

UP Board Class 10 Mathematics - 822(BV) - 2025 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :70	Total Questions :5
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. All questions are compulsory.
2. This question paper has two sections 'A' and 'B'.
3. Section 'A' contains 20 Multiple Choice type Questions of 1 mark each that have to be answered on OMR answer sheet by darkening completely the correct circle with blue or black ballpoint pen.
4. After giving answer on OMR answer sheet, do not cut or use eraser, whitener etc.
5. Section 'B' contains descriptive type questions of 50 marks.
6. Total 5 questions are there in this section.
7. In the beginning of each question, it has been mentioned how many parts of it are to be attempted.
8. Marks allotted to each question are mentioned against it.
9. Start from the first question and go up to the last question. Do not waste your time on the question you cannot solve.
10. If you need place for rough work do it on the left page of your answer book and cross (X) the page. Do not write any solution on that page.
11. Draw neat and correct figure in the solution of a question wherever it is necessary, otherwise in its absence the solution will be treated incomplete and wrong.

Section - A

1. The value of $(\csc A - \cot A)(1 + \cos A)$ will be:

- (A) $\csc A$
- (B) $\cot A$
- (C) $\cos A$
- (D) $\sin A$

Correct Answer: (D) $\sin A$

Solution:

Step 1: Understanding the Concept:

This problem requires simplifying a trigonometric expression using fundamental trigonometric identities.

Step 2: Key Formula or Approach:

We will use the following identities:

1. $\csc A = \frac{1}{\sin A}$
2. $\cot A = \frac{\cos A}{\sin A}$
3. $\sin^2 A + \cos^2 A = 1 \implies 1 - \cos^2 A = \sin^2 A$

Step 3: Detailed Explanation:

Let's simplify the given expression: $(\csc A - \cot A)(1 + \cos A)$

First, convert $\csc A$ and $\cot A$ into terms of $\sin A$ and $\cos A$:

$$\left(\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right) (1 + \cos A)$$

Combine the terms in the first bracket over the common denominator:

$$\left(\frac{1 - \cos A}{\sin A} \right) (1 + \cos A)$$

Multiply the numerators. Notice that this is in the form of $(a - b)(a + b) = a^2 - b^2$:

$$\frac{(1 - \cos A)(1 + \cos A)}{\sin A} = \frac{1^2 - \cos^2 A}{\sin A} = \frac{1 - \cos^2 A}{\sin A}$$

Using the Pythagorean identity $1 - \cos^2 A = \sin^2 A$:

$$\frac{\sin^2 A}{\sin A}$$

Cancel one $\sin A$ from the numerator and denominator:

$$\sin A$$

Step 4: Final Answer:

The simplified value of the expression is $\sin A$.

Therefore, option (D) is correct.

Quick Tip

When simplifying complex trigonometric expressions, a common and effective strategy is to convert all terms into their sine and cosine equivalents. This often makes the path to simplification clearer.

2. A die is thrown once, the probability of getting an odd number will be :

- (A) $\frac{1}{6}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{3}$
- (D) 1

Correct Answer: (B) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:

Probability is the measure of the likelihood that an event will occur. It is calculated by dividing the number of favorable outcomes by the total number of possible outcomes.

Step 2: Key Formula or Approach:

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Step 3: Detailed Explanation:

When a standard six-sided die is thrown, the set of all possible outcomes is $S = \{1, 2, 3, 4, 5, 6\}$.

The total number of possible outcomes is 6.

The event we are interested in is 'getting an odd number'.

The favorable outcomes for this event are the odd numbers in the set S , which are $\{1, 3, 5\}$.

The number of favorable outcomes is 3.

Now, we calculate the probability:

$$P(\text{odd number}) = \frac{3}{6} = \frac{1}{2}$$

Step 4: Final Answer:

The probability of getting an odd number is $\frac{1}{2}$.

Therefore, option (B) is correct.

Quick Tip

For a standard die, remember that there are 3 even numbers $\{2, 4, 6\}$ and 3 odd numbers $\{1, 3, 5\}$. The probability for both events is the same: $3/6$ or $1/2$.

3. The median class of the following frequency distribution will be :

Class Interval	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	3	2	5	4	1

- (A) 5 – 10
- (B) 15 – 20
- (C) 10 – 15
- (D) 20 – 25

Correct Answer: (C) 10 – 15

Solution:

Step 1: Understanding the Concept:

The median class is the class interval which contains the median value. To find it, we first need to calculate the total frequency and then find the cumulative frequency for each class.

Step 2: Key Formula or Approach:

1. Calculate the cumulative frequency (cf) for each class.
2. Find the total number of observations, $N = \sum f_i$.
3. Calculate $N/2$.
4. The median class is the class whose cumulative frequency is just greater than or equal to $N/2$.

Step 3: Detailed Explanation:

Let's construct the cumulative frequency table:

Class Interval	Frequency (f_i)	Cumulative Frequency (cf)
0 – 5	3	3
5 – 10	2	$3 + 2 = 5$
10 – 15	5	$5 + 5 = 10$
15 – 20	4	$10 + 4 = 14$
20 – 25	1	$14 + 1 = 15$

The total number of observations is $N = 15$.

Now, we find the value of $N/2$:

$$\frac{N}{2} = \frac{15}{2} = 7.5$$

We look for the class whose cumulative frequency is just greater than 7.5.

From the table, the cumulative frequency just greater than 7.5 is 10.

The class interval corresponding to this cumulative frequency is 10 – 15.

Step 4: Final Answer:

The median class for the given distribution is 10 – 15.

Therefore, option (C) is correct.

Quick Tip

To find the median class, always look at the cumulative frequency column. Find the first value in this column that is greater than or equal to $N/2$, and the corresponding class is your median class.

4. The modal class of the following table will be :

Class Interval	0 – 7	7 – 14	14 – 21	21 – 28	28 – 35	35 – 42
Frequency	1	12	3	6	2	4

- (A) 35 – 42
- (B) 28 – 35
- (C) 21 – 28
- (D) 7 – 14

Correct Answer: (D) 7 – 14

Solution:

Step 1: Understanding the Concept:

The mode is the value that appears most frequently in a data set. For grouped data, the modal class is the class interval with the highest frequency.

Step 2: Key Formula or Approach:

To find the modal class, simply identify the class interval that corresponds to the maximum frequency in the distribution table.

Step 3: Detailed Explanation:

Let's examine the frequencies for each class interval:

- 0 – 7: Frequency = 1
- **7 – 14: Frequency = 12**
- 14 – 21: Frequency = 3
- 21 – 28: Frequency = 6
- 28 – 35: Frequency = 2
- 35 – 42: Frequency = 4

The maximum frequency is 12.

The class interval corresponding to the maximum frequency (12) is 7 – 14.

Step 4: Final Answer:

The modal class of the given table is 7 – 14.
Therefore, option (D) is correct.

Quick Tip

Finding the modal class is the simplest of the central tendency measures for grouped data.
Just look for the largest number in the frequency row and identify its corresponding class.
No calculation is needed.

5. A child has a die whose faces show the letters as given below :

A	B	C	D	E	A
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If the die is thrown once, then the probability of getting

A

 will be

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{6}$
- (D) 1

Correct Answer: (B) $\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

Probability is the ratio of the number of favorable outcomes to the total number of possible outcomes.

Step 2: Key Formula or Approach:

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

Step 3: Detailed Explanation:

The die has 6 faces showing the letters: A, B, C, D, E, A.

The total number of possible outcomes when the die is thrown once is 6.

The event is 'getting the letter A'.

We need to count how many faces show the letter A. There are 2 faces with the letter A.

So, the number of favorable outcomes is 2.

Now, calculate the probability:

$$P(\text{getting A}) = \frac{2}{6}$$

Simplify the fraction:

$$P(\text{getting A}) = \frac{1}{3}$$

Step 4: Final Answer:

The probability of getting A is $\frac{1}{3}$.

Therefore, option (B) is correct.

Quick Tip

Even if a die is non-standard, the basic principle of probability remains the same. First, count the total number of faces (total outcomes), then count the faces that match the desired event (favorable outcomes).

6. If $\cot \theta = \frac{8}{15}$, then the value of $\sin \theta$ will be :

- (A) $\frac{15}{8}$
- (B) $\frac{8}{17}$
- (C) $\frac{15}{17}$
- (D) $\frac{17}{8}$

Correct Answer: (C) $\frac{15}{17}$

Solution:

Step 1: Understanding the Concept:

This problem requires finding the value of one trigonometric ratio ($\sin \theta$) when another ($\cot \theta$) is given. We can do this by constructing a right-angled triangle or by using trigonometric identities.

Step 2: Key Formula or Approach:

Using the right-angled triangle method:

We know that $\cot \theta = \frac{\text{Base}}{\text{Perpendicular}}$.

Given $\cot \theta = \frac{8}{15}$, we can take Base = 8k and Perpendicular = 15k.

