UP Board Class 10 Mathematics - 822 (BZ) - 2025 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**70	**Total questions :**35

General Instructions

Instruction:

i) *All* questions are compulsory. Marks allotted to each question are given in the margin.

ii) In numerical questions, give all the steps of calculation.

iii) Give relevant answers to the questions.

iv) Give chemical equations, wherever necessary.

Q1. HCF of the numbers 96 and 404 is 4. Value of their LCM will be:

(A) 1616

(B) 2424

(C) 3636

(D) 9696

Correct Answer: (D) 9696

Solution:

Step 1: Recall the relationship between HCF and LCM.

We know that:

$$\text{HCF} \times \text{LCM} = \text{Product of the two numbers}$$

Step 2: Substitute the values.

Here, numbers = 96 and 404.

$$\text{Product} = 96 \times 404$$

$$\text{HCF} = 4$$

Step 3: Find the product.

$$96 \times 404 = 38784$$

Step 4: Find the LCM.

$$\text{LCM} = \frac{\text{Product}}{\text{HCF}} = \frac{38784}{4} = 9696$$

Final Answer:

$$\boxed{9696}$$

Quick Tip

Always use the relation $\text{HCF} \times \text{LCM} = \text{Product of the numbers}$ to quickly solve LCM/HCF questions.

Q2. The distance of the point $(2, 5)$ from the origin will be:

(A) $\sqrt{21}$ units

(B) $\sqrt{23}$ units

(C) $\sqrt{29}$ units

(D) $\sqrt{31}$ units

Correct Answer: (C) $\sqrt{29}$

Solution:

Step 1: Recall the distance formula.

Distance from origin $(0, 0)$ to a point (x, y):

$$d = \sqrt{x^2 + y^2}$$

Step 2: Substitute the values.

For point $(2, 5)$:

$$d = \sqrt{2^2 + 5^2} = \sqrt{4 + 25}$$

Step 3: Simplify.

$$d = \sqrt{29}$$

Final Answer:

$$\boxed{\sqrt{29}}$$

Quick Tip

Always apply the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. For distance from origin, it reduces to $\sqrt{x^2 + y^2}$.

Q3. The value of $\sqrt{2} + \tan 45^\circ$ will be:

(A) $\sqrt{2} - 1$

(B) $\sqrt{2}$

(C) $\sqrt{2} + 1$

(D) $\sqrt{2} + 2$

Correct Answer: (C) $\sqrt{2} + 1$

Solution:

Step 1: Recall the trigonometric value.

$$\tan 45^\circ = 1$$

Step 2: Substitute the value.

$$\sqrt{2} + \tan 45^\circ = \sqrt{2} + 1$$

Step 3: Simplify.

This matches option (C).

Final Answer:

$$\boxed{\sqrt{2} + 1}$$

Quick Tip

Remember the standard trigonometric ratio: $\tan 45^\circ = 1$. Substitution makes these problems straightforward.

Q4. The value of $\sec^2 A - \tan^2 A$ will be:

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (A) 1

Solution:

Step 1: Recall the trigonometric identity.

$$\sec^2 A = 1 + \tan^2 A$$

Step 2: Substitute into the given expression.

$$\sec^2 A - \tan^2 A = (1 + \tan^2 A) - \tan^2 A$$

Step 3: Simplify.

$$\sec^2 A - \tan^2 A = 1$$

Final Answer:

$$\boxed{1}$$

Quick Tip

Always remember the Pythagorean identity: $\sec^2 A = 1 + \tan^2 A$. It directly solves such problems.

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Q5. In the equation $2x + 3y = 11$, if $x = 1$, the value of y will be:

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

Solution:

Step 1: Substitute $x = 1$ into the equation.

$$2(1) + 3y = 11$$

Step 2: Simplify.

$$2 + 3y = 11$$

$$3y = 11 - 2 = 9$$

Step 3: Solve for y.

$$y = \frac{9}{3} = 3$$

Final Answer:

$$\boxed{3}$$

Quick Tip

Always substitute carefully and simplify step by step in linear equations.

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Q6. The value of a term of an A.P. $21, 18, 15, \ldots$ is -81. Then the term will be:

(A) 31st

(B) 33rd

(C) 35th

(D) 37th

Correct Answer: (B) 33rd

Solution:

Step 1: Recall the formula for the n-th term of an A.P.

$$a_n = a + (n-1)d$$

where a is the first term, d is the common difference, and a_n is the n-th term.

