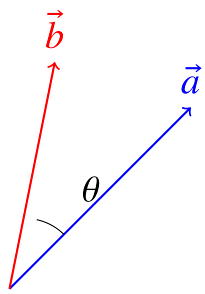
(A) 18

(B) 12

(C) 16

(D) 20

Q70. The angle between the vectors $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ is:



(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{6}$

Q71. The equation of the circle with centre at $(1, -2)$ and radius 3 is:

(A) $x^2 + y^2 + 2x - 4y - 4 = 0$

(B) $x^2 + y^2 - 2x + 4y + 4 = 0$

(C) $x^2 + y^2 - 2x - 4y - 4 = 0$

(D) $x^2 + y^2 - 2x + 4y - 4 = 0$

Q72. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, then $P(A|B)$ equals:

(A) 0.5

(B) 0.2

(C) 0.8

(D) 0.4

Q73. The value of $\int_0^1 \frac{dx}{1+x^2}$ is:

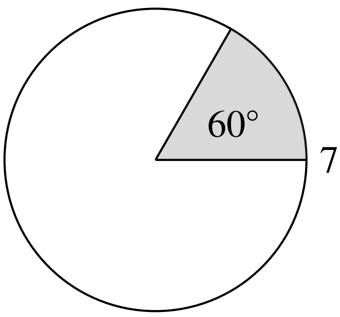
(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) 1

(D) 0

Q74. The area of a sector of a circle of radius 7 cm and angle 60° is:



(A) $\frac{49\pi}{6}$

(B) $\frac{49\pi}{3}$

(C) $\frac{7\pi}{6}$

(D) 14π

Q75. If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then A^{100} equals:

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Q76. If $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \times \vec{c} = \vec{0}$, then which of the following is true?

(A) $\vec{a} \perp \vec{b}$ and $\vec{a} \parallel \vec{c}$

(B) $\vec{a} \parallel \vec{b}$ and $\vec{a} \perp \vec{c}$

(C) $\vec{a} \perp \vec{b}$ and $\vec{a} \perp \vec{c}$

(D) $\vec{a} \parallel \vec{b}$ and $\vec{a} \parallel \vec{c}$

Q77. The slope of the line making an angle of 45° with the positive x-axis is:

(A) $\sqrt{3}$

(B) $\frac{1}{\sqrt{3}}$

(C) 0

(D) 1

Q78. The standard deviation of the data $1, 2, 3, 4, 5$ is:

(A) $\sqrt{2}$

(B) 2

(C) $\sqrt{3}$

(D) 3

Q79. The value of c in Rolle's theorem for $f(x) = x^2 - 4x + 3$ on $[1, 3]$ is:

(A) 2

(B) 1

(C) 3

(D) 2.5

Q80. The length of the latus rectum of the parabola $y^2 = 12x$ is:

(A) 6

(B) 3

(C) 12

(D) 24

Detailed Solutions

Q1.

Solution

Concept:
The standard trigonometric limit states that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$. When dealing with a variable coefficient like $\sin(kx)$, we can multiply and divide by k to reshape the expression into this fundamental form.

Solution:

(a) Given the expression $\lim_{x\to 0}\frac{\sin 3x}{x}$.

(b) To match the argument of the sine function in the denominator, multiply and divide the expression by 3:

$$\lim_{x\to 0}\frac{\sin 3x}{x} = \lim_{x\to 0}\left(3\cdot\frac{\sin 3x}{3x}\right)$$

(c) Substitute $u = 3x$. As $x \to 0$, it follows that $u \to 0$.

(d) Using the properties of limits, factor out the constant:

$$3\cdot\lim_{u\to 0}\frac{\sin u}{u} = 3\cdot 1 = 3$$

(e) This confirms that the limit of the given function evaluates directly to 3.

Final Answer: 3

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept:
The distance d between two coordinates (x_1, y_1) and (x_2, y_2) in a Cartesian coordinate system can be derived using the Pythagorean theorem, formulated as $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Solution:

(a) Identify the given points as $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (5, 5)$.

(b) Substitute these values into the standard distance formula:

$$d = \sqrt{(5 - 1)^2 + (5 - 2)^2}$$

(c) Simplify the differences inside the parentheses:

$$d = \sqrt{(4)^2 + (3)^2}$$

(d) Calculate the squares of both integers:

$$d = \sqrt{16 + 9} = \sqrt{25}$$

(e) Taking the principal square root gives $d = 5$. Thus, the physical distance separating the two coordinates is exactly 5 units.

Final Answer: 5

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept:
Matrix multiplication of a square matrix A with itself, denoted as A^2, is computed by taking the dot product of the rows of the first matrix with the columns of the second matrix.
Solution:

(a) Write the matrix multiplication explicitly for $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$:

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(b) Calculate the element in the first row, first column: $(1)(1) + (2)(3) = 1 + 6 = 7$.

(c) Calculate the element in the first row, second column: $(1)(2) + (2)(4) = 2 + 8 = 10$.

(d) Calculate the element in the second row, first column: $(3)(1) + (4)(3) = 3 + 12 = 15$.

(e) Calculate the element in the second row, second column: $(3)(2) + (4)(4) = 6 + 16 = 22$.

(f) Combine the elements back into matrix form to yield $\begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$.

Final Answer: $\begin{bmatrix} 7 & 10\ 15 & 22 \end{bmatrix}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 3

Q4.

Solution

Concept:
The dot product (scalar product) of two vectors $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ is given by multiplying corresponding spatial components and summing the results: $\vec{a}\cdot\vec{b} = a_1b_1+a_2b_2+a_3b_3$.
Solution:

(a) Given vectors are $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$.

(b) Extract the individual components: $a_1 = 2, a_2 = 3, a_3 = -1$ and $b_1 = 1, b_2 = -2, b_3 = 2$.

(c) Set up the component-wise multiplication:

$$\vec{a}\cdot\vec{b} = (2)(1) + (3)(-2) + (-1)(2)$$

(d) Evaluate each individual product:

$$\vec{a}\cdot\vec{b} = 2 - 6 - 2$$

(e) Combine the terms to find the scalar total: $2 - 6 - 2 = -6$.

Final Answer: -6

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept:
The double-angle identity for cosines allows us to determine $\cos 2\theta$ directly from $\sin\theta$ using the trigonometric formula $\cos 2\theta = 1 - 2\sin^2\theta$.

Solution:

(a) We are given that $\sin\theta = \frac{3}{5}$.

(b) Write down the relevant double-angle identity:

$$\cos 2\theta = 1 - 2\sin^2\theta$$

(c) Substitute the known value of $\sin\theta$ into the formula:

$$\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2$$

(d) Square the fraction:

$$\cos 2\theta = 1 - 2\left(\frac{9}{25}\right) = 1 - \frac{18}{25}$$

(e) Find a common denominator to subtract the values:

$$\cos 2\theta = \frac{25 - 18}{25} = \frac{7}{25}$$

Final Answer: $7_{\overline{25}}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept:

The arithmetic mean of an arithmetic sequence is the sum of all terms divided by the total count n. The sum of the first n natural numbers is evaluated using the formula $S_n = \frac{n(n+1)}{2}$.

Solution:

(a) We need to determine the mean of the first 10 natural numbers ($n = 10$).

(b) Calculate the total sum of these integers using the summation formula:

$$\sum_{i=1}^{10} i = \frac{10(10+1)}{2} = \frac{10 \times 11}{2} = 55$$

(c) The mathematical formula for the arithmetic mean is Mean $= \frac{\text{Sum}}{n}$.

(d) Substitute our calculated sum and count into the equation:

$$\text{Mean} = \frac{55}{10} = 5.5$$

(e) Thus, the central value of this numerical set is exactly 5.5.

Final Answer: 5.5

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:

To find the derivative of a composite function like $y = f(g(x))$, we apply the Chain Rule: $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$. For $f(u) = \ln u$, the derivative is $\frac{1}{u}$.

Solution:

(a) Identify the outer function as $\ln(u)$ and the inner function as $u = \cos x$.

(b) Differentiate the outer logarithmic layer with respect to its inner function:

$$\frac{d}{du}(\ln u) = \frac{1}{u} \implies \frac{1}{\cos x}$$

(c) Find the derivative of the inner function with respect to x:

$$\frac{d}{dx}(\cos x) = -\sin x$$

(d) Combine both parts using the Chain Rule:

$$\frac{dy}{dx} = \frac{1}{\cos x} \cdot (-\sin x) = -\frac{\sin x}{\cos x}$$

(e) Use the basic trigonometric identity $\frac{\sin x}{\cos x} = \tan x$ to simplify the fraction to $-\tan x$.

Final Answer: -tan x

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept:
The slope m of a straight line connecting two separate coordinate points (x_1, y_1) and (x_2, y_2) is defined as the ratio of the vertical change to the horizontal change: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
Solution:

(a) Identify the given points from the problem: $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 7)$.

(b) Substitute these coordinate values into the rise-over-run slope equation:

$$m = \frac{7 - 3}{4 - 2}$$

(c) Simplify the numerator by subtracting the y-values: $7 - 3 = 4$.

(d) Simplify the denominator by subtracting the x-values: $4 - 2 = 2$.

(e) Divide the final numbers to find the slope:

$$m = \frac{4}{2} = 2$$

Final Answer: 2

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept:

The sum of the first n terms of an Arithmetic Progression (AP) is computed using $S_n = \frac{n}{2}[2a + (n-1)d]$, where a denotes the first term and d represents the common difference.

Solution:

(a) Analyze the given progression: $3, 7, 11, \ldots$ to find the parameters.

(b) The initial term is $a = 3$. The common difference is found by subtracting consecutive terms: $d = 7 - 3 = 4$. We are summing $n = 15$ terms.

(c) Set up the calculation within the arithmetic summation formula:

$$S_{15} = \frac{15}{2}[2(3) + (15-1)4]$$

(d) Simplify the arithmetic operations inside the square brackets:

$$S_{15} = \frac{15}{2}[6 + (14)4] = \frac{15}{2}[6 + 56] = \frac{15}{2}[62]$$

(e) Divide 62 by 2 and multiply by 15:

$$S_{15} = 15 \times 31 = 465$$

Final Answer: 465

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 9

Q10.

