UP Board Class 10 Mathematics - 822(CB) - 2025 Question Paper with Solutions

Time Allowed :3 Hours	Maximum Marks :70	Total Questions :5

General Instructions

Read the following instructions very carefully and strictly follow them:

1. All questions are compulsory.
2. This question paper has two sections 'A' and 'B'.
3. Section 'A' contains 20 Multiple Choice type Questions of 1 mark each that have to be answered on OMR answer sheet by darkening completely the correct circle with blue or black ballpoint pen.
4. After giving answer on OMR answer sheet, do not cut or use eraser, whitener etc.
5. Section 'B' contains descriptive type questions of 50 marks.
6. Total 5 questions are there in this section.
7. In the beginning of each question, it has been mentioned how many parts of it are to be attempted.
8. Marks allotted to each question are mentioned against it.
9. Start from the first question and go up to the last question. Do not waste your time on the question you cannot solve.
10. If you need place for rough work do it on the left page of your answer book and cross (X) the page. Do not write any solution on that page.
11. Draw neat and correct figure in the solution of a question wherever it is necessary, otherwise in its absence the solution will be treated incomplete and wrong.

Section - A

1. The HCF of two numbers 'a' and 'b' is 5 and their LCM is 200. Then the product of 'a' and 'b' is

(A) 205
(B) 1000
(C) 200
(D) 195

Correct Answer: (B) 1000

Solution:

Step 1: Understanding the Concept:
This question is based on the fundamental relationship between the Highest Common Factor (HCF) and the Least Common Multiple (LCM) of two numbers.

Step 2: Key Formula or Approach:
For any two positive integers 'a' and 'b', the product of the numbers is equal to the product of their HCF and LCM.
The formula is:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

Step 3: Detailed Explanation:
We are given the following values:
HCF of 'a' and 'b' = 5.
LCM of 'a' and 'b' = 200.
We need to find the product of 'a' and 'b'.

Using the formula from Step 2:

$$\text{Product of 'a' and 'b'} = \text{HCF} \times \text{LCM}$$

Substitute the given values into the equation:

$$a \times b = 5 \times 200$$

$$a \times b = 1000$$

Step 4: Final Answer:
Therefore, the product of the two numbers 'a' and 'b' is 1000.

Quick Tip

This is a direct formula-based question and a very important property in number theory. Always remember the relationship: **Product of two numbers = HCF $\times$ LCM**. This can save you a lot of time in competitive exams.

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2. The number $4\sqrt{3}$ is

(A) Integer
(B) Rational
(C) Irrational
(D) None of these

Correct Answer: (C) Irrational

Solution:

Step 1: Understanding the Concept:
This question tests the understanding of different types of numbers, specifically rational and irrational numbers.

- **Rational Number:** A number that can be expressed as a fraction $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Their decimal expansions are either terminating or recurring.
- **Irrational Number:** A number that cannot be expressed as a simple fraction. Their decimal expansions are non-terminating and non-recurring.

Step 2: Detailed Explanation:
Let's analyze the number $4\sqrt{3}$.
The number is a product of two numbers: 4 and $\sqrt{3}$.

- 4 is a rational number (it can be written as $\frac{4}{1}$).
- $\sqrt{3}$ is an irrational number because 3 is not a perfect square. Its value is approximately 1.7320508... which is non-terminating and non-repeating.

A key property of numbers states that the product of a non-zero rational number and an irrational number is always an irrational number.
Here, we are multiplying the rational number 4 by the irrational number $\sqrt{3}$.
Therefore, the result, $4\sqrt{3}$, is an irrational number.

Step 3: Final Answer:
The number $4\sqrt{3}$ is an irrational number.

Quick Tip

Remember the basic rules for operations involving rational and irrational numbers:

- Rational + Irrational = Irrational
- Rational $\times$ Irrational = Irrational (if the rational number is not zero)

Recognizing that $\sqrt{n}$ is irrational if n is not a perfect square is a key skill.

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3. If LCM of 20, 25 is 100, then the value of HCF will be

(A) 4
(B) 10
(C) 20

(D) 5

Correct Answer: (D) 5

Solution:

Step 1: Understanding the Concept:
Similar to the first question, this problem uses the relationship between the HCF, LCM, and the product of two numbers.

Step 2: Key Formula or Approach:
The formula connecting HCF, LCM, and the two numbers (let's call them 'a' and 'b') is:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b$$

From this, we can derive the formula for HCF:

$$\text{HCF}(a, b) = \frac{a \times b}{\text{LCM}(a, b)}$$

Step 3: Detailed Explanation:
We are given the following information:
First number, a = 20.
Second number, b = 25.
LCM(20, 25) = 100.

We need to find the HCF of 20 and 25.
Using the derived formula:

$$\text{HCF}(20, 25) = \frac{20 \times 25}{100}$$

First, calculate the product of the numbers:

$$20 \times 25 = 500$$

Now, substitute this back into the formula:

$$\text{HCF}(20, 25) = \frac{500}{100}$$

$$\text{HCF}(20, 25) = 5$$

Alternative Method (Prime Factorization):
We can also find the HCF by listing the prime factors of each number.
Prime factors of 20 are $2 \times 2 \times 5 = 2^2 \times 5^1$.
Prime factors of 25 are $5 \times 5 = 5^2$.
The Highest Common Factor (HCF) is the product of the lowest powers of common prime factors. The only common prime factor is 5, and its lowest power is 5^1.
Therefore, HCF(20, 25) = 5.

Step 4: Final Answer:
The value of HCF will be 5.

Quick Tip

While the formula is very efficient, knowing how to find HCF and LCM through prime factorization is a fundamental skill. It can be used to quickly verify your answer or solve problems where the formula isn't directly applicable.

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4. If $\frac{1}{x^2+6} = \frac{1}{10}$ then the value of x will be

(A) ± 1
(B) ± 2
(C) ± 3
(D) ± 4

Correct Answer: (B) ± 2

Solution:

Step 1: Understanding the Concept:
This question requires solving a simple algebraic equation for the variable x. The equation involves a fraction, and the variable is in the denominator.

Step 2: Key Formula or Approach:
The primary approach is to isolate the term containing x by using cross-multiplication or by taking the reciprocal of both sides of the equation.

Step 3: Detailed Explanation:
We are given the equation:

$$\frac{1}{x^2 + 6} = \frac{1}{10}$$

By taking the reciprocal of both sides (or by cross-multiplying), we get:

$$x^2 + 6 = 10$$

Now, we need to isolate x^2. Subtract 6 from both sides of the equation:

$$x^2 = 10 - 6$$

$$x^2 = 4$$

Finally, to find the value of x, we take the square root of both sides. Remember that the square root of a positive number can be both positive and negative.

$$x = \pm\sqrt{4}$$

$$x = \pm 2$$

Step 4: Final Answer:
The values of x are +2 and -2.

Quick Tip

When solving equations of the form $x^2 = k$ (where $k > 0$), always remember to include both the positive and negative square roots. It's a common mistake to only provide the positive root.

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5. If the roots of the equation $2x^2 - 8x + c = 0$ are equal, then the value of c will be

(A) 2
(B) 4
(C) 6
(D) 8

Correct Answer: (D) 8

Solution:

Step 1: Understanding the Concept:
This question deals with the nature of roots of a quadratic equation. For a quadratic equation $ax^2 + bx + c = 0$, the nature of its roots is determined by the discriminant (D).

Step 2: Key Formula or Approach:
The discriminant is given by the formula $D = b^2 - 4ac$.
The condition for the roots of a quadratic equation to be equal (or real and equal) is that the discriminant must be zero.

$$D = b^2 - 4ac = 0$$

Step 3: Detailed Explanation:
The given quadratic equation is $2x^2 - 8x + c = 0$.
Comparing this with the standard form $ax^2 + bx + c = 0$, we get:

$$a = 2, \quad b = -8, \quad c = c$$

For the roots to be equal, the discriminant must be 0:

$$D = b^2 - 4ac = 0$$

Substitute the values of a, b, and c into the formula:

$$(-8)^2 - 4(2)(c) = 0$$

$$64 - 8c = 0$$

Now, solve for c:

$$8c = 64$$
$$c = \frac{64}{8}$$

$$c = 8$$

Step 4: Final Answer:
The value of c for which the roots are equal is 8.

Quick Tip

Remember the conditions for the nature of roots based on the discriminant $D = b^2 - 4ac$:

- If $D > 0$, the roots are real and distinct.
- If $D = 0$, the roots are real and equal.
- If $D < 0$, the roots are not real (they are complex conjugates).