Step 2: Identify the values.

Here, $a = 21$, $d = 18 - 21 = -3$, and $a_n = -81$.

Step 3: Substitute into the formula.

$$-81 = 21 + (n-1)(-3)$$

Step 4: Simplify.

$$-81 = 21 - 3n + 3$$

$$-81 = 24 - 3n$$

$$-81 - 24 = -3n$$

$$-105 = -3n$$

$$n = \frac{105}{3} = 35$$

Oops — correction: let's carefully recompute.

Recheck Step 4:

$$-81 = 21 + (n-1)(-3)$$

$$-81 = 21 - 3n + 3$$

$$-81 = 24 - 3n$$

$$-81 - 24 = -3n$$

$$-105 = -3n$$

$$n = 35$$

So the correct term is the 35th, not 33rd.

Final Answer:

$$\boxed{35^{th}}$$

Quick Tip

Always double-check the arithmetic in A.P. problems. Using the $a_n = a + (n-1)d$ formula systematically avoids errors.

Q7. There are 2 blue, 3 white, and 4 red balls in a bag. One ball is taken out randomly from this bag. The probability that the ball is not red will be:

(A) $\frac{5}{9}$
(B) $\frac{4}{9}$
(C) $\frac{1}{3}$
(D) $\frac{2}{9}$

Correct Answer: (A) $\frac{5}{9}$

Solution:

Step 1: Count the total balls.

Total balls = $2 + 3 + 4 = 9$.

Step 2: Count the favorable outcomes.

Balls that are not red = Blue + White = $2 + 3 = 5$.

Step 3: Apply probability formula.

$$P(\text{Not Red}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{5}{9}$$

Final Answer:

$$\boxed{\frac{5}{9}}$$

Quick Tip

When asked for "not" probability, simply subtract the unfavorable cases from the total, or use complement rule: $P(\text{Not A}) = 1 - P(A)$.

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Q8. The probability that an event will happen surely is:

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Recall probability basics.

The probability of any event lies between 0 and 1.

Step 2: Certain event.

If an event is certain (sure to happen), its probability is the maximum possible value.

Step 3: Conclusion.

Therefore, the probability of a sure event is:

$$P(\text{Sure Event}) = 1$$

Final Answer:

$$\boxed{1}$$

Quick Tip

Remember: Impossible event probability = 0, Sure event probability = 1.

Q9. In the given figure, if $DE \parallel BC$, then the value of $\frac{DE}{BC}$ will be:

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

Correct Answer: (B) $\frac{1}{2}$

Solution:

Step 1: Recall Basic Proportionality Theorem (Thales' theorem).

If a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally.

So,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Step 2: Substitute the values from the figure.

Here, $AD = 3$, $DB = 6$, $AE = 2$, and $EC = 4$.

$$\frac{AD}{DB} = \frac{3}{6} = \frac{1}{2}, \quad \frac{AE}{EC} = \frac{2}{4} = \frac{1}{2}$$

So, triangles ADE and ABC are similar.

Step 3: Ratio of corresponding sides.

Since $\triangle ADE \sim \triangle ABC$,

$$\frac{DE}{BC} = \frac{AD}{AB} = \frac{3}{3+6} = \frac{3}{9} = \frac{1}{3}$$

Wait — let's recheck carefully.

Step 3 (Revised):

For similar triangles:

$$\frac{DE}{BC} = \frac{AD}{AB}$$

Now, $AD = 3$, $AB = AD + DB = 3 + 6 = 9$.

$$\frac{DE}{BC} = \frac{3}{9} = \frac{1}{3}$$

Final Answer:

$$\boxed{\frac{1}{3}}$$

Quick Tip

In problems with parallel lines inside triangles, always apply the Basic Proportionality Theorem: $\triangle ADE \sim \triangle ABC$. Use side ratios carefully to avoid mistakes.

Q10. From a point Q, the length of tangent to a circle is $24\,\text{cm}$ and the distance of Q from the centre is $25\,\text{cm}$. The radius of the circle is:

(A) 7 cm

(B) 12 cm

(C) 15 cm

(D) 24.5 cm

Correct Answer: (C) 15 cm

Solution:

Step 1: Apply Pythagoras theorem.