Solution

Concept:
To differentiate a variable base raised to a variable power, $f(x) = x^x$, we employ logarithmic differentiation by taking the natural logarithm of both sides to convert the exponent into a product.
Solution:

(a) Let $y = x^x$. Take the natural logarithm of both sides:

$$\ln y = \ln(x^x) \implies \ln y = x \ln x$$

(b) Perform implicit differentiation with respect to x on both sides using the product rule on the right:

$$\frac{1}{y}\frac{dy}{dx} = (1)(\ln x) + x\left(\frac{1}{x}\right) = \ln x + 1$$

(c) Multiply through by y to solve explicitly for the derivative function $f'(x)$:

$$f'(x) = y(\ln x + 1) = x^x(\ln x + 1)$$

(d) Substitute $x = 1$ to evaluate the value at the given point:

$$f'(1) = 1^1(\ln 1 + 1) = 1(0 + 1) = 1$$

Final Answer: 1

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept:
To evaluate trigonometric values for non-standard angles such as 15°, we can express the angle as the difference between two well-known standard angles, namely 45° and 30°, and then apply the standard tangent subtraction formula.

Solution:

(a) Express the angle 15° as a difference of two standard angles whose trigonometric values are readily known: $15° = 45° - 30°$.

(b) Recall the standard trigonometric subtraction identity for the tangent function:

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(c) Substitute $A = 45°$ and $B = 30°$ directly into this identity:

$$\tan 15° = \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$$

(d) Insert the known values $\tan 45° = 1$ and $\tan 30° = \frac{1}{\sqrt{3}}$ into the expression:

$$\tan 15° = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

(e) Rationalize the denominator by multiplying both the numerator and the denominator by the conjugate expression $\sqrt{3} - 1$:

$$\tan 15° = \frac{(\sqrt{3} - 1)^2}{(\sqrt{3} + 1)(\sqrt{3} - 1)} = \frac{3 + 1 - 2\sqrt{3}}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

Final Answer: 2-$\sqrt{3}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:
The coefficients of the variables x, y, and z in the general linear equation of a plane $ax + by + cz = d$ represent the components of a vector normal to that plane. A unit normal vector is obtained by dividing this vector by its magnitude.

Solution:

(a) Identify the given scalar equation of the plane from the problem statement, which is $x + y + z = 1$.

(b) Extract the coefficients of x, y, and z to write down the normal vector $\vec{n}$ to the plane:

$$\vec{n} = 1\hat{i} + 1\hat{j} + 1\hat{k} = \hat{i} + \hat{j} + \hat{k}$$

(c) Calculate the scalar magnitude of this normal vector using the standard Euclidean norm formula:

$$|\vec{n}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

(d) Determine the unit normal vector $\hat{n}$ by dividing the normal vector by its calculated magnitude:

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$$

(e) This unit vector points directly perpendicular to the entire surface area of the given plane.

Final Answer: $+ \; + \; \frac{}{\sqrt{3}}$

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 12

Q13.

Solution

Concept:
To find the indefinite integral of a product of an algebraic function and an exponential function, we utilize the integration by parts technique, which is formulated mathematically as $\int u\,dv = uv - \int v\,du$.

Solution:

(a) State the given indefinite integral that needs evaluation: $\int xe^x\,dx$.

(b) Select the functions for integration by parts using the LIATE rule. Let the algebraic term be $u = x$ and the exponential term be $dv = e^x\,dx$.

(c) Compute the differential of u and find the antiderivative of dv:

$$du = dx \quad \text{and} \quad v = \int e^x\,dx = e^x$$

(d) Apply the integration by parts formula directly with these terms:

$$\int xe^x\,dx = xe^x - \int e^x\,dx$$

(e) Evaluate the remaining basic integral on the right side and append the constant of integration C:

$$\int xe^x\,dx = xe^x - e^x + C$$

Final Answer: x e$^x - e^x + C$

Answer: (D)

Go Back to Question 13

Q14.

Solution

Concept:
The general second-degree equation representing a circle in a two-dimensional Cartesian coordinate system is expressed as $x^2 + y^2 + 2gx + 2fy + c = 0$, where the geometric center of the circle is located at the coordinates $(-g, -f)$.
Solution:

(a) Write down the given equation of the circle from the problem description:

$$x^2 + y^2 - 4x + 6y - 3 = 0$$

(b) Compare this given expression side-by-side with the standard general form to determine the parameters:

$$2g = -4 \implies g = -2$$

$$2f = 6 \implies f = 3$$

(c) Recall that the formula for the coordinates of the center point is defined as $(-g, -f)$.

(d) Substitute the values of g and f calculated above into the center coordinate expression:

$$\text{Center} = (-(-2), -(3)) = (2, -3)$$

(e) Thus, the central point of the circle is located precisely at $(2, -3)$ in the coordinate plane.

Final Answer: (2, -3)

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 14

Q15.

Solution

Concept:

For a standard quadratic equation written in the form $ax^2 + bx + c = 0$, the sum of its roots is given by $\alpha + \beta = -\frac{b}{a}$ and the product of its roots is $\alpha\beta = \frac{c}{a}$. These relationships can be rearranged to find the sum of the squares of the roots.

Solution:

(a) Identify the coefficients from the given quadratic equation $x^2 - 5x + 6 = 0$: $a = 1$, $b = -5$, and $c = 6$.

(b) Determine the sum of the roots using the coefficient relationship:

$$\alpha + \beta = -\frac{-5}{1} = 5$$

(c) Determine the product of the roots using the constant term relationship:

$$\alpha\beta = \frac{6}{1} = 6$$

(d) Use the algebraic identity for the sum of squares to rewrite the required expression:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

(e) Substitute the calculated numeric sum and product into this identity:

$$\alpha^2 + \beta^2 = (5)^2 - 2(6) = 25 - 12 = 13$$

Final Answer: 13

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 15

Q16.

Solution

Concept:
When an integrand contains both a function and its derivative, the method of integration by substitution is highly effective. Substituting $u = f(x)$ transforms the differential element via $du = f'(x)\,dx$, simplifying the integral into a standard form.

Solution:

(a) State the indefinite integral to be calculated: $\int \frac{dx}{x \ln x}$.

(b) Observe that the derivative of the natural logarithm function, $\frac{d}{dx}(\ln x) = \frac{1}{x}$, is present as a factor within the integrand.

(c) Choose the substitution variable $u = \ln x$. Differentiating both sides gives:

$$du = \frac{1}{x}\,dx$$

(d) Substitute these expressions back into the original integral to transform it into terms of u:

$$\int \frac{1}{\ln x} \cdot \left(\frac{1}{x}\,dx\right) = \int \frac{1}{u}\,du$$

The integration of this basic reciprocal form results in a natural logarithm:

$$\int \frac{1}{u}\,du = \ln|u| + C$$

(e) Substitute the original expression for u back into the result to obtain $\ln|\ln x| + C$.

Final Answer: $\ln|\ln x| + C$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept:

The fundamental complementary identity for inverse trigonometric functions states that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for any value of x within the domain $[-1, 1]$. This relationship allows for a direct comparison between variable terms.

Solution:

(a) We are given the inverse trigonometric equation:

$$\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$$

(b) Isolate the term containing y on one side of the equation:

$$\cos^{-1} y = \frac{\pi}{2} - \cos^{-1} x$$

(c) Utilize the well-known inverse trigonometric identity $\frac{\pi}{2} - \cos^{-1} x = \sin^{-1} x$:

$$\cos^{-1} y = \sin^{-1} x$$

(d) Apply the cosine function to both sides of the equation to eliminate the inverse cosine on the left:

$$y = \cos(\sin^{-1} x)$$

(e) Use the standard geometric identity $\cos(\sin^{-1} x) = \sqrt{1 - x^2}$ to relate the variables:

$$y = \sqrt{1 - x^2}$$

(f) Square both sides of the equation and rearrange terms:

$$y^2 = 1 - x^2 \implies x^2 + y^2 = 1$$

Final Answer: 1

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 17

Q18.

Solution

Concept:
The distance d between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ located in a three-dimensional Euclidean space is calculated using the generalized distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.
Solution:

(a) Identify the coordinates of the two given points: $(x_1, y_1, z_1) = (1, 2, 3)$ and $(x_2, y_2, z_2) = (4, 6, 9)$.

(b) Substitute these coordinate values into the three-dimensional distance formula:

$$d = \sqrt{(4-1)^2 + (6-2)^2 + (9-3)^2}$$

(c) Evaluate the differences within each set of parentheses:

$$d = \sqrt{(3)^2 + (4)^2 + (6)^2}$$

(d) Compute the squares of each individual integer:

$$d = \sqrt{9 + 16 + 36}$$

(e) Sum the values inside the square root to find the final radicand:

$$d = \sqrt{61}$$

Final Answer: $\sqrt{61}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 18

Q19.

Solution

Concept:
To determine the local maximum of a function, we find its critical points where the first derivative equals zero, $f'(x) = 0$. We then apply the second derivative test; if $f''(x) < 0$ at a critical point, that point is a local maximum.
Solution:

(a) Given the cubic function: $f(x) = 2x^3 - 9x^2 + 12x - 3$.

(b) Compute the first derivative with respect to x:

$$f'(x) = 6x^2 - 18x + 12$$

(c) Set the first derivative to zero to locate all critical points:

$$6(x^2 - 3x + 2) = 0 \implies 6(x-1)(x-2) = 0 \implies x = 1, x = 2$$

(d) Compute the second derivative of the function:

$$f''(x) = 12x - 18$$

(e) Evaluate the second derivative at both critical points:

$$\text{At } x = 1 : f''(1) = 12(1) - 18 = -6 < 0 \quad \text{(Local Maximum)}$$

$$\text{At } x = 2 : f''(2) = 12(2) - 18 = 6 > 0 \quad \text{(Local Minimum)}$$

(f) Calculate the local maximum value by evaluating the original function at $x = 1$:

$$f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 3 = 2 - 9 + 12 - 3 = 2$$

Final Answer: 2 at x = 1

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 19

Q20.

Solution

Concept:
The area of a triangle defined on a coordinate plane can be determined using the standard geometric formula Area $= \frac{1}{2} \times \text{base} \times \text{height}$ when the base and height align perfectly along perpendicular coordinate axes.

Solution:

(a) Identify the given vertices of the triangle from the coordinate diagram: $O(0,0)$, $A(4,0)$, and $B(0,3)$.

