UP Board Class 10 Mathematics - 822(BV) - 2025 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :70	Total Questions :5

General Instructions

Read the following instructions very carefully and strictly follow them:

1. All questions are compulsory.
2. This question paper has two sections 'A' and 'B'.
3. Section 'A' contains 20 Multiple Choice type Questions of 1 mark each that have to be answered on OMR answer sheet by darkening completely the correct circle with blue or black ballpoint pen.
4. After giving answer on OMR answer sheet, do not cut or use eraser, whitener etc.
5. Section 'B' contains descriptive type questions of 50 marks.
6. Total 5 questions are there in this section.
7. In the beginning of each question, it has been mentioned how many parts of it are to be attempted.
8. Marks allotted to each question are mentioned against it.
9. Start from the first question and go up to the last question. Do not waste your time on the question you cannot solve.
10. If you need place for rough work do it on the left page of your answer book and cross (X) the page. Do not write any solution on that page.
11. Draw neat and correct figure in the solution of a question wherever it is necessary, otherwise in its absence the solution will be treated incomplete and wrong.

Section - A

1. The sum of a rational number and an irrational number will be:

(A) Rational number
(B) Natural number
(C) Whole number
(D) Irrational number

Correct Answer: (D) Irrational number

Solution:

Step 1: Understanding the Concept:
This question is based on the properties of real numbers.
A rational number is one that can be written in the form p/q, where p and q are integers and $q \neq 0$ (e.g., 2, 3/4, -5).
An irrational number is one that cannot be written in the form p/q (e.g., $\sqrt{2}$, $\sqrt{3}$, π).

Step 2: Detailed Explanation:
The sum of a rational number and an irrational number is always an irrational number.
We can prove this by contradiction:
Let 'r' be a rational number and 'i' be an irrational number.
Let's assume their sum, $r + i = q$, is a rational number.
Then, $i = q - r$.
Since the difference of two rational numbers (q and r) is also a rational number, this implies that 'i' must be a rational number.
This contradicts our original assumption that 'i' is an irrational number.
Therefore, our initial belief that their sum is rational is false.
Hence, the sum of a rational and an irrational number is always irrational.
Example: 2 (rational) + $\sqrt{3}$ (irrational) = $2+\sqrt{3}$ (irrational).

Step 3: Final Answer:
The sum of a rational number and an irrational number is always an irrational number.
Therefore, option (D) is correct.

Quick Tip

Remember this rule: (Rational $\pm$ Irrational) = Irrational. Similarly, the product or quotient of a non-zero rational number and an irrational number is also irrational.

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2. The prime factorization of the number 156 will be:

(A) $2 \times 3 \times 13$
(B) $2^2 \times 3 \times 13$
(C) $2^2 \times 3 \times 11$
(D) $2 \times 3^2 \times 13$

Correct Answer: (B) $2^2 \times 3 \times 13$

Solution:

Step 1: Understanding the Concept:
Prime factorization is the process of expressing a number as the product of its prime factors.

Step 2: Key Formula or Approach:
We start dividing the given number by the smallest prime number and continue until we get 1.

Step 3: Detailed Explanation:
To find the prime factorization of 156, we divide it sequentially by prime numbers:

$$156 \div 2 = 78$$
$$78 \div 2 = 39$$
$$39 \div 3 = 13$$

13 is itself a prime number, so we divide it by 13:

$$13 \div 13 = 1$$

Thus, the prime factors of 156 are $2, 2, 3$, and 13.
Writing it as a product:

$$156 = 2 \times 2 \times 3 \times 13$$

Expressing in exponential form:

$$156 = 2^2 \times 3 \times 13$$

Step 4: Final Answer:
The prime factorization of 156 is $2^2 \times 3 \times 13$, which matches option (B).

Quick Tip

Always start division with the smallest prime number (2), then move to 3, 5, and so on. This reduces the chances of making mistakes.

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3. If $\frac{3x+5y}{3x-5y} = \frac{7}{3}$, then the value of $x : y$ will be:

(A) 25 : 6
(B) 5 : 3
(C) 3 : 5
(D) 7 : 3

Correct Answer: (A) 25 : 6

Solution:

Step 1: Understanding the Concept:
We need to find the ratio of x to y by solving the given equation. We can solve this using

cross-multiplication or the rule of Componendo and Dividendo.

Step 2: Key Formula or Approach:
Method 1: Cross-multiplication
Cross-multiply the numerator and denominator in the given equation.

$$\frac{3x+5y}{3x-5y} = \frac{7}{3}$$
$$3(3x+5y) = 7(3x-5y)$$

Step 3: Detailed Explanation:
Solve the equation:

$$9x + 15y = 21x - 35y$$

Bring the terms with x to one side and the terms with y to the other:

$$15y + 35y = 21x - 9x$$
$$50y = 12x$$

We need to find the value of $x : y$ or x/y, so rearrange the equation:

$$\frac{x}{y} = \frac{50}{12}$$

Simplify the fraction:

$$\frac{x}{y} = \frac{25}{6}$$

Thus, $x : y = 25 : 6$.

Step 4: Final Answer:
The value of $x : y$ is $25 : 6$.
Therefore, option (A) is correct.

Quick Tip

For such questions, the Componendo and Dividendo rule can be a faster method. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. Here, applying it in reverse, $\frac{3x}{5y} = \frac{7+3}{7-3} = \frac{10}{4} = \frac{5}{2}$, which gives $\frac{x}{y} = \frac{25}{6}$.

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4. The discriminant of the equation $3x^2 - 2x + \frac{1}{3} = 0$ will be:

(A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:
The discriminant determines the nature of the roots of a quadratic equation.
For a standard quadratic equation $ax^2 + bx + c = 0$, the discriminant is denoted by D.

Step 2: Key Formula or Approach:
The formula for the discriminant is:

$$D = b^2 - 4ac$$

Step 3: Detailed Explanation:
Comparing the given equation $3x^2 - 2x + \frac{1}{3} = 0$ with the standard equation $ax^2 + bx + c = 0$, we get the coefficients:

$$a = 3, \quad b = -2, \quad c = \frac{1}{3}$$

Substitute these values into the discriminant formula:

$$D = (-2)^2 - 4(3)\left(\frac{1}{3}\right)$$
$$D = 4 - 4(1)$$
$$D = 4 - 4$$
$$D = 0$$

Step 4: Final Answer:
The discriminant of the given equation is 0.
Therefore, option (A) is correct.

Quick Tip

When the discriminant (D) is 0, the quadratic equation has two equal and real roots. If D ¿ 0, there are two distinct real roots, and if D ¡ 0, there are no real roots.

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5. The mean of the following table will be:

Class-Interval	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
Frequency	2	4	7	3	4

(A) 6.0
(B) 6.3
(C) 7.0
(D) 7.3

Correct Answer: (D) 7.3

Solution:

Step 1: Understanding the Concept:
To find the mean of grouped data, we use the mid-point (class mark) of each class and calculate the mean using the direct method.

Step 2: Key Formula or Approach:
The formula for the mean ($\bar{x}$) is:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Where f_i is the frequency and x_i is the class mark.
Class Mark (x_i) = (Upper class limit + Lower class limit) / 2

Step 3: Detailed Explanation:
We create a table for the calculations:

Class-Interval	**Frequency** (f_i)	**Class Mark** (x_i)	$f_i x_i$
2 – 4	2	$(2+4)/2 = 3$	$2 \times 3 = 6$
4 – 6	4	$(4+6)/2 = 5$	$4 \times 5 = 20$
6 – 8	7	$(6+8)/2 = 7$	$7 \times 7 = 49$
8 – 10	3	$(8+10)/2 = 9$	$3 \times 9 = 27$
10 – 12	4	$(10+12)/2 = 11$	$4 \times 11 = 44$
Total	$\sum f_i = 20$		$\sum f_i x_i = 146$

Now, we calculate the mean using the formula:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{146}{20}$$
$$\bar{x} = 7.3$$

Step 4: Final Answer:
The mean of the table is 7.3.
Therefore, option (D) is correct.