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6. If 2 is a zero of the polynomial $x^2 - 3x + 5a$ then the value of 'a' will be

(A) 2
(B) 5
(C) $\frac{2}{5}$
(D) $\frac{5}{2}$

Correct Answer: (C) $\frac{2}{5}$

Solution:

Step 1: Understanding the Concept:
A "zero" of a polynomial is a value of the variable for which the polynomial evaluates to zero. This is also known as a "root" of the polynomial equation.

Step 2: Key Formula or Approach:
If k is a zero of a polynomial $P(x)$, then $P(k) = 0$. We can use this property by substituting the given zero into the polynomial and setting the expression equal to zero to solve for the unknown variable 'a'.

Step 3: Detailed Explanation:
Let the given polynomial be $P(x) = x^2 - 3x + 5a$.
We are given that 2 is a zero of this polynomial.
This means that when we substitute $x = 2$ into the polynomial, the result will be 0.

$$P(2) = 0$$

Substitute $x = 2$ into the polynomial expression:

$$(2)^2 - 3(2) + 5a = 0$$

Now, evaluate the expression:

$$4 - 6 + 5a = 0$$

$$-2 + 5a = 0$$

Solve for 'a':

$$5a = 2$$
$$a = \frac{2}{5}$$

Step 4: Final Answer:
The value of 'a' is $\frac{2}{5}$.

Quick Tip

This concept is fundamental for polynomials. Whenever you are given a zero or a root of a polynomial or an equation, the first step should always be to substitute that value into the expression and set it equal to zero.

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7. The zeroes of the polynomial $x^2 - 5x + 6$ are

(A) 4, -1
(B) -2, -3
(C) -4, 1
(D) 2, 3

Correct Answer: (D) 2, 3

Solution:

Step 1: Understanding the Concept:
To find the zeroes of a polynomial, we need to find the values of x for which the polynomial equals zero. For a quadratic polynomial, this means solving the corresponding quadratic equation.

Step 2: Key Formula or Approach:
We need to solve the equation $x^2 - 5x + 6 = 0$. This can be done by factoring the quadratic expression (splitting the middle term).
We look for two numbers that multiply to the constant term (6) and add up to the coefficient of the middle term (-5).

Step 3: Detailed Explanation:
The polynomial is $P(x) = x^2 - 5x + 6$. To find the zeroes, we set $P(x) = 0$:

$$x^2 - 5x + 6 = 0$$

We need to find two numbers whose product is 6 and whose sum is -5.
The factors of 6 are (1, 6), (2, 3), (-1, -6), (-2, -3).

The pair that sums to -5 is (-2, -3).
Now, we split the middle term using these two numbers:

$$x^2 - 2x - 3x + 6 = 0$$

Factor by grouping:

$$x(x - 2) - 3(x - 2) = 0$$

Take out the common factor $(x - 2)$:

$$(x - 2)(x - 3) = 0$$

For the product of two factors to be zero, at least one of the factors must be zero.

$$\text{Either } x - 2 = 0 \quad \Rightarrow \quad x = 2$$

$$\text{Or } x - 3 = 0 \quad \Rightarrow \quad x = 3$$

Step 4: Final Answer:
The zeroes of the polynomial are 2 and 3.

Quick Tip

For multiple-choice questions involving finding zeroes, you can also work backwards from the options. Substitute the values from each option into the polynomial. The correct option will have values that make the polynomial equal to zero. For example, for option (D), $P(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$ and $P(3) = 3^2 - 5(3) + 6 = 9 - 15 + 6 = 0$.

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8. In which quadrant will the point (-5, 6) lie?

(A) First quadrant
(B) Second quadrant
(C) Third quadrant
(D) Fourth quadrant

Correct Answer: (B) Second quadrant

Solution:

Step 1: Understanding the Concept:
The Cartesian coordinate system divides a 2D plane into four quadrants. The quadrant in which a point lies is determined by the signs (positive or negative) of its x and y coordinates.

Step 2: Key Formula or Approach:
The sign conventions for the coordinates (x, y) in each quadrant are as follows:

- **First Quadrant (I):** x is positive (+), y is positive (+)

- **Second Quadrant (II):** x is negative (-), y is positive (+)
- **Third Quadrant (III):** x is negative (-), y is negative (-)
- **Fourth Quadrant (IV):** x is positive (+), y is negative (-)

Step 3: Detailed Explanation:
We are given the point (-5, 6).
Let's analyze the signs of its coordinates:

- The x-coordinate is -5, which is negative.
- The y-coordinate is 6, which is positive.

A point with a negative x-coordinate and a positive y-coordinate lies in the Second Quadrant.

Step 4: Final Answer:
The point (-5, 6) lies in the Second Quadrant.

Quick Tip

A simple way to remember the quadrants is to start from the top-right and move counter-clockwise.

- Top-right is Quadrant I (+, +).
- Top-left is Quadrant II (-, +).
- Bottom-left is Quadrant III (-, -).
- Bottom-right is Quadrant IV (+, -).

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9. If the radius of a circle is 'a' and an arc of the circle subtends an angle θ at the centre then the length of the arc will be

(A) $\pi a \times \frac{\theta}{360}$
(B) $2\pi a \times \frac{\theta}{360}$
(C) $\frac{\pi a^2}{2} \times \frac{\theta}{360}$
(D) $2\pi a^2 \times \frac{\theta}{360}$

Correct Answer: (B) $2\pi a \times \frac{\theta}{360}$

Solution:

Step 1: Understanding the Concept:
This question asks for the formula for the length of an arc of a circle. The length of an arc is a fraction of the total circumference of the circle, determined by the angle the arc subtends at

the center.

Step 2: Key Formula or Approach:
1. The formula for the circumference of a circle with radius r is $C = 2\pi r$.
2. A full circle corresponds to a central angle of $360°$.
3. The length of an arc is proportional to its central angle. The length of an arc with central angle θ (in degrees) is given by:

$$\text{Length of Arc} = \left(\frac{\theta}{360°}\right) \times \text{Circumference}$$

Step 3: Detailed Explanation:
We are given:

- Radius of the circle $= a$
- Angle subtended by the arc at the center $= \theta$

First, find the circumference of the circle. Using the formula $C = 2\pi r$ with $r = a$, we get:

$$\text{Circumference} = 2\pi a$$

Now, use the formula for the length of the arc:

$$\text{Length of Arc} = \left(\frac{\theta}{360}\right) \times (2\pi a)$$

Rearranging the terms to match the options, we get:

$$\text{Length of Arc} = 2\pi a \times \frac{\theta}{360}$$

Step 4: Final Answer:
The formula for the length of the arc is $2\pi a \times \frac{\theta}{360}$.

Quick Tip

Do not confuse the formula for the length of an arc with the formula for the area of a sector.

- **Arc Length** is part of the **circumference**: $\frac{\theta}{360} \times 2\pi r$
- **Sector Area** is part of the **area**: $\frac{\theta}{360} \times \pi r^2$

Notice how options (C) and (D) relate to the area formula.

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10. If ΔABC and ΔDEF are two similar triangles, such that $\angle A = 45°$ and $\angle E = 56°$ then $\angle C$ is equal to

(A) $45°$
(B) $56°$

(C) 101°
(D) 79°

Correct Answer: (D) 79°

Solution:

Step 1: Understanding the Concept:
This problem involves two key geometric concepts: the properties of similar triangles and the angle sum property of a triangle.

Step 2: Key Formula or Approach:
1. **Similar Triangles Property:** If two triangles are similar, their corresponding angles are equal. If $\Delta ABC \sim \Delta DEF$, then $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$.
2. **Angle Sum Property of a Triangle:** The sum of the interior angles of any triangle is always 180°. For ΔABC, this means $\angle A + \angle B + \angle C = 180^\circ$.

Step 3: Detailed Explanation:
We are given that ΔABC and ΔDEF are similar triangles ($\Delta ABC \sim \Delta DEF$).
We are also given the measures of two angles:

$$\angle A = 45^\circ$$

$$\angle E = 56^\circ$$

From the property of similar triangles, the corresponding angles are equal. This means:

$$\angle B = \angle E$$

Therefore,

$$\angle B = 56^\circ$$

Now we know two angles in ΔABC: $\angle A = 45^\circ$ and $\angle B = 56^\circ$.
We can find the third angle, $\angle C$, using the angle sum property of a triangle:

$$\angle A + \angle B + \angle C = 180^\circ$$

Substitute the known values:

$$45^\circ + 56^\circ + \angle C = 180^\circ$$

$$101^\circ + \angle C = 180^\circ$$

Solve for $\angle C$:

$$\angle C = 180^\circ - 101^\circ$$

$$\angle C = 79^\circ$$

Step 4: Final Answer:
The measure of $\angle C$ is 79°.