(b) Notice that vertex $A(4,0)$ lies entirely on the horizontal x-axis, meaning the base of the triangle stretches from $(0,0)$ to $(4,0)$.

$$\text{Length of base} = 4 - 0 = 4 \text{ units}$$

(c) Notice that vertex $B(0,3)$ lies entirely on the vertical y-axis, meaning the height of the triangle stretches from $(0,0)$ to $(0,3)$.

$$\text{Length of height} = 3 - 0 = 3 \text{ units}$$

(d) Since the x-axis and y-axis intersect at a right angle, this forms a right-angled triangle.

(e) Substitute these lengths into the standard triangular area formula:

$$\text{Area} = \frac{1}{2} \times 4 \times 3 = \frac{12}{2} = 6 \text{ square units}$$

Final Answer: 6

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 20

Q21.

Solution

Concept:
The fundamental property of combinations states that if two combinations of the same set size are equal, namely ${}^nC_x = {}^nC_y$, then either the selection sizes are identical ($x = y$) or they are complementary ($x + y = n$). This symmetric property arises because choosing x items is equivalent to leaving $n - x$ items behind.
Solution:

(a) Identify the given equation involving combinations from the problem statement:

$$ {}^nC_5 = {}^nC_9 $$

(b) Note that the selection sizes are distinct since $5 \neq 9$. Therefore, we must apply the complementary relationship rule for combinations.

(c) According to the standard combinatorial identity:

$$ \text{If } {}^nC_x = {}^nC_y \text{ and } x \neq y, \text{ then } x + y = n $$

(d) Assign the given parameters to the variables where $x = 5$ and $y = 9$.

(e) Substitute these values directly into the addition identity to find the value of n:

$$ n = 5 + 9 = 14 $$

(f) This confirms that choosing 5 objects out of 14 yields the exact same number of possibilities as selecting 9 objects.

Final Answer: 14

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept:
The order of an ordinary differential equation is defined as the highest derivative present in the equation. The degree of a differential equation is the power or exponent to which the highest-order derivative is raised, provided the equation can be expressed as a polynomial in terms of its derivatives.

Solution:

(a) Write down the ordinary differential equation given in the problem text:

$$y' + y^2 = x^3$$

(b) Rewrite the derivative term using standard Leibniz notation to clearly identify its composition:

$$\frac{dy}{dx} + y^2 = x^3$$

(c) Inspect the equation to determine the highest derivative present. The term $\frac{dy}{dx}$ represents a first-order derivative. Thus, the order of the equation is 1.

(d) Look at the exponent associated with this highest-order derivative. The term $\left(\frac{dy}{dx}\right)$ is raised to the power of 1.

(e) Since the equation is already in a polynomial form with respect to its derivatives, the degree is equal to this power, which is 1.

Final Answer: 1, 1

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 22

Q23.

Solution

Concept:
To find the value of a triple-angle trigonometric function, we can apply the standard sine triple-angle identity: $\sin 3A = 3\sin A - 4\sin^3 A$. This identity allows us to compute the output value directly using only the value of $\sin A$.

Solution:

(a) Identify the given numeric value from the problem statement:

$$\sin A = \frac{1}{2}$$

(b) Recall the standard triple-angle trigonometric formula for the sine function:

$$\sin 3A = 3\sin A - 4\sin^3 A$$

(c) Substitute the fraction value $\frac{1}{2}$ into the algebraic identity in place of $\sin A$:

$$\sin 3A = 3\left(\frac{1}{2}\right) - 4\left(\frac{1}{2}\right)^3$$

(d) Evaluate the cubic power in the second term of the expression:

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

(e) Multiply out the constants and simplify the resulting fractions:

$$\sin 3A = \frac{3}{2} - 4\left(\frac{1}{8}\right) = \frac{3}{2} - \frac{1}{2}$$

(f) Combine the terms over the common denominator to obtain the final integer result:

$$\sin 3A = \frac{3-1}{2} = \frac{2}{2} = 1$$

Final Answer: 1

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 23

Q24.

Solution

Concept:
The direction ratios of a straight line passing through two distinct spatial points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are proportional to the coordinate differences between those points, calculated as $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$. Any scalar multiple of these differences also serves as valid direction ratios.

Solution:

(a) Identify the coordinates of the two given spatial points:

$$P = (1, 2, 3) \quad \text{and} \quad Q = (4, 5, 6)$$

(b) Compute the differences between the corresponding coordinates of Q and P:

$$\Delta x = 4 - 1 = 3$$

$$\Delta y = 5 - 2 = 3$$

$$\Delta z = 6 - 3 = 3$$

(c) Write down the primary set of calculated direction ratios based on these differences: $(3, 3, 3)$.

(d) Since direction ratios represent a directional orientation, they can be scaled by dividing by a common factor. Dividing each component by 3 gives the simplest ratio set:

$$\left(\frac{3}{3}, \frac{3}{3}, \frac{3}{3}\right) = (1, 1, 1)$$

(e) Match this result with the options provided in the question. The option list includes the unsimplified coordinate differences $(3, 3, 3)$.

Final Answer: (3,3,3)

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 24

Q25.

Solution

Concept:
The geometric area bounded by a continuous curve $y = f(x)$, vertical boundary lines $x = a$ and $x = b$, and the horizontal x-axis is calculated by evaluating the definite integral $\int_a^b f(x)\,dx$. This accumulates the infinitesimal rectangular strips under the curve.

Solution:

(a) Identify the bounding curve function and the interval limits from the given description:

$$f(x) = x^2, \quad a = 0, \quad b = 2$$

(b) Set up the definite area integral using these integration limits:

$$\text{Area} = \int_0^2 x^2\,dx$$

(c) Find the standard algebraic antiderivative of the function x^2 using the power rule:

$$\int x^2\,dx = \frac{x^3}{3}$$

(d) Apply the Fundamental Theorem of Calculus to evaluate this antiderivative at the boundary limits:

$$\text{Area} = \left[\frac{x^3}{3}\right]_0^2$$

(e) Substitute the upper limit 2 and lower limit 0 into the expression:

$$\text{Area} = \frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3} - 0 = \frac{8}{3}$$

Final Answer: 8_3

Answer: (C)

Go Back to Question 25

Q26.

Solution

Concept:
The equation of a tangent line to a standard circle $x^2+y^2 = r^2$ at a specific point on its circumference (x_1, y_1) can be determined directly using the linearized transformation formula $xx_1 + yy_1 = r^2$.
Solution:

(a) Write down the standard circle equation given in the problem statement:

$$x^2 + y^2 = 25$$

(b) Identify the specific point of tangency on the circle: $(x_1, y_1) = (3, 4)$.

(c) Recall the standard point-form tangent line equation for a circle at the origin:

$$xx_1 + yy_1 = r^2$$

(d) Substitute the point values $x_1 = 3$ and $y_1 = 4$ along with the radius squared constant $r^2 = 25$ directly into the linear equation:

$$x(3) + y(4) = 25$$

(e) Arrange the terms into standard linear format:

$$3x + 4y = 25$$

(f) This line touches the circle exactly at $(3, 4)$ and is perpendicular to the radial line connecting the origin to that point.

Final Answer: 3x + 4y = 25

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 26

Q27.

Solution

Concept:
The complex cube roots of unity, denoted as 1, ω, and ω^2, satisfy two core algebraic properties: the sum of all three roots is zero ($1 + \omega + \omega^2 = 0$), and the cube of the primary root equals one ($\omega^3 = 1$). These relations allow for the simplification of complex polynomials.

Solution:

(a) State the algebraic expression that requires simplification:

$$(1 + \omega)(1 + \omega^2)$$

(b) Use the fundamental identity $1 + \omega + \omega^2 = 0$ to express the binomial terms in simpler single-term forms:

$$1 + \omega = -\omega^2$$

$$1 + \omega^2 = -\omega$$

(c) Substitute these simplified single terms back into the original product expression:

$$(1 + \omega)(1 + \omega^2) = (-\omega^2)(-\omega)$$

(d) Multiply the terms together, taking into account that the product of two negative signs results in a positive sign:

$$(-\omega^2)(-\omega) = \omega^3$$

(e) Apply the second core property of the cube roots of unity, which states that $\omega^3 = 1$:

$$\omega^3 = 1$$

Final Answer: 1

$\boxed{\textbf{Answer: (D)}}$

Go Back to Question 27

Q28.

Solution

Concept:
When evaluating limits that result in an indeterminate form of type $\frac{0}{0}$, we can use L'Hôpital's Rule. This rule states that the limit of a fraction of functions is equal to the limit of the fraction of their derivatives, provided the limit exists.

Solution:

(a) State the limit expression to be evaluated:

$$\lim_{x\to 0} \frac{\tan x - x}{x^3}$$

(b) Substitute $x = 0$ into the expression to check its form. Since $\tan 0 - 0 = 0$ and $0^3 = 0$, it produces an indeterminate form of $\frac{0}{0}$.

(c) Apply L'Hôpital's Rule by differentiating the numerator and denominator with respect to x:

$$\lim_{x\to 0} \frac{\sec^2 x - 1}{3x^2}$$

(d) Use the trigonometric identity $\sec^2 x - 1 = \tan^2 x$ to rewrite the numerator:

$$\lim_{x\to 0} \frac{\tan^2 x}{3x^2} = \frac{1}{3} \lim_{x\to 0} \left(\frac{\tan x}{x}\right)^2$$

(e) Recall the standard fundamental trigonometric limit identity, which states that $\lim_{x\to 0} \frac{\tan x}{x} = 1$.

(f) Substitute this limit value back into our simplified expression:

$$\frac{1}{3} \cdot (1)^2 = \frac{1}{3}$$

Final Answer: 1_3

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 28

Q29.

Solution

Concept:
Right-angle trigonometry connects angles and side lengths in triangles. The tangent of an angle in a right triangle is defined as the ratio of the length of the opposite side to the length of the adjacent side ($\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$).

Solution:

(a) Identify the given elements from the word problem and geometric diagram:

$$\text{Adjacent distance from foot } (AB) = 20 \text{ m}$$

$$\text{Angle of elevation } (\theta) = 30^\circ$$

$$\text{Height of the tower } (BC) = h$$

(b) Formulate the trigonometric relation using the tangent function for right triangle ABC:

$$\tan 30^\circ = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{h}{20}$$

(c) Recall the exact radical value for the tangent of a 30° angle:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

(d) Equate the two expressions to set up a linear equation for h:

$$\frac{1}{\sqrt{3}} = \frac{h}{20}$$

(e) Isolate the variable h by multiplying both sides by 20:

$$h = \frac{20}{\sqrt{3}} \text{ metres}$$

Final Answer: $20\frac{}{\sqrt{3}}$

$\boxed{\textbf{Answer: (D)}}$

Go Back to Question 29

Q30.

