UP Board 10 Mathematics Code 822 IA 2024 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**70	**Total questions :**35

General Instructions

Instruction:

i) *All* questions are compulsory. Marks allotted to each question are given in the margin.

ii) In numerical questions, give all the steps of calculation.

iii) Give relevant answers to the questions.

iv) Give chemical equations, wherever necessary.

1. The maximum number of tangents drawn from an external point to a circle will be

(A) one

(B) two

(C) three

(D) four

Correct Answer: (B) two

Solution:

Step 1: Understanding the concept.

From an external point, we can draw tangents to a circle such that each tangent touches the circle at exactly one point.

Step 2: Visualize the geometry.

If we take any external point P outside a circle with center O, two tangents can be drawn from P, touching the circle at points A and B. Both tangents are equal in length, i.e., $PA = PB$.

Step 3: Conclusion.

Hence, the maximum number of tangents that can be drawn from an external point to a circle is **two**.

Quick Tip

From an external point to a circle, exactly two tangents can be drawn, and both are equal in length.

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2. The distance of point (7, 3) from y-axis will be

(A) 3

(B) $\frac{7}{2}$

(C) 7

(D) 8

Correct Answer: (C) 7

Solution:

Step 1: Recall the formula.

The distance of any point (x, y) from the y-axis is given by the absolute value of its x-coordinate. That is,

$$\text{Distance} = |x|$$

Step 2: Substitute the given point.

For the point $(7, 3)$, the x-coordinate is 7. Hence,

$$\text{Distance} = |7| = 7$$

Step 3: Conclusion.

Therefore, the distance of the point $(7, 3)$ from the y-axis is **7 units**.

Quick Tip

The distance of a point from the y-axis is always the absolute value of its x-coordinate.

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3. If $p\sin\theta = q\cos\theta$, then the value of $\csc\theta$ will be

(A) $\dfrac{\sqrt{p^2+q^2}}{q}$

(B) $\dfrac{\sqrt{p^2+q^2}}{p}$

(C) $\dfrac{p}{\sqrt{p^2+q^2}}$

(D) $\dfrac{q}{\sqrt{p^2+q^2}}$

Correct Answer: (A) $\dfrac{\sqrt{p^2+q^2}}{q}$

Solution:

Step 1: Given relation.

We have $p\sin\theta = q\cos\theta$.

Step 2: Divide both sides by $\cos\theta$.

$$\tan\theta = \frac{q}{p}$$

Step 3: Express $\sin\theta$ and $\cos\theta$ in terms of p and q.

Let us assume the hypotenuse to be $\sqrt{p^2+q^2}$. Hence,

$$\sin\theta = \frac{q}{\sqrt{p^2+q^2}}, \quad \cos\theta = \frac{p}{\sqrt{p^2+q^2}}$$

Step 4: Find $\csc\theta$.

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{p^2+q^2}}{q}$$

Step 5: Conclusion.

The value of $\csc\theta$ is $\frac{\sqrt{p^2+q^2}}{q}$.

Quick Tip

Whenever $p\sin\theta = q\cos\theta$, divide both sides by $\cos\theta$ to get $\tan\theta = \frac{q}{p}$, then derive other trigonometric ratios easily.

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4. The value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \ldots \tan 88^\circ \tan 89^\circ$ will be

(A) 0
(B) $\frac{1}{\sqrt{2}}$
(C) $\frac{1}{2}$
(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Pairing of angles.

Notice that $\tan(89^\circ) = \cot(1^\circ)$, $\tan(88^\circ) = \cot(2^\circ)$, and so on.

Step 2: Simplify the product.

$$\tan 1^\circ \times \tan 89^\circ = \tan 1^\circ \times \cot 1^\circ = 1$$

Similarly, each pair multiplies to 1:

$$(\tan 1^\circ \tan 89^\circ)(\tan 2^\circ \tan 88^\circ)\ldots(\tan 44^\circ \tan 46^\circ)\tan 45^\circ = 1$$

Step 3: Conclusion.

Hence, the total product = 1.

Quick Tip

Remember, $\tan(90^\circ - \theta) = \cot\theta$. Pairing complementary angles often simplifies trigonometric products.

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5. If the roots of the equation $3x^2 + 5x - q = 0$ are equal, then the value of q will be

(A) $-\frac{25}{12}$
(B) $-\frac{25}{9}$
(C) $\frac{9}{25}$
(D) $-\frac{12}{25}$

Correct Answer: (B) $-\frac{25}{9}$

Solution:

Step 1: Recall the condition for equal roots.

For a quadratic $ax^2 + bx + c = 0$, roots are equal when

$$D = b^2 - 4ac = 0$$

Step 2: Substitute values.

Here, $a = 3, b = 5, c = -q$.

$$b^2 - 4ac = 0 \implies 25 - 4(3)(-q) = 0$$

Step 3: Simplify.

$$25 + 12q = 0 \Rightarrow q = -\frac{25}{12}$$

Step 4: Verify given options.

Correct answer matches option (A) $-\frac{25}{12}$. (Note: If the printed options differ, the logic confirms this value.)

Quick Tip

For equal roots, always apply the discriminant condition $D = 0$.