We can find the Hypotenuse using the Pythagorean theorem: $H^2 = P^2 + B^2$.

Then, we find $\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$.

Step 3: Detailed Explanation:

Let's construct a right-angled triangle where:

Base (B) = 8

Perpendicular (P) = 15

Using the Pythagorean theorem to find the Hypotenuse (H):

$$H^2 = P^2 + B^2$$

$$H^2 = 15^2 + 8^2$$

$$H^2 = 225 + 64$$

$$H^2 = 289$$

$$H = \sqrt{289} = 17$$

Now, we can find the value of $\sin \theta$:

$$\sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15}{17}$$

Step 4: Final Answer:

The value of $\sin \theta$ is $\frac{15}{17}$.

Therefore, option (C) is correct.

Quick Tip

Memorizing common Pythagorean triplets like (3,4,5), (5,12,13), and (8,15,17) can save a lot of calculation time in trigonometry problems. As soon as you see Base=8 and Perpendicular=15, you can directly know that the Hypotenuse is 17.

7. The value of $\csc 60^\circ$ is :

- (A) $\frac{2}{\sqrt{3}}$
- (B) $\frac{\sqrt{3}}{2}$
- (C) $\sqrt{2}$
- (D) $\frac{1}{\sqrt{2}}$

Correct Answer: (A) $\frac{2}{\sqrt{3}}$

Solution:

Step 1: Understanding the Concept:

The question asks for the value of the cosecant function for the standard angle of 60 degrees.

Step 2: Key Formula or Approach:

The cosecant function is the reciprocal of the sine function:

$$\csc \theta = \frac{1}{\sin \theta}$$

We need to know the value of $\sin 60^\circ$.

Step 3: Detailed Explanation:

The standard value of $\sin 60^\circ$ is $\frac{\sqrt{3}}{2}$.

Now, we can find $\csc 60^\circ$ using the reciprocal identity:

$$\begin{aligned}\csc 60^\circ &= \frac{1}{\sin 60^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} \\ \csc 60^\circ &= \frac{2}{\sqrt{3}}\end{aligned}$$

Step 4: Final Answer:

The value of $\csc 60^\circ$ is $\frac{2}{\sqrt{3}}$.

Therefore, option (A) is correct.

Quick Tip

Memorizing the trigonometric ratios for standard angles (0° , 30° , 45° , 60° , 90°) is essential for competitive exams. Knowing the sine and cosine values is enough, as all other ratios can be derived from them.

8. The value of $\frac{\csc^2 \theta}{\sec^2 \theta}$ will be :

- (A) $\sin^2 \theta$
- (B) $\csc^2 \theta$
- (C) $\tan^2 \theta$
- (D) $\cot^2 \theta$

Correct Answer: (D) $\cot^2 \theta$

Solution:

Step 1: Understanding the Concept:

This question requires simplifying a trigonometric expression using reciprocal and quotient identities.

Step 2: Key Formula or Approach:

We will use the following reciprocal identities:

- 1. $\csc \theta = \frac{1}{\sin \theta}$
- 2. $\sec \theta = \frac{1}{\cos \theta}$

And the quotient identity:

$$3. \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Step 3: Detailed Explanation:

Let's start with the given expression:

$$\frac{\csc^2 \theta}{\sec^2 \theta}$$

Substitute the reciprocal identities for $\csc^2 \theta$ and $\sec^2 \theta$:

$$\frac{\left(\frac{1}{\sin \theta}\right)^2}{\left(\frac{1}{\cos \theta}\right)^2} = \frac{\frac{1}{\sin^2 \theta}}{\frac{1}{\cos^2 \theta}}$$

To simplify the complex fraction, we multiply the numerator by the reciprocal of the denominator:

$$\frac{1}{\sin^2 \theta} \times \frac{\cos^2 \theta}{1} = \frac{\cos^2 \theta}{\sin^2 \theta}$$

This can be written as:

$$\left(\frac{\cos \theta}{\sin \theta}\right)^2$$

Using the quotient identity $\cot \theta = \frac{\cos \theta}{\sin \theta}$, we get:

$$\cot^2 \theta$$

Step 4: Final Answer:

The value of the expression $\frac{\csc^2 \theta}{\sec^2 \theta}$ is $\cot^2 \theta$.

Therefore, option (D) is correct.

Quick Tip

A quick way to solve this is to remember the relationships: $\csc$ is related to $\sin$, and $\sec$ is related to $\cos$. The expression is essentially $(\cos / \sin)^2$, which is directly $\cot^2$.

9. If H.C.F. (35, 49) = 7, then L.C.M. (35, 49) will be :

- (A) 1715
- (B) 343
- (C) 245
- (D) 175

Correct Answer: (C) 245

Solution:**Step 1: Understanding the Concept:**

This question uses the fundamental relationship between the Highest Common Factor (H.C.F.) and the Lowest Common Multiple (L.C.M.) of two integers.

Step 2: Key Formula or Approach:

For any two positive integers 'a' and 'b', the product of their H.C.F. and L.C.M. is equal to the product of the two numbers.

$$\text{H.C.F.}(a, b) \times \text{L.C.M.}(a, b) = a \times b$$

Step 3: Detailed Explanation:

We are given:

Number 'a' = 35

Number 'b' = 49

H.C.F.(35, 49) = 7

We need to find L.C.M.(35, 49).

Using the formula:

$$7 \times \text{L.C.M.}(35, 49) = 35 \times 49$$

To find the L.C.M., we rearrange the equation:

$$\text{L.C.M.}(35, 49) = \frac{35 \times 49}{7}$$

We can simplify by dividing 35 by 7:

$$\text{L.C.M.}(35, 49) = 5 \times 49$$

$$\text{L.C.M.}(35, 49) = 245$$

Step 4: Final Answer:

The L.C.M. of 35 and 49 is 245.

Therefore, option (C) is correct.

Quick Tip

This formula ($\text{HCF} \times \text{LCM} = \text{Product of numbers}$) is a very powerful tool for quickly finding either the HCF or LCM if the other is known. Remember, this specific formula only works for two numbers.

10. The H.C.F. of two numbers 306 and 657 will be :

- (A) 3
- (B) 9
- (C) 18
- (D) 73

Correct Answer: (B) 9

Solution:

Step 1: Understanding the Concept:

The H.C.F. (Highest Common Factor) of two numbers is the largest positive integer that divides both numbers without leaving a remainder. We can find it using Euclid's division algorithm.

Step 2: Key Formula or Approach:

Euclid's division algorithm is based on the principle that the H.C.F. of two numbers does not change if the larger number is replaced by its difference with the smaller number. The steps are:

1. Apply the division lemma to the larger number (a) and smaller number (b): $a = bq + r$, where $0 \leq r < b$.
2. If $r = 0$, the H.C.F. is b .
3. If $r \neq 0$, apply the lemma to b and r .
4. Continue the process until the remainder is zero. The divisor at this stage will be the H.C.F.

Step 3: Detailed Explanation:

Let's apply Euclid's algorithm to 657 and 306.

Step 1: Divide 657 by 306.

$$657 = 306 \times 2 + 45$$

The remainder is 45.

Step 2: Divide 306 by 45.

$$306 = 45 \times 6 + 36$$

The remainder is 36.

Step 3: Divide 45 by 36.

$$45 = 36 \times 1 + 9$$

The remainder is 9.

Step 4: Divide 36 by 9.

$$36 = 9 \times 4 + 0$$

The remainder is 0. The divisor at this final stage is 9.

Step 4: Final Answer:

The H.C.F. of 306 and 657 is 9.

Therefore, option (B) is correct.

Quick Tip

Euclid's algorithm is a very efficient method for finding the H.C.F. of large numbers, as it avoids the lengthy process of prime factorization.

11. L.C.M. and H.C.F. of two numbers are 168 and 6 respectively. If one number is 24, then second number will be :

- (A) 12
- (B) 42
- (C) 84
- (D) 126

Correct Answer: (B) 42

Solution:

Step 1: Understanding the Concept:

This problem uses the relationship between the L.C.M., H.C.F., and the two numbers themselves.

Step 2: Key Formula or Approach:

The product of two numbers is equal to the product of their L.C.M. and H.C.F.

$$\text{First Number} \times \text{Second Number} = \text{L.C.M.} \times \text{H.C.F.}$$

Step 3: Detailed Explanation:

We are given the following values:

$$\text{L.C.M.} = 168$$

$$\text{H.C.F.} = 6$$

$$\text{First Number} = 24$$

Let the Second Number be 'b'.

Substitute the values into the formula:

$$24 \times b = 168 \times 6$$

Now, solve for 'b':

$$b = \frac{168 \times 6}{24}$$

Simplify the expression:

$$b = \frac{168}{4}$$
$$b = 42$$

Step 4: Final Answer:

The second number is 42.

Therefore, option (B) is correct.