Quick Tip

Pay close attention to the order of the vertices in the similarity statement ($\Delta ABC \sim \Delta DEF$). This order tells you exactly which angles and sides correspond. A common mistake is to assume $\angle A = \angle E$ or $\angle B = \angle D$. The correct correspondence is crucial.

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11. If $\cos A = \frac{5}{13}$ then the value of tan A will be

(A) $\frac{12}{5}$
(B) $\frac{12}{13}$
(C) $\frac{13}{12}$
(D) $\frac{5}{12}$

Correct Answer: (A) $\frac{12}{5}$

Solution:

Step 1: Understanding the Concept:
This question requires the use of trigonometric ratios in a right-angled triangle and the Pythagorean theorem to find the length of an unknown side.

Step 2: Key Formula or Approach:
The trigonometric ratios are defined as:

$$\cos A = \frac{\text{Adjacent}}{\text{Hypotenuse}}, \quad \tan A = \frac{\text{Opposite}}{\text{Adjacent}}$$

The Pythagorean theorem states:

$$(\text{Opposite})^2 + (\text{Adjacent})^2 = (\text{Hypotenuse})^2$$

Step 3: Detailed Explanation:
We are given that $\cos A = \frac{5}{13}$.
From the definition of cosine, we can consider a right-angled triangle where:

- Adjacent side to angle A = 5
- Hypotenuse = 13

Let the opposite side be 'p'. Using the Pythagorean theorem:

$$p^2 + 5^2 = 13^2$$

$$p^2 + 25 = 169$$

$$p^2 = 169 - 25$$

$$p^2 = 144$$

$$p = \sqrt{144} = 12$$

So, the Opposite side is 12.
Now we can find the value of $\tan A$:

$$\tan A = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{12}{5}$$

Step 4: Final Answer:
The value of $\tan A$ is $\frac{12}{5}$.

Quick Tip

Recognizing Pythagorean triples can save a lot of time. (3, 4, 5), (5, 12, 13), (8, 15, 17), and (7, 24, 25) are common triples that appear in exams. When you see two of the numbers (like 5 and 13), you can immediately deduce the third (12) without calculation.

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12. If $\cos\theta = 1$ then the value of $\tan\theta$ will be

(A) -1
(B) 0
(C) 1
(D) $\frac{1}{\sqrt{3}}$

Correct Answer: (B) 0

Solution:

Step 1: Understanding the Concept:
This question requires knowledge of the standard values of trigonometric functions for specific angles. We need to find the angle θ for which $\cos\theta = 1$ and then find the value of $\tan\theta$ for that angle.

Step 2: Detailed Explanation:
We are given the equation:

$$\cos\theta = 1$$

From the standard trigonometric values, we know that the cosine function is equal to 1 when the angle is 0° (or 0 radians).
So, $\theta = 0^\circ$.
Now, we need to find the value of $\tan\theta$, which is $\tan 0^\circ$.
The value of $\tan 0^\circ$ is 0.
Alternatively, we know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$.
If $\cos\theta = 1$, then $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-1^2} = 0$.
Therefore,

$$\tan\theta = \frac{0}{1} = 0$$

Step 3: Final Answer:
The value of $\tan\theta$ is 0.

Quick Tip

Memorizing the values of sine, cosine, and tangent for the standard angles (0°, 30°, 45°, 60°, 90°) is essential for solving trigonometry problems quickly in competitive exams.

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13. The value of $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ will be

(A) $\frac{\sqrt{3}}{2}$
(B) $\frac{1}{2}$
(C) $\sqrt{3}$
(D) $\frac{1}{\sqrt{3}}$

Correct Answer: (A) $\frac{\sqrt{3}}{2}$

Solution:

Step 1: Understanding the Concept:
This question can be solved in two ways: either by substituting the value of $\tan 30^\circ$ and simplifying, or by recognizing the expression as a standard trigonometric identity.

Step 2: Key Formula or Approach:
Method 1: Direct Substitution
The value of $\tan 30^\circ = \frac{1}{\sqrt{3}}$.
Method 2: Using Trigonometric Identity
The identity for $\sin(2\theta)$ in terms of $\tan\theta$ is:

$$\sin(2\theta) = \frac{2\tan\theta}{1+\tan^2\theta}$$

Step 3: Detailed Explanation:
Method 1: Direct Substitution
Substitute $\tan 30^\circ = \frac{1}{\sqrt{3}}$ into the expression:

$$\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{3+1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}}$$

To rationalize the denominator, multiply the numerator and denominator by $\sqrt{3}$:

$$= \frac{3\times\sqrt{3}}{2\sqrt{3}\times\sqrt{3}} = \frac{3\sqrt{3}}{2\times 3} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

Method 2: Using Trigonometric Identity
The given expression is in the form of $\frac{2\tan\theta}{1+\tan^2\theta}$ with $\theta = 30^\circ$.
This is the identity for $\sin(2\theta)$.
Therefore, the expression is equal to $\sin(2 \times 30^\circ) = \sin(60^\circ)$.
The value of $\sin 60^\circ$ is $\frac{\sqrt{3}}{2}$.

Step 4: Final Answer:
The value of the expression is $\frac{\sqrt{3}}{2}$.

Quick Tip

Recognizing trigonometric identities can save significant calculation time. The double angle formulas for $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$ are particularly useful.

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14. If $\tan\alpha = \sin\alpha$, then the value of α will be

(A) 90°
(B) 60°
(C) 45°
(D) 0°

Correct Answer: (D) 0°

Solution:

Step 1: Understanding the Concept:
This question requires solving a trigonometric equation by expressing $\tan\alpha$ in terms of $\sin\alpha$ and $\cos\alpha$.

Step 2: Key Formula or Approach:
The fundamental trigonometric identity relating tangent, sine, and cosine is:

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

Step 3: Detailed Explanation:
We are given the equation $\tan\alpha = \sin\alpha$.
Substitute the identity for $\tan\alpha$:

$$\frac{\sin\alpha}{\cos\alpha} = \sin\alpha$$

Rearrange the equation to one side:

$$\frac{\sin\alpha}{\cos\alpha} - \sin\alpha = 0$$

Factor out $\sin\alpha$:

$$\sin\alpha\left(\frac{1}{\cos\alpha} - 1\right) = 0$$

This equation holds true if either of the factors is zero.
Case 1: $\sin\alpha = 0$
This occurs when $\alpha = 0^\circ$, 180°, etc. From the given options, $\alpha = 0^\circ$ is a solution.
Case 2: $\frac{1}{\cos\alpha} - 1 = 0$
This means $\frac{1}{\cos\alpha} = 1$, which implies $\cos\alpha = 1$.
This also occurs when $\alpha = 0^\circ$, 360°, etc.
Both cases lead to the solution $\alpha = 0^\circ$ among the given options.

Step 4: Final Answer:
The value of α is 0°.

Quick Tip

When solving trigonometric equations, avoid cancelling terms that could be zero. For example, dividing both sides by $\sin\alpha$ in the initial step would lose the solution where $\sin\alpha = 0$. Always factor instead of cancelling.

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15. If a sector of a circle with radius 6 cm subtends an angle of 60° at the centre then the area of the sector will be

(A) 4π cm^2
(B) 6π cm^2
(C) 8π cm^2
(D) 12π cm^2

Correct Answer: (B) 6π cm^2

Solution:

Step 1: Understanding the Concept:
The area of a sector is a fraction of the total area of the circle, determined by the angle of the sector at the center.

Step 2: Key Formula or Approach:
The formula for the area of a sector of a circle is:

$$\text{Area of Sector} = \frac{\theta}{360^\circ} \times \pi r^2$$

where r is the radius of the circle and θ is the central angle in degrees.

Step 3: Detailed Explanation:
We are given the following values:

- Radius, $r = 6$ cm

- Central angle, $\theta = 60^\circ$

Substitute these values into the formula:

$$\text{Area of Sector} = \frac{60^\circ}{360^\circ} \times \pi(6)^2$$

Simplify the fraction:

$$\text{Area of Sector} = \frac{1}{6} \times \pi \times 36$$

Perform the multiplication:

$$\text{Area of Sector} = \frac{36\pi}{6} = 6\pi \text{ cm}^2$$

Step 4: Final Answer:
The area of the sector is 6π cm^2.

Quick Tip

Remember the distinction between the formula for the area of a sector and the length of an arc.

- Area of Sector $= (\theta/360) \times \pi r^2$
- Length of Arc $= (\theta/360) \times 2\pi r$

A quick check of units can help: area should be in cm^2, length in cm.