In a right-angled triangle,

$$OQ^2 = OP^2 + PQ^2$$

where O = centre, P = point of tangency, Q = external point.

Step 2: Substitute values.

$$25^2 = r^2 + 24^2$$

Step 3: Simplify.

$$625 = r^2 + 576$$

$$r^2 = 625 - 576 = 49$$

$$r = 7\,\text{cm}$$

Oops — correction: I mistakenly selected option (C). The correct radius is **7 cm**.

Final Answer:

$$\boxed{7\,\text{cm}}$$

Quick Tip

Use the Pythagoras theorem: $(\text{distance from centre})^2 = (\text{radius})^2 + (\text{tangent length})^2$.

—

Q11. The angle of a sector of a circle of radius $6\,\text{cm}$ is 30°. The measure of corresponding arc will be:

(A) $\frac{\pi}{2}$ cm

(B) π cm

(C) $\frac{3\pi}{2}$ cm

(D) 2π cm

Correct Answer: (B) π cm

Solution:

Step 1: Recall arc length formula.

$$l = \frac{\theta}{360^\circ} \times 2\pi r$$

Step 2: Substitute values.

$$l = \frac{30}{360} \times 2\pi \times 6$$

Step 3: Simplify.

$$l = \frac{1}{12} \times 12\pi = \pi$$

Final Answer:

$$\boxed{\pi\,\text{cm}}$$

Quick Tip

Arc length is always proportional to the central angle: $l = \frac{\theta}{360^\circ} \times$ circumference.

—

Q12. The height of a solid cylinder of radius $3\,\text{cm}$ is $5\,\text{cm}$. A hemisphere of the same radius is placed on its vertex. The total surface area of this solid will be:

(A) $33\pi\,\text{cm}^2$

(B) $53\pi\,\text{cm}^2$

(C) $55\pi\,\text{cm}^2$

(D) $57\pi\,\text{cm}^2$

Correct Answer: (B) $53\pi\,\text{cm}^2$

Solution:

Step 1: Surface area formula.

Total Surface Area = CSA of cylinder + CSA of hemisphere + base area of cylinder

Step 2: Cylinder CSA.

$$\text{CSA of cylinder} = 2\pi rh = 2\pi(3)(5) = 30\pi$$

Step 3: Hemisphere CSA.

$$\text{CSA of hemisphere} = 2\pi r^2 = 2\pi(3^2) = 18\pi$$

Step 4: Base area of cylinder.

$$\pi r^2 = \pi(3^2) = 9\pi$$

Step 5: Total.

$$30\pi + 18\pi + 9\pi = 57\pi$$

So the correct answer is (D) $57\pi\,\text{cm}^2$, not (B).

Final Answer:

$$\boxed{57\pi\,\text{cm}^2}$$

Quick Tip

Be careful: when a hemisphere is placed on a cylinder, we exclude the common base but include the cylinder base.

—

Q13. The contact point of a tangent to the circle is joined to the centre. The angle at the contact point between tangent and radius will be:

(A) 90°

(B) 60°

(C) 45°

(D) 30°

Correct Answer: (A) 90°

Solution:

Step 1: Recall tangent property.

At the point of contact, the tangent is always perpendicular to the radius.

Step 2: Conclusion.

$$\angle(\text{radius, tangent}) = 90^\circ$$

Final Answer:

$$\boxed{90^\circ}$$

Quick Tip

Always remember: A tangent to a circle makes a right angle with the radius at the point of contact.

Q14. The value of $\dfrac{1+\tan^2 A}{1+\cot^2 A}$ will be:

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) $\tan^2 A$

Correct Answer: (D) $\tan^2 A$

Solution:

Step 1: Recall identities.

$$1 + \tan^2 A = \sec^2 A, \quad 1 + \cot^2 A = \csc^2 A$$

Step 2: Rewrite expression.

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\csc^2 A}$$

Step 3: Simplify.

$$\frac{\sec^2 A}{\csc^2 A} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Final Answer:

$$\boxed{\tan^2 A}$$

Quick Tip

Use Pythagorean identities $\sec^2 A = 1 + \tan^2 A$ and $\csc^2 A = 1 + \cot^2 A$ to simplify ratios easily.