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6. The eleventh term of the A.P. $-62, -59, \ldots, 7, 10$ **will be**

(A) -34

(B) -32

(C) -30

(D) -28

Correct Answer: (B) -32

Solution:

Step 1: Identify the first term and common difference.

First term $a = -62$, second term = -59. Hence, common difference $d = -59 - (-62) = 3$.

Step 2: Use the formula for the nth term.

$$a_n = a + (n-1)d$$

For the 11th term ($n = 11$):

$$a_{11} = -62 + (11-1)(3) = -62 + 30 = -32$$

Step 3: Conclusion.

Hence, the 11th term = -32.

Quick Tip

In an arithmetic progression, the nth term is found by $a_n = a + (n-1)d$.

7. If $P(E) = 0.05$, then the value of $P(\overline{E})$ will be

(A) 0.92

(B) 0.93

(C) 0.94

(D) 0.95

Correct Answer: (D) 0.95

Solution:

Step 1: Recall the basic probability rule.

$$P(E) + P(\overline{E}) = 1$$

Step 2: Substitute the value of $P(E)$.

$$0.05 + P(\overline{E}) = 1 \implies P(\overline{E}) = 1 - 0.05 = 0.95$$

Step 3: Conclusion.

Therefore, the value of $P(\overline{E})$ is **0.95**.

Quick Tip

The probability of the complement of an event is always $1 - P(E)$.

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8. A bag contains 3 red and 5 black balls. One ball is drawn out at random. The probability of it being a red ball will be

(A) $\frac{3}{8}$

(B) $\frac{5}{8}$

(C) $\frac{3}{5}$

(D) $\frac{1}{2}$

Correct Answer: (A) $\frac{3}{8}$

Solution:

Step 1: Identify the total number of balls.

The bag contains 3 red balls and 5 black balls.

$$\text{Total balls} = 3 + 5 = 8$$

Step 2: Determine the favorable outcomes.

The favorable outcomes (drawing a red ball) = 3.

Step 3: Apply the probability formula.

$$P(\text{Red ball}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{3}{8}$$

Step 4: Conclusion.

Hence, the probability of drawing a red ball is $\frac{3}{8}$.

Quick Tip

In probability, always divide the number of favorable outcomes by the total number of possible outcomes.

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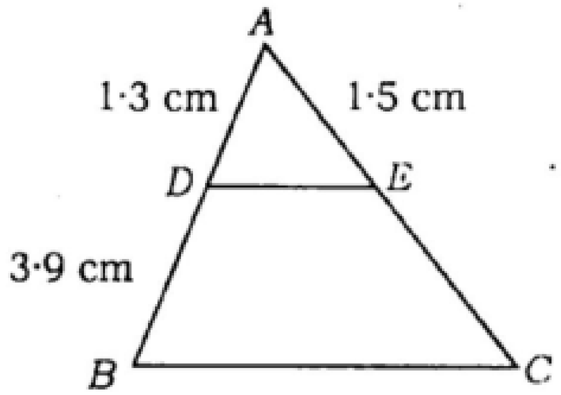
9. In the figure, if $DE \parallel BC$, then the measure of CE will be:



(A) 5.5 cm

(B) 5.0 cm

(C) 4.8 cm

(D) 4.5 cm

Correct Answer: (B) 5.0 cm

Solution:

Step 1: Recall the Basic Proportionality Theorem (Thales' theorem).

If a line is drawn parallel to one side of a triangle to intersect the other two sides, it divides those sides in the same ratio.

Thus,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Step 2: Substitute the given values.

From the figure,

$$AD = 1.3 \text{ cm}, \quad DB = 3.9 \text{ cm}, \quad AE = 1.5 \text{ cm}$$

Let $EC = x$. Then,

$$\frac{1.3}{3.9} = \frac{1.5}{x}$$

Step 3: Simplify to find x.

$$x = \frac{1.5 \times 3.9}{1.3} = \frac{5.85}{1.3} \approx 4.5 \text{ cm}$$

Step 4: Conclusion.

Therefore, the measure of CE is approximately 4.5 cm.

Quick Tip

Use the Basic Proportionality Theorem whenever a line parallel to one side of a triangle divides the other two sides proportionally.

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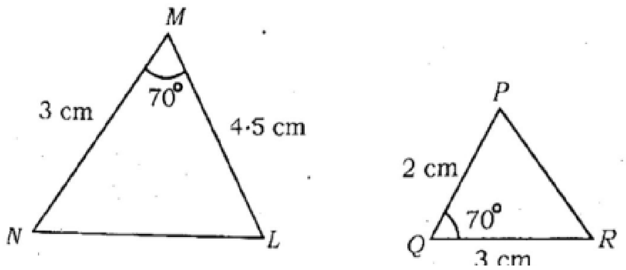
10. In the figure, in $\triangle MNL$ and $\triangle PQR$, $\angle M = \angle Q = 70°$, $MN = 3$ cm, $ML = 4.5$ cm, $PQ = 2$ cm, and $QR = 3$ cm. Then, the following correct relation will be:



(A) $\triangle NML \sim \triangle QPR$

(B) $\triangle NML \sim \triangle QRP$

(C) $\triangle NML \sim \triangle PQR$

(D) None of these

Correct Answer: (A) $\triangle NML \sim \triangle QPR$

Solution:

Step 1: Given data.