Solution

Concept:
The scalar equation of a plane passing through a specific point (x_0, y_0, z_0) with a known normal vector $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$ is given by the vector dot product equation $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$.
Solution:

(a) Identify the given coordinates of the point on the plane: $(x_0, y_0, z_0) = (1, 2, 3)$.

(b) Identify the components of the perpendicular normal vector $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$, which gives the coefficients: $a = 2$, $b = -3$, and $c = 4$.

(c) Substitute these point coordinates and coefficients into the point-normal plane formula:

$$2(x - 1) - 3(y - 2) + 4(z - 3) = 0$$

(d) Expand the terms by distributing the coefficients across the parentheses:

$$2x - 2 - 3y + 6 + 4z - 12 = 0$$

(e) Group the variable terms together and combine the scalar constants:

$$2x - 3y + 4z - 8 = 0$$

(f) Move the constant to the right side to get the standard form:

$$2x - 3y + 4z = 8$$

Final Answer: 2x - 3y + 4z = 8

Answer: (B)

Go Back to Question 30

Q31.

Solution

Concept:
The order of a differential equation represents the highest derivative appearing in it. The degree of a differential equation represents the power of the highest derivative present when the differential coefficients are free from radicals and fractions.

Solution:

(a) Write down the given differential equation:

$$(y'')^2 + (y')^3 + y = 0$$

(b) Identify the derivatives in the equation, where $y'' = \frac{d^2y}{dx^2}$ is the second derivative and $y' = \frac{dy}{dx}$ is the first derivative.

(c) Determine the highest order derivative present in this expression, which is y''. Thus, the order is 2.

(d) Look at the exponent to which the highest order derivative is raised. Here, the term $(y'')^2$ is raised to the power of 2.

(e) Since the equation is a polynomial in terms of its derivatives, the degree is the power of this highest derivative, which is 2.

Final Answer: 2, 2

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept:

For an ellipse in standard form, the relationship between the semi-major axis a, the semi-minor axis b, and the eccentricity e describes how elongated the conic section is. The eccentricity formula for a horizontal ellipse is given by $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution:

(a) Convert the given equation of the ellipse into standard form by dividing both sides by 144:

$$\frac{9x^2}{144} + \frac{16y^2}{144} = 1 \implies \frac{x^2}{16} + \frac{y^2}{9} = 1$$

(b) Compare this with the standard equation of a horizontal ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a^2 > b^2$.

(c) Identify the parameters: $a^2 = 16$ and $b^2 = 9$.

(d) Use the standard eccentricity formula for an ellipse:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

(e) Substitute the values into the formula:

$$e = \sqrt{1 - \frac{9}{16}} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$$

Final Answer: $\sqrt{7}_4$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 32

Q33.

Solution

Concept:

A matrix M is symmetric if $M^T = M$ and skew-symmetric if $M^T = -M$. Using matrix transpose properties, specifically $(A - B)^T = A^T - B^T$ and $(AB)^T = B^T A^T$, we can evaluate the properties of combined matrix expressions.

Solution:

(a) Identify the properties of the given symmetric matrices A and B, which mean $A^T = A$ and $B^T = B$.

(b) Let the matrix expression be $M = AB - BA$. Take the transpose of the entire matrix combination:

$$M^T = (AB - BA)^T$$

(c) Apply the subtraction rule for matrix transposes:

$$M^T = (AB)^T - (BA)^T$$

(d) Apply the reversal law of multiplication for transposes:

$$M^T = B^T A^T - A^T B^T$$

(e) Substitute $A^T = A$ and $B^T = B$ into the expanded expression:

$$M^T = BA - AB$$

(f) Factor out a negative sign to compare it with the original matrix M:

$$M^T = -(AB - BA) = -M$$

(g) Since $M^T = -M$, the resulting matrix is skew-symmetric.

Final Answer: skew-symmetric

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept:
To find the absolute maximum value of a continuous function on a closed interval $[a, b]$, calculate the values of the function at its critical points where $f'(x) = 0$ inside the interval and evaluate the function at the boundary endpoints.
Solution:

(a) Write down the function and its closed interval: $f(x) = x^3 - 6x^2 + 9x + 1$ on $[0, 4]$.

(b) Compute the first derivative of the function to find critical numbers:

$$f'(x) = 3x^2 - 12x + 9$$

(c) Set the derivative to zero and solve the quadratic equation:

$$3(x^2 - 4x + 3) = 0 \implies 3(x - 1)(x - 3) = 0 \implies x = 1, x = 3$$

(d) Both critical values lie within the interval $[0, 4]$. Evaluate the function at the endpoints and critical points:

$$f(0) = 0^3 - 6(0)^2 + 9(0) + 1 = 1$$

$$f(1) = 1^3 - 6(1)^2 + 9(1) + 1 = 5$$

$$f(3) = 3^3 - 6(3)^2 + 9(3) + 1 = 1$$

$$f(4) = 4^3 - 6(4)^2 + 9(4) + 1 = 17$$

(e) Compare the computed values. The absolute maximum value is 17 at $x = 4$.

Final Answer: 17

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 34

Q35.

Solution

Concept:
Double-angle trigonometric expressions can be computed directly using tangent relationships. The standard identity that converts the double-angle sine value using only the tangent function is $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$.

Solution:

(a) State the given value from the problem text: $\tan\theta = 2$.

(b) Recall the double-angle formula for the sine function expressed in terms of tangent:

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

(c) Substitute the numeric value of $\tan\theta$ directly into this rational expression:

$$\sin 2\theta = \frac{2(2)}{1+(2)^2}$$

(d) Calculate the values in the numerator and denominator:

$$\text{Numerator} = 2 \times 2 = 4$$

$$\text{Denominator} = 1 + 4 = 5$$

(e) Write the final simplified fraction:

$$\sin 2\theta = \frac{4}{5}$$

Final Answer: $4_{\overline{5}}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 35

Q36.

Solution

Concept:
The angle θ between a line with direction vector $\vec{b}$ and a plane with normal vector $\vec{n}$ is computed using the sine relation $\sin\theta = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$. When the line vector and normal vector are parallel, the line is perpendicular to the plane.

Solution:

(a) Extract the direction vector of the line from the parameter equation: $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

(b) Extract the coefficients from the Cartesian plane equation $1x + 1y + 1z = 5$ to get the normal vector: $\vec{n} = \hat{i} + \hat{j} + \hat{k}$.

(c) Observe that the line direction vector $\vec{b}$ and the normal vector $\vec{n}$ are identical, meaning they point in the exact same direction.

(d) Since the line is parallel to the normal vector of the plane, it must run directly perpendicular to the plane surface itself.

(e) Therefore, the geometric angle between the line path and the plane surface is 90°.

Final Answer: 90°

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 36

Q37.

Solution

Concept:
The absolute value function $f(x) = |x|$ changes behavior at the origin. Continuity requires that the left-hand limit, right-hand limit, and function value are all equal. Differentiability requires that the slope matches from both sides.

Solution:

(a) Examine the continuity of $f(x) = |x|$ at $x = 0$. Evaluate the limit from both sides:

$$\lim_{x \to 0^-} (-x) = 0, \quad \lim_{x \to 0^+} (x) = 0, \quad f(0) = 0$$

Since all values match, the function is continuous at the origin.

(b) Examine the differentiability at $x = 0$ by finding the left-hand and right-hand derivatives.

(c) Calculate the left-hand derivative ($x < 0$):

$$f'(0^-) = \lim_{h \to 0^-} \frac{-h - 0}{h} = -1$$

(d) Calculate the right-hand derivative ($x > 0$):

$$f'(0^+) = \lim_{h \to 0^+} \frac{h - 0}{h} = 1$$

(e) Since the left-hand slope (-1) does not equal the right-hand slope (1), the derivative does not exist at $x = 0$. The function has a sharp point there, making it continuous but not differentiable.

Final Answer: continuous but not differentiable

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept:
By definition, a parabola is the locus of a moving point whose distance from a fixed point (focus) is equal to its perpendicular distance from a fixed straight line (directrix). We use the distance formula to equate these geometric properties.

Solution:

(a) Let an arbitrary moving point on the locus be denoted by coordinates $P(x, y)$.

(b) The given fixed point is the focus $F(2, 0)$. Use the coordinate distance formula to find the distance from P to F:

$$PF = \sqrt{(x-2)^2 + (y-0)^2}$$

(c) The given fixed line is the y-axis, which has the line equation $x = 0$. The perpendicular distance from $P(x, y)$ to this line is simply $|x|$.

(d) Equate the two geometric distances according to the definition given in the prompt:

$$\sqrt{(x-2)^2 + y^2} = |x|$$

(e) Square both sides of the equation to eliminate the radical sign:

$$(x-2)^2 + y^2 = x^2$$

(f) Expand the squared binomial term and cancel common elements:

$$x^2 - 4x + 4 + y^2 = x^2 \implies y^2 - 4x + 4 = 0$$

(g) Rearrange the final terms into standard form:

$$y^2 = 4(x-1)$$

Final Answer: $y^2 = 4(x-1)$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 38

Q39.

Solution

Concept:
The sum of the first n terms of a geometric progression (GP) with a starting term a and a common ratio r is calculated using the algebraic summation formula $S_n = \frac{a(1-r^n)}{1-r}$.
Solution:

(a) Identify the parameters of the given geometric series: $1, -\frac{1}{2}, \frac{1}{4}, \ldots$

(b) The first term is $a = 1$. Find the common ratio by dividing the second term by the first term:

$$r = \frac{-1/2}{1} = -\frac{1}{2}$$

(c) Set the total number of terms to be added: $n = 10$.

(d) Substitute these variables into the geometric series sum formula:

$$S_{10} = \frac{1 \cdot \left[1 - \left(-\frac{1}{2}\right)^{10}\right]}{1 - \left(-\frac{1}{2}\right)}$$

(e) Evaluate the exponent and solve the fraction arithmetic:

$$\left(-\frac{1}{2}\right)^{10} = \frac{1}{1024} \implies 1 - \frac{1}{1024} = \frac{1023}{1024}$$

$$\text{Denominator} = 1 + \frac{1}{2} = \frac{3}{2}$$

(f) Multiply the numerator fraction by the reciprocal of the denominator:

$$S_{10} = \frac{1023}{1024} \times \frac{2}{3} = \frac{341}{512}$$

Final Answer: $341\frac{}{512}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 39

Q40.