Quick Tip

When calculating the mean, create the calculation table carefully. Ensure accuracy in calculating the class marks (x_i) and the products $f_i x_i$. It's a good practice to double-check the sums.

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6. In $\triangle ABC$, $DE \parallel BC$ such that $AD = 1.5$ cm, $DB = 3$ cm and $AE = 2$ cm. The measure of AC will be:

(A) 6.0 cm
(B) 4.5 cm
(C) 4.0 cm
(D) 3.0 cm

Correct Answer: (A) 6.0 cm

Solution:

Step 1: Understanding the Concept:
This question is based on the similarity of triangles and the Basic Proportionality Theorem (Thales' Theorem).
According to this theorem, if a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, then the other two sides are divided in the same ratio.

Step 2: Key Formula or Approach:
According to the Basic Proportionality Theorem (Thales' Theorem), if in $\triangle ABC$, $DE \parallel BC$, then:

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Step 3: Detailed Explanation:
The given values are:
$AD = 1.5$ cm
$DB = 3$ cm
$AE = 2$ cm

Substituting these values into the formula:

$$\frac{1.5}{3} = \frac{2}{EC}$$

Now, solve for EC:

$$1.5 \times EC = 3 \times 2$$

$$1.5 \times EC = 6$$

$$EC = \frac{6}{1.5}$$

$$EC = \frac{60}{15} = 4 \text{ cm}$$

We need to find the length of AC. From the figure, it is clear that:

$$AC = AE + EC$$

$$AC = 2 \text{ cm} + 4 \text{ cm} = 6 \text{ cm}$$

Step 4: Final Answer:
The measure of AC is 6.0 cm.
Therefore, option (A) is correct.

> **Quick Tip**
>
> The Basic Proportionality Theorem (Thales' Theorem) and its converse are extremely important for solving problems related to parallel lines in triangles. Always substitute the given values into the formula correctly.

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7. The distance between the points (2, 3) and (4, 1) will be:

(A) 2 units
(B) $2\sqrt{2}$ units
(C) $\sqrt{2}$ units
(D) 3 units

Correct Answer: (B) $2\sqrt{2}$ units

Solution:

Step 1: Understanding the Concept:
This question is based on coordinate geometry and involves using the distance formula to find the distance between two points.

Step 2: Key Formula or Approach:
The formula for the distance (d) between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 3: Detailed Explanation:
The given points are $(2, 3)$ and $(4, 1)$.
Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 1)$.

Substitute these values into the distance formula:

$$d = \sqrt{(4-2)^2 + (1-3)^2}$$
$$d = \sqrt{(2)^2 + (-2)^2}$$
$$d = \sqrt{4+4}$$
$$d = \sqrt{8}$$

Simplifying $\sqrt{8}$:

$$d = \sqrt{4 \times 2} = 2\sqrt{2}$$

Hence, the distance is $2\sqrt{2}$ units.

Step 4: Final Answer:
The distance between the points (2, 3) and (4, 1) is $2\sqrt{2}$ units.
Therefore, option (B) is correct.

> **Quick Tip**
>
> When using the distance formula, be careful with the signs of the coordinates, especially during subtraction. Remember that the square of any number is always positive, so $(-2)^2 = 4$.

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8. If the roots of the quadratic equation $3x^2 - 12x + m = 0$ are equal, then the value of m will be:

(A) 4
(B) 6
(C) 12
(D) 14

Correct Answer: (C) 12

Solution:

Step 1: Understanding the Concept:
The roots of a quadratic equation $ax^2+bx+c = 0$ are equal if and only if its discriminant is zero.

Step 2: Key Formula or Approach:
The condition for equal roots is:
Discriminant, $D = b^2 - 4ac = 0$

Step 3: Detailed Explanation:
Comparing the given equation $3x^2 - 12x + m = 0$ with the standard quadratic equation $ax^2 + bx + c = 0$:
$a = 3$
$b = -12$
$c = m$
Since the roots are equal, we use the condition $D = 0$:

$$b^2 - 4ac = 0$$
$$(-12)^2 - 4(3)(m) = 0$$
$$144 - 12m = 0$$
$$144 = 12m$$
$$m = \frac{144}{12}$$
$$m = 12$$

Step 4: Final Answer:
The value of m is 12.
Therefore, option (C) is correct.

> Quick Tip
>
> The nature of the roots of a quadratic equation depends on the discriminant (D):
> - $D > 0$: Two distinct real roots.
> - $D = 0$: Two equal real roots.
> - $D < 0$: No real roots.

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9. The HCF of 15 and 25 is 5, so their LCM will be:

(A) 150
(B) 75
(C) 125
(D) 100

Correct Answer: (B) 75

Solution:

Step 1: Understanding the Concept:
This question is based on the relationship between the HCF (Highest Common Factor) and LCM (Lowest Common Multiple) of two numbers.

Step 2: Key Formula or Approach:
For two positive integers 'a' and 'b', the formula for the relationship between their HCF and LCM is:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Step 3: Detailed Explanation:
The given values are:
First number (a) = 15
Second number (b) = 25
HCF(15, 25) = 5

Using the formula:

$$5 \times \text{LCM}(15, 25) = 15 \times 25$$
$$\text{LCM}(15, 25) = \frac{15 \times 25}{5}$$
$$\text{LCM}(15, 25) = 3 \times 25$$
$$\text{LCM}(15, 25) = 75$$

Step 4: Final Answer:
The LCM of 15 and 25 is 75.
Therefore, option (B) is correct.

Quick Tip

This formula (Product of two numbers = HCF × LCM) is applicable only for two numbers. It is not valid for three or more numbers.

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10. For the A.P. 3, 1, -1, -3, ..., the common difference will be:

(A) -2
(B) 1
(C) 2
(D) 5

Correct Answer: (A) -2

Solution:

Step 1: Understanding the Concept:
In an Arithmetic Progression (A.P.), the common difference is the constant difference between

any two consecutive terms.

Step 2: Key Formula or Approach:
The formula for the common difference (d) is:

$$d = a_2 - a_1$$

where a_2 is the second term and a_1 is the first term.

Step 3: Detailed Explanation:
The given A.P. is: 3, 1, -1, -3, ...
Here, the first term $a_1 = 3$.
The second term $a_2 = 1$.
Calculate the common difference using the formula:

$$d = 1 - 3$$
$$d = -2$$

We can also use the next two terms to verify:

$$d = a_3 - a_2 = -1 - 1 = -2$$

Since the difference is constant, the common difference is -2.

Step 4: Final Answer:
The common difference for the given A.P. is -2.
Therefore, option (A) is correct.

Quick Tip

When calculating the common difference, always subtract the preceding term from the succeeding term ($a_{n+1} - a_n$). If you subtract the succeeding term from the preceding term, the sign will be incorrect.

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11. The median class of the following table will be:

Class-Interval	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	2	7	3	10	4

(A) 0 – 5
(B) 5 – 10
(C) 10 – 15
(D) 15 – 20

Correct Answer: (D) 15 – 20

Solution:

Step 1: Understanding the Concept:
The median class is the class interval in which the median lies. To find it, we first need to calculate the cumulative frequency.

Step 2: Key Formula or Approach:
1. Find the sum of all frequencies ($N = \sum f_i$).
2. Calculate $N/2$.
3. Find the class interval whose cumulative frequency is just greater than $N/2$. This is the median class.