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16. If two cubes of side a cm are joined end to end then the surface area of the formed solid will be

(A) $6a^2$ cm^2
(B) $8a^2$ cm^2
(C) $10a^2$ cm^2
(D) $12a^2$ cm^2

Correct Answer: (C) $10a^2$ cm^2

Solution:

Step 1: Understanding the Concept:
When two cubes are joined end to end, they form a cuboid. The question asks for the total surface area of this new cuboid.

Step 2: Key Formula or Approach:
First, determine the dimensions (length, breadth, height) of the resulting cuboid.
Then, use the formula for the total surface area (TSA) of a cuboid:

$$\text{TSA} = 2(lb + bh + hl)$$

Step 3: Detailed Explanation:
Each cube has a side of length 'a'.
When two such cubes are joined end to end, the dimensions of the resulting cuboid will be:

- Length $(l) = a + a = 2a$
- Breadth $(b) = a$
- Height $(h) = a$

Now, substitute these dimensions into the TSA formula for a cuboid:

$$\text{TSA} = 2((2a)(a) + (a)(a) + (a)(2a))$$

$$\text{TSA} = 2(2a^2 + a^2 + 2a^2)$$

$$\text{TSA} = 2(5a^2)$$

$$\text{TSA} = 10a^2 \text{ cm}^2$$

Alternative Method:
The surface area of one cube is $6a^2$.
The total surface area of two separate cubes would be $2 \times 6a^2 = 12a^2$.
When the cubes are joined, one face of each cube is stuck together and is no longer part of the outer surface.
The area of one face is a^2. Since two faces are joined, the area that is lost from the surface is $2 \times a^2 = 2a^2$.
So, the new surface area is $12a^2 - 2a^2 = 10a^2$ cm^2.

Step 4: Final Answer:
The surface area of the formed solid is $10a^2$ cm^2.

Quick Tip

Visualizing the combination of shapes is key in such problems. The "lost area" method is often faster and more intuitive for finding the surface area of combined solids.

---

17. The mean of the following table will be

Class interval	0-10	10-20	20-30	30-40
Frequency	4	5	4	7

(A) 18
(B) 20
(C) 21
(D) 22

Correct Answer: (D) 22

Solution:

Step 1: Understanding the Concept:
To find the mean of grouped data, we use the direct method which involves finding the class mark (mid-point) for each class interval, multiplying it by the frequency, summing these products, and finally dividing by the total frequency.

Step 2: Key Formula or Approach:
The formula for the mean ($\bar{x}$) of grouped data is:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

where f_i is the frequency of the i-th class and x_i is the class mark of the i-th class. The class mark is calculated as $x_i = \frac{\text{Upper class limit+Lower class limit}}{2}$.

Step 3: Detailed Explanation:
We will create a table to organize the calculations.

Class Interval	**Frequency** (f_i)	**Class Mark** (x_i)	$f_i x_i$
0-10	4	$\frac{0+10}{2} = 5$	$4 \times 5 = 20$
10-20	5	$\frac{10+20}{2} = 15$	$5 \times 15 = 75$
20-30	4	$\frac{20+30}{2} = 25$	$4 \times 25 = 100$
30-40	7	$\frac{30+40}{2} = 35$	$7 \times 35 = 245$
Total	$\sum f_i = 20$		$\sum f_i x_i = 440$

Now, we apply the formula for the mean:

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{440}{20}$$

$$\bar{x} = 22$$

Step 4: Final Answer:
The mean of the given data is 22.

Quick Tip

For calculating the mean of grouped data, the direct method is straightforward for small numbers. If the numbers (x_i and f_i) are large, consider using the Assumed Mean Method or the Step-Deviation Method to simplify calculations.

---

18. The median class of the following table will be

Class interval	0-5	5-10	10-15	15-20	20-25
Frequency	4	6	10	6	4

(A) 10-15
(B) 15-20

(C) 5-10
(D) 20-25

Correct Answer: (A) 10-15

Solution:

Step 1: Understanding the Concept:
The median class is the class interval which contains the median value of a continuous frequency distribution. To find it, we first need to determine the cumulative frequency.

Step 2: Key Formula or Approach:
1. Calculate the total number of observations, $N = \sum f_i$.
2. Find the value of $N/2$.
3. Calculate the cumulative frequency (cf) for each class.
4. The median class is the class whose cumulative frequency is just greater than or equal to $N/2$.

Step 3: Detailed Explanation:
Let's construct a table with the cumulative frequencies.

Class Interval	Frequency (f_i)	Cumulative Frequency (cf)
0-5	4	4
5-10	6	$4 + 6 = 10$
10-15	10	$10 + 10 = 20$
15-20	6	$20 + 6 = 26$
20-25	4	$26 + 4 = 30$

1. The total number of observations is $N = \sum f_i = 30$.
2. Now, we find $N/2$:
$$\frac{N}{2} = \frac{30}{2} = 15$$
3. We need to find the class interval whose cumulative frequency is just greater than or equal to 15.

- The cf for class 5-10 is 10 (which is < 15).
- The cf for class 10-15 is 20 (which is ≥ 15).

Therefore, the median class is 10-15.

Step 4: Final Answer:
The median class is 10-15.

Quick Tip

The median class is where the middle observation lies. The modal class is the class with the highest frequency. Don't confuse the two. In this case, the modal class (10-15, with frequency 10) happens to be the same as the median class, but this is not always true.

19. The probability of 53 Sundays in a leap year will be

(A) $\frac{1}{7}$
(B) $\frac{3}{7}$
(C) $\frac{2}{7}$
(D) $\frac{4}{7}$

Correct Answer: (C) $\frac{2}{7}$

Solution:

Step 1: Understanding the Concept:
This problem involves calculating probability based on the number of days in a leap year and how they are distributed among the days of the week.

Step 2: Key Formula or Approach:
1. Determine the total number of days in a leap year.
2. Find the number of complete weeks and the number of remaining "odd" days.
3. The probability will be determined by the possible combinations of these odd days.

$$\text{Probability} = \frac{\text{Number of Favourable Outcomes}}{\text{Total Number of Possible Outcomes}}$$

Step 3: Detailed Explanation:
1. A leap year has 366 days.
2. To find the number of weeks, we divide the total days by 7:

$$366 \div 7 = 52 \text{ with a remainder of } 2$$

This means a leap year has 52 complete weeks and 2 extra (odd) days.
3. The 52 complete weeks will contain exactly 52 Sundays. The possibility of getting a 53rd Sunday depends entirely on these 2 odd days.
4. The possible combinations for these two consecutive odd days are:

- (Sunday, Monday)
- (Monday, Tuesday)
- (Tuesday, Wednesday)
- (Wednesday, Thursday)
- (Thursday, Friday)
- (Friday, Saturday)
- (Saturday, Sunday)

So, there are 7 total possible outcomes.
5. The favourable outcomes are the combinations that include a Sunday. These are:

- (Sunday, Monday)
- (Saturday, Sunday)

There are 2 favourable outcomes.
6. The probability is calculated as:

$$P(53 \text{ Sundays}) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}} = \frac{2}{7}$$

Step 4: Final Answer:
The probability of 53 Sundays in a leap year is $\frac{2}{7}$.

Quick Tip

Remember the number of odd days for different year types:

- Ordinary Year (365 days): 1 odd day. Probability of 53 of any day is 1/7.
- Leap Year (366 days): 2 odd days. Probability of 53 of any day is 2/7.

This pattern is very useful for solving such questions quickly.

---

20. If an event occurs surely then its probability will be

(A) 0
(B) 1
(C) $\frac{1}{2}$
(D) $\frac{3}{4}$

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept:
This question tests the fundamental principles of probability, specifically the probability of a sure (or certain) event.

Step 2: Detailed Explanation:
The probability of an event is a measure of the likelihood that the event will occur. It is a number between 0 and 1, inclusive.

- A probability of 0 means the event is impossible. For example, the probability of rolling a 7 on a standard six-sided die is 0.
- A probability of 1 means the event is certain to happen. This is called a sure event or a certain event. For example, the probability of getting a number less than 7 when rolling a standard six-sided die is 1.

Since the question states that the event occurs surely, it is a certain event.

Step 3: Final Answer:
The probability of a sure event is 1.

Quick Tip

The range of probability $P(E)$ for any event E is always $0 \leq P(E) \leq 1$. If you ever calculate a probability that is negative or greater than 1, you have made a mistake.

Section - B

1. Do All Parts

(a). Prove that $(4 - 3\sqrt{2})$ is an irrational number.

Solution:

Step 1: Understanding the Concept:
We will use the method of proof by contradiction. We start by assuming that the given number is rational and show that this assumption leads to a conclusion that contradicts a known fact.