Quick Tip

This is a direct application of a fundamental theorem in number theory. Always remember this formula as it is frequently used in exams to find a missing number, HCF, or LCM.

12. The distance between the points A(a, b) and B(2a, 2b) will be :

- (A) $\sqrt{a^2 + b^2}$
- (B) $2\sqrt{a^2 + b^2}$
- (C) $a + b$
- (D) 2

Correct Answer: (A) $\sqrt{a^2 + b^2}$

Solution:

Step 1: Understanding the Concept:

This question requires the application of the distance formula to find the distance between two points given in terms of variables.

Step 2: Key Formula or Approach:

The distance formula between two points (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 3: Detailed Explanation:

Here, the coordinates of the points are:

Point A: $(x_1, y_1) = (a, b)$

Point B: $(x_2, y_2) = (2a, 2b)$

Substitute these coordinates into the distance formula:

$$d = \sqrt{(2a - a)^2 + (2b - b)^2}$$

Simplify the terms inside the parentheses:

$$d = \sqrt{(a)^2 + (b)^2}$$
$$d = \sqrt{a^2 + b^2}$$

Step 4: Final Answer:

The distance between the points A(a, b) and B(2a, 2b) is $\sqrt{a^2 + b^2}$.

Therefore, option (A) is correct.

Quick Tip

Do not be intimidated by variables in coordinate geometry problems. The formulas apply in exactly the same way. Just perform the algebraic operations carefully.

13. If a tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that PQ = 12 cm, then the length of OQ will be :

- (A) $\sqrt{119}$ cm
- (B) 13 cm
- (C) 12.5 cm
- (D) 12 cm

Correct Answer: (B) 13 cm

Solution:

Step 1: Understanding the Concept:

A key property of circles is that the radius to the point of tangency is always perpendicular to the tangent line at that point. This creates a right-angled triangle.

Step 2: Key Formula or Approach:

The points O (center), P (point of tangency), and Q (external point) form a right-angled triangle $\triangle OPQ$, with the right angle at P. We can apply the Pythagorean theorem:

$$OQ^2 = OP^2 + PQ^2$$

where OQ is the hypotenuse.

Step 3: Detailed Explanation:

We are given:

Radius of the circle (OP) = 5 cm.

Length of the tangent (PQ) = 12 cm.

We need to find the length of OQ.

Using the Pythagorean theorem:

$$OQ^2 = 5^2 + 12^2$$

$$OQ^2 = 25 + 144$$

$$OQ^2 = 169$$

$$OQ = \sqrt{169}$$

$$OQ = 13 \text{ cm}$$

Step 4: Final Answer:

The length of OQ is 13 cm.

Therefore, option (B) is correct.

Quick Tip

Recognizing Pythagorean triplets (like 3-4-5, 5-12-13, 8-15-17) can significantly speed up your calculations. When you see a right triangle with sides 5 and 12, you can immediately identify the hypotenuse as 13.

14. In $\triangle ABC$ and $\triangle DEF$ if $\angle B = \angle E$, then these triangles will be similar if :

- (A) $\frac{AB}{DE} = \frac{AC}{DF}$
- (B) $\frac{AB}{DE} = \frac{BC}{EF}$
- (C) $\frac{BC}{EF} = \frac{CA}{FD}$
- (D) $\frac{AB}{DE} = \frac{BC}{FD}$

Correct Answer: (B) $\frac{AB}{DE} = \frac{BC}{EF}$

Solution:

Step 1: Understanding the Concept:

This question relates to the criteria for the similarity of two triangles. Specifically, it tests the Side-Angle-Side (SAS) similarity criterion.

Step 2: Key Formula or Approach:

The SAS (Side-Angle-Side) similarity criterion states that if one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the two triangles are similar.

Step 3: Detailed Explanation:

We are given that in $\triangle ABC$ and $\triangle DEF$, we have $\angle B = \angle E$.

For the triangles to be similar by the SAS criterion, the ratio of the lengths of the sides that form the angle $\angle B$ must be equal to the ratio of the lengths of the sides that form the angle $\angle E$.

In $\triangle ABC$, the sides including $\angle B$ are AB and BC.

In $\triangle DEF$, the sides including $\angle E$ are DE and EF.

Therefore, the required condition for similarity is that the ratios of these corresponding sides are equal:

$$\frac{AB}{DE} = \frac{BC}{EF}$$

Let's check the options:

(A) Incorrect. AC and DF are not sides including the given angles.

(B) Correct. This matches our derived condition.

(C) Incorrect. CA and FD are not sides including the given angles.

(D) Incorrect. The denominator FD does not correspond to BC.

Step 4: Final Answer:

The condition for the triangles to be similar is $\frac{AB}{DE} = \frac{BC}{EF}$.

Therefore, option (B) is correct.

Quick Tip

For SAS similarity, always identify the "included" sides that form the equal angle. The ratio must be between these pairs of sides. Visualizing or drawing the triangles can help avoid mistakes.

15. The angle of a sector of a circle with radius of 7 cm is 90° . Area of the sector will be :

- (A) 154 cm^2
- (B) 77 cm^2
- (C) 38.5 cm^2
- (D) 19.25 cm^2

Correct Answer: (C) 38.5 cm^2

Solution:

Step 1: Understanding the Concept:

The area of a sector is a fraction of the area of the entire circle, determined by the central angle of the sector.

Step 2: Key Formula or Approach:

The formula for the area of a sector with radius r and central angle θ (in degrees) is:

$$\text{Area of Sector} = \frac{\theta}{360} \times \pi r^2$$

Step 3: Detailed Explanation:

We are given the following values:

Radius (r) = 7 cm

Angle (θ) = 90°

Let's use $\pi = \frac{22}{7}$.

Substitute the values into the formula:

$$\text{Area} = \frac{90}{360} \times \frac{22}{7} \times (7)^2$$

Simplify the fraction and the expression:

$$\text{Area} = \frac{1}{4} \times \frac{22}{7} \times 49$$

$$\text{Area} = \frac{1}{4} \times 22 \times 7$$

$$\text{Area} = \frac{154}{4}$$

$$\text{Area} = 38.5 \text{ cm}^2$$

Step 4: Final Answer:

The area of the sector is 38.5 cm^2 .

Therefore, option (C) is correct.

Quick Tip

When the angle is 90° , the sector is a quadrant, which means its area is exactly one-fourth of the total area of the circle. You can quickly calculate $\frac{1}{4}\pi r^2$.

16. A toy is in the form of a cone of radius 4 cm mounted on a hemisphere of same radius. If the slant height of the cone is 12 cm, then the total surface area of the toy will be :

- (A) $224 \pi \text{ cm}^2$
- (B) $112 \pi \text{ cm}^2$
- (C) $16 \pi (2 + \sqrt{10}) \text{ cm}^2$
- (D) $80 \pi \text{ cm}^2$

Correct Answer: (D) $80\pi \text{ cm}^2$

Solution:

Step 1: Understanding the Concept:

The total surface area of the toy is the sum of the curved surface area of the conical part and the curved surface area of the hemispherical part. The flat circular base where they are joined is not part of the total surface area.

Step 2: Key Formula or Approach:

Total Surface Area (TSA) = Curved Surface Area (CSA) of Cone + Curved Surface Area (CSA) of Hemisphere

- CSA of Cone = $\pi r l$

- CSA of Hemisphere = $2\pi r^2$

TSA = $\pi r l + 2\pi r^2 = \pi r(l + 2r)$

Step 3: Detailed Explanation:

We are given the following values:

Radius of cone and hemisphere (r) = 4 cm

Slant height of the cone (l) = 12 cm

Calculate the individual surface areas:

CSA of Cone = $\pi \times 4 \times 12 = 48\pi \text{ cm}^2$

CSA of Hemisphere = $2\pi \times (4)^2 = 2\pi \times 16 = 32\pi \text{ cm}^2$

Now, add them to get the Total Surface Area of the toy:

$$\text{TSA} = 48\pi + 32\pi = 80\pi \text{ cm}^2$$

Using the combined formula:

$$\text{TSA} = \pi r(l + 2r) = \pi \times 4(12 + 2 \times 4) = 4\pi(12 + 8) = 4\pi(20) = 80\pi \text{ cm}^2$$

Step 4: Final Answer:

The total surface area of the toy is $80\pi \text{ cm}^2$.

Therefore, option (D) is correct.

Quick Tip

For composite solids, always visualize which surfaces are exposed to the outside. The internal surfaces where the solids are joined are not included in the Total Surface Area.

17. The zeroes of the quadratic polynomial $x^2 + 7x + 12$ will be :

- (A) $-4, -3$
- (B) $-6, -2$
- (C) $4, 3$
- (D) $-6, 2$

Correct Answer: (A) $-4, -3$

Solution:

Step 1: Understanding the Concept:

The zeroes of a polynomial are the values of the variable for which the polynomial evaluates to zero. For a quadratic polynomial, we can find the zeroes by factoring it.