Step 3: Detailed Explanation:
We calculate the cumulative frequency (c.f.) for the given table:

Class-Interval	Frequency (f_i)	Cumulative Frequency (c.f.)
0 – 5	2	2
5 – 10	7	$2 + 7 = 9$
10 – 15	3	$9 + 3 = 12$
15 – 20	10	$12 + 10 = 22$
20 – 25	4	$22 + 4 = 26$

Total frequency, $N = \sum f_i = 26$.
Now, we calculate $N/2$:

$$\frac{N}{2} = \frac{26}{2} = 13$$

We need to find the class whose cumulative frequency is just greater than 13.
From the table, we see that the cumulative frequency 22 is just greater than 13.
The corresponding class interval for this cumulative frequency is 15 – 20.

Step 4: Final Answer:
Hence, the median class is 15 – 20.
Therefore, option (D) is correct.

Quick Tip

To find the median class, creating a cumulative frequency table is the first and most important step. Always compare the value of $N/2$ with the cumulative frequency column, not the frequency column.

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12. If $\tan A = 1$, then the value of $2 \sin A \cos A$ will be:

(A) 0
(B) $\frac{1}{2}$
(C) 1
(D) 2

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Concept:
This question is based on trigonometry. We need to find the value of angle A from the given value of $\tan A$ and then substitute it into the given expression.

Step 2: Key Formula or Approach:
Method 1: By finding the value of the angle
We know that $\tan 45^\circ = 1$.
Therefore, if $\tan A = 1$, then $A = 45^\circ$.
Now we will substitute this value into the expression $2 \sin A \cos A$.

Method 2: Using the identity
We can use the trigonometric identity $\sin 2A = 2 \sin A \cos A$.

Step 3: Detailed Explanation:
Solution by Method 1:
Given, $\tan A = 1$.
We know that $\tan 45^\circ = 1$, so $A = 45^\circ$.
Now, substituting $A = 45^\circ$ into the expression $2 \sin A \cos A$:

$$2 \sin 45^\circ \cos 45^\circ$$

We know that $\sin 45^\circ = \frac{1}{\sqrt{2}}$ and $\cos 45^\circ = \frac{1}{\sqrt{2}}$.

$$= 2 \times \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{\sqrt{2}}\right)$$
$$= 2 \times \frac{1}{2}$$
$$= 1$$

Step 4: Final Answer:
The value of $2 \sin A \cos A$ is 1.
Therefore, option (C) is correct.

Quick Tip

Remembering the identity $\sin 2A = 2\sin A\cos A$ helps solve such problems very quickly. From $\tan A = 1$, we get $A = 45°$, so $\sin(2 \times 45°) = \sin(90°) = 1$.

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13. From a point Q, 25 cm away from the center of a circle, the length of the tangent to the circle is 24 cm. The radius of the circle will be:

(A) 7 cm
(B) 12 cm
(C) 15 cm
(D) 24.5 cm

Correct Answer: (A) 7 cm

Solution:

Step 1: Understanding the Concept:
The tangent at any point of a circle is perpendicular to the radius through the point of contact. This property forms a right-angled triangle with the center, the external point, and the point of contact as vertices.

Step 2: Key Formula or Approach:
Use Pythagoras' theorem. If O is the center, Q is the external point, and P is the point of contact, then $\triangle OPQ$ is a right-angled triangle with $\angle OPQ = 90°$.
According to Pythagoras' theorem:

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2$$
$$OQ^2 = OP^2 + PQ^2$$

Where OQ is the distance of the external point from the center, OP is the radius, and PQ is the length of the tangent.

Step 3: Detailed Explanation:
The given values are:
Distance of point Q from the center, $OQ = 25$ cm.
Length of the tangent, $PQ = 24$ cm.
Radius of the circle, $OP = r$ (to be found).

Substituting the values into the Pythagoras' theorem formula:

$$25^2 = r^2 + 24^2$$
$$625 = r^2 + 576$$

Solve for r^2:

$$r^2 = 625 - 576$$
$$r^2 = 49$$
$$r = \sqrt{49}$$
$$r = 7 \text{ cm}$$

Step 4: Final Answer:
The radius of the circle is 7 cm.
Therefore, option (A) is correct.

Quick Tip

Remembering Pythagorean triplets like (3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17) saves time in competitive exams. Here, we saw the (7, 24, 25) triplet.

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14. The height of a tower is 20 m. The length of its shadow formed on the ground is $20\sqrt{3}$ m; the value of the angle of elevation will be:

(A) 30°
(B) 45°
(C) 60°
(D) 90°

Correct Answer: (A) 30°

Solution:

Step 1: Understanding the Concept:
This question is based on the applications of trigonometry (height and distance).
The angle of elevation is the angle between the horizontal line and the line of sight to an object above the horizontal.
Here, the height of the tower (perpendicular) and the length of the shadow (base) form a right-angled triangle.

Step 2: Key Formula or Approach:
We will use the trigonometric ratio $\tan\theta$:

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Height of the tower}}{\text{Length of the shadow}}$$

Step 3: Detailed Explanation:
The given values are:

Height of the tower (Perpendicular) = 20 m
Length of the shadow (Base) = $20\sqrt{3}$ m
Let the angle of elevation be θ.

Substituting the values into the formula:

$$\tan\theta = \frac{20}{20\sqrt{3}}$$
$$\tan\theta = \frac{1}{\sqrt{3}}$$

We know that $\tan 30^\circ = \frac{1}{\sqrt{3}}$.
Therefore, $\theta = 30^\circ$.

Step 4: Final Answer:
The value of the angle of elevation is 30°.
Therefore, option (A) is correct.

Quick Tip

For problems of height and distance, drawing a clear diagram is always helpful. It helps you identify which side is the perpendicular, base, and hypotenuse, and which trigonometric ratio to use.

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15. An arc of a circle of radius 14 cm subtends an angle of 30° at its center. The area of the corresponding sector will be:

(A) $\frac{55}{3}$ cm^2
(B) $\frac{77}{3}$ cm^2
(C) $\frac{154}{3}$ cm^2
(D) $\frac{165}{3}$ cm^2

Correct Answer: (C) $\frac{154}{3}$ cm^2

Solution:

Step 1: Understanding the Concept:
The area of a sector of a circle is the portion of the area of the circle enclosed by two radii and the corresponding arc.

Step 2: Key Formula or Approach:
The formula for the area of a sector with radius r and angle θ (in degrees) is:

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

Step 3: Detailed Explanation:
The given values are:
Radius $r = 14$ cm
Angle $\theta = 30^\circ$
Using the value of π as $\frac{22}{7}$.

Substituting the values into the formula:

$$\text{Area} = \frac{30}{360} \times \frac{22}{7} \times (14)^2$$
$$\text{Area} = \frac{1}{12} \times \frac{22}{7} \times 14 \times 14$$
$$\text{Area} = \frac{1}{12} \times 22 \times 2 \times 14$$
$$\text{Area} = \frac{1}{6} \times 22 \times 14$$
$$\text{Area} = \frac{1}{3} \times 11 \times 14$$
$$\text{Area} = \frac{154}{3} \text{ cm}^2$$

Step 4: Final Answer:
The area of the corresponding sector is $\frac{154}{3}$ cm^2.
Therefore, option (C) is correct.

Quick Tip

When calculating the area of a sector, cancel out the numbers first to simplify the calculation, instead of multiplying them till the end. This avoids dealing with large numbers and reduces the chance of errors.

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16. The shape of a joker's cap is:

(A) Cylindrical
(B) Conical
(C) Triangular
(D) Rectangular

Correct Answer: (B) Conical

Solution:

Step 1: Understanding the Concept:
This question is based on general knowledge and recognition of common 3D shapes.