Solution

Concept:

The area bounded by a non-negative function curve $y = f(x)$ and the x-axis from $x = a$ to $x = b$ is given by evaluating the definite integral $\int_a^b f(x)\,dx$. Since $\sin x \geq 0$ on the interval $[0, \pi]$, the area matches the standard integral value.

Solution:

(a) Set up the definite integral expression to calculate the area under the sine curve:

$$\text{Area} = \int_0^{\pi} \sin x\,dx$$

(b) Recall the standard trigonometric antiderivative for the sine function, which is $-\cos x$:

$$\text{Area} = \Big[-\cos x\Big]_0^{\pi}$$

(c) Apply the Fundamental Theorem of Calculus by substituting the upper and lower limits:

$$\text{Area} = (-\cos\pi) - (-\cos 0)$$

(d) Substitute the known values for the cosine function at these points, where $\cos\pi = -1$ and $\cos 0 = 1$:

$$\text{Area} = (-(-1)) - (-1) = 1 + 1 = 2$$

Final Answer: 2

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 40

Q41.

Solution

Concept:
The principal value branch of the inverse tangent function $\tan^{-1} x$ is restricted to the open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. To determine the principal angle, we identify an angle θ within this valid domain such that $\tan\theta$ matches the given ratio.

Solution:

(a) Write down the expression to evaluate: $\theta = \tan^{-1}(-1)$.

(b) Recall the general identity for negative arguments in inverse tangent calculations, which states that $\tan^{-1}(-x) = -\tan^{-1} x$.

(c) Apply this identity directly to the problem to factor out the negative sign: $\theta = -\tan^{-1}(1)$.

(d) Identify the standard angle in the first quadrant whose tangent equals 1, which is $\frac{\pi}{4}$ radians because $\tan\left(\frac{\pi}{4}\right) = 1$.

(e) Substitute this value back to get the final result: $\theta = -\frac{\pi}{4}$. This value lies correctly within the principal interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Final Answer: -$\pi_{\overline{4}}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept:
The vector cross product between two three-dimensional vectors $\vec{a}$ and $\vec{b}$ determines a vector perpendicular to both. It is evaluated by computing the determinant of a 3×3 matrix where the first row consists of the standard unit basis vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

Solution:

(a) Set up the cross product formula as a matrix determinant using the given vectors $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix}$$

(b) Expand the determinant along the first row:

$$\vec{a} \times \vec{b} = \hat{\imath} \begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} - \hat{\jmath} \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + \hat{k} \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix}$$

(c) Calculate the 2×2 minor determinants:

$$\hat{\imath} \text{ component} : (1)(3) - (1)(-1) = 3 + 1 = 4$$

$$\hat{\jmath} \text{ component} : -[(1)(3) - (1)(2)] = -[3 - 2] = -1$$

$$\hat{k} \text{ component} : (1)(-1) - (1)(2) = -1 - 2 = -3$$

(d) Combine the components to get the final vector: $4\hat{\imath} - \hat{\jmath} - 3\hat{k}$.

Final Answer: 4 - - 3

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept:
To find the definite integral of a squared trigonometric function, use a double-angle reduction identity. Specifically, replace $\sin^2 x$ with the standard linear relationship $\frac{1-\cos 2x}{2}$ to find its antiderivative before substituting limits.

Solution:

(a) Use the double-angle trigonometric formula to rewrite the integrand:

$$\int_0^{\pi/2} \sin^2 x \, dx = \int_0^{\pi/2} \frac{1-\cos 2x}{2} \, dx$$

(b) Separate the components and find the general algebraic antiderivative:

$$\int \frac{1-\cos 2x}{2} \, dx = \frac{1}{2}x - \frac{\sin 2x}{4}$$

(c) Apply the limits of integration from 0 to $\frac{\pi}{2}$:

$$\left[\frac{1}{2}x - \frac{\sin 2x}{4}\right]_0^{\pi/2}$$

(d) Substitute the upper limit value $x = \frac{\pi}{2}$ into the expression:

$$\left(\frac{1}{2}\left(\frac{\pi}{2}\right) - \frac{\sin(\pi)}{4}\right) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

(e) Substitute the lower limit value $x = 0$ into the expression:

$$\left(\frac{1}{2}(0) - \frac{\sin(0)}{4}\right) = 0 - 0 = 0$$

(f) Subtract the lower limit result from the upper limit result to find the total area: $\frac{\pi}{4}$.

Final Answer: $\pi_{\overline{4}}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 43

Q44.

Solution

Concept:
The equation of a line passing through two distinct points (x_1, y_1) and (x_2, y_2) can be determined using the two-point formula, which is expressed as $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.
Solution:

(a) Identify the given coordinate values from the problem statement: $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (3, 4)$.

(b) Compute the numeric slope m of the line using the vertical change divided by horizontal change:

$$m = \frac{4-2}{3-1} = \frac{2}{2} = 1$$

(c) Set up the equation using the point-slope form with the first coordinate pair:

$$y - 2 = 1(x - 1)$$

(d) Simplify and rearrange the algebraic terms into standard linear form:

$$y - 2 = x - 1 \implies x - y + 1 = 0$$

Final Answer: x - y + 1 = 0

Answer: (C)

Go Back to Question 44

Q45.

Solution

Concept:

For a system of two linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ to possess infinitely many solutions, the lines must coincide completely. This requires their corresponding coefficients to be directly proportional: $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

Solution:

(a) Match the given system equations $1x + 2y = 3$ and $4x + 8y = k$ to standard form parameters:

$$a_1 = 1, \quad b_1 = 2, \quad c_1 = 3$$

$$a_2 = 4, \quad b_2 = 8, \quad c_2 = k$$

(b) Set up the equality ratio condition for coincident lines:

$$\frac{1}{4} = \frac{2}{8} = \frac{3}{k}$$

(c) Observe that the coefficient scaling factor between the first and second equations is exactly $\frac{1}{4}$.

(d) Solve for the unknown constant k by setting up the final ratio equivalence:

$$\frac{1}{4} = \frac{3}{k} \implies k = 3 \times 4 = 12$$

Final Answer: 12

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 45

Q46.

Solution

Concept:
Integrals containing an exponential factor combined with functional terms often fit the special integration rule $\int e^x [f(x) + f'(x)]\, dx = e^x f(x) + C$. We solve this by identifying the primary function and verifying its derivative.

Solution:

(a) Write down the given expression: $\int e^x (\sec x + \sec x \tan x)\, dx$.

(b) Compare the integrand structure to the standard exponential template formula:

$$e^x [f(x) + f'(x)]$$

(c) Define the candidate function as $f(x) = \sec x$.

(d) Differentiate this function with respect to x to check if it matches the remaining term:

$$f'(x) = \frac{d}{dx}(\sec x) = \sec x \tan x$$

(e) Since the second term matches the derivative exactly, apply the integration rule directly:

$$\int e^x (\sec x + \sec x \tan x)\, dx = e^x \sec x + C$$

Final Answer: $e^x \sec x + C$

Answer: (B)

Go Back to Question 46

Q47.

Solution

Concept:
Trigonometric equations can be expanded by squaring operations to uncover hidden double-angle relationships. Squaring both sides uses the fundamental identities $\sin^2 x + \cos^2 x = 1$ and $\sin 2x = 2\sin x \cos x$.

Solution:

(a) Write down the initial trigonometric equation: $\sin x + \cos x = 1$.

(b) Square both sides of the equation to create product combinations:

$$(\sin x + \cos x)^2 = 1^2$$

(c) Expand the algebraic trinomial on the left side:

$$\sin^2 x + \cos^2 x + 2\sin x \cos x = 1$$

(d) Substitute the standard Pythagorean identity $\sin^2 x + \cos^2 x = 1$ into the equation:

$$1 + 2\sin x \cos x = 1$$

(e) Subtract 1 from both sides to isolate the product term:

$$2\sin x \cos x = 0$$

(f) Replace the isolated product term with its standard double-angle identity:

$$\sin 2x = 0$$

Final Answer: 0

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 47

Q48.

Solution

Concept:

Repeated independent trials with two possible outcomes follow the binomial probability distribution formula, expressed as $P(X = k) = \binom{n}{k} p^k q^{n-k}$, where n is total trials, k is successes, p is success probability, and $q = 1 - p$.

Solution:

(a) Identify the binomial parameters for flipping a fair coin: total trials $n = 4$ and desired successes $k = 2$.

(b) Define the individual probabilities for a fair coin: success probability $p = \frac{1}{2}$ and failure probability $q = 1 - p = \frac{1}{2}$.

(c) Substitute these numbers into the binomial formula:

$$P(X = 2) = \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{4-2}$$

(d) Evaluate the combination coefficient:

$$\binom{4}{2} = \frac{4 \times 3}{2 \times 1} = 6$$

(e) Compute the exponential values:

$$\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

(f) Multiply the results together:

$$P(X = 2) = 6 \times \frac{1}{16} = \frac{6}{16} = \frac{3}{8}$$

Final Answer: $3_{\overline{8}}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 48

Q49.

Solution

Concept:
To find the derivative of a composite inverse trigonometric function, apply the standard derivative rule $\frac{d}{dx}[\sin^{-1} u] = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$ using the calculus chain rule technique.
Solution:

(a) Define the inner composite function term: $u = 2x$.

(b) Compute the derivative of this inner term with respect to x: $\frac{du}{dx} = 2$.

(c) Set up the standard calculus derivative formula for the outer inverse sine function:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$$

(d) Substitute $u = 2x$ and $\frac{du}{dx} = 2$ back into the formula:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2$$

(e) Simplify the expression under the square root and arrange the numerator:

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

Final Answer: $2\overline{\sqrt{1-4x^2}}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept:

The shortest perpendicular distance d between two parallel straight lines written in standard form $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is computed using the absolute formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$.

Solution:

(a) Convert the given equations from slope-intercept form to standard linear form:

$$y = 2x + 3 \implies 2x - y + 3 = 0$$

$$y = 2x + 7 \implies 2x - y + 7 = 0$$

(b) Identify the corresponding coefficient values: $A = 2$, $B = -1$, $C_1 = 3$, and $C_2 = 7$.