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Q15. An arc of a circle of radius $7\,\text{cm}$ subtends an angle of 60° at the centre. The area of the sector will be:

(A) $\frac{77}{4}\,\text{cm}^2$

(B) $\frac{77}{3}\,\text{cm}^2$

(C) $\frac{77}{2}\,\text{cm}^2$

(D) $77\,\text{cm}^2$

Correct Answer: (A) $\frac{77}{4}\,\text{cm}^2$

Solution:

Step 1: Recall area of sector formula.

$$\text{Area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

Step 2: Substitute values.

$$= \frac{60}{360} \times \pi \times (7^2)$$
$$= \frac{1}{6} \times \pi \times 49$$

Step 3: Simplify.

$$= \frac{49\pi}{6}$$

Taking $\pi = \frac{22}{7}$:

$$= \frac{49 \times 22}{6 \times 7} = \frac{154}{6} = \frac{77}{3}\,\text{cm}^2$$

Correction — the correct answer is (B) $\frac{77}{3}$.

Final Answer:

$$\boxed{\frac{77}{3}\,\text{cm}^2}$$

Quick Tip

Always check if π needs to be taken as $\frac{22}{7}$. For exact forms, leave answers in terms of π.

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Q16. The surface area of a sphere is $452\frac{4}{7}\,\text{cm}^2$. Its radius will be:

(A) 6 cm

(B) 9 cm

(C) 12 cm

(D) 15 cm

Correct Answer: (B) 9 cm

Solution:

Step 1: Recall surface area formula.

$$\text{Surface Area} = 4\pi r^2$$

Step 2: Convert mixed fraction.

$$452\tfrac{4}{7} = \frac{3168}{7}\,\text{cm}^2$$

Step 3: Solve for r.

$$4\pi r^2 = \frac{3168}{7}$$

Take $\pi = \frac{22}{7}$:

$$4 \times \frac{22}{7} \times r^2 = \frac{3168}{7}$$

$$\frac{88}{7} r^2 = \frac{3168}{7}$$

$$88r^2 = 3168$$

$$r^2 = 36$$

$$r = 6\,\text{cm}$$

Correction — the correct answer is (A) 6 cm, not 9 cm.

Final Answer:

$$\boxed{6\,\text{cm}}$$

Quick Tip

Always first convert mixed fractions to improper fractions before substituting.

Q17. The product of zeroes of the quadratic polynomial $p(x) = 4x^2 - 4x - 1$ will be:

(A) -1

(B) $-\frac{1}{4}$

(C) $\frac{1}{4}$

(D) 1

Correct Answer: (B) $-\frac{1}{4}$

Solution:

Step 1: Recall the product of roots formula.

For a quadratic $ax^2 + bx + c$, product of zeroes = $\dfrac{c}{a}$.

Step 2: Identify coefficients.

Here, $a = 4$, $b = -4$, $c = -1$.

Step 3: Apply the formula.

$$\alpha\beta = \frac{c}{a} = \frac{-1}{4} = -\frac{1}{4}$$

Final Answer:

$$\boxed{-\frac{1}{4}}$$

Quick Tip

Always remember: Sum of roots = $-b/a$, Product of roots = c/a.

—

Q18. The discriminant of the quadratic equation $3x^2 - 4x + \frac{4}{3} = 0$ will be:

(A) 0

(B) 1

(C) 2

(D) 4

Correct Answer: (A) 0

Solution:

Step 1: Recall discriminant formula.

$$D = b^2 - 4ac$$

Step 2: Identify coefficients.

Here, $a = 3$, $b = -4$, $c = \frac{4}{3}$.

Step 3: Substitute values.

$$D = (-4)^2 - 4(3)\left(\frac{4}{3}\right)$$

$$D = 16 - 16 = 0$$

Final Answer:

$$\boxed{0}$$

Quick Tip

If $D = 0$, the quadratic has equal real roots.

Q19. The mean from the following table will be:

Class-interval	Frequency
$0 - 4$	4
$4 - 8$	6
$8 - 12$	6
$12 - 16$	5
$16 - 20$	4

(A) 8.32

(B) 8.76

(C) 9.84

(D) 10.36

Correct Answer: (D) 10.36

Solution:

Step 1: Find midpoints of each class.

$$\text{Midpoints} = 2,\ 6,\ 10,\ 14,\ 18$$

Step 2: Multiply frequency and midpoint.

$$f \times x = 4 \times 2 + 6 \times 6 + 6 \times 10 + 5 \times 14 + 4 \times 18$$

$$= 8 + 36 + 60 + 70 + 72 = 246$$

Step 3: Sum of frequencies.

$$\Sigma f = 4 + 6 + 6 + 5 + 4 = 25$$

Step 4: Calculate mean.

$$\bar{x} = \frac{\Sigma f x}{\Sigma f} = \frac{246}{25} = 9.84$$

Correction: The correct answer is (C) 9.84, not (D).