Step 2: Detailed Explanation:
A joker's cap, also known as a birthday cap, is typically shaped like a cone.
A cone is a three-dimensional (3D) geometric shape that tapers smoothly from a flat base (often, though not necessarily, circular) to a point called the apex or vertex.
(A) Cylindrical: The shape of a cylinder (like a can).
(C) Triangular: This is a 2D shape.
(D) Rectangular: This is also a 2D shape.
Therefore, the correct 3D shape of a joker's cap is conical.

Step 3: Final Answer:
The shape of a joker's cap is conical.
Therefore, option (B) is correct.

> Quick Tip
>
> Practice identifying the geometric shapes of objects around you. This helps you differentiate between 2D (two-dimensional) and 3D (three-dimensional) shapes.

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17. The assumed mean method and the step-deviation method are simplified forms of:

(A) Indirect method
(B) Direct method
(C) Combined method
(D) Simple method

Correct Answer: (B) Direct method

Solution:

Step 1: Understanding the Concept:
This question is about understanding the relationship between the different methods of calculating the mean of grouped data.
There are three main methods for calculating the mean: the direct method, the assumed mean method, and the step-deviation method.

Step 2: Detailed Explanation:
Direct Method ($\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$): This is the basic method of calculating the mean. However, when the values of x_i and f_i are large, the calculation becomes complex.
Assumed Mean Method ($\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$): This is a simplified form of the direct method. In it, we choose an assumed mean ('a') and use deviations ($d_i = x_i - a$) to make the calculation easier.
Step-Deviation Method ($\bar{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$): This is an even more simplified form of the assumed mean method, used when the class size ('h') is uniform. It further simplifies the calculation by dividing the deviations by the class size ($u_i = d_i/h$).
Thus, both the assumed mean and step-deviation methods are ways to simplify the calculations of the basic **Direct Method**. They are simplified forms of the direct method.

Step 3: Final Answer:
The assumed mean method and the step-deviation method are simplified forms of the direct method.
Therefore, option (B) is correct.

> Quick Tip
>
> Think of the three methods as a progression: the direct method is the original, the assumed mean method simplifies it, and the step-deviation method simplifies it even further (when the class size is uniform).

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18. If the mean of some observations is 15 and the median is 24, then the mode will be:

(A) 49
(B) 47
(C) 46
(D) 42

Correct Answer: (D) 42

Solution:

Step 1: Understanding the Concept:
This question is based on the empirical relationship between mean, median, and mode. This relationship is particularly useful for moderately skewed distributions.

Step 2: Key Formula or Approach:
The formula for the empirical relationship between mean, median, and mode is:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Step 3: Detailed Explanation:
The given values are:
Mean = 15
Median = 24

Substituting these values into the formula:

$$\text{Mode} = 3 \times (24) - 2 \times (15)$$
$$\text{Mode} = 72 - 30$$
$$\text{Mode} = 42$$

Step 4: Final Answer:
The value of the mode is 42.
Therefore, option (D) is correct.

> Quick Tip
>
> Remember this important empirical relationship (Mode = 3 Median - 2 Mean). It is often asked in competitive exams where two measures are given and the third has to be found.

---

19. When a die is thrown once, the probability of getting an even number will be:

(A) $\frac{2}{3}$
(B) $\frac{1}{2}$
(C) $\frac{1}{6}$
(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:
Probability is the measure of the likelihood of an event occurring.

Step 2: Key Formula or Approach:
The formula for the probability (P) of an event is:

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Step 3: Detailed Explanation:
When a die is thrown, the total possible outcomes are: $\{1, 2, 3, 4, 5, 6\}$.
Therefore, the total number of possible outcomes = 6.
The favorable outcomes for the event of getting an even number are: $\{2, 4, 6\}$.
Therefore, the number of favorable outcomes = 3.

Now, we calculate the probability:

$$P(\text{Even number}) = \frac{3}{6}$$
$$P(\text{Even number}) = \frac{1}{2}$$

Step 4: Final Answer:
The probability of getting an even number is $\frac{1}{2}$.
Therefore, option (B) is correct.

> **Quick Tip**
>
> For problems related to a die, always remember the total of 6 possible outcomes. Even numbers are (2, 4, 6), odd numbers are (1, 3, 5), and prime numbers are (2, 3, 5).

---

20. From a well-shuffled deck of 52 cards, if one card is drawn at random, the probability of the card being a queen will be:

(A) $\frac{1}{4}$
(B) $\frac{1}{13}$
(C) $\frac{1}{26}$
(D) $\frac{1}{52}$

Correct Answer: (B) $\frac{1}{13}$

Solution:

Step 1: Understanding the Concept:
This question is based on the probability of drawing a card from a standard deck of playing cards.

Step 2: Key Formula or Approach:
The formula for probability:

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Step 3: Detailed Explanation:
The total number of cards in a standard deck = 52.
Therefore, the total number of possible outcomes = 52.
The favorable outcomes for the event of drawing a queen.
A deck has 4 queens (one of spades, one of hearts, one of clubs, and one of diamonds).
Therefore, the number of favorable outcomes = 4.

Now, we calculate the probability:

$$P(\text{Queen}) = \frac{4}{52}$$

Simplifying the fraction:

$$P(\text{Queen}) = \frac{1}{13}$$

Step 4: Final Answer:
The probability of drawing a queen is $\frac{1}{13}$.
Therefore, option (B) is correct.

> **Quick Tip**
>
> Remember the structure of a standard deck of cards: 52 cards, 4 suits (spades, hearts, clubs, diamonds), 13 cards in each suit (A, 2-10, J, Q, K). There are 26 red and 26 black cards.

Section - B

Q(1) Do All Parts

(a) The height and the diameter of the base of a right circular cone are 48 cm and 28 cm respectively. Find the volume of the cone.

Solution:

Step 1: Understanding the Concept:
This question requires the calculation of the volume of a right circular cone using the given height and diameter.

Step 2: Key Formula or Approach:
The formula for the volume (V) of a cone is:

$$V = \frac{1}{3}\pi r^2 h$$

where r is the radius of the base and h is the height of the cone.

Step 3: Detailed Explanation:
First, we need to find the radius from the given diameter.
Given:
Height $(h) = 48$ cm
Diameter = 28 cm

The radius (r) is half of the diameter:

$$r = \frac{\text{Diameter}}{2} = \frac{28}{2} = 14 \text{ cm}$$

Now, substitute the values of r and h into the volume formula (using $\pi = \frac{22}{7}$):

$$V = \frac{1}{3} \times \frac{22}{7} \times (14)^2 \times 48$$
$$V = \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times 48$$

To simplify the calculation, we can cancel the terms:

$$V = \frac{1}{3} \times 22 \times (\frac{14}{7} \times 14) \times 48$$
$$V = \frac{1}{3} \times 22 \times (2 \times 14) \times 48$$
$$V = 22 \times 28 \times (\frac{48}{3})$$
$$V = 22 \times 28 \times 16$$
$$V = 616 \times 16$$
$$V = 9856 \text{ cm}^3$$

Step 4: Final Answer:
The volume of the cone is **9856 cm**^{3}.

> **Quick Tip**
>
> Always double-check if the question provides the radius or the diameter. A common mistake is to use the diameter value directly in the formula instead of the radius.

(b) Find the HCF of the 96 & 404 by the prime factorization method.

Solution:

Step 1: Understanding the Concept:
The HCF (Highest Common Factor) of two numbers is the largest number that divides both of them. The prime factorization method involves breaking down each number into a product of its prime factors.