(c) Substitute these numbers into the parallel line distance formula:

$$d = \frac{|3 - 7|}{\sqrt{2^2 + (-1)^2}}$$

(d) Simplify the numerator using absolute value: $|3 - 7| = |-4| = 4$.

(e) Simplify the denominator terms: $\sqrt{4 + 1} = \sqrt{5}$.

(f) Combine the terms to find the final distance: $d = \frac{4}{\sqrt{5}}$.

Final Answer: $4\frac{}{\sqrt{5}}$

Answer: (A)

Go Back to Question 50

Q51.

Solution

Concept:

The inverse of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by the formula $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, provided the determinant $|A| = ad - bc$ is non-zero. This operation scales the adjugate matrix by the reciprocal of the determinant.

Solution:

(a) Identify the entries of the given matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$, where $a = 1$, $b = 2$, $c = 0$, and $d = 3$.

(b) Calculate the determinant of the matrix:

$$|A| = (1)(3) - (2)(0) = 3 - 0 = 3$$

(c) Find the adjugate matrix by swapping the diagonal elements and changing the signs of the off-diagonal elements:

$$\text{adj}(A) = \begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix}$$

(d) Divide every element of the adjugate matrix by the determinant value to find the final inverse matrix:

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 3 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{3} \\ 0 & \frac{1}{3} \end{bmatrix}$$

Final Answer: $\begin{bmatrix} 1 & -\frac{2}{3}\, 0 & \frac{1}{3} \end{bmatrix}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 51

Q52.

Solution

Concept:
The minimum value of a quadratic function $f(x) = ax^2 + bx + c$ with $a > 0$ can be found either by rewriting the quadratic expression in vertex form by completing the square, or by using differential calculus to locate the critical point where the derivative equals zero.

Solution:

(a) Express the function: $f(x) = x^2 - 4x + 5$.

(b) Differentiate the function with respect to x to find its slope expression:

$$f'(x) = 2x - 4$$

(c) Set the derivative equal to zero to determine the critical value where the minimum occurs:

$$2x - 4 = 0 \implies 2x = 4 \implies x = 2$$

(d) Check the second derivative to confirm the nature of this point: $f''(x) = 2$, which is greater than zero, verifying a local minimum.

(e) Substitute $x = 2$ back into the original quadratic function to calculate the minimum value:

$$f(2) = (2)^2 - 4(2) + 5 = 4 - 8 + 5 = 1$$

Final Answer: 1

Answer: (B)

Go Back to Question 52

Q53.

Solution

Concept:
The arithmetic mean of a set of numbers is calculated by dividing the total sum of the terms by the number of terms. The first n even natural numbers form an arithmetic progression with a first term of 2 and a common difference of 2, whose sum is given by $n(n+1)$.
Solution:

(a) Identify the sequence of the first 20 even natural numbers: $2, 4, 6, \ldots, 40$. Here, the total number of terms is $n = 20$.

(b) Recall the algebraic formula for the sum of the first n even natural numbers:

$$\text{Sum} = n(n+1)$$

(c) Substitute $n = 20$ into the formula to find the complete sum of the sequence:

$$\text{Sum} = 20 \times (20+1) = 20 \times 21 = 420$$

(d) Calculate the arithmetic mean by dividing this total sum by the number of terms:

$$\text{Mean} = \frac{\text{Sum}}{n} = \frac{420}{20} = 21$$

Final Answer: 21

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 53

Q54.

Solution

Concept:

The statistical variance σ^2 measures the dispersion of a dataset and is calculated using the formula $\sigma^2 = \frac{\sum x_i^2}{n} - \mu^2$, where μ is the mean. Alternatively, the variance for the first n natural numbers can be found using the standard simplified formula $\sigma^2 = \frac{n^2-1}{12}$.

Solution:

(a) Identify the given number of terms from the problem statement, which is $n = 5$ for the dataset containing $1, 2, 3, 4, 5$.

(b) Substitute the value of n directly into the standard algebraic variance formula:

$$\sigma^2 = \frac{5^2 - 1}{12}$$

(c) Evaluate the exponent in the numerator:

$$5^2 = 25$$

(d) Subtract 1 from the squared result to simplify the numerator:

$$25 - 1 = 24$$

(e) Divide the simplified numerator by the denominator to get the final variance value:

$$\sigma^2 = \frac{24}{12} = 2$$

Final Answer: 2

Answer: (C)

Go Back to Question 54

Q55.

Solution

Concept:

To evaluate an integral of the form $\int \frac{dx}{ax^2+bx+c}$ where the denominator cannot be factored into real linear factors, we complete the square to transform it into the standard form $\int \frac{dx}{u^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$.

Solution:

(a) Write down the quadratic expression in the denominator: $x^2 + 4x + 5$.

(b) Complete the square by taking half of the x-coefficient, squaring it, and splitting the constant term:

$$x^2 + 4x + 5 = (x^2 + 4x + 4) + 1 = (x + 2)^2 + 1^2$$

(c) Rewrite the original indefinite integral using this completed square form:

$$\int \frac{dx}{(x + 2)^2 + 1^2}$$

(d) Use the standard integration identity $\int \frac{du}{u^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + C$ where $u = x + 2$ and $a = 1$:

$$\int \frac{dx}{(x + 2)^2 + 1^2} = \tan^{-1}(x + 2) + C$$

Final Answer: $\tan^{-1}(x + 2) + C$

Answer: (D)

Go Back to Question 55

Q56.

Solution

Concept:
A parabola with its vertex at the origin $(0, 0)$ and its focus situated on the positive y-axis at $(0, a)$ opens upwards. The standard geometric equation for such a vertical parabola is given by the formula $x^2 = 4ay$, where a represents the focal distance.

Solution:

(a) Identify the given coordinates of the focus from the problem statement: $(0, 2)$.

(b) Compare these coordinates to the standard vertical focus form $(0, a)$ to find the value of the focal parameter:

$$a = 2$$

(c) Recall the standard equation template for a parabola that opens vertically upwards:

$$x^2 = 4ay$$

(d) Substitute the value $a = 2$ directly into this standard equation template:

$$x^2 = 4(2)y$$

(e) Multiply the numerical constants to obtain the final equation:

$$x^2 = 8y$$

Final Answer: $x^2 = 8y$

Answer: (C)

Go Back to Question 56

Q57.

Solution

Concept:
The number of diagonals in a polygon with n vertices is determined by finding the total number of lines that can be drawn connecting any two vertices, given by $\binom{n}{2}$, and subtracting the n outer sides of the polygon. This simplifies to the formula $\frac{n(n-3)}{2}$.

Solution:

(a) Identify the type of polygon given: a decagon, which by definition has $n = 10$ vertices and sides.

(b) Recall the standard algebraic formula used to find the number of diagonals in any polygon:

$$\text{Diagonals} = \frac{n(n-3)}{2}$$

(c) Substitute the value $n = 10$ into the formula:

$$\text{Diagonals} = \frac{10 \times (10-3)}{2}$$

(d) Simplify the subtraction inside the parentheses:

$$10 - 3 = 7$$

(e) Multiply the values in the numerator and divide by 2 to reach the final answer:

$$\text{Diagonals} = \frac{10 \times 7}{2} = \frac{70}{2} = 35$$

Final Answer: 35

Answer: (C)

Go Back to Question 57

Q58.

Solution

Concept:
The shortest perpendicular distance d from a three-dimensional point (x_1, y_1, z_1) to a plane defined by the linear equation $Ax + By + Cz + D = 0$ is calculated using the standard formula $d = \frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}$.
Solution:

(a) Identify the coordinates of the point and coefficients of the plane from the problem:

$$(x_1, y_1, z_1) = (1, 2, 3)$$

$$A = 2, \quad B = -1, \quad C = 1, \quad D = 1$$

(b) Substitute these values into the numerator of the distance formula:

$$\text{Numerator} = |2(1) + (-1)(2) + 1(3) + 1| = |2 - 2 + 3 + 1| = 4$$

(c) Substitute the coefficients into the denominator of the formula:

$$\text{Denominator} = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{4 + 1 + 1} = \sqrt{6}$$

(d) Combine the components and rationalize the denominator:

$$d = \frac{4}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3}$$

Final Answer: $2\sqrt{6}_3$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 58

Q59.

Solution

Concept:

The permutation formula represents the number of unique ways to arrange r items chosen from a total set of n distinct items. It is defined algebraically as ${}^{n}P_{r} = \frac{n!}{(n-r)!}$, which expands to the product of r consecutive decreasing integers starting from n.

Solution:

(a) Write down the given equation involving permutations: ${}^{n}P_{3} = 120$.

(b) Expand the permutation expression into a product of three consecutive decreasing integers:

$$n(n-1)(n-2) = 120$$

(c) Find three consecutive integers whose product equals 120 by performing a prime factorization:

$$120 = 4 \times 5 \times 6$$

(d) Arrange these integer factors in strictly descending order to match the algebraic terms:

$$6 \times 5 \times 4 = 120$$

(e) Compare the largest term on both sides of the equation to find the value of n:

$$n = 6$$

Final Answer: 6

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 59

Q60.

Solution

Concept:

The probability of the union of two events A and B describes the likelihood that at least one of the events occurs. It is found using the fundamental Addition Rule of probability theory: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Solution:

(a) Identify the individual numerical probabilities given in the problem statement:

$$P(A) = 0.4, \quad P(B) = 0.5, \quad P(A \cap B) = 0.2$$

(b) Recall the standard addition formula for combining probabilities:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(c) Substitute the given numerical values directly into the formula:

$$P(A \cup B) = 0.4 + 0.5 - 0.2$$

(d) Add the first two terms together:

$$0.4 + 0.5 = 0.9$$

(e) Subtract the intersection value from this sum to reach the final probability:

$$P(A \cup B) = 0.9 - 0.2 = 0.7$$

Final Answer: 0.7

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 60

Q61.

Solution

Concept:

To determine the number of real roots of a cubic polynomial function, we analyze its local extrema using differentiation. By finding the values of the function at its critical points, we can understand how many times its continuous graph crosses the horizontal x-axis.