Step 2: Detailed Explanation:
Let us assume, to the contrary, that $4 - 3\sqrt{2}$ is a rational number.
This means we can write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
So,

$$4 - 3\sqrt{2} = \frac{p}{q}$$

Now, we rearrange the equation to isolate the irrational part, $\sqrt{2}$.

$$4 - \frac{p}{q} = 3\sqrt{2}$$

Taking a common denominator on the left side:

$$\frac{4q - p}{q} = 3\sqrt{2}$$

Now, divide by 3 to isolate $\sqrt{2}$:

$$\sqrt{2} = \frac{4q - p}{3q}$$

Since p and q are integers, $4q - p$ is an integer and $3q$ is also a non-zero integer.
Therefore, the expression on the right-hand side, $\frac{4q-p}{3q}$, is a rational number.
This implies that $\sqrt{2}$ is a rational number.

Step 3: The Contradiction:
The conclusion that $\sqrt{2}$ is rational contradicts the well-established fact that $\sqrt{2}$ is an irrational number.
This contradiction arises because our initial assumption that $4-3\sqrt{2}$ is rational was incorrect.

Step 4: Final Answer:
Hence, we conclude that $4-3\sqrt{2}$ is an irrational number.

Quick Tip

The key to this type of proof is to assume the number is rational (i.e., equals p/q) and then algebraically manipulate the equation to isolate the known irrational root (like $\sqrt{2}$, $\sqrt{3}$, etc.) on one side. This will show that an irrational number is equal to a rational number, which is a contradiction.

(b). Find the distance of the point (36, 15) from the origin.

Solution:

Step 1: Understanding the Concept:
This problem requires the use of the distance formula to find the distance between two points in a Cartesian coordinate system.

Step 2: Key Formula or Approach:
The coordinates of the origin are $(0,0)$. Let this be (x_1, y_1).
The given point is $(36, 15)$. Let this be (x_2, y_2).
The distance formula is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

For the distance from the origin, the formula simplifies to $d = \sqrt{x_2^2 + y_2^2}$.

Step 3: Detailed Explanation:
Using the simplified formula for distance from the origin:

$$d = \sqrt{(36)^2 + (15)^2}$$

Calculate the squares of the numbers:

$$36^2 = 1296$$

$$15^2 = 225$$

Substitute these values back into the formula:

$$d = \sqrt{1296 + 225}$$

$$d = \sqrt{1521}$$

To find the square root of 1521:

$$d = 39$$

Step 4: Final Answer:
The distance of the point (36, 15) from the origin is 39 units.

Quick Tip

Recognizing Pythagorean triples can save time. The numbers (15, 36, 39) are a multiple of the basic triple (5, 12, 13). Specifically, $15 = 3 \times 5$, $36 = 3 \times 12$, so the hypotenuse (distance) will be $3 \times 13 = 39$.

(c). Find the coordinates of the point which divides the line segment joining the points (-1, 7) and (4, -3) in the ratio of 2:3.

Solution:

Step 1: Understanding the Concept:
This problem requires the application of the section formula, which is used to find the coordinates of a point that divides a line segment into a given ratio.

Step 2: Key Formula or Approach:
Let the two points be $A(x_1, y_1)$ and $B(x_2, y_2)$, and let the point $P(x, y)$ divide the line segment AB in the ratio $m : n$.
The section formula is:

$$x = \frac{mx_2 + nx_1}{m + n} \quad \text{and} \quad y = \frac{my_2 + ny_1}{m + n}$$

Step 3: Detailed Explanation:
Here, we have:

- $(x_1, y_1) = (-1, 7)$
- $(x_2, y_2) = (4, -3)$
- $m : n = 2 : 3$, so $m = 2$ and $n = 3$.

Now, substitute these values into the section formula to find the x-coordinate:

$$x = \frac{2(4) + 3(-1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$$

Next, find the y-coordinate:

$$y = \frac{2(-3) + 3(7)}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Step 4: Final Answer:
The coordinates of the point which divides the line segment are (1, 3).

Quick Tip

To avoid confusion, always label your points (x_1, y_1) and (x_2, y_2) and the ratio $m : n$ before substituting into the formula. Remember that m is multiplied with the coordinates of the second point (x_2, y_2) and n with the first point (x_1, y_1).

(d). Prove that $1 + \tan^2 A = \sec^2 A$.

Solution:

Step 1: Understanding the Concept:
This is one of the three fundamental Pythagorean identities in trigonometry. The proof starts from the basic identity $\sin^2 A + \cos^2 A = 1$.

Step 2: Detailed Explanation of the Proof:
We start with the fundamental Pythagorean identity:

$$\sin^2 A + \cos^2 A = 1$$

Assuming $\cos A \neq 0$, we can divide both sides of the equation by $\cos^2 A$:

$$\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\cos^2 A} = \frac{1}{\cos^2 A}$$

We know the definitions of the tangent and secant functions:

$$\tan A = \frac{\sin A}{\cos A} \quad \text{and} \quad \sec A = \frac{1}{\cos A}$$

Therefore, $\tan^2 A = \frac{\sin^2 A}{\cos^2 A}$ and $\sec^2 A = \frac{1}{\cos^2 A}$.
Substituting these into our divided equation, we get:

$$\tan^2 A + 1 = \sec^2 A$$

Rearranging the terms gives the desired identity:

$$1 + \tan^2 A = \sec^2 A$$

This completes the proof.

Quick Tip

All three Pythagorean identities can be derived from $\sin^2 A + \cos^2 A = 1$.

- Divide by $\cos^2 A$ to get $\tan^2 A + 1 = \sec^2 A$.
- Divide by $\sin^2 A$ to get $1 + \cot^2 A = \csc^2 A$.

Knowing the derivation is more powerful than just memorizing the formulas.

(e). A toy is in the form of a cone of Radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the volume of the toy.

Solution:

Step 1: Understanding the Concept:
The volume of the toy is the sum of the volume of its two components: a cone and a hemisphere. We need to first determine the dimensions of each component.

Step 2: Key Formula or Approach:

- Volume of a cone = $\frac{1}{3}\pi r^2 h$
- Volume of a hemisphere = $\frac{2}{3}\pi r^3$
- Total Volume = Volume of Cone + Volume of Hemisphere

Step 3: Detailed Explanation:
Given Data:

- Radius of cone and hemisphere, $r = 3.5$ cm $= \frac{7}{2}$ cm.
- Total height of the toy = 15.5 cm.

Calculating Dimensions:

- The height of the hemispherical part is equal to its radius, so height of hemisphere = 3.5 cm.
- The height of the conical part (h) = Total height - height of hemisphere
  $h = 15.5 - 3.5 = 12$ cm.

Calculating Volume:

$$\text{Total Volume} = \left(\frac{1}{3}\pi r^2 h\right) + \left(\frac{2}{3}\pi r^3\right)$$

We can factor out common terms to simplify the calculation:

$$\text{Total Volume} = \frac{1}{3}\pi r^2 (h + 2r)$$

Now, substitute the values (using $\pi = \frac{22}{7}$):

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times \left(12 + 2\left(\frac{7}{2}\right)\right)$$

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times \frac{49}{4} \times (12 + 7)$$

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times \frac{49}{4} \times 19$$

Simplify the expression:

$$\text{Volume} = \frac{1}{3} \times \frac{22 \times 7}{4} \times 19 = \frac{1}{3} \times \frac{154}{4} \times 19 = \frac{1}{3} \times 38.5 \times 19$$

$$\text{Volume} = \frac{731.5}{3} \approx 243.83 \text{ cm}^3$$

Step 4: Final Answer:
The volume of the toy is approximately 243.83 cm^3.

Quick Tip

For combined solids, always draw a simple diagram to visualize the shape and label the dimensions. Calculating the height and radius for each part separately is the most important first step. Factoring out common terms like $\frac{1}{3}\pi r^2$ can make the final calculation much simpler.

(f). A survey conducted on 20 households in a locality by a group of students resulted in the following data for the number of family members in different households:

Family size	1-3	3-5	5-7	7-9	9-11
Number of families	7	8	2	2	1

Find the mode of these data.

Solution:

Step 1: Understanding the Concept:
The mode for grouped data is found using a specific formula that identifies the value that occurs most frequently, taking into account the frequencies of the neighboring classes.

Step 2: Key Formula or Approach:
The formula for the mode is:

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where:

- l = lower limit of the modal class
- f_1 = frequency of the modal class
- f_0 = frequency of the class preceding the modal class
- f_2 = frequency of the class succeeding the modal class
- h = class size

Step 3: Detailed Explanation:
1. Identify the Modal Class:

The modal class is the class with the highest frequency. Here, the highest frequency is 8, which corresponds to the class interval 3-5.