Step 2: Key Formula or Approach:
1. Find the prime factorization of each number.
2. Identify the common prime factors.
3. The HCF is the product of the lowest powers of these common prime factors.

Step 3: Detailed Explanation:
First, find the prime factorization of 96:

$$96 = 2 \times 48 = 2 \times 2 \times 24 = 2 \times 2 \times 2 \times 12 = 2 \times 2 \times 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$
$$96 = 2^5 \times 3^1$$

Next, find the prime factorization of 404:

$$404 = 2 \times 202 = 2 \times 2 \times 101$$
$$404 = 2^2 \times 101^1$$

(Note: 101 is a prime number).

Now, identify the common prime factors and their lowest powers:
The only common prime factor is 2.
The lowest power of 2 in both factorizations is 2^2.
Therefore, the HCF is the product of these lowest powers:

$$\text{HCF}(96, 404) = 2^2 = 4$$

Step 4: Final Answer:
The HCF of 96 and 404 is **4**.

Quick Tip

To find the HCF, take the lowest power of common factors. To find the LCM (Lowest Common Multiple), take the highest power of all factors.

(c) Find the co-ordinates of the point dividing the line segment joining the points (–1, 7) and (4, –3) in the ratio 2 : 3.

Solution:

Step 1: Understanding the Concept:
This question requires the use of the section formula, which is used to find the coordinates of a point that divides a line segment in a given ratio.

Step 2: Key Formula or Approach:
The section formula for a point $P(x, y)$ that divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ is:

$$P(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Step 3: Detailed Explanation:
Here, the given points and ratio are:
$(x_1, y_1) = (-1, 7)$
$(x_2, y_2) = (4, -3)$
Ratio $m : n = 2 : 3$, so $m = 2$ and $n = 3$.

Now, calculate the x-coordinate:

$$x = \frac{2(4) + 3(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

Next, calculate the y-coordinate:

$$y = \frac{2(-3) + 3(7)}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

So, the coordinates of the point are (1, 3).

Step 4: Final Answer:
The co-ordinates of the point are **(1, 3)**.

Quick Tip

Be careful with the signs of the coordinates and the order of m, n, x_1, x_2, y_1, y_2 when substituting into the section formula to avoid calculation errors.

(d) If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then find the value of $(\alpha + \beta)$.

Solution:

Step 1: Understanding the Concept:
This question requires finding the values of angles α and β from their given trigonometric ratios and then calculating their sum. We assume the angles are in the first quadrant (acute angles),

which is standard for such problems.

Step 2: Key Formula or Approach:
We need to recall the standard values of trigonometric functions for common angles (like 0°, 30°, 45°, 60°, 90°).

Step 3: Detailed Explanation:
First, find the value of α:
Given $\sin\alpha = \frac{1}{2}$.
We know that $\sin 30° = \frac{1}{2}$.
Therefore, $\alpha = 30°$.

Next, find the value of β:
Given $\cos\beta = \frac{1}{2}$.
We know that $\cos 60° = \frac{1}{2}$.
Therefore, $\beta = 60°$.

Now, find the value of $(\alpha + \beta)$:

$$\alpha + \beta = 30° + 60° = 90°$$

Step 4: Final Answer:
The value of $(\alpha + \beta)$ is **90°**.

> Quick Tip
>
> Memorizing the trigonometric values for standard angles (0°, 30°, 45°, 60°, 90°) is essential for quickly solving such problems in exams.

(e) Find the distance of point P (–6, 8) from the origin.

Solution:

Step 1: Understanding the Concept:
This question requires finding the distance between a given point and the origin (0, 0) in a Cartesian coordinate system using the distance formula.

Step 2: Key Formula or Approach:
The distance (d) of a point (x, y) from the origin $(0, 0)$ is given by a simplified version of the distance formula:

$$d = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

Step 3: Detailed Explanation:
The given point is P(–6, 8).
Here, $x = -6$ and $y = 8$.
Substitute these values into the formula:

$$d = \sqrt{(-6)^2 + (8)^2}$$
$$d = \sqrt{36 + 64}$$
$$d = \sqrt{100}$$
$$d = 10$$

The distance is 10 units.

Step 4: Final Answer:
The distance of point P(–6, 8) from the origin is **10 units**.

Quick Tip

The distance of any point (x, y) from the origin is simply the hypotenuse of a right-angled triangle with base $|x|$ and height $|y|$. This is a direct application of the Pythagorean theorem.

(f) Prove : $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\csc A - 1}{\csc A + 1}$

Solution:

Step 1: Understanding the Concept:
This question requires proving a trigonometric identity by manipulating one side of the equation (usually the more complex one) to show it is equivalent to the other side.

Step 2: Key Formula or Approach:
We will start with the Left Hand Side (LHS) and use the following fundamental identities:
1. $\cot A = \frac{\cos A}{\sin A}$
2. $\csc A = \frac{1}{\sin A}$

Step 3: Detailed Explanation:
Starting with the LHS:

$$\text{LHS} = \frac{\cot A - \cos A}{\cot A + \cos A}$$

Substitute $\cot A = \frac{\cos A}{\sin A}$ into the expression:

$$\text{LHS} = \frac{\frac{\cos A}{\sin A} - \cos A}{\frac{\cos A}{\sin A} + \cos A}$$

Factor out $\cos A$ from the numerator and the denominator:

$$\text{LHS} = \frac{\cos A \left(\frac{1}{\sin A} - 1\right)}{\cos A \left(\frac{1}{\sin A} + 1\right)}$$

Cancel the common factor $\cos A$:

$$\text{LHS} = \frac{\frac{1}{\sin A} - 1}{\frac{1}{\sin A} + 1}$$

Now, substitute $\csc A = \frac{1}{\sin A}$:

$$\text{LHS} = \frac{\csc A - 1}{\csc A + 1}$$

This is equal to the Right Hand Side (RHS).

$$\text{LHS} = \text{RHS}$$

Hence, proved.

Step 4: Final Answer:
The identity $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\csc A - 1}{\csc A + 1}$ is successfully proven by expressing $\cot A$ in terms of $\sin A$ and $\cos A$ and simplifying.

> **Quick Tip**
>
> When proving trigonometric identities, a good strategy is often to express all terms in the form of sin and cos. Then, use algebraic simplification and other identities to reach the desired expression.

Q(2) Do Any 5 Parts

(a) The 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Solution:

Step 1: Understanding the Concept:
This question is based on the properties of an Arithmetic Progression (A.P.). We will use the formula for the n^{th} term of an A.P. to set up an equation and solve for the common difference.

Step 2: Key Formula or Approach:
The formula for the nth term (a_n) of an A.P. is:

$$a_n = a + (n-1)d$$

where a is the first term and d is the common difference.

Step 3: Detailed Explanation:
First, let's write the expressions for the 17th term (a_{17}) and the 10th term (a_{10}):

$$a_{17} = a + (17-1)d = a + 16d$$
$$a_{10} = a + (10-1)d = a + 9d$$

According to the question, the 17th term exceeds the 10th term by 7. We can write this as an equation:

$$a_{17} = a_{10} + 7$$

Now, substitute the expressions for a_{17} and a_{10}:

$$a + 16d = (a + 9d) + 7$$

Subtract 'a' from both sides:

$$16d = 9d + 7$$

Solve for d:

$$16d - 9d = 7$$
$$7d = 7$$
$$d = \frac{7}{7} = 1$$

Step 4: Final Answer:
The common difference of the A.P. is **1**.

Quick Tip

For problems comparing two terms of an A.P., you can directly use the fact that the difference between the mth and nth term is $a_m - a_n = (m-n)d$. Here, $a_{17} - a_{10} = (17-10)d = 7d$. Since this difference is given as 7, we get $7d = 7$, which gives $d = 1$.