Final Answer:

$$\boxed{9.84}$$

Quick Tip

For grouped data, mean is always $\frac{\Sigma f x}{\Sigma f}$, where x is the class midpoint.

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Q20. The median class of the following table will be:

Class-interval	Frequency
$0 - 10$	6
$10 - 20$	8
$20 - 30$	10
$30 - 40$	9
$40 - 50$	12

(A) 10-20

(B) 20-30

(C) 30-40

(D) 40-50

Correct Answer: (B) 20-30

Solution:

Step 1: Total frequency.

$$N = 6 + 8 + 10 + 9 + 12 = 45$$

Step 2: Find median class.

Median class is the class containing $\frac{N}{2} = \frac{45}{2} = 22.5$.

Step 3: Cumulative frequencies.

$$\text{Cumulative frequencies} = 6,\ 14,\ 24,\ 33,\ 45$$

The 22.5^{th} value lies in the class $20 - 30$.

Final Answer:

$$\boxed{20 - 30}$$

Quick Tip

The median class is found by locating the $\frac{N}{2}$-th item using cumulative frequencies.

Part B

Q1. Do all the parts:

— **(a)** If a point $(0, 1)$ is equidistant from the points $(5, -3)$ and $(x, 6)$, then find the values of x.

Solution:

Using distance formula:

$$\sqrt{(0-5)^2 + (1+3)^2} = \sqrt{(0-x)^2 + (1-6)^2}$$

$$\sqrt{25+16} = \sqrt{x^2+25}$$

$$\sqrt{41} = \sqrt{x^2+25}$$

$$x^2 = 16 \quad \Rightarrow \quad x = \pm 4$$

Final Answer:

$$\boxed{x = 4 \text{ or } x = -4}$$

Quick Tip

Equidistant condition always gives equality of distances from the given point. Apply distance formula carefully.

— **(b)** Find the ratio in which the line segment joining the points $(-3, 10)$ and $(6, -8)$ is divided by the point $(-1, 6)$.

Solution:

Using section formula:

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

$$-1 = \frac{6m - 3n}{m+n}, \quad 6 = \frac{-8m + 10n}{m+n}$$

From first equation:

$$-1(m+n) = 6m - 3n \quad \Rightarrow \quad -m - n = 6m - 3n \quad \Rightarrow \quad -7m + 2n = 0$$

$$\frac{m}{n} = \frac{2}{7}$$

Final Answer:

$$\boxed{2:7}$$

Quick Tip

When finding ratios, always apply section formula separately for both x and y. They should give the same ratio.

— **(c)** Prove: $\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$.

Solution:

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{(1-\tan A)\tan A}{\tan A - 1}\right)^2$$

Since $(1-\tan A) = -(\tan A - 1)$,

$$= \left(\frac{-\tan A(\tan A - 1)}{\tan A - 1}\right)^2 = (-\tan A)^2 = \tan^2 A$$

Final Answer:

$$\boxed{\tan^2 A}$$

Quick Tip

Express $\cot A$ in terms of $\tan A$ and then simplify step by step.

— **(d)** Find the median from the following frequency distribution:

Class-interval	Frequency
$0 - 10$	7
$10 - 20$	9
$20 - 30$	19
$30 - 40$	15
$40 - 50$	7

Solution:

Step 1: Cumulative frequency.

CF = 7, 16, 35, 50, 57.

Step 2: Median class.

$N = 57 \quad \Rightarrow \quad \frac{N}{2} = 28.5$. Median class = 20-30.

Step 3: Apply formula.

$$\text{Median} = L + \left(\frac{\frac{N}{2} - CF}{f}\right) \times h$$

Here, $L = 20,\ CF = 16,\ f = 19,\ h = 10.$

$$\text{Median} = 20 + \frac{28.5 - 16}{19} \times 10 = 20 + 6.58 = 26.58$$

Final Answer:

$$\boxed{26.58}$$

Quick Tip

Use cumulative frequencies to locate the median class, then apply the formula systematically.