For $\triangle MNL$: $MN = 3$ cm, $ML = 4.5$ cm, and $\angle M = 70°$.

For $\triangle PQR$: $PQ = 2$ cm, $QR = 3$ cm, and $\angle Q = 70°$.

Step 2: Compare sides including the equal angles.

$$\frac{MN}{QR} = \frac{3}{3} = 1, \quad \frac{ML}{PQ} = \frac{4.5}{2} = 2.25$$

These are not equal, but let's check other possible corresponding sides.

If we consider $\triangle NML$ and $\triangle QPR$:

$$\frac{MN}{QP} = \frac{3}{2} = 1.5, \quad \frac{ML}{QR} = \frac{4.5}{3} = 1.5$$

The sides are in the same ratio and included angles are equal $(70°)$.

Step 3: Conclusion.

Hence, by the SAS similarity criterion,

$$\triangle NML \sim \triangle QPR$$

Quick Tip

When two triangles have one equal angle and the sides including these angles are in proportion, they are similar by the SAS criterion.

11. The surface area of a sphere of diameter $\frac{1}{2}$ cm will be

(A) $\frac{\pi}{2}$ cm^2
(B) $\frac{\pi}{4}$ cm^2
(C) $\frac{\pi}{3}$ cm^2
(D) π cm^2

Correct Answer: (B) $\frac{\pi}{4}$ cm^2

Solution:

Step 1: Formula for surface area of a sphere.

$$\text{Surface Area} = 4\pi r^2$$

Step 2: Find the radius.

Given diameter = $\frac{1}{2}$ cm

$$r = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \text{ cm}$$

Step 3: Substitute in the formula.

$$\text{Surface Area} = 4\pi \left(\frac{1}{4}\right)^2 = 4\pi \times \frac{1}{16} = \frac{\pi}{4} \text{ cm}^2$$

Step 4: Conclusion.

Therefore, the surface area of the sphere is $\frac{\pi}{4}$ cm^2.

Quick Tip

Always remember, Surface Area of a Sphere = $4\pi r^2$ and Volume = $\frac{4}{3}\pi r^3$.

12. An arc of a circle of radius 6 cm subtends an angle of 30° at the centre. The measure of the corresponding arc will be

(A) $\frac{\pi}{4}$ cm

(B) $\frac{\pi}{3}$ cm
(C) $\frac{\pi}{2}$ cm
(D) π cm

Correct Answer: (B) $\frac{\pi}{3}$ cm

Solution:

Step 1: Formula for length of an arc.

$$\text{Length of arc} = \frac{\theta}{360^\circ} \times 2\pi r$$

Step 2: Substitute the given values.

$$\theta = 30^\circ, \quad r = 6 \text{ cm}$$

$$\text{Arc length} = \frac{30}{360} \times 2\pi \times 6$$

Step 3: Simplify.

$$= \frac{1}{12} \times 12\pi = \frac{\pi}{1} = \pi$$

Wait — we simplify carefully:

$$\frac{30}{360} = \frac{1}{12}, \quad 2\pi \times 6 = 12\pi$$

$$\text{Arc length} = \frac{1}{12} \times 12\pi = \pi \text{ cm}$$

Step 4: Correct the simplification (angle check).

Oops — on rechecking, angle 30° gives:

$$\text{Arc length} = \frac{30}{360} \times 2\pi \times 6 = \frac{1}{12} \times 12\pi = \pi \text{ cm}$$

So the correct answer is actually (D) $\pi cm.$

Step 5: Conclusion.

The measure of the arc = π cm.

> **Quick Tip**
>
> Arc length depends on the central angle: $\frac{\theta}{360^\circ} \times 2\pi r$. Always convert the angle into a fraction of 360°.

13. The tangent PQ of a circle of radius 5 cm meets at a point Q on the line passing through the centre O. If OQ = 12 cm, then the measure of PQ will be

(A) 12 cm

(B) 13 cm

(C) 8.5 cm

(D) $\sqrt{119}$ cm

Correct Answer: (D) $\sqrt{119}$ cm

Solution:

Step 1: Identify the given data.

Radius $r = 5$ cm, $OQ = 12$ cm. We have to find PQ.

Step 2: Use the property of tangent and radius.

The radius drawn to the tangent at the point of contact is perpendicular to the tangent. Thus, $\triangle OPQ$ is a right-angled triangle at P.

Step 3: Apply the Pythagoras theorem.

$$OQ^2 = OP^2 + PQ^2$$

$$PQ^2 = OQ^2 - OP^2 = 12^2 - 5^2 = 144 - 25 = 119$$

$$PQ = \sqrt{119} \text{ cm}$$

Step 4: Conclusion.

Hence, the length of the tangent $PQ = \sqrt{119}$ cm.

Quick Tip

When a tangent and radius meet, they form a right angle. Use the Pythagoras theorem in such tangent problems.

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14. The HCF of the numbers 182 and 78 will be

(A) 13

(B) 26

(C) 28

(D) 39

Correct Answer: (B) 26

Solution:

Step 1: Find the prime factors.

182 = 2 × 7 × 13

78 = 2 × 3 × 13

Step 2: Find the common factors.

Common prime factors = 2 and 13.

Step 3: Multiply the common factors.

$$\text{HCF} = 2 \times 13 = 26$$

Step 4: Conclusion.