(b) Find the roots of the equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.

Solution:

Step 1: Understanding the Concept:
This is a quadratic equation of the form $ax^2 + bx + c = 0$. We can find its roots by either using the quadratic formula or by factoring the equation (splitting the middle term).

Step 2: Key Formula or Approach:
We will use the factorization method. We need to find two numbers that multiply to ac and add up to b.
Here, $a = \sqrt{2}$, $b = 7$, and $c = 5\sqrt{2}$.
The product $ac = (\sqrt{2})(5\sqrt{2}) = 5 \times 2 = 10$.
We need two numbers that multiply to 10 and add to 7. These numbers are 2 and 5.

Step 3: Detailed Explanation:
Split the middle term (7x) into 2x and 5x:

$$\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = 0$$

Now, factor by grouping. We can write 2 as $(\sqrt{2} \times \sqrt{2})$:

$$\sqrt{2}x^2 + \sqrt{2}\sqrt{2}x + 5x + 5\sqrt{2} = 0$$

Factor out the common terms from the first two and the last two terms:

$$\sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0$$

Now, factor out the common binomial term $(x + \sqrt{2})$:

$$(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$$

To find the roots, set each factor to zero:
Either $\sqrt{2}x + 5 = 0 \implies \sqrt{2}x = -5 \implies x = -\frac{5}{\sqrt{2}}$
Or $x + \sqrt{2} = 0 \implies x = -\sqrt{2}$

Step 4: Final Answer:
The roots of the equation are $-\sqrt{2}$ and $-\frac{5}{\sqrt{2}}$.

Quick Tip

When factoring quadratic equations involving square roots, look for ways to express whole numbers as products of square roots (e.g., $2 = \sqrt{2} \times \sqrt{2}$). This often reveals the common factors.

(c) D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Prove that $CA^2 = CB \times CD$.

Solution:

Step 1: Understanding the Concept:
This problem requires the use of the Angle-Angle (AA) similarity criterion for triangles. If two angles of one triangle are equal to two corresponding angles of another triangle, then the two triangles are similar.

Step 2: Key Formula or Approach:
1. Identify two triangles that can be proven similar.
2. Use the AA similarity criterion.
3. Use the property that the ratios of corresponding sides of similar triangles are equal.

Step 3: Detailed Explanation:
Let's consider $\triangle ADC$ and $\triangle BAC$.
In these two triangles, we have:
1. $\angle ADC = \angle BAC$ (This is given in the problem).
2. $\angle ACD = \angle BCA$ (This is the common angle $\angle C$).

Since two angles of $\triangle ADC$ are equal to two corresponding angles of $\triangle BAC$, the two triangles are similar by the AA similarity criterion.

$$\triangle ADC \sim \triangle BAC$$

Because the triangles are similar, the ratio of their corresponding sides must be equal. Let's match the corresponding vertices: A↔B, D↔A, C↔C.

$$\frac{AD}{BA} = \frac{DC}{AC} = \frac{AC}{BC}$$

Taking the last two parts of the ratio:

$$\frac{DC}{AC} = \frac{AC}{BC}$$

In a more intuitive order for this problem:

$$\frac{CA}{CB} = \frac{CD}{CA}$$

Now, cross-multiply to get the desired result:

$$CA \times CA = CB \times CD$$
$$CA^2 = CB \times CD$$

Hence, proved.

Step 4: Final Answer:
By proving that $\triangle ADC \sim \triangle BAC$ using the AA similarity criterion, we establish that the ratio of corresponding sides is equal, which leads to the result $CA^2 = CB \times CD$.

> **Quick Tip**
>
> When proving similarity, it is very important to write the similarity relation with the corresponding vertices in the correct order (e.g., $\triangle ADC \sim \triangle BAC$). This helps in correctly identifying the pairs of corresponding sides.

(d) A quadrilateral ABCD is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$.

Solution:

Step 1: Understanding the Concept:
This question is based on a property of tangents drawn to a circle from an external point. A quadrilateral that circumscribes a circle is called a tangential quadrilateral.

Step 2: Key Formula or Approach:
The key theorem is: The lengths of tangents drawn from an external point to a circle are equal.

Step 3: Detailed Explanation:
Let the circle touch the sides AB, BC, CD, and DA of the quadrilateral at points P, Q, R, and S, respectively.
ABCD is the quadrilateral and A, B, C, D are the external points from which tangents are drawn to the circle.

Using the theorem about tangents from an external point:
From point A: $AP = AS$ —(1)
From point B: $BP = BQ$ —(2)
From point C: $CR = CQ$ —(3)
From point D: $DR = DS$ —(4)

Now, add these four equations:

$$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$$

By observing the figure, we can group the segments to form the sides of the quadrilateral:
$AP + BP = AB$
$CR + DR = CD$
$AS + DS = AD$

$BQ + CQ = BC$

Substitute these into the combined equation:

$$AB + CD = AD + BC$$

Hence, proved. This is known as Pitot's theorem.

Step 4: Final Answer:
By applying the theorem that tangents from an external point to a circle are equal in length and summing up the lengths of the tangent segments, we have proven that for a quadrilateral circumscribing a circle, $AB + CD = AD + BC$.

Quick Tip

This is a standard theorem for tangential quadrilaterals. The key to the proof is to label the points of tangency and add the four equations of equal tangent lengths. The rearrangement of terms is crucial to get the final result.

(e) If the median of given frequency distribution is 28.5, then find the value of 'x' and 'y'. (Given n = 60)

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	5	x	20	15	y	5

Solution:

Step 1: Understanding the Concept:
We are given the median and the total frequency of a distribution with two missing frequencies. We will use the formula for the median of grouped data and the sum of frequencies to create two simultaneous equations to solve for x and y.

Step 2: Key Formula or Approach:
1. Sum of frequencies: $\sum f_i = n$
2. Median formula: Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$ where l = lower limit of median class, n = total frequency, cf = cumulative frequency of the class preceding the median class, f = frequency of the median class, and h = class size.

Step 3: Detailed Explanation:
First, let's create the cumulative frequency (cf) table:

Class interval	Frequency (f_i)	Cumulative Frequency (cf)
0 – 10	5	5
10 – 20	x	5 + x
20 – 30	20	25 + x
30 – 40	15	40 + x
40 – 50	y	40 + x + y
50 – 60	5	45 + x + y

Equation 1: From the total frequency
Given $n = 60$. The sum of frequencies from our table is $45 + x + y$.

$$45 + x + y = 60$$
$$x + y = 15$$

—(1)

Equation 2: From the median formula
Given Median = 28.5. This value lies in the class interval 20 – 30.
Therefore, the median class is 20 – 30.
From this class, we get:
Lower limit $(l) = 20$
Frequency of median class $(f) = 20$
Class size $(h) = 10$
Total frequency $(n) = 60$, so $\frac{n}{2} = 30$.
Cumulative frequency of the class preceding the median class $(cf) = 5 + x$.

Substitute these values into the median formula:

$$28.5 = 20 + \left(\frac{30 - (5 + x)}{20}\right) \times 10$$
$$28.5 - 20 = \left(\frac{30 - 5 - x}{20}\right) \times 10$$
$$8.5 = \left(\frac{25 - x}{2}\right)$$
$$8.5 \times 2 = 25 - x$$
$$17 = 25 - x$$
$$x = 25 - 17 = 8$$

Now, substitute $x = 8$ into equation (1):

$$8 + y = 15$$
$$y = 15 - 8 = 7$$

Step 4: Final Answer:
The values of the missing frequencies are $\mathbf{x = 8}$ and $\mathbf{y = 7}$.