— **(e)** Prove that $3\sqrt{2}$ is an irrational number.

Solution:

We know $\sqrt{2}$ is irrational. A rational number multiplied by an irrational number gives an irrational number.

Since 3 is rational,

$$3\sqrt{2} \text{ is irrational}.$$

Final Answer:

$$\boxed{3\sqrt{2} \text{ is irrational}}$$

Quick Tip

Product of rational and irrational number (non-zero rational) is always irrational.

— **(f)** Find the LCM of the numbers 120 and 315.

Solution:

Prime factorization:

$$120 = 2^3 \times 3 \times 5$$

$$315 = 3^2 \times 5 \times 7$$

Taking highest powers:

$$\text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520$$

Final Answer:

$$\boxed{2520}$$

Quick Tip

For LCM, take the highest power of each prime appearing in the factorization.

Q2. Do any five parts:

— **(a)** Find the mode from the following table:

Class-interval	Frequency
$0 - 10$	6
$10 - 20$	11
$20 - 30$	18
$30 - 40$	21
$40 - 50$	15

Solution:

The modal class is the class with highest frequency = $30 - 40$.

Formula:

$$\text{Mode} = L + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Here, $L = 30, f_1 = 21, f_0 = 18, f_2 = 15, h = 10.$

$$\text{Mode} = 30 + \left(\frac{21 - 18}{2(21) - 18 - 15}\right) \times 10 = 30 + \frac{3}{9} \times 10 = 30 + 3.33 = 33.33$$

Final Answer:

$$\boxed{33.33}$$

Quick Tip

The mode lies in the class with highest frequency. Use the formula with f_0, f_1, f_2.

— **(b)** Prove that the lengths of tangents drawn from an external point to a circle are equal.

Solution:

Let P be an external point, PT and PS be tangents touching the circle at T and S. Join O (centre) to T, S, P.

In right triangles $\triangle OPT$ and $\triangle OPS$:

$$OP = OP, \quad OT = OS, \quad \angle OTP = \angle OSP = 90^\circ$$

By RHS congruence, $\triangle OPT \cong \triangle OPS$.

$$\Rightarrow PT = PS$$

Final Answer:

$$\boxed{PT = PS}$$

Quick Tip

Tangents from an external point to a circle are always equal in length.

— **(c)** Prove that triangles with sides $3, 6, 8$ and $4.5, 9, 12$ are similar.

Solution:

Check ratios:

$$\frac{3}{4.5} = \frac{6}{9} = \frac{8}{12} = \frac{2}{3}$$

Since all corresponding sides are in same ratio, triangles are similar by SSS criterion.

Final Answer:

$$\boxed{\text{Triangles are similar}}$$

Quick Tip

To prove similarity, show all sides are in proportion (SSS similarity criterion).

— **(d)** By adding 1 to numerator and denominator of a fraction, it becomes $\frac{4}{5}$. If 5 is subtracted from numerator and denominator, fraction becomes $\frac{1}{2}$. Find the fraction.

Solution:

Let fraction = $\frac{x}{y}$.

Condition 1:

$$\frac{x+1}{y+1} = \frac{4}{5} \quad \Rightarrow \quad 5(x+1) = 4(y+1) \quad \Rightarrow \quad 5x - 4y = -1$$

Condition 2:

$$\frac{x-5}{y-5} = \frac{1}{2} \quad \Rightarrow \quad 2(x-5) = y-5 \quad \Rightarrow \quad 2x - y = 5$$

Solve:

$$5x - 4y = -1, \quad 2x - y = 5 \;\Rightarrow\; y = 2x - 5$$

Substitute:

$$5x - 4(2x-5) = -1 \quad \Rightarrow \quad 5x - 8x + 20 = -1$$

$$-3x = -21 \quad \Rightarrow \quad x = 7,\; y = 9$$

Fraction = $\frac{7}{9}$.

Final Answer:

$$\boxed{\frac{7}{9}}$$

Quick Tip

Always form equations from conditions and solve using substitution or elimination.

— **(e)** Solve the equation: $9x^2 - 3x - 2 = 0$.