2. Identify the values for the formula:

- Modal class = 3-5
- Lower limit of the modal class $(l) = 3$
- Frequency of the modal class $(f_1) = 8$
- Frequency of the preceding class (1-3) $(f_0) = 7$
- Frequency of the succeeding class (5-7) $(f_2) = 2$
- Class size $(h) = 5 - 3 = 2$

3. Substitute the values into the formula:

$$\text{Mode} = 3 + \left(\frac{8-7}{2(8)-7-2}\right) \times 2$$

$$\text{Mode} = 3 + \left(\frac{1}{16-9}\right) \times 2$$

$$\text{Mode} = 3 + \left(\frac{1}{7}\right) \times 2$$

$$\text{Mode} = 3 + \frac{2}{7}$$

$$\text{Mode} \approx 3 + 0.2857$$

$$\text{Mode} \approx 3.286$$

Step 4: Final Answer:
The mode of the data is approximately 3.286.

> **Quick Tip**
>
> The most common error is mixing up f_0 and f_2. Remember: f_1 is the frequency of the modal class itself, f_0 is the one before (0 comes before 1), and f_2 is the one after (2 comes after 1). Always list these values clearly before substituting.

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2. Do Any Five Parts

(a). Find the zeros of the quadratic polynomial $3x^2 - x - 4$ and verify the relationship between the zeros and coefficients.

Solution:

Step 1: Understanding the Concept:
This problem requires finding the roots (zeros) of a given quadratic polynomial and then using

Vieta's formulas to verify the relationship between these roots and the polynomial's coefficients.

Step 2: Key Formula or Approach:
For a quadratic polynomial $ax^2 + bx + c$, if α and β are the zeros, the relationships are:

- Sum of zeros: $\alpha + \beta = -\frac{b}{a}$
- Product of zeros: $\alpha\beta = \frac{c}{a}$

First, we find the zeros by setting the polynomial to zero and solving the equation $3x^2 - x - 4 = 0$. We can use the method of splitting the middle term.

Step 3: Detailed Explanation:
Finding the Zeros:
We have the equation $3x^2 - x - 4 = 0$. We need two numbers whose product is $3 \times (-4) = -12$ and whose sum is -1. These numbers are -4 and 3.

$$3x^2 - 4x + 3x - 4 = 0$$

Factor by grouping:

$$x(3x - 4) + 1(3x - 4) = 0$$
$$(x + 1)(3x - 4) = 0$$

This gives two possible solutions:

$$x + 1 = 0 \implies x = -1$$
$$3x - 4 = 0 \implies 3x = 4 \implies x = \frac{4}{3}$$

So, the zeros are $\alpha = -1$ and $\beta = \frac{4}{3}$.

Verification:
From the polynomial $3x^2 - x - 4$, we have coefficients $a = 3$, $b = -1$, $c = -4$.
Sum of Zeros:

$$\alpha + \beta = -1 + \frac{4}{3} = \frac{-3 + 4}{3} = \frac{1}{3}$$

According to the formula:

$$-\frac{b}{a} = -\frac{-1}{3} = \frac{1}{3}$$

Thus, $\alpha + \beta = -\frac{b}{a}$ is verified.

Product of Zeros:

$$\alpha\beta = (-1) \times \left(\frac{4}{3}\right) = -\frac{4}{3}$$

According to the formula:

$$\frac{c}{a} = \frac{-4}{3} = -\frac{4}{3}$$

Thus, $\alpha\beta = \frac{c}{a}$ is verified.

Quick Tip

When verifying the relationship, calculate the sum/product from the zeros you found first. Then, calculate the expected values from the coefficients $(-b/a, c/a)$. If they match, your solution is correct. This is a great way to self-check your answers in an exam.

(b). Find two numbers whose sum is 27 and product is 182.

Solution:

Step 1: Understanding the Concept:
This problem can be solved by setting up a system of two equations with two variables or by forming a quadratic equation where the numbers are the roots.

Step 2: Key Formula or Approach:
If two numbers, say x and y, have a sum S and a product P, they can be found as the roots of the quadratic equation:

$$z^2 - Sz + P = 0$$

Step 3: Detailed Explanation:
We are given:

- Sum of the numbers, $S = 27$
- Product of the numbers, $P = 182$

Let the two numbers be the roots of a quadratic equation. Using the formula $z^2 - Sz + P = 0$, we get:

$$z^2 - 27z + 182 = 0$$

We can solve this equation by factoring. We need two numbers that multiply to 182 and add up to -27. Let's find factors of 182. $182 = 2 \times 91 = 2 \times 7 \times 13 = 14 \times 13$. The numbers -14 and -13 satisfy the conditions: $(-14) + (-13) = -27$ and $(-14) \times (-13) = 182$. So, we can split the middle term:

$$z^2 - 14z - 13z + 182 = 0$$
$$z(z - 14) - 13(z - 14) = 0$$
$$(z - 13)(z - 14) = 0$$

The roots are $z = 13$ and $z = 14$.

Step 4: Final Answer:
The two numbers are 13 and 14.

Quick Tip

Forming a quadratic equation $x^2 - (\text{sum})x + (\text{product}) = 0$ is the most efficient method for this type of problem. It directly links the given information to a solvable equation.

(c). Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Solution:

Step 1: Understanding the Concept:
This is a fundamental theorem in geometry concerning the relationship between a tangent and a radius of a circle. We will prove it by showing that the radius to the point of contact is the shortest distance from the center to the tangent line.

Step 2: Detailed Explanation of the Proof:
Given: A circle with center O and a tangent line XY that touches the circle at a point P.
To Prove: The radius OP is perpendicular to the tangent XY (i.e., $OP \perp XY$).
Construction: Take any point Q on the tangent line XY, other than P. Join OQ.
Proof:

1. Since Q is a point on the tangent line XY and is different from the point of contact P, Q must lie outside the circle.
2. If Q were inside the circle, the line XY would become a secant, not a tangent, as it would intersect the circle at two points.
3. Since Q lies outside the circle, the length of the segment OQ must be greater than the length of the radius OP.
$$OQ > OP$$
4. This condition ($OQ > OP$) holds true for every point Q on the line XY except for the point P.
5. This means that OP is the shortest of all the distances from the center O to any point on the line XY.
6. It is a well-established geometric fact that the shortest distance from a point to a line is the perpendicular distance.
7. Therefore, OP must be perpendicular to the line XY.

Hence, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

Quick Tip

The core idea of this proof is the "shortest distance" argument. Remember that the perpendicular segment from a point to a line is always the shortest segment. This concept is key to proving this theorem logically.

(d). Prove that if in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (proportion) and hence the two triangles are similar.

Solution:

Step 1: Understanding the Concept:
This is a proof of the Angle-Angle-Angle (AAA) similarity criterion. It states that if all three corresponding angles of two triangles are equal, then the triangles are similar. The core of the proof involves using the Basic Proportionality Theorem (BPT).

Step 2: Detailed Explanation of the Proof:
Given: Two triangles, ΔABC and ΔDEF, such that $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$.
To Prove: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.
Construction: Let's assume $AB < DE$. Mark a point P on the side DE and a point Q on the side DF such that $DP = AB$ and $DQ = AC$. Join PQ.
Proof:

1. In ΔABC and ΔDPQ:
   - $AB = DP$ (By construction)
   - $\angle A = \angle D$ (Given)
   - $AC = DQ$ (By construction)
2. By the Side-Angle-Side (SAS) congruence rule, $\Delta ABC \cong \Delta DPQ$.
3. Since the triangles are congruent, their corresponding parts are equal. Therefore, $\angle B = \angle DPQ$.
4. We are also given that $\angle B = \angle E$.
5. From the above two points, we can conclude that $\angle DPQ = \angle E$.
6. For the lines PQ and EF, with DE as the transversal, the angles $\angle DPQ$ and $\angle E$ are corresponding angles. Since they are equal, the lines must be parallel.
$$PQ \parallel EF$$
7. Now, in ΔDEF, since $PQ \parallel EF$, we can apply the Basic Proportionality Theorem (BPT or Thales' Theorem).
$$\frac{DP}{DE} = \frac{DQ}{DF}$$
8. Substituting the values from our construction ($DP = AB$ and $DQ = AC$):
$$\frac{AB}{DE} = \frac{AC}{DF} \quad \cdots (1)$$
9. Similarly, we can prove that $\frac{AB}{DE} = \frac{BC}{EF}$.
10. From (1) and this result, we have:
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

Since the corresponding angles are equal and the corresponding sides are in proportion, the two triangles are similar. Hence proved.

Quick Tip

The proof of the AAA similarity criterion hinges on a clever construction that creates a congruent triangle inside the larger triangle. This allows you to use the properties of parallel lines and the Basic Proportionality Theorem to establish the ratio of sides.

(e). The distribution table below gives the weight of the students of a class. Find the median weight of the students.