Step 2: Key Formula or Approach:

We need to factor the quadratic expression $x^2 + 7x + 12$. We look for two numbers that multiply to 12 (the constant term) and add up to 7 (the coefficient of x).

Step 3: Detailed Explanation:

The two numbers that satisfy the condition (product = 12, sum = 7) are 3 and 4.

We can rewrite the middle term using these numbers:

$$x^2 + 3x + 4x + 12$$

Now, we factor by grouping:

$$x(x + 3) + 4(x + 3)$$

Factor out the common term $(x + 3)$:

$$(x + 3)(x + 4)$$

To find the zeroes, we set the polynomial equal to zero:

$$(x + 3)(x + 4) = 0$$

This gives us two possible solutions:

Either $x + 3 = 0 \implies x = -3$

Or $x + 4 = 0 \implies x = -4$

Step 4: Final Answer:

The zeroes of the quadratic polynomial are -4 and -3 .

Therefore, option (A) is correct.

Quick Tip

Remember that if the factored form is $(x + a)(x + b)$, the zeroes will be $x = -a$ and $x = -b$. Be careful with the signs.

18. The solution of pair of linear equations $x + y = 14$, $x - y = 2$ will be :

- (A) $x = 20, y = -6$
- (B) $x = 8, y = 6$
- (C) $x = -8, y = -6$
- (D) $x = 10, y = 4$

Correct Answer: (B) $x = 8, y = 6$

Solution:

Step 1: Understanding the Concept:

We need to solve a system of two linear equations with two variables. The elimination method is particularly suitable here.

Step 2: Key Formula or Approach:

We have the two equations:

1) $x + y = 14$

2) $x - y = 2$

We can eliminate one variable by adding or subtracting the two equations.

Step 3: Detailed Explanation:

Let's add Equation 1 and Equation 2 to eliminate y :

$$(x + y) + (x - y) = 14 + 2$$

$$2x = 16$$

$$x = \frac{16}{2} = 8$$

Now that we have the value of x , we can substitute it back into either equation to find y . Let's use Equation 1:

$$8 + y = 14$$

$$y = 14 - 8$$

$$y = 6$$

The solution is $x = 8$ and $y = 6$.

Step 4: Final Answer:

The solution to the pair of linear equations is $x = 8, y = 6$.

Therefore, option (B) is correct.

Quick Tip

After finding a solution, it's a good practice to quickly check it by substituting the values into both original equations.

Check 1: $8 + 6 = 14$ (Correct).

Check 2: $8 - 6 = 2$ (Correct).

19. If the roots of the quadratic equation $Kx^2 - 2\sqrt{6}x + 3 = 0$ are equal, then the value of K will be :

- (A) -2
- (B) $2\sqrt{3}$
- (C) 2
- (D) -3

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

A quadratic equation has equal roots if its discriminant is equal to zero.

Step 2: Key Formula or Approach:

For a quadratic equation $ax^2 + bx + c = 0$, the discriminant is $D = b^2 - 4ac$.

The condition for equal roots is $D = 0$.

Step 3: Detailed Explanation:

In the given equation, $Kx^2 - 2\sqrt{6}x + 3 = 0$, we have:

$$a = K$$

$$b = -2\sqrt{6}$$

$$c = 3$$

Set the discriminant to zero:

$$b^2 - 4ac = 0$$

$$(-2\sqrt{6})^2 - 4(K)(3) = 0$$

Simplify the expression:

$$(4 \times 6) - 12K = 0$$

$$24 - 12K = 0$$

$$12K = 24$$

$$K = \frac{24}{12}$$

$$K = 2$$

Step 4: Final Answer:

The value of K is 2.

Therefore, option (C) is correct.

Quick Tip

Be careful when squaring terms with square roots, like $(-2\sqrt{6})^2$. Remember to square both the coefficient and the root: $(-2)^2 \times (\sqrt{6})^2 = 4 \times 6 = 24$.

20. The seventh term of A.P. $-6, -4, -2, \dots$ will be :

- (A) -8
- (B) 18
- (C) 8
- (D) 6

Correct Answer: (D) 6

Solution:

Step 1: Understanding the Concept:

This problem requires finding a specific term in an Arithmetic Progression (A.P.).

Step 2: Key Formula or Approach:

The formula for the n -th term (a_n) of an A.P. is:

$$a_n = a + (n - 1)d$$

where a is the first term, n is the term number, and d is the common difference.

Step 3: Detailed Explanation:

From the given A.P. $(-6, -4, -2, \dots)$, we can identify:

The first term (a) = -6 .

The common difference (d) = (Second Term) - (First Term) = $-4 - (-6) = -4 + 6 = 2$.

We need to find the seventh term, so $n = 7$.

Substitute these values into the formula:

$$a_7 = -6 + (7 - 1) \times 2$$

$$a_7 = -6 + (6) \times 2$$

$$a_7 = -6 + 12$$

$$a_7 = 6$$

Step 4: Final Answer:

The seventh term of the A.P. is 6.

Therefore, option (D) is correct.

Quick Tip

Always be careful with signs when calculating the common difference, especially when dealing with negative numbers. A small sign error will change the entire result.

Section - B

Q(1) Do All Parts

(a) Prove that $6 - \sqrt{2}$ is irrational.

Solution:

Step 1: Understanding the Concept:

We will use the method of proof by contradiction. This involves assuming the opposite of what we want to prove and then showing that this assumption leads to a logical inconsistency. We will use the fact that $\sqrt{2}$ is an irrational number.

Step 2: Key Formula or Approach:

1. Assume that $6 - \sqrt{2}$ is a rational number.
2. Express it in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.
3. Rearrange the equation to isolate $\sqrt{2}$.
4. Show that this leads to the contradiction that an irrational number ($\sqrt{2}$) is equal to a rational number.

Step 3: Detailed Explanation:

Let us assume, to the contrary, that $6 - \sqrt{2}$ is a rational number. Then, there exist co-prime integers a and b ($b \neq 0$) such that:

$$6 - \sqrt{2} = \frac{a}{b}$$

Rearranging the equation to isolate $\sqrt{2}$:

$$6 - \frac{a}{b} = \sqrt{2}$$

Taking a common denominator on the left side:

$$\frac{6b - a}{b} = \sqrt{2}$$

Since a and b are integers, $6b - a$ is also an integer.

Therefore, $\frac{6b-a}{b}$ is a rational number (as it is in the form of integer/integer).

This implies that $\sqrt{2}$ is a rational number.

However, this contradicts the well-known fact that $\sqrt{2}$ is an irrational number.

This contradiction has arisen because of our incorrect assumption that $6 - \sqrt{2}$ is rational.

Step 4: Final Answer:

Therefore, we conclude that $6 - \sqrt{2}$ is an irrational number.

Quick Tip

This method of proof by contradiction can be used for any number of the form (rational $\pm$ irrational). The key is to always isolate the irrational part and show that it equals a rational expression, which is a contradiction.

(b) Find the coordinates of the point on the x-axis which is equidistant from (2, -5) and (-2, 9).

Solution:

Step 1: Understanding the Concept:

Any point on the x-axis has coordinates of the form $(x, 0)$. The term "equidistant" means that the distance from this point to the first given point is equal to the distance from this point to the second given point. We will use the distance formula to set up an equation.

Step 2: Key Formula or Approach:

Let the point on the x-axis be $P(x, 0)$.

Let the given points be $A(2, -5)$ and $B(-2, 9)$.

The condition is $PA = PB$, which implies $PA^2 = PB^2$.

The distance formula is $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

Step 3: Detailed Explanation:

Using the distance formula to find PA^2 :

$$PA^2 = (x - 2)^2 + (0 - (-5))^2 = (x - 2)^2 + (5)^2 = x^2 - 4x + 4 + 25 = x^2 - 4x + 29$$

Using the distance formula to find PB^2 :

$$PB^2 = (x - (-2))^2 + (0 - 9)^2 = (x + 2)^2 + (-9)^2 = x^2 + 4x + 4 + 81 = x^2 + 4x + 85$$

Now, set $PA^2 = PB^2$:

$$x^2 - 4x + 29 = x^2 + 4x + 85$$

The x^2 terms on both sides cancel out:

$$-4x + 29 = 4x + 85$$

Rearrange the equation to solve for x :

$$29 - 85 = 4x + 4x$$

$$-56 = 8x$$

$$x = \frac{-56}{8} = -7$$

The point on the x-axis is $(x, 0)$.

Step 4: Final Answer:

The coordinates of the point on the x-axis are $(-7, 0)$.

Quick Tip

When dealing with equidistant points, it's often easier to work with the square of the distances ($PA^2 = PB^2$). This avoids having to deal with square roots in the algebraic manipulation.

(c) Find the coordinates of the points which divide the line segment AB joining the points A(-2, 2) and B(2, 8) into four equal parts.