Quick Tip

When solving for two missing frequencies, one equation will almost always come from the sum of frequencies ($\sum f_i = n$). The second will come from the formula for the given measure of central tendency (mean, median, or mode).

(f) 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether it is defective or not. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

Solution:

Step 1: Understanding the Concept:
This is a basic probability problem. Probability is the ratio of the number of favorable outcomes to the total number of possible outcomes.

Step 2: Key Formula or Approach:
The formula for probability of an event (E) is:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Step 3: Detailed Explanation:
First, we determine the total number of pens.
Number of defective pens = 12
Number of good pens = 132
Total number of pens = Number of defective pens + Number of good pens

$$\text{Total number of pens} = 12 + 132 = 144$$

This is the total number of possible outcomes.

Next, we determine the number of favorable outcomes.
The event is "the pen taken out is a good one".
Number of good pens = 132
This is the number of favorable outcomes.

Now, calculate the probability:

$$P(\text{good pen}) = \frac{\text{Number of good pens}}{\text{Total number of pens}} = \frac{132}{144}$$

Simplify the fraction. Both numbers are divisible by 12.

$$P(\text{good pen}) = \frac{132 \div 12}{144 \div 12} = \frac{11}{12}$$

Step 4: Final Answer:
The probability that the pen taken out is a good one is $\frac{11}{12}$.

Quick Tip

In probability questions, always start by identifying two key numbers: the total number of possible outcomes (the denominator) and the number of outcomes that satisfy the event's condition (the numerator).

---

3. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Solution:

Step 1: Understanding the Concept:
This is a word problem that can be solved by setting up a pair of linear equations in two variables representing the numerator and the denominator of the fraction.

Step 2: Key Formula or Approach:
Let the fraction be $\frac{x}{y}$, where x is the numerator and y is the denominator.
We will translate the two given conditions into two separate linear equations.

Step 3: Detailed Explanation:
Condition 1: "A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator."
This gives us the equation:

$$\frac{x-1}{y} = \frac{1}{3}$$

Cross-multiplying, we get:

$$3(x-1) = y$$
$$3x - 3 = y$$
$$3x - y = 3$$

— (Equation 1)

Condition 2: "it becomes $\frac{1}{4}$ when 8 is added to its denominator."
This gives us the equation:

$$\frac{x}{y+8} = \frac{1}{4}$$

Cross-multiplying, we get:

$$4x = y + 8$$
$$4x - y = 8$$

— (Equation 2)

Now we have a system of two linear equations:
1) $3x - y = 3$
2) $4x - y = 8$

To solve this, we can subtract Equation 1 from Equation 2:

$$(4x - y) - (3x - y) = 8 - 3$$
$$4x - y - 3x + y = 5$$
$$x = 5$$

Now, substitute the value of $x = 5$ into Equation 1 to find y:

$$3(5) - y = 3$$
$$15 - y = 3$$
$$y = 15 - 3 = 12$$

So, the numerator is 5 and the denominator is 12.

Step 4: Final Answer:
The required fraction is $\frac{5}{12}$.

Quick Tip

Always check your answer by substituting the values back into the original conditions.
Condition 1: $\frac{5-1}{12} = \frac{4}{12} = \frac{1}{3}$ (Correct).
Condition 2: $\frac{5}{12+8} = \frac{5}{20} = \frac{1}{4}$ (Correct).

---

3. (OR) A plane left 30 minutes later than the scheduled time and in order to reach its destination 1500 km away on time, it has to increase its speed 250 km/hr from its usual speed. Find the usual speed of the plane.

Solution:

Step 1: Understanding the Concept:
This problem relates distance, speed, and time. We will form a quadratic equation based on

the relationship Time $= \frac{\text{Distance}}{\text{Speed}}$. The key is that the difference in the time taken at the usual speed and the increased speed is 30 minutes.

Step 2: Key Formula or Approach:
Let the usual speed of the plane be x km/hr.
Let the distance be $D = 1500$ km.
Usual time taken $(T_1) = \frac{1500}{x}$ hours.
Increased speed $= (x + 250)$ km/hr.
New time taken $(T_2) = \frac{1500}{x+250}$ hours.
The plane is late by 30 minutes, which is $\frac{30}{60} = \frac{1}{2}$ hour.
This means the usual time is longer than the new time by $\frac{1}{2}$ hour.
So, the equation is: $T_1 - T_2 = \frac{1}{2}$.

Step 3: Detailed Explanation:
Set up the equation based on the time difference:

$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

Factor out 1500:

$$1500\left(\frac{1}{x} - \frac{1}{x+250}\right) = \frac{1}{2}$$

$$1500\left(\frac{(x+250)-x}{x(x+250)}\right) = \frac{1}{2}$$

$$1500\left(\frac{250}{x^2+250x}\right) = \frac{1}{2}$$

Cross-multiply:

$$1500 \times 250 \times 2 = x^2 + 250x$$

$$750000 = x^2 + 250x$$

Rearrange into a standard quadratic equation:

$$x^2 + 250x - 750000 = 0$$

We can solve this by factoring. We need two numbers that multiply to -750000 and have a difference of 250. These numbers are 1000 and -750.

$$x^2 + 1000x - 750x - 750000 = 0$$

$$x(x+1000) - 750(x+1000) = 0$$

$$(x-750)(x+1000) = 0$$

This gives two possible values for x: $x = 750$ or $x = -1000$.
Since speed cannot be negative, we discard $x = -1000$.

Step 4: Final Answer:
The usual speed of the plane is **750 km/hr**.

Quick Tip

In time-speed-distance problems, ensure all units are consistent. If speed is in km/hr, time must be in hours. Convert minutes to hours (e.g., 30 minutes = 0.5 hours) before setting up the equation.

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4. Two poles of same height are standing on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and distance of the point from the poles.

Solution:

Step 1: Understanding the Concept:
This problem is an application of trigonometry involving heights and distances. We will use the tangent trigonometric ratio to set up two equations with two variables (the height of the poles and the distance of the point from one pole).

Step 2: Key Formula or Approach:
Let h be the height of the two poles. Let AB and CD be the poles.
Let P be the point on the road between the poles. The width of the road is BD = 80 m.
Let the distance of the point P from pole AB be x meters (i.e., BP = x).
Then the distance of the point P from pole CD will be $(80 - x)$ meters (i.e., DP = $80 - x$).
The angles of elevation are $\angle APB = 60^\circ$ and $\angle CPD = 30^\circ$.
We will use the formula: $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$.

Step 3: Detailed Explanation:
From the right-angled $\triangle APB$:

$$\tan 60^\circ = \frac{AB}{BP} = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{x} \implies h = x\sqrt{3}$$

— (Equation 1)

From the right-angled $\triangle CPD$:

$$\tan 30^\circ = \frac{CD}{DP} = \frac{h}{80 - x}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{80 - x} \implies h = \frac{80 - x}{\sqrt{3}}$$

— (Equation 2)

Now, we equate the two expressions for h from Equation 1 and Equation 2:

$$x\sqrt{3} = \frac{80 - x}{\sqrt{3}}$$

Cross-multiply:

$$x\sqrt{3} \times \sqrt{3} = 80 - x$$
$$3x = 80 - x$$
$$4x = 80$$
$$x = 20 \text{ m}$$

This is the distance of the point from the first pole (AB).
The distance from the second pole (CD) is $80 - x = 80 - 20 = 60$ m.

Now, find the height h using Equation 1:

$$h = x\sqrt{3} = 20\sqrt{3} \text{ m}$$

Step 4: Final Answer:
The height of the poles is $\mathbf{20\sqrt{3}}$ **m**, and the distances of the point from the poles are **20 m** and **60 m**.