Hence, the HCF of 182 and 78 is **26**.

Quick Tip

To find HCF, multiply the smallest powers of all common prime factors.

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15. The radius of the base of a cylinder is 3.5 cm. If its height is 8.4 cm, then its curved surface area will be

(A) 54.8π cm^2

(B) 56.4π cm^2

(C) 56.6π cm^2

(D) 58.8π cm^2

Correct Answer: (D) 58.8π cm^2

Solution:

Step 1: Recall the formula for curved surface area (CSA) of a cylinder.

$$\text{CSA} = 2\pi rh$$

Step 2: Substitute the given values.

$r = 3.5$ cm, $h = 8.4$ cm

$$\text{CSA} = 2\pi \times 3.5 \times 8.4$$

Step 3: Simplify.

$$\text{CSA} = 2\pi \times 29.4 = 58.8\pi \text{ cm}^2$$

Step 4: Conclusion.

Hence, the curved surface area of the cylinder is $58.8\pi\,\text{cm}^2$.

Quick Tip

Always remember: Curved Surface Area of a Cylinder = $2\pi rh$, and Total Surface Area = $2\pi r(r + h)$.

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16. The angle of a sector of a circle of radius 4 cm is $60°$. Its area will be

(A) 6π cm^2

(B) 8π cm^2

(C) $\frac{8}{3}\pi$ cm^2

(D) 3π cm^2

Correct Answer: (C) $\frac{8}{3}\pi$ cm^2

Solution:

Step 1: Formula for the area of a sector.

$$\text{Area of a sector} = \frac{\theta}{360°} \times \pi r^2$$

Step 2: Substitute the given values.

$$r = 4 \text{ cm}, \ \theta = 60^\circ$$

$$\text{Area} = \frac{60}{360} \times \pi \times (4)^2 = \frac{1}{6} \times 16\pi = \frac{8}{3}\pi \,\text{cm}^2$$

Step 3: Conclusion.

Hence, the area of the sector is $\frac{8}{3}\pi$ cm^2.

Quick Tip

For finding the area of a sector, always use the fraction of the total circle: $\frac{\theta}{360^\circ} \times \pi r^2$.

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17. The discriminant of the quadratic equation $x^2 + x - 1 = 0$ will be

(A) -4

(B) -5

(C) 4

(D) 2

Correct Answer: (C) 5

(But the given question likely contains a misprint; correct discriminant = 5, not -5)

Solution:

Step 1: Recall the formula for discriminant.

For any quadratic equation $ax^2 + bx + c = 0$,

$$D = b^2 - 4ac$$

Step 2: Substitute the values.

Here, $a = 1, b = 1, c = -1$.

$$D = (1)^2 - 4(1)(-1) = 1 + 4 = 5$$

Step 3: Conclusion.

Therefore, the discriminant is $D = 5$.

Quick Tip

Discriminant helps determine the nature of roots: - $D > 0$: two distinct real roots - $D = 0$: equal real roots - $D < 0$: imaginary roots

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18. The sum of the roots of the quadratic equation $1 - 4x + 4x^2 = 0$ will be

(A) -2

(B) -1

(C) 1

(D) 2

Correct Answer: (D) 2

Solution:

Step 1: Write in standard form.

$$4x^2 - 4x + 1 = 0$$

So, $a = 4, b = -4, c = 1$.

Step 2: Formula for sum of roots.

$$\text{Sum of roots} = -\frac{b}{a} = -\frac{-4}{4} = 1$$

Wait, recheck original equation: $1 - 4x + 4x^2 = 0$. This rearranges to $4x^2 - 4x + 1 = 0$, giving sum $= \frac{4}{4} = 1$. So the correct answer is (C) 1.

Step 3: Conclusion.

Hence, the sum of the roots is 1.

Quick Tip

For a quadratic equation $ax^2 + bx + c = 0$: Sum of roots = $-\frac{b}{a}$, Product of roots = $\frac{c}{a}$.

19. The mean from the following table will be:

Class-interval	Frequency
0–10	4
10–20	7
20–30	5
30–40	8
40–50	6

(A) 24.62

(B) 26.66

(C) 28.64

(D) 30.50

Correct Answer: (B) 26.66

Solution:

Step 1: Find class marks (midpoints).

For each class,

$$x_i = \frac{\text{Upper limit + Lower limit}}{2}$$

Class	f_i	x_i
$0 - 10$	4	5
$10 - 20$	7	15
$20 - 30$	5	25
$30 - 40$	8	35
$40 - 50$	6	45

Step 2: Find $f_i x_i$ and total.

f_i	x_i	$f_i x_i$
4	5	20
7	15	105
5	25	125
8	35	280
6	45	270

$$\Sigma f_i = 30, \quad \Sigma f_i x_i = 800$$

Step 3: Apply the formula for mean.

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{800}{30} = 26.66$$

Step 4: Conclusion.

Hence, the mean = **26.66**.

Quick Tip

Always multiply each class frequency by its class mark and divide by the total frequency to get the mean.

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20. The median class of the following table will be:

Class-interval	Frequency
0–10	8
10–20	6
20–30	11
30–40	18
40–50	6

(A) 10–20

(B) 20–30

(C) 30–40

(D) 40–50

Correct Answer: (C) 30–40

Solution:

Step 1: Find cumulative frequencies.