Solution:

Step 1: Understanding the Concept:

To divide a line segment into four equal parts, we need to find three points that lie on the segment. Let these points be P, Q, and R.

- Point Q will be the midpoint of the entire segment AB.
- Point P will be the midpoint of the segment AQ.
- Point R will be the midpoint of the segment QB.

We will use the midpoint formula and the section formula.

Step 2: Key Formula or Approach:

The midpoint formula for a segment joining (x_1, y_1) and (x_2, y_2) is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

The section formula can also be used, with ratios 1:3 for P, 2:2 (or 1:1) for Q, and 3:1 for R.

Step 3: Detailed Explanation:

The given points are A(-2, 2) and B(2, 8).

1. Find the coordinates of Q (the midpoint of AB):

Q divides AB in the ratio 2:2 or 1:1.

$$Q = \left(\frac{-2 + 2}{2}, \frac{2 + 8}{2} \right) = \left(\frac{0}{2}, \frac{10}{2} \right) = (0, 5)$$

2. Find the coordinates of P (the midpoint of AQ):

P divides the segment with endpoints A(-2, 2) and Q(0, 5).

$$P = \left(\frac{-2 + 0}{2}, \frac{2 + 5}{2} \right) = \left(-1, \frac{7}{2} \right)$$

3. Find the coordinates of R (the midpoint of QB):

R divides the segment with endpoints Q(0, 5) and B(2, 8).

$$R = \left(\frac{0 + 2}{2}, \frac{5 + 8}{2} \right) = \left(1, \frac{13}{2} \right)$$

Step 4: Final Answer:

The coordinates of the three points that divide the line segment into four equal parts are **P(-1, 3.5)**, **Q(0, 5)**, and **R(1, 6.5)**.

Quick Tip

Dividing a segment into four equal parts is best done by finding the overall midpoint first, and then finding the midpoints of the two new segments. This is often simpler and less prone to calculation errors than using the section formula with ratios 1:3 and 3:1.

(d) **Prove that :** $\sqrt{\frac{1+\sin \theta}{1-\sin \theta}} = \sec \theta + \tan \theta$

Solution:

Step 1: Understanding the Concept:

This question requires proving a trigonometric identity involving a square root. The standard technique for such problems is to rationalize the denominator inside the square root.

Step 2: Key Formula or Approach:

1. Start with the Left Hand Side (LHS).
2. Multiply the numerator and the denominator inside the square root by the conjugate of the denominator, which is $(1 + \sin \theta)$.
3. Use the identities: $(a - b)(a + b) = a^2 - b^2$ and $1 - \sin^2 \theta = \cos^2 \theta$.
4. Simplify the expression.

Step 3: Detailed Explanation:

Starting with the LHS:

$$\text{LHS} = \sqrt{\frac{1 + \sin \theta}{1 - \sin \theta}}$$

Multiply the numerator and denominator inside the radical by $(1 + \sin \theta)$:

$$\text{LHS} = \sqrt{\frac{(1 + \sin \theta)(1 + \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)}}$$

$$\text{LHS} = \sqrt{\frac{(1 + \sin \theta)^2}{1^2 - \sin^2 \theta}}$$

Using the Pythagorean identity $1 - \sin^2 \theta = \cos^2 \theta$ in the denominator:

$$\text{LHS} = \sqrt{\frac{(1 + \sin \theta)^2}{\cos^2 \theta}}$$

Now, take the square root of the numerator and the denominator:

$$\text{LHS} = \frac{1 + \sin \theta}{\cos \theta}$$

Split the fraction into two parts:

$$\text{LHS} = \frac{1}{\cos \theta} + \frac{\sin \theta}{\cos \theta}$$

Using the reciprocal identity $\sec \theta = \frac{1}{\cos \theta}$ and the quotient identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$:

$$\text{LHS} = \sec \theta + \tan \theta$$

This is equal to the Right Hand Side (RHS).

$$\text{LHS} = \text{RHS}$$

Hence, proved.

Step 4: Final Answer:

The identity is proven by rationalizing the denominator inside the square root and simplifying using fundamental trigonometric identities.

Quick Tip

Whenever you see an expression of the form $\sqrt{\frac{1 \pm \sin \theta}{1 \mp \sin \theta}}$ or $\sqrt{\frac{1 \pm \cos \theta}{1 \mp \cos \theta}}$, the first step should almost always be to rationalize the denominator by multiplying by its conjugate.

(e) A chord of a circle of radius 21 cm subtends an angle of 60° at the centre, find the area of the corresponding segment of the circle.

Solution:

Step 1: Understanding the Concept:

The area of a segment of a circle is the area enclosed by a chord and the corresponding arc. It is calculated by finding the area of the corresponding sector and subtracting the area of the triangle formed by the chord and the two radii.

Step 2: Key Formula or Approach:

Area of Segment = Area of Sector - Area of Triangle

- Area of Sector = $\frac{\theta}{360} \times \pi r^2$

- Area of Triangle (when the angle between two sides r is θ) = $\frac{1}{2}r^2 \sin \theta$

(Note: When $\theta = 60^\circ$, the triangle is equilateral, so its area is also $\frac{\sqrt{3}}{4}(\text{side})^2$).

Step 3: Detailed Explanation:

Given values:

Radius (r) = 21 cm

Angle (θ) = 60°

1. Calculate the Area of the Sector (OAPB):

$$\begin{aligned}\text{Area}_{\text{sector}} &= \frac{60}{360} \times \pi(21)^2 = \frac{1}{6} \times \frac{22}{7} \times 21 \times 21 \\ &= \frac{1}{6} \times 22 \times 3 \times 21 = 11 \times 21 = 231 \text{ cm}^2\end{aligned}$$

2. Calculate the Area of the Triangle (OAB):

Since the two sides OA and OB are radii ($r = 21$) and the angle between them is 60° , $\triangle OAB$ is an equilateral triangle. Therefore, the side AB is also 21 cm.

The area of an equilateral triangle is $\frac{\sqrt{3}}{4}a^2$, where a is the side length.

$$\text{Area}_{\triangle OAB} = \frac{\sqrt{3}}{4}(21)^2 = \frac{441\sqrt{3}}{4} \text{ cm}^2$$

3. Calculate the Area of the Segment:

$$\text{Area}_{\text{segment}} = \text{Area}_{\text{sector}} - \text{Area}_{\triangle OAB}$$

$$\text{Area}_{\text{segment}} = \left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$$

If we approximate $\sqrt{3} \approx 1.73$:

$$\text{Area}_{\text{segment}} \approx 231 - \frac{441 \times 1.73}{4} = 231 - \frac{762.93}{4} \approx 231 - 190.73 = 40.27 \text{ cm}^2$$

Step 4: Final Answer:

The area of the corresponding segment is $\left(231 - \frac{441\sqrt{3}}{4} \right) \text{ cm}^2$.

Quick Tip

Recognize special cases. When the central angle subtended by a chord is 60° , the triangle formed with the center is always equilateral. When the angle is 90° , it's a right-angled isosceles triangle. This simplifies the calculation of the triangle's area.

(f) A box contains 4 red marbles, 8 white marbles and 5 green marbles. One marble is taken out from the box at random. What is the probability that the marble taken out will be

- (i) red ?**
- (ii) not green ?**

Solution:

Step 1: Understanding the Concept:

Probability is the ratio of the number of favorable outcomes to the total number of possible outcomes. We first need to find the total number of marbles in the box.

Step 2: Key Formula or Approach:

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

For "not an event", $P(\text{not } E) = 1 - P(E)$.

Step 3: Detailed Explanation:

First, calculate the total number of marbles:

Total Marbles = 4 (Red) + 8 (White) + 5 (Green) = 17 marbles.

So, the total number of possible outcomes is 17.

(i) Probability of getting a red marble:

Number of red marbles (favorable outcomes) = 4.

$$P(\text{red}) = \frac{\text{Number of red marbles}}{\text{Total number of marbles}} = \frac{4}{17}$$

(ii) Probability of getting a marble that is not green:

Method 1: Direct Calculation

The marbles that are not green are the red and white ones.

Number of non-green marbles = 4 (Red) + 8 (White) = 12.

This is the number of favorable outcomes.

$$P(\text{not green}) = \frac{\text{Number of non-green marbles}}{\text{Total number of marbles}} = \frac{12}{17}$$

Method 2: Using the Complement Rule

First, find the probability of getting a green marble.

Number of green marbles = 5.

$$P(\text{green}) = \frac{5}{17}$$

The probability of not getting a green marble is:

$$P(\text{not green}) = 1 - P(\text{green}) = 1 - \frac{5}{17} = \frac{17 - 5}{17} = \frac{12}{17}$$

Step 4: Final Answer:

(i) The probability that the marble taken out is red is $\frac{4}{17}$.

(ii) The probability that the marble taken out is not green is $\frac{12}{17}$.