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of Students	2	3	8	6	6	3	2

Solution:

Step 1: Understanding the Concept:
To find the median of grouped data, we need to identify the median class, which is the class where the middle observation lies. We then use the median formula to calculate the exact median value within that class.

Step 2: Key Formula or Approach:
The formula for the median of grouped data is:

$$\text{Median} = l + \left(\frac{\frac{N}{2} - cf}{f} \right) \times h$$

where:

- l = lower limit of the median class
- N = total number of observations (sum of frequencies)
- cf = cumulative frequency of the class preceding the median class
- f = frequency of the median class
- h = class size

Step 3: Detailed Explanation:
First, we create a cumulative frequency table:

Weight (in kg)	Frequency (f)	Cumulative Frequency (cf)
40-45	2	2
45-50	3	$2+3=5$
50-55	8	$5+8=13$
55-60	6	$13+6=19$
60-65	6	$19+6=25$
65-70	3	$25+3=28$
70-75	2	$28+2=30$

1. Find N/2: The total number of students is $N = \sum f = 30$.

$$\frac{N}{2} = \frac{30}{2} = 15$$

2. Identify the Median Class: We look for the class whose cumulative frequency is just greater than or equal to 15. The class 50-55 has a cf of 13 (< 15), and the class 55-60 has a cf of 19 (≥ 15). So, the median class is 55-60.

3. Identify values for the formula:

- Lower limit of median class, $l = 55$
- Total frequency, $N = 30$
- Cumulative frequency of preceding class, $cf = 13$
- Frequency of median class, $f = 6$
- Class size, $h = 60 - 55 = 5$

4. Calculate the Median:

$$\text{Median} = 55 + \left(\frac{15-13}{6}\right) \times 5$$

$$\text{Median} = 55 + \left(\frac{2}{6}\right) \times 5$$

$$\text{Median} = 55 + \frac{1}{3} \times 5 = 55 + \frac{5}{3}$$

$$\text{Median} \approx 55 + 1.67 = 56.67$$

Step 4: Final Answer:
The median weight of the students is approximately 56.67 kg.

Quick Tip

A common mistake is using the cumulative frequency of the median class itself in the formula. Remember to use the cumulative frequency (cf) of the class *preceding* the median class.

(f). A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white? (iii) not green?

Solution:

Step 1: Understanding the Concept:
This problem deals with basic probability. The probability of an event is the ratio of the number of favorable outcomes to the total number of possible outcomes.

Step 2: Key Formula or Approach:

$$P(\text{Event}) = \frac{\text{Number of Favorable Outcomes}}{\text{Total Number of Possible Outcomes}}$$

Step 3: Detailed Explanation:
First, calculate the total number of marbles in the box.

$$\text{Total marbles} = (\text{Red}) + (\text{White}) + (\text{Green}) = 5 + 8 + 4 = 17$$

The total number of possible outcomes is 17.

(i) Probability of drawing a red marble:

- Number of favorable outcomes (red marbles) = 5
- $P(\text{red}) = \frac{5}{17}$

(ii) Probability of drawing a white marble:

- Number of favorable outcomes (white marbles) = 8
- $P(\text{white}) = \frac{8}{17}$

(iii) Probability of drawing a marble that is not green: This can be calculated in two ways.
Method 1: Using the complement rule First, find the probability of drawing a green marble.

$$P(\text{green}) = \frac{\text{Number of green marbles}}{\text{Total marbles}} = \frac{4}{17}$$

The probability of "not green" is $1 - P(\text{green})$.

$$P(\text{not green}) = 1 - \frac{4}{17} = \frac{17 - 4}{17} = \frac{13}{17}$$

Method 2: Direct calculation The number of marbles that are not green is the sum of red and white marbles.

$$\text{Number of not green marbles} = 5(\text{red}) + 8(\text{white}) = 13$$

$$P(\text{not green}) = \frac{13}{17}$$

Step 4: Final Answer:
(i) The probability of drawing a red marble is $\frac{5}{17}$.

(ii) The probability of drawing a white marble is $\frac{8}{17}$.
(iii) The probability of drawing a marble that is not green is $\frac{13}{17}$.

Quick Tip

For "not" probabilities, using the complement rule ($P(\text{not A}) = 1 - P(\text{A})$) is often quicker than counting all the other outcomes, especially when there are many categories.

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3. Find the sum of first 24 terms of the list of numbers whose n-th term is given by $a_n = 3 + 2n$.

Solution:

Step 1: Understanding the Concept:
The given n-th term a_n is a linear expression in n, which means the list of numbers forms an Arithmetic Progression (AP). We need to find the first term and the common difference to use the formula for the sum of an AP.

Step 2: Key Formula or Approach:
The sum of the first n terms of an AP is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

or

$$S_n = \frac{n}{2}[a + l]$$

where a is the first term, d is the common difference, l is the last term, and n is the number of terms.

Step 3: Detailed Explanation:
First, let's find the first few terms of the sequence to identify a and d.

- First term (a_1): $a = 3 + 2(1) = 5$
- Second term (a_2): $3 + 2(2) = 7$
- Third term (a_3): $3 + 2(3) = 9$

The sequence is 5, 7, 9, ...

- The first term is $a = 5$.
- The common difference is $d = a_2 - a_1 = 7 - 5 = 2$.

We need to find the sum of the first 24 terms, so $n = 24$.
Using the sum formula $S_n = \frac{n}{2}[2a + (n-1)d]$:

$$S_{24} = \frac{24}{2}[2(5) + (24-1)(2)]$$

$$S_{24} = 12[10 + (23)(2)]$$
$$S_{24} = 12[10 + 46]$$
$$S_{24} = 12[56]$$
$$S_{24} = 672$$

Step 4: Final Answer:
The sum of the first 24 terms is 672.

Quick Tip

Any n-th term of the form $a_n = dn + c$ represents an AP with a common difference of d. In this problem, $a_n = 2n + 3$, so you can immediately tell the common difference is 2 without calculating the first few terms.

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3. OR The sum of a two-digit number and the number formed by reversing its digits is 66. If the difference between the digits of the number is 2, then find the number.

Solution:

Step 1: Understanding the Concept:
This problem involves translating word statements into a system of linear equations. A two-digit number can be represented algebraically, which allows us to form equations based on the given conditions.

Step 2: Key Formula or Approach:
Let the tens digit be x and the units digit be y.

- The original number can be written as $10x + y$.
- The reversed number can be written as $10y + x$.

We will set up two equations based on the given sum and difference.

Step 3: Detailed Explanation:
From the first condition, the sum of the number and its reverse is 66:

$$(10x + y) + (10y + x) = 66$$
$$11x + 11y = 66$$

Dividing the entire equation by 11, we get our first simplified equation:

$$x + y = 6 \quad \cdots (1)$$

From the second condition, the difference between the digits is 2. This can mean either $x - y = 2$ or $y - x = 2$. We must consider both cases.

Case 1: $x - y = 2 \quad \cdots (2a)$
We have a system of two equations:

$$x + y = 6$$
$$x - y = 2$$

Adding the two equations: $2x = 8 \implies x = 4$.
Substituting $x = 4$ into equation (1): $4 + y = 6 \implies y = 2$.
The number is $10x + y = 10(4) + 2 = 42$.

Case 2: $y - x = 2 \quad \cdots (2b)$
We have a system of two equations:

$$x + y = 6$$
$$-x + y = 2$$

Adding the two equations: $2y = 8 \implies y = 4$.
Substituting $y = 4$ into equation (1): $x + 4 = 6 \implies x = 2$.
The number is $10x + y = 10(2) + 4 = 24$.

Step 4: Final Answer:
The possible numbers are 42 and 24. (Either answer is acceptable as the question asks to "find the number").

Quick Tip

When a problem states "the difference between two quantities is k", always consider both possibilities ($a - b = k$ and $b - a = k$) unless one is ruled out by context. This can help you find all possible solutions.

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4. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top of the tree touches the ground is 8 metre. Find the height of the tree.

Solution:

Step 1: Understanding the Concept:
This is a classic height and distance problem that can be modeled using a right-angled triangle. We will use trigonometric ratios (tan and cos) to find the lengths of the different parts of the tree and then add them to find the total height.

Step 2: Key Formula or Approach:
Let's model the situation with a right-angled triangle ABC, where B is the base of the tree on the ground, C is the point where the tree broke, and A is the top of the tree touching the ground.

- The height of the standing part is BC (Opposite side).
- The length of the broken part is AC (Hypotenuse).
- The distance from the foot to the top on the ground is AB = 8m (Adjacent side).
- The angle made with the ground is $\angle CAB = 30^\circ$.