Solution:

(a) Define the cubic function as $f(x) = x^3 - 3x + 1$ and find its first derivative with respect to x to identify its stationary turning points:

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1)$$

(b) Setting the derivative equal to zero gives the critical numbers where the graph turns:

$$3(x^2 - 1) = 0 \implies x = 1 \text{ or } x = -1$$

(c) Evaluate the original polynomial function at the local maximum location $x = -1$:

$$f(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$$

(d) Evaluate the original polynomial function at the local minimum location $x = 1$:

$$f(1) = (1)^3 - 3(1) + 1 = 1 - 3 + 1 = -1$$

(e) Since the local maximum value is positive and the local minimum value is negative, the continuous graph must cross the x-axis exactly three times.

Final Answer: 3

Answer: (B)

Go Back to Question 61

Q62.

Solution

Concept:
A horizontal hyperbola centered at the origin has the standard equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The parameter a represents the distance from the center to each vertex, while the parameter c represents the distance to each focus, satisfying the geometric relation $c^2 = a^2 + b^2$.

Solution:

(a) Identify the vertices given as $(\pm 3, 0)$, which tells us that the hyperbola is horizontal and centered at the origin with:

$$a = 3 \implies a^2 = 9$$

(b) Identify the foci given as $(\pm 5, 0)$, which provides the focal distance parameter:

$$c = 5 \implies c^2 = 25$$

(c) Use the fundamental hyperbolic relationship to calculate the remaining denominator value b^2:

$$c^2 = a^2 + b^2 \implies 25 = 9 + b^2 \implies b^2 = 16$$

(d) Substitute the calculated values of a^2 and b^2 into the standard equation template:

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Final Answer: $\mathrm{x}^2\frac{}{9-\frac{y^2}{16}=1}$

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 62

Q63.

Solution

Concept:
According to the binomial theorem, the general term in the expansion of $(1 + x)^n$ is given by $T_{r+1} = \binom{n}{r}x^r$. Therefore, the coefficient of the x^2 term corresponds directly to the combinatorial value $\binom{n}{2}$, which represents selecting 2 items from n.

Solution:

(a) Write down the binomial expression for the coefficient of x^2 and set it equal to the given value:

$$\binom{n}{2} = 28$$

(b) Expand the combinatorial notation into its algebraic product form:

$$\frac{n(n-1)}{2} = 28$$

(c) Clear the fraction by multiplying both sides of the equation by 2:

$$n(n-1) = 56$$

(d) Express the number 56 as a product of two consecutive positive integers:

$$8 \times 7 = 56$$

(e) Match the corresponding factors on both sides to solve for the integer value:

$$n = 8$$

Final Answer: 8

Answer: (A)

Go Back to Question 63

Q64.

Solution

Concept:
The geometric area of a parallelogram spanned by two adjacent vector sides $\vec{a}$ and $\vec{b}$ is equal to the magnitude of their vector cross product, $|\vec{a} \times \vec{b}|$. For two-dimensional vectors, this simplifies to the absolute value of their determinant.

Solution:

(a) Identify the components of the two adjacent vectors given in the problem:

$$\vec{a} = 2\hat{i} + 3\hat{j} + 0\hat{k}, \quad \vec{b} = \hat{i} + 4\hat{j} + 0\hat{k}$$

(b) Set up the vector cross product using a three-by-three determinant matrix:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 1 & 4 & 0 \end{vmatrix}$$

(c) Expand the determinant along the third column to find the resulting vector:

$$\vec{a} \times \vec{b} = \hat{k} \cdot [(2)(4) - (3)(1)] = \hat{k} \cdot (8 - 3) = 5\hat{k}$$

(d) Compute the magnitude of this cross product vector to find the area:

$$\text{Area} = |5\hat{k}| = 5$$

Final Answer: 5

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 64

Q65.

Solution

Concept:
The equation of a tangent line to a standard parabola $y^2 = 4ax$ at a specific given point (x_1, y_1) lying on its curve can be found directly using the substitution rule $y^2 \to yy_1$ and $x \to \frac{x+x_1}{2}$.
Solution:

(a) Write down the given equation of the parabola: $y^2 = 8x$.

(b) Compare it to the standard form $y^2 = 4ax$ to identify the parameter value:

$$4a = 8 \implies a = 2$$

(c) Recall the point-form equation for a tangent line to a parabola:

$$yy_1 = 2a(x + x_1)$$

(d) Substitute the coordinates of the given point $(x_1, y_1) = (2, 4)$ and $2a = 4$ into the formula:

$$y(4) = 4(x + 2)$$

(e) Simplify the equation by dividing both sides by 4 and rearranging all terms to one side:

$$4y = 4x + 8 \implies y = x + 2 \implies x - y + 2 = 0$$

Final Answer: x - y + 2 = 0

Answer: (C)

Go Back to Question 65

Q66.

Solution

Concept:
In statistical data analysis, the mode is defined as the specific value that appears with the highest frequency in a given dataset. A dataset can have one mode, multiple modes, or no mode at all if all elements appear equally.

Solution:

(a) Organize and inspect the given numerical dataset: $2, 3, 3, 4, 5, 5, 5, 6, 7$.

(b) Count the individual frequency of occurrence for each unique number in the list:

- The number 2 occurs exactly 1 time.
- The number 3 occurs exactly 2 times.
- The number 4 occurs exactly 1 time.
- The number 5 occurs exactly 3 times.
- The number 6 occurs exactly 1 time.
- The number 7 occurs exactly 1 time.

(c) Compare the frequency counts to determine which number appears most often.

(d) Since the number 5 has the maximum frequency of 3, it represents the mode of this dataset.

Final Answer: 5

Answer: (B)

Go Back to Question 66

Q67.

Solution

Concept:
To differentiate a product of two functions, we apply the product rule of calculus, which states that $\frac{d}{dx}[u(x)v(x)] = u'(x)v(x) + u(x)v'(x)$. Here, we use the derivative rules $\frac{d}{dx}(e^x) = e^x$ and $\frac{d}{dx}(\sin x) = \cos x$.

Solution:

(a) Identify the two function factors in the expression: $u(x) = e^x$ and $v(x) = \sin x$.

(b) Set up the differentiation using the product rule formula:

$$\frac{d}{dx}(e^x \sin x) = \left[\frac{d}{dx}(e^x)\right] \cdot \sin x + e^x \cdot \left[\frac{d}{dx}(\sin x)\right]$$

(c) Compute the derivative of each individual function factor:

$$\frac{d}{dx}(e^x) = e^x, \quad \frac{d}{dx}(\sin x) = \cos x$$

(d) Substitute these derivatives back into the expanded product rule formula:

$$\frac{d}{dx}(e^x \sin x) = e^x \sin x + e^x \cos x$$

(e) Factor out the common exponential term e^x to obtain the final simplified answer:

$$\frac{d}{dx}(e^x \sin x) = e^x(\sin x + \cos x)$$

Final Answer: $\mathrm{e}^x(\sin x + \cos x)$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 67

Q68.

Solution

Concept:

Two lines are perpendicular if the product of their slopes equals -1. The slope of a line given by $Ax + By + C = 0$ is $m = -\frac{A}{B}$, so a perpendicular line will have a slope of $\frac{B}{A}$, leading to a general form of $Bx - Ay + k = 0$.

Solution:

(a) Express the given line equation: $3x + 4y = 7$. Its slope is $m_1 = -\frac{3}{4}$.

(b) Determine the slope m_2 of any line perpendicular to it using the condition $m_1 \cdot m_2 = -1$:

$$-\frac{3}{4} \cdot m_2 = -1 \implies m_2 = \frac{4}{3}$$

(c) Write the point-slope form of the line passing through $(1, 2)$ with slope $\frac{4}{3}$:

$$y - 2 = \frac{4}{3}(x - 1)$$

(d) Eliminate the fraction by multiplying the entire equation by 3:

$$3(y - 2) = 4(x - 1) \implies 3y - 6 = 4x - 4$$

(e) Rearrange all the terms into standard linear form:

$$4x - 3y + 2 = 0$$

Final Answer: 4x - 3y + 2 = 0

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 68

Q69.

Solution

Concept:
A determinant can be evaluated by expanding along any row or column. Alternatively, we can use row operations to simplify the matrix. If two rows or columns become proportional, the value of the determinant is automatically zero.

Solution:

(a) Write down the given matrix determinant equation:

$$\begin{vmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & x \end{vmatrix} = 0$$

(b) Perform the row operations $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_2$ to simplify the entries:

$$R_2 - R_1 = \begin{bmatrix} 8-2 & 10-4 & 12-6 \end{bmatrix} = \begin{bmatrix} 6 & 6 & 6 \end{bmatrix}$$

$$R_3 - R_2 = \begin{bmatrix} 14-8 & 16-10 & x-12 \end{bmatrix} = \begin{bmatrix} 6 & 6 & x-12 \end{bmatrix}$$

(c) Substitute these simplified rows back into the determinant:

$$\begin{vmatrix} 2 & 4 & 6 \\ 6 & 6 & 6 \\ 6 & 6 & x-12 \end{vmatrix} = 0$$

(d) For the determinant to be zero, the second and third rows can be identical:

$$x - 12 = 6 \implies x = 18$$

Final Answer: 18

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 69

Q70.

Solution

Concept:

The angle θ between two non-zero vectors $\vec{a}$ and $\vec{b}$ is determined using the dot product formula $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, which can be rearranged to express the cosine of the angle as $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$.

Solution:

(a) Compute the dot product of the two vectors $\vec{a} = 1\hat{i} + 1\hat{j} + 0\hat{k}$ and $\vec{b} = 0\hat{i} + 1\hat{j} + 1\hat{k}$:

$$\vec{a} \cdot \vec{b} = (1)(0) + (1)(1) + (0)(1) = 0 + 1 + 0 = 1$$

(b) Compute the geometric magnitude of the first vector $\vec{a}$:

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}$$

(c) Compute the geometric magnitude of the second vector $\vec{b}$:

$$|\vec{b}| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}$$

(d) Substitute these values into the cosine angle formula:

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$$

(e) Find the principal angle whose cosine value equals $\frac{1}{2}$:

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Final Answer: π_3

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 70

Q71.

Solution

Concept:
The standard equation of a circle with a center located at (h, k) and a radius of r is given by $(x-h)^2 + (y-k)^2 = r^2$. Expanding this expression and gathering like terms translates it into the general quadratic equation of a circle.
Solution:

(a) Identify the given parameters for the circle where the center coordinates are $(h, k) = (1, -2)$ and the radius length is $r = 3$.