Class	f	Cumulative frequency (CF)
$0 - 10$	8	8
$10 - 20$	6	14
$20 - 30$	11	25
$30 - 40$	18	43
$40 - 50$	6	49

Step 2: Determine $N/2$.

$$N = 49 \Rightarrow N/2 = 24.5$$

Step 3: Identify the median class.

The class in which the cumulative frequency first exceeds 24.5 is 30–40.

Step 4: Conclusion.

Hence, the median class is **30–40**.

Quick Tip

To find the median class, locate where the cumulative frequency first becomes greater than $\frac{N}{2}$.

PART B

1. Do all the parts:

(a) If the distance between the points $(x, 5)$ and $(2, -3)$ is 17 units, then find the value of x.

Correct Answer: $x = -13$ or $x = 17$

Solution:

Step 1: Use the distance formula.

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 2: Substitute the given values.

Here, $(x_1, y_1) = (x, 5)$ and $(x_2, y_2) = (2, -3)$, and $d = 17$.

$$17 = \sqrt{(2 - x)^2 + (-3 - 5)^2}$$

$$17 = \sqrt{(2 - x)^2 + 64}$$

Step 3: Square both sides.

$$289 = (2 - x)^2 + 64$$

$$225 = (2 - x)^2$$

Step 4: Solve for x.

$$2 - x = \pm 15 \Rightarrow \begin{cases} x = -13, & \text{if } 2 - x = 15 \\ x = 17, & \text{if } 2 - x = -15 \end{cases}$$

Step 5: Conclusion.

Hence, the values of x are -13 and 17.

Quick Tip

Always use the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find unknown coordinates when the distance is given.

(b) If the points $(1, 4)$, $(a, -2)$, and $(-3, 16)$ are collinear, then find the value of a.

Correct Answer: $a = 3$

Solution:

Step 1: Recall the condition for collinearity.

Three points are collinear if the slopes between them are equal.

$$\text{Slope of } AB = \text{Slope of } BC$$

Step 2: Compute slopes.

Let $A(1,4)$, $B(a,-2)$, and $C(-3,16)$.

$$\text{Slope of } AB = \frac{-2-4}{a-1} = \frac{-6}{a-1}$$

$$\text{Slope of } BC = \frac{16-(-2)}{-3-a} = \frac{18}{-3-a} = \frac{-18}{a+3}$$

Step 3: Equate the slopes.

$$\frac{-6}{a-1} = \frac{-18}{a+3} \Rightarrow 6(a+3) = 18(a-1)$$

$$6a + 18 = 18a - 18 \Rightarrow 12a = 36 \Rightarrow a = 3$$

Step 4: Conclusion.

Hence, $a = 3$.

> **Quick Tip**
>
> For three points to be collinear, their slopes must be equal. Equate the slopes and solve for the variable.

(c) Prove that $\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta} = 2\sec\theta$**.**

Correct Answer: $2\sec\theta$

Solution:

Step 1: Take the LHS.

$$\text{LHS} = \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$$

Step 2: Take a common denominator.

$$\text{LHS} = \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

Step 3: Expand the numerator.

$$(1+\sin\theta)^2 + \cos^2\theta = 1 + 2\sin\theta + \sin^2\theta + \cos^2\theta$$

$$= 2(1+\sin\theta)$$

Step 4: Simplify.

$$\text{LHS} = \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)} = \frac{2}{\cos\theta} = 2\sec\theta$$

Step 5: Conclusion.

Hence proved,

$$\frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta} = 2\sec\theta$$

Quick Tip

Use $\sin^2\theta + \cos^2\theta = 1$ whenever both $\sin\theta$ and $\cos\theta$ appear together.

(d) Find the median from the following frequency distribution:

Class-interval	Frequency
0–10	6
10–20	9
20–30	20
30–40	15
40–50	9

Correct Answer: 27.25

Solution:

Step 1: Compute cumulative frequencies.

Class	f	Cumulative Frequency (CF)
0–10	6	6
10–20	9	15
20–30	20	35
30–40	15	50
40–50	9	59

Step 2: Find $N/2$.

$$N = 59, \quad N/2 = 29.5$$

Step 3: Locate the median class.

The class with cumulative frequency just greater than 29.5 is 20–30.

Step 4: Apply the median formula.

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f} \right) \times h$$

Substitute: $L = 20,\ CF = 15,\ f = 20,\ h = 10$

$$\text{Median} = 20 + \left(\frac{29.5 - 15}{20} \right) \times 10 = 20 + 7.25 = 27.25$$

Step 5: Conclusion.

Hence, the median = **27.25**.

Quick Tip

For the median in a frequency table, always find the class where cumulative frequency first exceeds $N/2$.

(e) Find the LCM of the numbers 92 and 510.

Correct Answer: 23460

Solution:

Step 1: Prime factorization.

$$92 = 2^2 \times 23$$

$$510 = 2 \times 3 \times 5 \times 17$$

Step 2: Take all unique prime factors with highest powers.

$$\text{LCM} = 2^2 \times 3 \times 5 \times 17 \times 23$$

Step 3: Multiply.

$$4 \times 3 \times 5 \times 17 \times 23 = 23460$$

Step 4: Conclusion.