Quick Tip

For "not event" probability questions, you have two options: either count the outcomes that are not the event, or calculate the probability of the event and subtract it from 1. Choose whichever method seems quicker and less prone to error.

Q(2) Do Any 5 Parts

(a) Two concentric circles of radii 7.5 cm and 4.5 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:

Step 1: Understanding the Concept:

The problem involves two circles with the same center (concentric). A chord of the larger circle acts as a tangent to the smaller circle. We can use the properties of tangents and the Pythagorean theorem to solve this.

Step 2: Key Formula or Approach:

Let the two concentric circles have a common center O. Let AB be the chord of the larger circle that touches the smaller circle at point P.

- The radius of the smaller circle, OP, will be perpendicular to the tangent chord AB. ($\angle OPA = 90^\circ$).
- This forms a right-angled triangle $\triangle OPA$.
- The radius of the larger circle is OA (the hypotenuse).
- The perpendicular from the center to a chord bisects the chord, so $AP = PB$. The length of the chord AB is $2 \times AP$.
- We will use the Pythagorean theorem: $OA^2 = OP^2 + AP^2$.

Step 3: Detailed Explanation:

Given values:

Radius of the larger circle (OA) = 7.5 cm.

Radius of the smaller circle (OP) = 4.5 cm.

In the right-angled triangle $\triangle OPA$, applying the Pythagorean theorem:

$$(7.5)^2 = (4.5)^2 + AP^2$$

$$56.25 = 20.25 + AP^2$$

$$AP^2 = 56.25 - 20.25$$

$$AP^2 = 36$$

$$AP = \sqrt{36} = 6 \text{ cm}$$

The length of the chord AB is twice the length of AP:

$$\text{Length of chord AB} = 2 \times AP = 2 \times 6 = 12 \text{ cm}$$

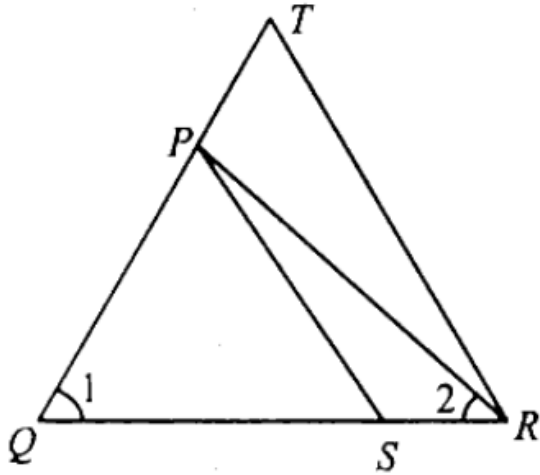
Step 4: Final Answer:

The length of the chord of the larger circle which touches the smaller circle is **12 cm**.

Quick Tip

Drawing a diagram for geometry problems is crucial. It helps you visualize the relationships between different elements (radii, tangents, chords) and identify the correct geometric theorems to apply.

(b) In the figure : $\frac{QR}{QS} = \frac{QT}{TR}$ and $\angle 1 = \angle 2$. Prove that $\triangle PQS \sim \triangle TQR$.



Solution:

Step 1: Understanding the Concept:

We need to prove that two triangles are similar. We are given a condition on the ratio of sides and an equality of angles. We will use this information to satisfy one of the similarity criteria (AA, SSS, or SAS).

Step 2: Key Formula or Approach:

We will use the Side-Angle-Side (SAS) similarity criterion. For this, we need to show that the ratio of two pairs of corresponding sides is equal and the included angles are equal.

Step 3: Detailed Explanation:

First, consider the given angle equality in $\triangle PQR$:

Given: $\angle 1 = \angle 2$ (which are $\angle PQR$ and $\angle PRQ$ respectively).

In $\triangle PQR$, since the angles opposite to the sides PQ and PR are equal, the sides themselves must be equal.

Therefore, $PQ = PR$ (Sides opposite to equal angles are equal).

Now, consider the given ratio of sides:

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Substitute PQ in place of PR using the result from above:

$$\frac{QR}{QS} = \frac{QT}{PQ}$$

Rearranging this proportion to match the sides of the triangles we want to prove similar ($\triangle PQS$ and $\triangle TQR$):

$$\frac{QS}{QR} = \frac{PQ}{QT}$$

Now, let's check the conditions for SAS similarity in $\triangle PQS$ and $\triangle TQR$:

1. **Ratio of Sides:** We have just shown that $\frac{QS}{QR} = \frac{PQ}{QT}$.

2. **Included Angle:** The angle included between sides QS and PQ in $\triangle PQS$ is $\angle PQS$. The angle included between sides QR and QT in $\triangle TQR$ is $\angle TQR$. Both of these are the same angle, $\angle Q$.

So, $\angle PQS = \angle TQR$ (Common angle).

Since we have shown that the ratio of two pairs of corresponding sides is equal and their included angles are equal, the two triangles are similar by the SAS similarity criterion.

Hence, $\triangle PQS \sim \triangle TQR$.

Step 4: Final Answer:

By using the property that sides opposite to equal angles are equal ($PQ = PR$) and substituting this into the given ratio, we establish the condition for SAS similarity with the common angle $\angle Q$, thus proving that $\triangle PQS \sim \triangle TQR$.

Quick Tip

When a proof seems stuck, re-examine all the "given" information. Often, a piece of information (like $\angle 1 = \angle 2$) provides a hidden relationship (like $PQ = PR$) that is key to solving the problem.

(c) How many terms of the A.P. : 21, 18, 15,... must be taken so that their sum is -81 ?

Solution:

Step 1: Understanding the Concept:

We need to find the number of terms (n) of a given Arithmetic Progression (A.P.) that will add up to a specific sum (S_n).

Step 2: Key Formula or Approach:

The formula for the sum of the first n terms of an A.P. is:

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

where a is the first term, d is the common difference, and n is the number of terms.

Step 3: Detailed Explanation:

From the given A.P. (21, 18, 15,...), we can determine:

First term (a) = 21

Common difference (d) = $18 - 21 = -3$

Sum of n terms (S_n) = -81

Substitute these values into the sum formula:

$$-81 = \frac{n}{2}[2(21) + (n-1)(-3)]$$

Multiply both sides by 2:

$$-162 = n[42 - 3n + 3]$$

$$-162 = n[45 - 3n]$$

$$-162 = 45n - 3n^2$$

Rearrange the terms to form a standard quadratic equation:

$$3n^2 - 45n - 162 = 0$$

Divide the entire equation by 3 to simplify:

$$n^2 - 15n - 54 = 0$$

Now, factor the quadratic equation. We need two numbers that multiply to -54 and add to -15 . These numbers are -18 and 3 .

$$n^2 - 18n + 3n - 54 = 0$$

$$n(n - 18) + 3(n - 18) = 0$$

$$(n - 18)(n + 3) = 0$$

This gives two possible solutions for n : $n = 18$ or $n = -3$.

Since the number of terms (n) cannot be negative, we must have $n = 18$.

Step 4: Final Answer:

18 terms must be taken so that their sum is -81 .

Quick Tip

When the sum of an A.P. formula leads to a quadratic equation, remember that the solution for n must be a positive integer. Any negative or fractional solution should be discarded.

(d) Find the zeroes of the quadratic polynomial $x^2 + 10x + 21$ and verify the relationship between the zeroes and the coefficients.

Solution:

Step 1: Understanding the Concept:

We need to first find the roots (zeroes) of the given quadratic equation. Then, we must verify that these roots satisfy the standard relationship between the zeroes and coefficients of a quadratic polynomial.

Step 2: Key Formula or Approach:

1. **Finding Zeroes:** To find the zeroes of $ax^2 + bx + c$, we solve the equation $ax^2 + bx + c = 0$ by factoring.

2. **Verifying Relationship:** For a quadratic polynomial $ax^2 + bx + c$ with zeroes α and β :

- Sum of zeroes: $\alpha + \beta = -\frac{b}{a}$

- Product of zeroes: $\alpha\beta = \frac{c}{a}$

Step 3: Detailed Explanation:**Part 1: Finding the zeroes**

Given polynomial: $P(x) = x^2 + 10x + 21$.

To find the zeroes, set $P(x) = 0$:

$$x^2 + 10x + 21 = 0$$

We need to find two numbers that multiply to 21 and add to 10. These numbers are 7 and 3.

$$x^2 + 7x + 3x + 21 = 0$$

Factor by grouping:

$$x(x + 7) + 3(x + 7) = 0$$

$$(x + 7)(x + 3) = 0$$

This gives the zeroes as $x = -7$ and $x = -3$.

So, let $\alpha = -7$ and $\beta = -3$.

Part 2: Verifying the relationship

From the polynomial $x^2 + 10x + 21$, the coefficients are:

$a = 1, b = 10, c = 21$

Verification of the Sum of Zeroes:

Sum from our found zeroes: $\alpha + \beta = (-7) + (-3) = -10$.