Solution:

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here, $a = 9, b = -3, c = -2$.

$$D = (-3)^2 - 4(9)(-2) = 9 + 72 = 81$$

$$x = \frac{-(-3) \pm \sqrt{81}}{18} = \frac{3 \pm 9}{18}$$

$$x = \frac{12}{18} = \tfrac{2}{3}, \quad x = \frac{-6}{18} = -\tfrac{1}{3}$$

Final Answer:

$$\boxed{x = \tfrac{2}{3}, \ -\tfrac{1}{3}}$$

Quick Tip

Check discriminant first. If perfect square, roots are rational.

— **(f)** Find the A.P. whose third term is 16 and seventh term is 12 more than fifth term.

Solution:

General term: $a_n = a + (n-1)d$.

Third term:

$$a + 2d = 16 \quad ...(1)$$

Seventh term = fifth term + 12:

$$a + 6d = (a + 4d) + 12 \quad \Rightarrow \quad 2d = 12 \quad \Rightarrow d = 6$$

From (1):

$$a + 2(6) = 16 \quad \Rightarrow \quad a = 4$$

Thus A.P. = $4, 10, 16, 22, \ldots$.

Final Answer:

$$\boxed{4, \ 10, \ 16, \ 22, \ldots}$$

Quick Tip

Always use given term conditions to form equations in a and d.

Q3. Solve the following pair of equations:

$$5x + y = 3, \quad 6x - 5y = \frac{1}{2}$$

Solution:

Step 1: Express y from first equation.

$$y = 3 - 5x$$

Step 2: Substitute into second equation.

$$6x - 5(3 - 5x) = \frac{1}{2}$$

$$6x - 15 + 25x = \frac{1}{2}$$

$$31x - 15 = \frac{1}{2}$$

Step 3: Simplify.

$$31x = \frac{1}{2} + 15 = \frac{1}{2} + \frac{30}{2} = \frac{31}{2}$$

$$x = \frac{31}{62} = \frac{1}{2}$$

Step 4: Find y.

$$y = 3 - 5\left(\frac{1}{2}\right) = 3 - \frac{5}{2} = \frac{1}{2}$$

Final Answer:

$$\boxed{x = \frac{1}{2},\ y = \frac{1}{2}}$$

Quick Tip

For solving pairs of linear equations, substitution method is effective when one equation can be easily expressed in terms of a variable.

—

OR

The area of a rectangle gets reduced by $28\,m^2$, if its length is increased by 2 m and breadth is reduced by 2 m. If the length is reduced by 1 m and breadth is increased by 2 m, the area is increased by $33\,m^2$. Find the area of the rectangle.

Solution:

Step 1: Let length = l, breadth = b.

Area = lb.

Condition 1:

$$(l+2)(b-2) = lb - 28$$

$$lb - 2l + 2b - 4 = lb - 28$$

$$-2l + 2b = -24 \quad \Rightarrow \quad l - b = 12 \quad ...(1)$$

Condition 2:

$$(l-1)(b+2) = lb + 33$$

$$lb + 2l - b - 2 = lb + 33$$

$$2l - b = 35 \quad ...(2)$$

Step 2: Solve equations.

From (1): $l = b + 12$.

Substitute in (2):

$$2(b+12) - b = 35$$

$$2b + 24 - b = 35$$

$$b + 24 = 35 \quad \Rightarrow \quad b = 11$$

$$l = b + 12 = 23$$

Step 3: Find area.

$$\text{Area} = l \times b = 23 \times 11 = 253$$

Final Answer:

$$\boxed{253\,m^2}$$

Quick Tip

Translate word problems into equations systematically. Rectangle problems often reduce to simultaneous equations.

Q4. The angle of elevation of the top of a 20 m high building from a point O on the horizontal plane is $30°$. There is a flagstaff on the top of the building. The angle of elevation of the top of flagstaff is $45°$. Find the height of the flagstaff and measure of the distance of foot of building from the point O.

Solution:

Step 1: Let distance from O to foot of building = x, height of flagstaff = h.

For building of height 20 m,

$$\tan 30° = \frac{20}{x} \quad \Rightarrow \quad x = \frac{20}{\frac{1}{\sqrt{3}}} = 20\sqrt{3}$$

Step 2: Apply to total height (building + flagstaff).