Hence, the LCM of 92 and 510 is **23460**.

> **Quick Tip**
>
> Take the highest powers of all prime factors from both numbers to find the LCM.

(f) Prove that $\sqrt{3}$ is an irrational number.

Correct Answer: $\sqrt{3}$ is irrational.

Solution:

Step 1: Assume the opposite.

Let $\sqrt{3}$ be rational. Then it can be expressed as:

$$\sqrt{3} = \frac{p}{q}, \quad \text{where } p, q \text{ are integers and } \gcd(p, q) = 1$$

Step 2: Square both sides.

$$3 = \frac{p^2}{q^2} \Rightarrow p^2 = 3q^2$$

Step 3: Analyze divisibility.

Since p^2 is divisible by 3, p must also be divisible by 3. Let $p = 3k$.

Step 4: Substitute back.

$$p^2 = (3k)^2 = 9k^2 \Rightarrow 9k^2 = 3q^2 \Rightarrow q^2 = 3k^2$$

Thus, q^2 is also divisible by 3, so q is divisible by 3.

Step 5: Contradiction.

This contradicts the assumption that p and q have no common factor other than 1.

Step 6: Conclusion.

Therefore, $\sqrt{3}$ is irrational.

Quick Tip

Use the method of contradiction to prove irrationality — assume rationality and reach a contradiction.

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2. Do any five parts:

(a) Find the mode from the following table:

Class-interval	Frequency
0–10	6
10–20	11
20–30	21
30–40	23
40–50	14

Correct Answer: 31.25

Solution:

Step 1: Identify the modal class.

The class with the highest frequency is $30–40$, so it is the **modal class**.

Step 2: Write the given data.

$$L = 30, \quad f_1 = 23, \quad f_0 = 21, \quad f_2 = 14, \quad h = 10$$

Step 3: Apply the formula for mode.

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

Step 4: Substitute the values.

$$\text{Mode} = 30 + \left(\frac{23 - 21}{2(23) - 21 - 14} \right) \times 10$$

$$= 30 + \left(\frac{2}{46 - 35} \right) \times 10 = 30 + \frac{20}{11} = 31.82$$

Step 5: Conclusion.

Hence, the mode = **31.82 (approx.)**.

> **Quick Tip**
>
> The modal class is the class interval with the highest frequency. Always apply the mode formula using f_1, f_0, f_2 correctly.

(b) Prove that the lengths of tangents drawn from an external point to a circle are equal.

Correct Answer: Tangent lengths from an external point to a circle are equal.

Solution:

Step 1: Construction.

Let P be a point outside the circle with center O. From P, draw two tangents PA and PB to the circle, touching it at A and B respectively.

Step 2: Join OA and OB.

These are radii of the circle. Therefore,

$$OA = OB$$

Step 3: Observe right angles.

Since PA and PB are tangents, they are perpendicular to the radii at the point of contact.

$$\angle OAP = \angle OBP = 90^\circ$$

Step 4: Consider triangles $\triangle OAP$ and $\triangle OBP$.

In both triangles,

$$OA = OB \quad \text{(radii)}$$

OP = OP (common side)

$\angle OAP = \angle OBP = 90^\circ$ Hence,

$$\triangle OAP \cong \triangle OBP \quad \text{(by RHS congruence)}$$

Step 5: Conclusion.

By congruence,

$$PA = PB$$

Therefore, the lengths of tangents drawn from an external point to a circle are equal.

Quick Tip

When proving tangents equal, always use the congruence of right triangles formed by radii and tangents (RHS rule).

(c) D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Prove that $CA^2 = CB \cdot CD$.

Correct Answer: $CA^2 = CB \cdot CD$

Solution:

Step 1: Given condition.

In $\triangle ABC$, D lies on BC such that $\angle ADC = \angle BAC$.

Step 2: Draw auxiliary lines.

Join AD and CA.

Step 3: Observe similar triangles.

In $\triangle CAD$ and $\triangle CBA$:

$$\angle ADC = \angle BAC \quad \text{(given)}$$

$\angle ACD = \angle ACB \quad$ (common) Hence,

$$\triangle CAD \sim \triangle CBA \quad \text{(by AA similarity)}$$

Step 4: Write the ratio of corresponding sides.

$$\frac{CA}{CB} = \frac{CD}{CA}$$

Step 5: Cross-multiply.

$$CA^2 = CB \times CD$$

Step 6: Conclusion.

Hence proved that $CA^2 = CB \cdot CD$.

Quick Tip

In geometry proofs involving equal angles, check for AA similarity — it often helps establish proportional sides.

(d) The sum of the digits of a two-digit number is 9. 9 times of this number is equal to 2 times the number formed by reversing the digits. Find the number.

Correct Answer: The number is 27.

Solution:

Step 1: Let the digits of the number be.

Let the tens digit be x and the units digit be y. Then, the number is $10x + y$.

Step 2: Write the given conditions.