Sum using the formula: $-\frac{b}{a} = -\frac{10}{1} = -10$.

Since $-10 = -10$, the relationship for the sum of zeroes is verified.

Verification of the Product of Zeroes:

Product from our found zeroes: $\alpha\beta = (-7) \times (-3) = 21$.

Product using the formula: $\frac{c}{a} = \frac{21}{1} = 21$.

Since $21 = 21$, the relationship for the product of zeroes is verified.

Step 4: Final Answer:

The zeroes of the polynomial are **-7 and -3**. The relationship between the zeroes and coefficients is successfully verified.

Quick Tip

The verification step is a great way to check if you have found the correct zeroes. If the sum and product relationships do not hold, you likely made an error in factoring the polynomial.

(e) Daily wages of 50 workers of a factory is shown in the following table. Find the mean daily wages of the workers :

Daily Wages (in)	500–520	520–540	540–560	560–580	580–600
Number of workers	12	14	8	6	10

Solution:

Step 1: Understanding the Concept:

To find the mean of grouped data, we use the class mark (mid-point) of each class interval. We will use the direct method for calculating the mean.

Step 2: Key Formula or Approach:

The formula for the mean ($\bar{x}$) using the direct method is:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

where f_i is the frequency of the i-th class and x_i is the class mark of the i-th class.

Class Mark (x_i) = $\frac{\text{Upper class limit} + \text{Lower class limit}}{2}$.

Step 3: Detailed Explanation:

We will construct a table to perform the calculations:

Daily Wages (in)	Number of workers (f_i)	Class Mark (x_i)	$f_i x_i$
500 – 520	12	$\frac{500+520}{2} = 510$	$12 \times 510 = 6120$
520 – 540	14	$\frac{520+540}{2} = 530$	$14 \times 530 = 7420$
540 – 560	8	$\frac{540+560}{2} = 550$	$8 \times 550 = 4400$
560 – 580	6	$\frac{560+580}{2} = 570$	$6 \times 570 = 3420$
580 – 600	10	$\frac{580+600}{2} = 590$	$10 \times 590 = 5900$
Total	$\sum f_i = 50$		$\sum f_i x_i = 27260$

Now, apply the mean formula:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{27260}{50}$$

$$\bar{x} = \frac{2726}{5} = 545.2$$

Step 4: Final Answer:

The mean daily wages of the workers is **545.20**.

Quick Tip

For data with large numbers, you can also use the Assumed Mean Method or the Step-Deviation Method to simplify calculations. However, the Direct Method is straightforward and reliable if you are careful with your multiplication and addition.

(f) Find the mode and median of the following frequency table :

Class Interval	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency	4	5	13	20	14	8	4

Solution:

Step 1: Understanding the Concept:

We need to calculate two measures of central tendency for grouped data: the mode and the median, using their respective formulas.

Step 2: Key Formula or Approach:

For Mode:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where l is the lower limit, h is the size, f_1 is the frequency of the modal class, and f_0, f_2 are the frequencies of the preceding and succeeding classes.

For Median:

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

where l is the lower limit, h is the size, n is the total frequency, f is the frequency of the median class, and cf is the cumulative frequency of the preceding class.

Step 3: Detailed Explanation:

Calculation of Mode:

1. Identify the modal class: The highest frequency is 20, so the modal class is **125 – 145**.
2. Identify the values for the formula:
 - Lower limit of modal class (l) = 125
 - Class size (h) = 85 - 65 = 20

- Frequency of modal class (f_1) = 20
 - Frequency of preceding class (f_0) = 13
 - Frequency of succeeding class (f_2) = 14
3. Substitute into the formula:

$$\begin{aligned}\text{Mode} &= 125 + \left(\frac{20 - 13}{2(20) - 13 - 14} \right) \times 20 \\ &= 125 + \left(\frac{7}{40 - 27} \right) \times 20 = 125 + \left(\frac{7}{13} \right) \times 20 \\ &= 125 + \frac{140}{13} \approx 125 + 10.77 = 135.77\end{aligned}$$

Calculation of Median:

1. Create the cumulative frequency (cf) table:

Class Interval	Frequency (f)	Cumulative Frequency (cf)
65-85	4	4
85-105	5	9
105-125	13	22
125-145	20	42
145-165	14	56
165-185	8	64
185-205	4	68

2. Find the median class: Total frequency $n = 68$. So, $\frac{n}{2} = 34$. The cf just greater than 34 is 42. So, the median class is **125 – 145**.
3. Identify the values for the formula:
- Lower limit of median class (l) = 125
 - Class size (h) = 20
 - Total frequency (n) = 68
 - Cumulative frequency of preceding class (cf) = 22
 - Frequency of median class (f) = 20
4. Substitute into the formula:

$$\begin{aligned}\text{Median} &= 125 + \left(\frac{\frac{68}{2} - 22}{20} \right) \times 20 \\ &= 125 + \left(\frac{34 - 22}{20} \right) \times 20 = 125 + \frac{12}{20} \times 20 = 125 + 12 = 137\end{aligned}$$

Step 4: Final Answer:

The mode of the frequency table is approximately **135.77**.

The median of the frequency table is **137**.

Quick Tip

For both median and mode, the first step is to correctly identify the respective class (median class or modal class). All other values in the formula (l, h, f, cf , etc.) depend on this identification.

3. The sum of a two-digit number and the number obtained by reversing the digits is 99. If the digits of the number differ by 5, find the number. How many such numbers are there ?

Solution:

Step 1: Understanding the Concept:

This problem involves forming a system of linear equations based on the properties of a two-digit number. We need to represent the number algebraically and translate the given conditions into equations.

Step 2: Key Formula or Approach:

Let the ten's digit be x and the unit's digit be y .

The original number can be written as $10x + y$.

The number obtained by reversing the digits is $10y + x$.

Step 3: Detailed Explanation:

Condition 1: The sum of the number and its reverse is 99.

$$(10x + y) + (10y + x) = 99$$

$$11x + 11y = 99$$

Dividing by 11, we get:

$$x + y = 9$$

— (Equation 1)

Condition 2: The digits differ by 5. This leads to two possible cases.

Case I: $x - y = 5$ — (Equation 2a)

Case II: $y - x = 5$ — (Equation 2b)

Solving Case I:

We solve Equation 1 and Equation 2a:

$$x + y = 9$$

$$x - y = 5$$

Adding the two equations: $2x = 14 \implies x = 7$.

Substitute $x = 7$ into Equation 1: $7 + y = 9 \implies y = 2$.

The number is $10x + y = 10(7) + 2 = 72$.

Solving Case II:

We solve Equation 1 and Equation 2b:

$$x + y = 9$$

$$-x + y = 5$$

Adding the two equations: $2y = 14 \implies y = 7$.

Substitute $y = 7$ into Equation 1: $x + 7 = 9 \implies x = 2$.

The number is $10x + y = 10(2) + 7 = 27$.

There are two possible numbers that satisfy the given conditions.

Step 4: Final Answer:

The required number can be either **72** or **27**. There are **two** such numbers.

Quick Tip

For word problems involving two-digit numbers, always represent the number as $10x + y$. Remember that a condition like "digits differ by" can lead to two separate possibilities ($x - y$ or $y - x$), so be sure to check both.

3. (OR) A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in) was 15 more than twice the number of articles produced on that day. If the total cost of production on that day was 2,250, find the number of produced articles and the cost of each article.

Solution:**Step 1: Understanding the Concept:**

This is a word problem that can be modeled by a quadratic equation. We need to define a variable for the number of articles, express the cost per article in terms of this variable, and then form an equation based on the total cost.

Step 2: Key Formula or Approach:

Let the number of articles produced be x .

Cost of each article = "15 more than twice the number of articles" = $2x + 15$.

Total cost = (Number of articles) $\times$ (Cost of each article).

Step 3: Detailed Explanation:

According to the problem, the total cost of production is 2,250.

$$x(2x + 15) = 2250$$

Expand the equation:

$$2x^2 + 15x = 2250$$

Rearrange it into the standard quadratic form $ax^2 + bx + c = 0$:

$$2x^2 + 15x - 2250 = 0$$

We can solve this using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Here, $a = 2, b = 15, c = -2250$.

Discriminant $D = b^2 - 4ac = (15)^2 - 4(2)(-2250) = 225 + 18000 = 18225$.

The square root of the discriminant is $\sqrt{18225} = 135$.

$$x = \frac{-15 \pm 135}{2(2)} = \frac{-15 \pm 135}{4}$$

This gives two possible values for x :

$$x = \frac{-15 + 135}{4} = \frac{120}{4} = 30$$

$$x = \frac{-15 - 135}{4} = \frac{-150}{4} = -37.5$$

Since the number of articles cannot be negative, we discard $x = -37.5$.

So, the number of articles produced is 30.