> **Quick Tip**
>
> Drawing a clear, labelled diagram is the most important first step for solving heights and distances problems. It helps you visualize the triangles and apply the correct trigonometric ratios.

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4. (OR) From a point on the ground, the angles of elevation of the bottom and the top of the tower fixed on a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Solution:

Step 1: Understanding the Concept:
This is another problem on heights and distances. We have a building with a tower on top. We will use the tangent ratio for two different right-angled triangles to find the required height.

Step 2: Key Formula or Approach:
Let BC be the building and CD be the tower on top. Let A be the point on the ground.
Height of the building, BC = 20 m.
Let the height of the tower, CD = h m.
Let the distance of the point A from the base of the building, AB = x m.
The angle of elevation of the bottom of the tower is $\angle CAB = 45°$.
The angle of elevation of the top of the tower is $\angle DAB = 60°$.
We will use $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$.

Step 3: Detailed Explanation:
From the right-angled $\triangle ABC$:

$$\tan 45° = \frac{BC}{AB} = \frac{20}{x}$$

Since $\tan 45° = 1$:

$$1 = \frac{20}{x} \implies x = 20 \text{ m}$$

— (Equation 1)

From the right-angled $\triangle ABD$:
The total height of the perpendicular is BD = BC + CD = $20 + h$.

$$\tan 60° = \frac{BD}{AB} = \frac{20+h}{x}$$

Since $\tan 60° = \sqrt{3}$:

$$\sqrt{3} = \frac{20+h}{x}$$
$$x\sqrt{3} = 20 + h$$

— (Equation 2)

Now, substitute the value of $x = 20$ from Equation 1 into Equation 2:

$$20\sqrt{3} = 20 + h$$

Solve for h:

$$h = 20\sqrt{3} - 20$$
$$h = 20(\sqrt{3} - 1) \text{ m}$$

We can use the value $\sqrt{3} \approx 1.732$:

$$h = 20(1.732 - 1) = 20(0.732) = 14.64 \text{ m}$$

Step 4: Final Answer:
The height of the tower is **20($\sqrt{3}$ - 1) m**, which is approximately **14.64 m**.

Quick Tip

In problems with two angles of elevation from the same point, you will always get two right-angled triangles sharing a common base or height. Use this common side to equate the two equations you form.

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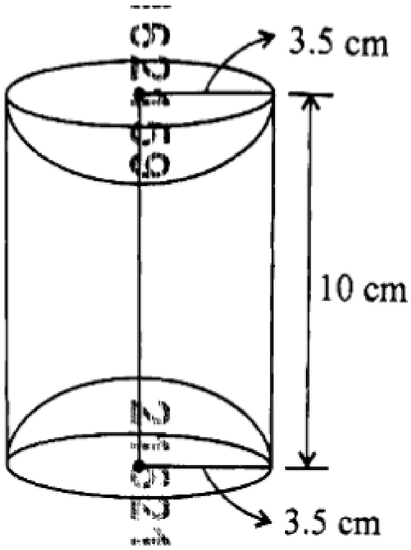
5. A wooden toy was made by scooping out a hemisphere from each end of a solid cylinder as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm, find the total surface area of the toy. (Use $\pi = \frac{22}{7}$)



Solution:

Step 1: Understanding the Concept:
The total surface area of the toy is the sum of the curved surface area of the cylinder and the curved surface areas of the two hemispheres that have been scooped out. The flat circular bases of the cylinder have been removed, so we do not include their area.

Step 2: Key Formula or Approach:
Total Surface Area (TSA) of the toy = (Curved Surface Area of Cylinder) + 2 × (Curved Surface Area of Hemisphere)
- CSA of Cylinder = $2\pi rh$
- CSA of Hemisphere = $2\pi r^2$
So, TSA = $2\pi rh + 2 \times (2\pi r^2) = 2\pi rh + 4\pi r^2 = 2\pi r(h + 2r)$

Step 3: Detailed Explanation:
Given values:
Height of the cylinder $(h) = 10$ cm
Radius of the base $(r) = 3.5$ cm $= \frac{7}{2}$ cm

Now, calculate the required areas:
Curved Surface Area of Cylinder:

$$\text{CSA}_{cyl} = 2\pi rh = 2 \times \frac{22}{7} \times 3.5 \times 10 = 2 \times \frac{22}{7} \times \frac{7}{2} \times 10 = 22 \times 10 = 220 \text{ cm}^2$$

Curved Surface Area of two Hemispheres:

$$2 \times \text{CSA}_{hemi} = 2 \times (2\pi r^2) = 4\pi r^2 = 4 \times \frac{22}{7} \times (3.5)^2 = 4 \times \frac{22}{7} \times 3.5 \times 3.5$$
$$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 22 \times 7 = 154 \text{ cm}^2$$

Total Surface Area of the toy:

$$\text{TSA}_{toy} = \text{CSA}_{cyl} + 2 \times \text{CSA}_{hemi} = 220 + 154 = 374 \text{ cm}^2$$

Alternatively, using the combined formula:

$$\text{TSA} = 2\pi r(h + 2r) = 2 \times \frac{22}{7} \times 3.5(10 + 2 \times 3.5)$$
$$= 2 \times \frac{22}{7} \times \frac{7}{2}(10 + 7) = 22 \times 17 = 374 \text{ cm}^2$$

Step 4: Final Answer:
The total surface area of the toy is **374 cm^2**.

Quick Tip

For combined solids, visualize the surfaces that are exposed. In this case, scooping out the hemispheres exposes their inner curved surfaces while removing the cylinder's flat top and bottom. So, we add the CSA of hemispheres and CSA of the cylinder.

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5. (OR) A solid is in the shape of a cone standing on a hemisphere, the radii of the cone and hemisphere are equal and its measure is 3 cm. If the height of the solid is 6 cm, then find the volume of the solid.

Solution:

Step 1: Understanding the Concept:
The solid is a combination of a cone and a hemisphere. The total volume of the solid will be the sum of the volume of the cone and the volume of the hemisphere.

Step 2: Key Formula or Approach:
Volume of the solid = Volume of Cone + Volume of Hemisphere
- Volume of Cone = $\frac{1}{3}\pi r^2 h_{cone}$

- Volume of Hemisphere = $\frac{2}{3}\pi r^3$

Step 3: Detailed Explanation:
Given values:
Radius of cone = Radius of hemisphere (r) = 3 cm
Total height of the solid = 6 cm

First, we need to find the height of the cone (h_{cone}).
The height of the hemispherical part is equal to its radius, so Height$_{hemi}$ = 3 cm.
Height of the cone = Total height - Height of hemisphere

$$h_{cone} = 6 - 3 = 3 \text{ cm}$$

Now, calculate the volumes:
Volume of Cone:

$$V_{cone} = \frac{1}{3}\pi r^2 h_{cone} = \frac{1}{3}\pi(3)^2(3) = \frac{1}{3}\pi \times 9 \times 3 = 9\pi \text{ cm}^3$$

Volume of Hemisphere:

$$V_{hemi} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3)^3 = \frac{2}{3}\pi \times 27 = 2 \times 9\pi = 18\pi \text{ cm}^3$$

Total Volume of the solid:

$$V_{solid} = V_{cone} + V_{hemi} = 9\pi + 18\pi = 27\pi \text{ cm}^3$$

If asked to use $\pi = \frac{22}{7}$:

$$V_{solid} = 27 \times \frac{22}{7} = \frac{594}{7} \approx 84.86 \text{ cm}^3$$

Step 4: Final Answer:
The volume of the solid is $\mathbf{27\pi}$ **cm**^{3}.

Quick Tip

For combined solids, remember that heights add up but radii must be common. In a cone-on-hemisphere problem, the total height is the sum of the cone's height and the hemisphere's radius.