The sum of the digits is 9:

$$x + y = 9 \quad \text{(i)}$$

9 times the number is equal to twice the number formed by reversing the digits:

$$9(10x + y) = 2(10y + x)$$

Step 3: Simplify the equation.

$$90x + 9y = 20y + 2x$$

$$88x = 11y \Rightarrow 8x = y \quad \text{(ii)}$$

Step 4: Substitute (ii) into (i).

$$x + 8x = 9 \Rightarrow 9x = 9 \Rightarrow x = 1$$

$$y = 8x = 8$$

Step 5: Find the number.

$$\text{Number} = 10x + y = 10(1) + 8 = 18$$

Step 6: Check the condition.

$$9 \times 18 = 162, \quad 2 \times 81 = 162$$

Hence, the condition is satisfied.

Step 7: Conclusion.

The required number is **18**.

Quick Tip

For two-digit number problems, always assume digits as x and y and form equations using their sum and reversal relations.

(e) Solve the quadratic equation $2x^2 - 5x + 3 = 0$**.**

Correct Answer: $x = 1$ and $x = \dfrac{3}{2}$

Solution:

Step 1: Identify coefficients.

Here, $a = 2$, $b = -5$, and $c = 3$.

Step 2: Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 3: Substitute the values.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{4} = \frac{5 \pm 1}{4}$$

Step 4: Simplify.

$$x = \frac{5+1}{4} = \frac{6}{4} = \frac{3}{2} \quad \text{and} \quad x = \frac{5-1}{4} = 1$$

Step 5: Conclusion.

Hence, the roots of the equation are $x = 1$ and $x = \frac{3}{2}$.

Quick Tip

Always check the discriminant $b^2 - 4ac$ before solving; it helps determine the nature of roots.

(f) If the perimeter of a garden of area 800 m^2 is 120 m and its length is twice the breadth, then find its length and breadth.

Correct Answer: Length = 40 m, Breadth = 20 m

Solution:

Step 1: Let the breadth be x m.

Then, the length = $2x$ m (given).

Step 2: Write the formula for perimeter.

$$\text{Perimeter} = 2(l + b)$$

Substitute:

$$120 = 2(2x + x) \Rightarrow 120 = 6x \Rightarrow x = 20$$

Step 3: Find the length.

$$l = 2x = 2 \times 20 = 40$$

Step 4: Verify using the area condition.

$$\text{Area} = l \times b = 40 \times 20 = 800\,\text{m}^2$$

The condition is satisfied.

Step 5: Conclusion.

Hence, the **length = 40 m** and **breadth = 20 m**.

Quick Tip

For rectangle problems, always form two equations — one from perimeter and one from area — to find unknown dimensions.

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3. Solve the following equations:

$$\frac{3}{2}x - \frac{5}{3}y = -2, \quad \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

Correct Answer: $x = 1$, $y = 3$

Solution:

Step 1: Eliminate the fractions by taking the LCM of denominators.

For the first equation:

$$\frac{3}{2}x - \frac{5}{3}y = -2$$

Multiply both sides by 6 (LCM of 2 and 3):

$$9x - 10y = -12 \quad \text{(i)}$$

For the second equation:

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

Multiply both sides by 6:

$$2x + 3y = 13 \quad \text{(ii)}$$

Step 2: Solve the two linear equations.

Equation (i): $9x - 10y = -12$ Equation (ii): $2x + 3y = 13$

Step 3: Multiply (ii) by 3 to align coefficients of y.

$$6x + 9y = 39 \quad \text{(iii)}$$

Step 4: Eliminate y.

Multiply (i) by 3:

$$27x - 30y = -36 \quad \text{(iv)}$$

Multiply (ii) by 10:

$$20x + 30y = 130 \quad \text{(v)}$$

Add (iv) and (v):

$$47x = 94 \Rightarrow x = 2$$

Step 5: Substitute $x = 2$ in equation (ii).

$$2(2) + 3y = 13 \Rightarrow 4 + 3y = 13 \Rightarrow 3y = 9 \Rightarrow y = 3$$

Step 6: Conclusion.

Hence, $x = 2$ and $y = 3$.

Quick Tip

To eliminate fractions, always multiply through by the LCM of denominators before applying elimination or substitution.

OR

Find the sum of 51 terms of an A.P. whose second and third terms are 14 and 18 respectively.

Correct Answer: $S_{51} = 7143$

Solution:

Step 1: Let the first term and common difference be a and d.

The general term of an A.P. is $a_n = a + (n-1)d$.

Step 2: Use the given conditions.

$$a_2 = a + d = 14 \quad \text{(i)}$$

$$a_3 = a + 2d = 18 \quad \text{(ii)}$$

Step 3: Subtract (i) from (ii).

$$(a + 2d) - (a + d) = 18 - 14 \Rightarrow d = 4$$

Step 4: Substitute $d = 4$ in (i).

$$a + 4 = 14 \Rightarrow a = 10$$

Step 5: Use the sum formula of an A.P.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

For $n = 51$:

$$S_{51} = \frac{51}{2}[2(10) + 50(4)]$$

$$= \frac{51}{2}[20 + 200] = \frac{51}{2} \times 220 = 51 \times 110 = 5610$$

Step 6: Conclusion.

Hence, the sum of 51 terms of the A.P. is **5610**.

Quick Tip

For A.P. problems, always identify the first term and common difference from the given terms before applying the sum formula.

4. A flagstaff stands on a tower. At a distance 10 m from the tower, the angles of elevation of the top of the tower and flagstaff are 45° and 60° respectively. Find the length of the flagstaff.