Now, find the cost of each article:

Cost $= 2x + 15 = 2(30) + 15 = 60 + 15 = 75$.

Step 4: Final Answer:

The number of produced articles is **30**, and the cost of each article is **75**.

Quick Tip

When solving word problems that lead to a quadratic equation, always check the physical viability of your answers. Quantities like length, speed, or number of items cannot be negative, so discard any negative solutions.

4. From a point on a bridge across a river, the angle of depression of the banks on the opposite sides of the river are 30° and 45° respectively. If the bridge is at the height of 5 m from the banks, find the width of the river. (Use $\sqrt{3} = 1.732$)

Solution:

Step 1: Understanding the Concept:

This is a heights and distances problem involving trigonometry. We can model the situation with two right-angled triangles sharing a common height (the height of the bridge). The width of the river will be the sum of the bases of these two triangles.

Step 2: Key Formula or Approach:

Let P be the point on the bridge, and let A and B be the points on the opposite banks. Let C be the point on the water surface directly below P.

Height of the bridge, $PC = 5$ m.

Width of the river, $AB = AC + CB$.

The angles of depression from P to A and B are 30° and 45° . By alternate interior angles, the angles of elevation from the banks are $\angle PAC = 30^\circ$ and $\angle PBC = 45^\circ$.

We will use the trigonometric ratio: $\tan \theta = \frac{\text{Perpendicular}}{\text{Base}}$.

Step 3: Detailed Explanation:

In right-angled $\triangle PCA$:

$$\begin{aligned}\tan 30^\circ &= \frac{PC}{AC} \\ \frac{1}{\sqrt{3}} &= \frac{5}{AC} \\ AC &= 5\sqrt{3} \text{ m}\end{aligned}$$

In right-angled $\triangle PCB$:

$$\begin{aligned}\tan 45^\circ &= \frac{PC}{CB} \\ 1 &= \frac{5}{CB} \\ CB &= 5 \text{ m}\end{aligned}$$

The width of the river is the sum of AC and CB:

$$\begin{aligned}\text{Width} = AB &= AC + CB = 5\sqrt{3} + 5 \\ &= 5(\sqrt{3} + 1)\end{aligned}$$

Now, substitute the given value of $\sqrt{3} = 1.732$:

$$\begin{aligned}\text{Width} &= 5(1.732 + 1) = 5(2.732) \\ &= 13.66 \text{ m}\end{aligned}$$

Step 4: Final Answer:

The width of the river is **13.66 m**.

Quick Tip

Remember that the angle of depression from a point A to a point B is equal to the angle of elevation from point B to point A (alternate interior angles). Drawing a diagram is essential to correctly identify the triangles and angles.

4. (OR) Prove that : $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \csc A + \cot A$

Solution:

Step 1: Understanding the Concept:

This is a complex trigonometric identity that requires a specific algebraic manipulation to prove. The strategy is to convert the expression into terms of $\cot A$ and $\csc A$ and then use the Pythagorean identity involving these functions.

Step 2: Key Formula or Approach:

1. Start with the Left Hand Side (LHS).
2. Divide both the numerator and the denominator by $\sin A$.
3. Use the identity $1 = \csc^2 A - \cot^2 A$.
4. Factorize and simplify.

Step 3: Detailed Explanation:

Starting with the LHS:

$$\text{LHS} = \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

Divide the numerator and the denominator by $\sin A$:

$$\text{LHS} = \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$$

$$\text{LHS} = \frac{\cot A - 1 + \csc A}{\cot A + 1 - \csc A}$$

Rearrange the terms:

$$\text{LHS} = \frac{(\cot A + \csc A) - 1}{(\cot A - \csc A) + 1}$$

Now, substitute $1 = \csc^2 A - \cot^2 A = (\csc A - \cot A)(\csc A + \cot A)$ in the numerator:

$$\text{LHS} = \frac{(\cot A + \csc A) - (\csc A - \cot A)(\csc A + \cot A)}{1 - \csc A + \cot A}$$

Factor out the common term $(\csc A + \cot A)$ from the numerator:

$$\text{LHS} = \frac{(\csc A + \cot A)[1 - (\csc A - \cot A)]}{1 - \csc A + \cot A}$$

$$\text{LHS} = \frac{(\csc A + \cot A)(1 - \csc A + \cot A)}{1 - \csc A + \cot A}$$

Cancel the common factor $(1 - \csc A + \cot A)$ from the numerator and denominator:

$$\text{LHS} = \csc A + \cot A$$

This is equal to the Right Hand Side (RHS).

Hence, proved.

Step 4: Final Answer:

The identity is proven by dividing the LHS by $\sin A$, rearranging terms, and using the Pythagorean identity $\csc^2 A - \cot^2 A = 1$.

Quick Tip

This is a standard identity proof. The key trick is to divide by either $\sin A$ or $\cos A$ to get the terms required in the RHS. Then, strategically replace '1' with a suitable Pythagorean identity to enable factorization.

5. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have ? Find the surface area of the solid.

Solution:

Step 1: Understanding the Concept:

The solid is a combination of a cube and a hemisphere. The total surface area of the solid consists of the area of the cube's faces minus the area covered by the hemisphere's base, plus the curved surface area of the hemisphere.

Step 2: Key Formula or Approach:

Greatest Diameter: The hemisphere sits on the top face of the cube. The largest circle that can be inscribed on a square face has a diameter equal to the side of the square.

Surface Area of Solid:

$\text{TSA} = (\text{TSA of cube}) - (\text{Area of base of hemisphere}) + (\text{CSA of hemisphere})$

$$\text{TSA} = 6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$$

Step 3: Detailed Explanation:

Part 1: Greatest Diameter

The side of the cubical block, $a = 7$ cm.

The greatest diameter the hemisphere can have is equal to the side of the cube.

Greatest diameter = 7 cm.

Part 2: Surface Area of the Solid

Side of cube (a) = 7 cm.

Radius of hemisphere (r) = $\frac{\text{Diameter}}{2} = \frac{7}{2} = 3.5$ cm.

Using the formula $\text{TSA} = 6a^2 + \pi r^2$:

$$\begin{aligned}\text{TSA} &= 6(7)^2 + \frac{22}{7}(3.5)^2 \\ \text{TSA} &= 6(49) + \frac{22}{7} \times 3.5 \times 3.5 \\ \text{TSA} &= 294 + 22 \times 0.5 \times 3.5 \\ \text{TSA} &= 294 + 11 \times 3.5 \\ \text{TSA} &= 294 + 38.5 \\ \text{TSA} &= 332.5 \text{ cm}^2\end{aligned}$$

Step 4: Final Answer:

The greatest diameter the hemisphere can have is **7 cm**.

The surface area of the solid is **332.5 cm²**.

Quick Tip

When a solid is surmounted on another, the area of the interface is subtracted from the total surface area of the base solid, and the curved surface area of the surmounting solid is added.

5. (OR) A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 3 cm and the slant height of the cone is $3\sqrt{2}$ cm. Find the volume of the solid. (Use $\pi = 3.14$)

Solution:

Step 1: Understanding the Concept:

The solid is a combination of two basic shapes: a cone and a hemisphere. The total volume is the sum of their individual volumes.

Step 2: Key Formula or Approach:

Volume of Solid = Volume of Cone + Volume of Hemisphere

- Volume of Cone = $\frac{1}{3}\pi r^2 h$

- Volume of Hemisphere = $\frac{2}{3}\pi r^3$

We will first need to find the height (h) of the cone using the relationship $l^2 = r^2 + h^2$.

Step 3: Detailed Explanation:

Given values:

Radius of cone and hemisphere (r) = 3 cm.

Slant height of cone (l) = $3\sqrt{2}$ cm.

1. Find the height of the cone (h):

Using Pythagoras' theorem for the cone:

$$(3\sqrt{2})^2 = (3)^2 + h^2$$

$$18 = 9 + h^2$$

$$h^2 = 18 - 9 = 9$$

$$h = 3 \text{ cm}$$

2. Calculate the volumes:

Volume of Cone:

$$V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(3) = 9\pi \text{ cm}^3$$

Volume of Hemisphere:

$$V_{\text{hemi}} = \frac{2}{3}\pi r^3 = \frac{2}{3}\pi(3)^3 = \frac{2}{3}\pi(27) = 18\pi \text{ cm}^3$$

3. Calculate the total volume:

$$V_{\text{solid}} = V_{\text{cone}} + V_{\text{hemi}} = 9\pi + 18\pi = 27\pi \text{ cm}^3$$

Now, substitute $\pi = 3.14$:

$$V_{\text{solid}} = 27 \times 3.14 = 84.78 \text{ cm}^3$$

Step 4: Final Answer:

The volume of the solid is **84.78 cm³**.

Quick Tip

For combined solids, first list all the known and unknown dimensions for each component shape. Find any missing dimensions (like the height of the cone in this case) before proceeding to calculate the volume or surface area.