Total height = $20 + h$.

$$\tan 45° = \frac{20+h}{x}$$
$$1 = \frac{20+h}{20\sqrt{3}} \quad \Rightarrow \quad 20 + h = 20\sqrt{3}$$
$$h = 20(\sqrt{3} - 1)$$

Step 3: Approximate value.

$$h = 20(1.732 - 1) = 20(0.732) = 14.64\,m$$

Final Answer:

$\boxed{\text{Height of flagstaff } = 20(\sqrt{3} - 1) \approx 14.64\,m}$, $\boxed{\text{Distance of building from O } = 20\sqrt{3} \approx 34.64\,m}$

Quick Tip

In such problems, always first find the horizontal distance using one triangle, then use it again in the larger triangle with flagstaff.

—

OR

The angles of depression from a point on the bridge of a river to opposite banks are $30°$ and $45°$ respectively. If the height of the bridge from the banks is $4.5\,m$, then find the breadth of the river.

Solution:

Step 1: Let breadth of river = AB.

Let point on bridge = P, banks = A, B, with $PA \perp AB$, height $PA = 4.5$.

Step 2: For angle of depression $30°$.

$$\tan 30° = \frac{PA}{AD} = \frac{4.5}{AD} \quad \Rightarrow \quad AD = \frac{4.5}{1/\sqrt{3}} = 4.5\sqrt{3}$$

Step 3: For angle of depression $45°$.

$$\tan 45° = \frac{PA}{DB} = \frac{4.5}{DB} \quad \Rightarrow \quad DB = 4.5$$

Step 4: Total breadth.

$$AB = AD + DB = 4.5\sqrt{3} + 4.5 = 4.5(\sqrt{3}+1)$$

$$= 4.5(1.732 + 1) = 4.5 \times 2.732 = 12.29\,m$$

Final Answer:

$$\boxed{AB = 4.5(\sqrt{3}+1) \approx 12.29\,m}$$

Quick Tip

Angles of depression are measured from the horizontal, so they form right triangles with the vertical height of the bridge.

Q5. (a) A maximum sphere is made by peeling a wooden cube of side 14 cm. Find the volume of the peeled wood.

Solution:

Step 1: Volume of cube.

$$V_{\text{cube}} = a^3 = 14^3 = 2744\,\text{cm}^3$$

Step 2: Volume of largest sphere inscribed in cube.

Radius of sphere = half of cube's side = $\frac{14}{2} = 7$ cm.

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7^3) = \frac{4}{3}\pi(343) = \frac{1372}{3}\pi$$

Using $\pi = \frac{22}{7}$:

$$V_{\text{sphere}} = \frac{1372}{3} \times \frac{22}{7} = \frac{30184}{21} \approx 1437.33\,\text{cm}^3$$

Step 3: Volume of peeled wood.

$$V_{\text{peeled}} = V_{\text{cube}} - V_{\text{sphere}} = 2744 - 1437.33 = 1306.67\,\text{cm}^3$$

Final Answer:

$$\boxed{1306.67\,\text{cm}^3 \text{ (approx)}}$$

Quick Tip

The largest sphere that can fit inside a cube has diameter equal to the side of the cube.

— OR

(b) In the figure, two concentric circles of radii 14 cm and 7 cm whose centre is O and arcs are $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ and $\angle AOB = 30°$. Find the area of the shaded portion.

Solution:

Step 1: Formula for area of sector.

$$\text{Area of sector} = \frac{\theta}{360^\circ}\pi r^2$$

Step 2: Outer sector (radius 14).

$$\text{Outer area} = \frac{30}{360}\pi(14^2) = \frac{1}{12}\pi(196) = \frac{196}{12}\pi = \frac{49}{3}\pi$$

Step 3: Inner sector (radius 7).

$$\text{Inner area} = \frac{30}{360}\pi(7^2) = \frac{1}{12}\pi(49) = \frac{49}{12}\pi$$

Step 4: Shaded area (difference).

$$\text{Shaded area} = \frac{49}{3}\pi - \frac{49}{12}\pi$$
$$= \left(\frac{196-49}{12}\right)\pi = \frac{147}{12}\pi = \frac{49}{4}\pi$$

Using $\pi = \frac{22}{7}$:

$$\text{Shaded area} = \frac{49}{4} \times \frac{22}{7} = \frac{154}{4} = 38.5\,\text{cm}^2$$

Final Answer:

$$\boxed{38.5\,\text{cm}^2}$$

Quick Tip

For ring-shaped sectors, subtract the area of smaller sector from the larger one.