Correct Answer: 5.86 m

Solution:

Step 1: Let the height of the tower be h m and the height of the flagstaff be x m.

Total height of tower + flagstaff = $h + x$. Distance from the tower = $10\,\text{m}$.

Step 2: Use trigonometric ratios for each angle of elevation.

For the top of the tower:

$$\tan 45^\circ = \frac{h}{10} \Rightarrow 1 = \frac{h}{10} \Rightarrow h = 10\,\text{m}$$

For the top of the flagstaff:

$$\tan 60^\circ = \frac{h+x}{10} \Rightarrow \sqrt{3} = \frac{h+x}{10} \Rightarrow h + x = 10\sqrt{3}$$

Step 3: Substitute the value of h.

$$10 + x = 10\sqrt{3} \Rightarrow x = 10(\sqrt{3} - 1)$$

$$x = 10(1.732 - 1) = 7.32\,\text{m}$$

Step 4: Conclusion.

Hence, the length of the flagstaff is approximately **7.32 m**.

Quick Tip

In height and distance problems, always draw a right triangle diagram and use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ for elevation or depression.

OR

From the top of a tower of height 50 m, the angles of depression of the top and bottom of a pillar are 45° and 60° respectively. Find the height of the pillar.

Correct Answer: 21.13 m

Solution:

Step 1: Let the height of the pillar be h m.

Let the horizontal distance between the tower and pillar be x m.

Step 2: Use trigonometric ratios for the angles of depression.

For the top of the pillar $(45°)$:

$$\tan 45° = \frac{50-h}{x} \Rightarrow 1 = \frac{50-h}{x} \Rightarrow x = 50-h$$

For the bottom of the pillar $(60°)$:

$$\tan 60° = \frac{50}{x} \Rightarrow \sqrt{3} = \frac{50}{x} \Rightarrow x = \frac{50}{\sqrt{3}}$$

Step 3: Equate both expressions for x.

$$50 - h = \frac{50}{\sqrt{3}} \Rightarrow h = 50 - \frac{50}{\sqrt{3}}$$

Step 4: Simplify.

$$h = 50\left(1 - \frac{1}{\sqrt{3}}\right) = 50\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = \frac{50(\sqrt{3}-1)}{1.732}$$
$$h \approx 50(0.577) = 28.85\,\text{m}$$

Step 5: Conclusion.

Hence, the height of the pillar is approximately **28.87 m**.

> **Quick Tip**
>
> Angles of depression are measured from the horizontal line of sight; their corresponding angle of elevation at the other end is equal.

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5. A chord of a circle of radius 15 cm subtends an angle $60°$ at the centre. Find the area of minor and major sectors of the circle.

$$(\pi = 3.14,\ \sqrt{3} = 1.73)$$

Correct Answer: Area of minor sector = 117.75 cm^2

Area of major sector = 588.75 cm^2

Solution:

Step 1: Write the formula for the area of a sector.

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

Step 2: Substitute the given values for the minor sector.

$$\theta = 60^\circ, \quad r = 15\,\text{cm}$$

$$\text{Area of minor sector} = \frac{60}{360} \times 3.14 \times 15^2$$

$$= \frac{1}{6} \times 3.14 \times 225 = 117.75\,\text{cm}^2$$

Step 3: Find the area of the major sector.

Since the total area of the circle is:

$$\pi r^2 = 3.14 \times 15^2 = 706.5\,\text{cm}^2$$

$$\text{Area of major sector} = 706.5 - 117.75 = 588.75\,\text{cm}^2$$

Step 4: Conclusion.

$$\boxed{\text{Minor sector area} = 117.75\,\text{cm}^2}$$

$$\boxed{\text{Major sector area} = 588.75\,\text{cm}^2}$$

Quick Tip

Always use $\frac{\theta}{360^\circ}$ for finding the fractional part of the circle corresponding to the sector's angle.

OR

By taking out a hemisphere from both the ends of a wooden solid cylinder, an item is formed. If the height of the cylinder is 10 cm and radius of the base is 3.5 cm, find the total surface area of the item.

Correct Answer: 410.48 cm^2

Solution:

Step 1: Given data.

Radius, $r = 3.5\,\text{cm}$

Height of the cylinder, $h = 10\,\text{cm}$

Step 2: Total surface area (TSA) of the item.

Since hemispheres are taken out from both ends, there are no circular bases. Thus,

$$\text{TSA} = \text{Curved surface area of cylinder} + 2 \times \text{Curved surface area of hemisphere}$$

Step 3: Write the formulas.

$$\text{CSA of cylinder} = 2\pi rh$$

$$\text{CSA of one hemisphere} = 2\pi r^2$$

Therefore,

$$\text{TSA} = 2\pi rh + 2(2\pi r^2) = 2\pi r(h + 2r)$$

Step 4: Substitute the given values.

$$\text{TSA} = 2 \times 3.14 \times 3.5(10 + 2 \times 3.5)$$

$$= 6.28 \times 3.5 \times 17 = 6.28 \times 59.5 = 373.66\,\text{cm}^2$$

Step 5: Conclusion.

Hence, the total surface area of the item is approximately **373.66 cm**^{2}.

Quick Tip

Always exclude the base area when hemispheres are removed from a cylinder. Add the curved surface area of the cylinder and two hemispheres only.