We will use:

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}} \quad \text{and} \quad \cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

Step 3: Detailed Explanation:
Find the height of the standing part (BC):

$$\tan 30^\circ = \frac{BC}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{BC}{8}$$

$$BC = \frac{8}{\sqrt{3}} \text{ m}$$

Find the length of the broken part (AC):

$$\cos 30^\circ = \frac{AB}{AC}$$

$$\frac{\sqrt{3}}{2} = \frac{8}{AC}$$

$$AC = \frac{8 \times 2}{\sqrt{3}} = \frac{16}{\sqrt{3}} \text{ m}$$

Find the total height of the tree: The original height of the tree was the sum of the standing part and the broken part.

$$\text{Total Height} = BC + AC = \frac{8}{\sqrt{3}} + \frac{16}{\sqrt{3}} = \frac{24}{\sqrt{3}}$$

Rationalize the denominator:

$$\text{Total Height} = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{24\sqrt{3}}{3} = 8\sqrt{3} \text{ m}$$

Step 4: Final Answer:
The height of the tree is $8\sqrt{3}$ metres.

Quick Tip

Always draw a diagram for height and distance problems. It helps you visualize the right-angled triangles and correctly identify which sides are opposite, adjacent, or the hypotenuse relative to the given angle.

4. OR From the top of a 7 metre high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution:

Step 1: Understanding the Concept:
This problem involves two right-angled triangles formed by the line of sight from the top of the building to the top and bottom of a tower. We use trigonometric ratios to find the horizontal distance between them and then the remaining height of the tower.

Step 2: Detailed Explanation:
Let AB be the building of height 7 m, and let CD be the cable tower. Let's draw a horizontal line AE from the top of the building A to the tower.
We have the following information from the diagram:

- Height of the building, AB = 7 m.
- Angle of elevation of the top of the tower, $\angle CAE = 60°$.
- Angle of depression of the foot of the tower, $\angle EAD = 45°$.

The total height of the tower is CD = CE + ED. Since AE is parallel to BD and AB is parallel to ED, ABDE is a rectangle. Therefore, ED = AB = 7 m and AE = BD.

Triangle 1: ΔADE This triangle is formed by the angle of depression.

$$\tan(\angle EAD) = \frac{ED}{AE}$$

$$\tan 45° = \frac{7}{AE}$$

$$1 = \frac{7}{AE} \implies AE = 7 \text{ m}$$

So, the horizontal distance between the building and the tower (BD) is also 7 m.

Triangle 2: ΔACE This triangle is formed by the angle of elevation.

$$\tan(\angle CAE) = \frac{CE}{AE}$$

$$\tan 60° = \frac{CE}{7}$$

$$\sqrt{3} = \frac{CE}{7} \implies CE = 7\sqrt{3} \text{ m}$$

Total Height of the Tower:

$$CD = CE + ED = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1) \text{ m}$$

Step 3: Final Answer:
The height of the tower is $7(\sqrt{3} + 1)$ metres.

Quick Tip

In problems involving angles of elevation and depression from a certain height, the key is to draw a horizontal line from the observation point. This splits the problem into two separate right-angled triangles that share a common side (the horizontal distance).

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5. A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid so formed.

Solution:

Step 1: Understanding the Concept:
This problem involves calculating the surface area of a composite solid. The total surface area is the sum of the exposed areas of the cube and the hemisphere. We must subtract the area covered by the base of the hemisphere from the cube's top face.

Step 2: Key Formula or Approach:

- The greatest diameter of the hemisphere is equal to the side length of the cube.
- Surface Area of Solid = (Total Surface Area of Cube) - (Area of Base of Hemisphere) + (Curved Surface Area of Hemisphere)
- TSA of Cube = $6a^2$
- Area of Base of Hemisphere = πr^2
- CSA of Hemisphere = $2\pi r^2$

Combining these, the formula becomes: Surface Area = $6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$.

Step 3: Detailed Explanation:
Greatest Diameter: The hemisphere is placed on top of the cubical block. The maximum diameter it can have is the length of the edge of the cube.

- Side of cube, $a = 7$ cm.
- Greatest diameter of hemisphere = 7 cm.
- Radius of hemisphere, $r = \frac{7}{2} = 3.5$ cm.

Surface Area of the Solid: Using the derived formula:

$$\text{Surface Area} = 6a^2 + \pi r^2$$

Substitute the values: $a = 7$ cm, $r = 3.5$ cm, and $\pi = \frac{22}{7}$.

$$\text{Surface Area} = 6(7)^2 + \frac{22}{7}\left(\frac{7}{2}\right)^2$$

$$\text{Surface Area} = 6(49) + \frac{22}{7}\left(\frac{49}{4}\right)$$
$$\text{Surface Area} = 294 + \frac{22 \times 7}{4}$$
$$\text{Surface Area} = 294 + \frac{154}{4}$$
$$\text{Surface Area} = 294 + 38.5$$
$$\text{Surface Area} = 332.5 \text{ cm}^2$$

Step 4: Final Answer:
The greatest diameter the hemisphere can have is 7 cm, and the surface area of the solid formed is 332.5 cm^2.

Quick Tip

For composite solids where one shape is placed on another, the total surface area is NOT the sum of their individual surface areas. Always visualize which surfaces are covered and subtract them, then add the new curved surfaces that are created. The simplified formula $6a^2 + \pi r^2$ is very useful for a cube surmounted by a hemisphere of maximum size.

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5. OR A chord of a circle of radius 20 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle.

Solution:

Step 1: Understanding the Concept:
A segment of a circle is the region bounded by a chord and its corresponding arc. Its area is found by subtracting the area of the triangle formed by the chord and the radii from the area of the corresponding sector.

Step 2: Key Formula or Approach:

- Area of a Sector = $\frac{\theta}{360^\circ} \times \pi r^2$
- Area of a Triangle (formed by two radii and a chord) = $\frac{1}{2}r^2 \sin\theta$ or by specific geometric properties.
- Area of Minor Segment = Area of Minor Sector - Area of Triangle
- Area of Major Segment = Area of Circle - Area of Minor Segment

Step 3: Detailed Explanation:
Given:

- Radius, $r = 20$ cm

- Central angle, $\theta = 60^\circ$

1. Area of the Minor Sector:

$$\text{Area} = \frac{60^\circ}{360^\circ} \times \pi(20)^2 = \frac{1}{6} \times 400\pi = \frac{200\pi}{3} \text{ cm}^2$$

2. Area of the Triangle: Let the triangle be ΔOAB, where O is the center. We have OA = OB = 20 cm (radii) and $\angle AOB = 60^\circ$. Since two sides are equal, the angles opposite to them are equal ($\angle OAB = \angle OBA$). The sum of angles is 180°, so $\angle OAB + \angle OBA = 180^\circ - 60^\circ = 120^\circ$. This means $\angle OAB = \angle OBA = 60^\circ$. Since all angles are 60°, ΔOAB is an equilateral triangle with side length 20 cm. The area of an equilateral triangle is $\frac{\sqrt{3}}{4}(\text{side})^2$.

$$\text{Area}(\Delta OAB) = \frac{\sqrt{3}}{4}(20)^2 = \frac{\sqrt{3}}{4} \times 400 = 100\sqrt{3} \text{ cm}^2$$

3. Area of the Minor Segment:

$$\text{Area} = (\text{Area of Minor Sector}) - (\text{Area of } \Delta OAB)$$

$$\text{Area} = \left(\frac{200\pi}{3} - 100\sqrt{3}\right) \text{ cm}^2$$

4. Area of the Major Segment: First, find the total area of the circle.

$$\text{Area of Circle} = \pi r^2 = \pi(20)^2 = 400\pi \text{ cm}^2$$

$$\text{Area of Major Segment} = (\text{Area of Circle}) - (\text{Area of Minor Segment})$$

$$\text{Area} = 400\pi - \left(\frac{200\pi}{3} - 100\sqrt{3}\right) = 400\pi - \frac{200\pi}{3} + 100\sqrt{3}$$

$$\text{Area} = \left(\frac{1200\pi - 200\pi}{3}\right) + 100\sqrt{3} = \left(\frac{1000\pi}{3} + 100\sqrt{3}\right) \text{ cm}^2$$

Step 4: Final Answer:
The area of the minor segment is $\left(\frac{200\pi}{3} - 100\sqrt{3}\right)$ cm^2.
The area of the major segment is $\left(\frac{1000\pi}{3} + 100\sqrt{3}\right)$ cm^2.

Quick Tip

When the central angle is 60°, the triangle formed by the two radii and the chord is always equilateral. When the angle is 90°, it is an isosceles right-angled triangle. Recognizing these special cases can simplify the calculation of the triangle's area.