(b) Substitute these coordinate values directly into the standard central form equation:

$$(x-1)^2 + (y-(-2))^2 = 3^2$$

(c) Simplify the signs within the second binomial term before performing algebraic expansion:

$$(x-1)^2 + (y+2)^2 = 9$$

(d) Expand both squared binomial terms using standard algebraic identities:

$$(x^2 - 2x + 1) + (y^2 + 4y + 4) = 9$$

(e) Group the quadratic terms, linear terms, and constant terms together on one side:

$$x^2 + y^2 - 2x + 4y + 1 + 4 - 9 = 0$$

(f) Combine the constants to achieve the final general form:

$$x^2 + y^2 - 2x + 4y - 4 = 0$$

Final Answer: $\mathrm{x}^2 + y^2 - 2x + 4y - 4 = 0$

$\boxed{\textbf{Answer: (D)}}$

Go Back to Question 71

Q72.

Solution

Concept:
Conditional probability defines the likelihood of an event A occurring given that event B has already taken place. It is computed mathematically by dividing the probability of the joint intersection of both events by the probability of the conditioning event.

Solution:

(a) Recall the standard definition and mathematical formula for conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(b) Identify the numerical probability values given for the individual components in the problem statement:

$$P(A) = 0.5, \quad P(B) = 0.4, \quad P(A \cap B) = 0.2$$

(c) Substitute the joint intersection probability and the conditioning event probability into the ratio:

$$P(A|B) = \frac{0.2}{0.4}$$

(d) Simplify the fractional expression by shifting the decimal points or dividing the numbers:

$$P(A|B) = \frac{2}{4} = 0.5$$

(e) Observe that since $P(A|B) = P(A)$, events A and B are statistically independent.

Final Answer: 0.5

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 72

Q73.

Solution

Concept:
The definite integral of a function is calculated by finding its general antiderivative and evaluating it at the boundaries. The standard integral formula states that the antiderivative of $\frac{1}{1+x^2}$ is the inverse tangent function, $\tan^{-1} x$.

Solution:

(a) Recall the standard integration rule for the given algebraic rational function:

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

(b) Apply the Fundamental Theorem of Calculus to evaluate this antiderivative over the interval $[0, 1]$:

$$\int_0^1 \frac{dx}{1+x^2} = \left[\tan^{-1} x\right]_0^1$$

(c) Expand the bracketed expression at the upper boundary and lower boundary limits:

$$\left[\tan^{-1} x\right]_0^1 = \tan^{-1}(1) - \tan^{-1}(0)$$

(d) Determine the principal angles corresponding to these inverse trigonometric values:

$$\tan^{-1}(1) = \frac{\pi}{4}, \quad \tan^{-1}(0) = 0$$

(e) Complete the subtraction to find the exact numerical value of the definite area:

$$\frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Final Answer: $\pi_{\overline{4}}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 73

Q74.

Solution

Concept:

A sector is a portion of a circle enclosed by two radii and an arc. The area of a sector with a central angle measured in degrees is proportional to the total area of the circle, calculated using the formula Area = $\frac{\theta}{360^\circ} \times \pi r^2$.

Solution:

(a) Extract the given geometric measurements from the problem description:

$$\text{Radius } r = 7 \text{ cm}, \quad \text{Central Angle } \theta = 60^\circ$$

(b) Set up the standard sector area formula using these parameters:

$$\text{Area} = \frac{60^\circ}{360^\circ} \times \pi \times 7^2$$

(c) Reduce the angular fraction to its simplest fractional form:

$$\frac{60}{360} = \frac{1}{6}$$

(d) Compute the square of the radius length to find the total area coefficient:

$$7^2 = 49$$

(e) Combine the simplified fraction with the squared radius and the constant π:

$$\text{Area} = \frac{1}{6} \times \pi \times 49 = \frac{49\pi}{6}$$

Final Answer: $49\pi_{\overline{6}}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 74

Q75.

Solution

Concept:

To find a high power of a matrix, it is beneficial to compute its initial sequential powers to identify a recurring pattern. An identity matrix I satisfies the property that any positive integer power of I remains equal to I.

Solution:

(a) Start with the given matrix A and compute its square, A^2, using standard matrix multiplication rules:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(b) Calculate the individual elements of the product matrix row by row:

$$A^2 = \begin{bmatrix} (1)(1) + (0)(0) & (1)(0) + (0)(-1) \\ (0)(1) + (-1)(0) & (0)(0) + (-1)(-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(c) Since A^2 equals the identity matrix I, any higher even power of the matrix can be simplified:

$$A^{100} = (A^2)^{50} = I^{50}$$

(d) Recall that multiplying the identity matrix by itself repeatedly always yields I:

$$A^{100} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Final Answer: $\begin{bmatrix} 1 & 0 0 & 1 \end{bmatrix}$

$\boxed{\textbf{Answer: (A)}}$

Go Back to Question 75

Q76.

Solution

Concept:
The dot product and cross product convey relative geometric orientations between vectors. A dot product of zero indicates orthogonal perpendicular vectors, while a cross product equal to the zero vector implies parallel collinear vectors.
Solution:

(a) Analyze the first given vector condition, which involves a scalar dot product:

$$\vec{a} \cdot \vec{b} = 0$$

(b) For non-zero vectors, a vanishing dot product means that the angle between them is 90°, establishing that vector $\vec{a}$ is perpendicular to vector $\vec{b}$ ($\vec{a} \perp \vec{b}$).

(c) Analyze the second given vector condition, which involves a vector cross product:

$$\vec{a} \times \vec{c} = \vec{0}$$

(d) A vanishing cross product means that the vectors are collinear, establishing that vector $\vec{a}$ is parallel to vector $\vec{c}$ ($\vec{a} \parallel \vec{c}$).

(e) Combine both individual geometric facts together to state the complete accurate description.

Final Answer: $\perp \vec{b}$ and $\vec{a} \parallel \vec{c}$

Answer: (A)

Go Back to Question 76

Q77.

Solution

Concept:
The geometric slope m of a straight line measures its steepness relative to the horizontal plane. It is mathematically defined as the trigonometric tangent of the inclination angle θ that the line makes with the positive direction of the x-axis.

Solution:

(a) Identify the angle of inclination given in the problem statement:

$$\theta = 45^\circ$$

(b) Recall the fundamental relationship between a line's inclination angle and its mathematical slope:

$$m = \tan\theta$$

(c) Substitute the given angle value directly into the trigonometric tangent function:

$$m = \tan(45^\circ)$$

(d) Evaluate the exact value of $\tan(45^\circ)$ from standard trigonometric tables:

$$\tan(45^\circ) = 1$$

(e) Conclude that the slope of the line is exactly equal to 1.

Final Answer: 1

Answer: (D)

Go Back to Question 77

Q78.

Solution

Concept:
The standard deviation is a metric of statistical dispersion that reflects the spread of a dataset around its arithmetic mean. It is calculated by taking the square root of the variance, which is the average of the squared deviations from the mean.

Solution:

(a) List the elements of the given dataset: $1, 2, 3, 4, 5$.

(b) Calculate the arithmetic mean (μ) by summing all numbers and dividing by the total count:

$$\mu = \frac{1+2+3+4+5}{5} = \frac{15}{5} = 3$$

(c) Calculate the squared deviations from this calculated mean for each data point:

$$(1-3)^2 = 4, \quad (2-3)^2 = 1, \quad (3-3)^2 = 0, \quad (4-3)^2 = 1, \quad (5-3)^2 = 4$$

(d) Sum these squared values and divide by the count to find the variance (σ^2):

$$\sigma^2 = \frac{4+1+0+1+4}{5} = \frac{10}{5} = 2$$

(e) Take the square root of the variance to determine the standard deviation (σ):

$$\sigma = \sqrt{2}$$

Final Answer: $\sqrt{2}$

$\boxed{\textbf{Answer: (A)}}$

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Q79.

Solution

Concept:
Rolle's theorem states that if a function $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) with $f(a) = f(b)$, there exists at least one value c in (a, b) where the first derivative vanishes, meaning $f'(c) = 0$.
Solution:

(a) Verify the theorem boundaries for the polynomial $f(x) = x^2 - 4x + 3$ on the interval $[1, 3]$:

$$f(1) = 1 - 4 + 3 = 0, \quad f(3) = 9 - 12 + 3 = 0$$

(b) Differentiate the polynomial function with respect to x to find its slope formula:

$$f'(x) = 2x - 4$$

(c) Replace the independent variable x with the theorem parameter symbol c:

$$f'(c) = 2c - 4$$

(d) Set this derivative expression equal to zero to locate the stationary turning point:

$$2c - 4 = 0 \implies 2c = 4 \implies c = 2$$

(e) Check that the value $c = 2$ lies strictly within the open boundaries of $(1, 3)$.

Final Answer: 2

Answer: (A)

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Q80.

Solution

Concept:
For a standard parabola opening to the right with the structural equation form $y^2 = 4ax$, the latus rectum is defined as the focal chord perpendicular to the axis of symmetry. The total geometric length of this segment is equal to $4a$.

Solution:

(a) Write down the algebraic equation of the parabola provided in the problem statement:

$$y^2 = 12x$$

(b) State the standard structural equation template for a horizontal parabola centered at the origin:

$$y^2 = 4ax$$

(c) Compare the coefficients of the linear variable x from both equations to set up a direct equivalence:

$$4a = 12$$

(d) Recall that the total geometric length of the latus rectum is defined precisely by the coefficient parameter magnitude $|4a|$.

(e) Conclude directly from the step 3 comparison that the length of the latus rectum is 12.

Final Answer: 12

$\boxed{\textbf{Answer: (C)}}$

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	A	3	A	4	A	5	B
6	B	7	C	8	B	9	A	10	C
11	B	12	C	13	D	14	A	15	A
16	A	17	B	18	A	19	A	20	A
21	A	22	A	23	A	24	B	25	C
26	A	27	D	28	B	29	D	30	B
31	C	32	A	33	B	34	C	35	B
36	A	37	A	38	A	39	A	40	B
41	A	42	A	43	A	44	C	45	C
46	B	47	C	48	A	49	A	50	A
51	A	52	B	53	B	54	C	55	D
56	C	57	C	58	A	59	B	60	C
61	B	62	C	63	A	64	A	65	C
66	B	67	A	68	A	69	A	70	A
71	D	72	A	73	A	74	A	75	A
76	A	77	D	78	A	79	A	80	C