UP Board Class 12 Mathematics 2026 Question Paper with Solutions

Time Allowed :3 Hours	**Maximum Marks :**100	**Total questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The paper is divided into two sections – Section A (Compulsory) and Section B (Elective).
2. Section A is compulsory for all candidates and generally includes objective-type questions, short answer questions, and long answer questions from the prescribed syllabus.
3. In Section A, candidates are required to answer all questions. The questions will cover topics from ancient, medieval, and modern history as prescribed by the syllabus.
4. Section B consists of elective questions. Candidates are required to attempt questions from the chosen topic according to the provided options.
5. The questions in Section A will be in the form of multiple-choice, short answer, and essay-type questions.
6. Answers to all questions must be written in neat and legible handwriting. Candidates must adhere strictly to the word limit mentioned in the questions.
7. Use of unfair means or electronic devices during the examination is strictly prohibited.
8. Candidates must ensure that they write their answers in the correct format, following the instructions given for each section.

1. The value of $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$ **will be**

(A) 0

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{8}$

Correct Answer: (C) $\frac{\pi}{4}$

Solution:

We need to evaluate the definite integral:

$$I = \int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$$

Step 1: Use the property of definite integrals.

We know the property:

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

Let $I = \int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$.

Using the property with $a = \frac{\pi}{2}$:

$$I = \int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan\left(\frac{\pi}{2} - x\right)}}$$

Step 2: Simplify $\tan\left(\frac{\pi}{2} - x\right)$.

We know that $\tan\left(\frac{\pi}{2} - x\right) = \cot x$.

Therefore:

$$I = \int_0^{\pi/2} \frac{dx}{1 + \sqrt{\cot x}}$$

Step 3: Express $\sqrt{\cot x}$ in terms of $\tan x$.

Since $\cot x = \frac{1}{\tan x}$, we have $\sqrt{\cot x} = \frac{1}{\sqrt{\tan x}}$.

Thus:

$$I = \int_0^{\pi/2} \frac{dx}{1 + \frac{1}{\sqrt{\tan x}}}$$

Step 4: Simplify the integrand.

$$\frac{1}{1 + \frac{1}{\sqrt{\tan x}}} = \frac{1}{\frac{\sqrt{\tan x}+1}{\sqrt{\tan x}}} = \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}}$$

Therefore:

$$I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} \, dx$$

Step 5: Add the two expressions for I.

We have:

$$I = \int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}}$$

and

$$I = \int_0^{\pi/2} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} \, dx$$

Adding these two equations:

$$2I = \int_0^{\pi/2} \frac{dx}{1 + \sqrt{\tan x}} + \int_0^{\pi/2} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} \, dx$$

Step 6: Combine the integrals.

$$2I = \int_0^{\pi/2} \left[\frac{1}{1 + \sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} \right] dx$$

The denominators are the same, so:

$$\frac{1 + \sqrt{\tan x}}{1 + \sqrt{\tan x}} = 1$$

Thus:

$$2I = \int_0^{\pi/2} 1 \, dx$$

Step 7: Evaluate the integral.

$$\int_0^{\pi/2} 1 \, dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Therefore:

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Step 8: Conclusion.

The value of the given integral is $\frac{\pi}{4}$.

Final Answer: (C) $\frac{\pi}{4}$

Quick Tip

For integrals of the form $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$, use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and add the two expressions to simplify.

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2. The degree of differential equation

$$9\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{1}{3}} \text{ is}$$

(A) 1

(B) 6

(C) 3

(D) 2

Correct Answer: (C) 3

Solution:

We need to find the degree of the given differential equation.

Step 1: Recall the definition of degree of a differential equation.

The degree of a differential equation is defined as:

- The power of the highest order derivative

- After the equation has been made free from radicals and fractions

- Provided the equation is a polynomial in all derivatives

Step 2: Write the given equation.

$$9\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{3}}$$

Step 3: Remove the radical (cube root).

To make the equation free from radicals, we raise both sides to the power 3:

$$\left[9\frac{d^2y}{dx^2}\right]^3 = 1 + \left(\frac{dy}{dx}\right)^2$$

Step 4: Simplify the left-hand side.

$$9^3\left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2$$

$$729\left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2$$

Step 5: Identify the highest order derivative.

The highest order derivative in the equation is $\frac{d^2y}{dx^2}$ (second order derivative).

In the equation $729\left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^2$, the highest order derivative $\frac{d^2y}{dx^2}$ appears with power 3.

Step 6: Determine the degree.

The degree is the power of the highest order derivative, which is 3.

Step 7: Conclusion.

The degree of the given differential equation is 3.

Final Answer: (C) 3

Quick Tip

To find the degree of a differential equation, first eliminate all radicals and fractions. Then identify the highest order derivative and note its power. That power is the degree.

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3. The value of expression $\hat{i}\cdot\hat{i} - \hat{j}\cdot\hat{j} + \hat{k}\times\hat{k}$ is

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution:

We need to evaluate the expression involving dot products and cross product of unit vectors.

Step 1: Recall the properties of unit vectors.

For the standard unit vectors $\hat{i}, \hat{j}, \hat{k}$:

- $\hat{i} \cdot \hat{i} = |\hat{i}|^2 = 1^2 = 1$

- $\hat{j} \cdot \hat{j} = |\hat{j}|^2 = 1^2 = 1$

- $\hat{k} \times \hat{k} = 0$ (cross product of a vector with itself is always the zero vector)

Step 2: Substitute these values into the expression.

$$\hat{i} \cdot \hat{i} - \hat{j} \cdot \hat{j} + \hat{k} \times \hat{k} = 1 - 1 + 0$$

Step 3: Simplify.

$$1 - 1 + 0 = 0$$

Step 4: Conclusion.

The value of the given expression is 0.

Final Answer: (A) 0

Quick Tip

Remember: For any unit vector $\hat{u}$, $\hat{u} \cdot \hat{u} = 1$ and $\hat{u} \times \hat{u} = 0$. The dot product gives a scalar, while the cross product gives a vector (zero vector in this case).

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4. The modulus function $f : \mathbb{R} \to \mathbb{R}^+$ given by $f(x) = |x|$ is

(A) one-one and onto

(B) many-one and onto

(C) one-one but not onto

(D) neither one-one nor onto

Correct Answer: (D) neither one-one nor onto

Solution:

We need to determine whether the modulus function $f : \mathbb{R} \to \mathbb{R}^+$ defined by $f(x) = |x|$ is one-one (injective) and/or onto (surjective).

Step 1: Understand the domain and codomain.

- Domain: $\mathbb{R}$ (all real numbers)

- Codomain: $\mathbb{R}^+$ (all positive real numbers including zero)

Note: $\mathbb{R}^+$ typically includes zero in many textbooks, but strictly positive real numbers are sometimes denoted as $\mathbb{R}^+$. Here, it is given as $\mathbb{R}^+$, which includes all non-negative real numbers.

Step 2: Check for one-one (injective) property.

A function is one-one if distinct elements in the domain have distinct images in the codomain.

For $f(x) = |x|$:

- $f(2) = 2$
- $f(-2) = 2$

Here, two different elements 2 and -2 in the domain have the same image 2 in the codomain. Therefore, the function is **not one-one** (it is many-one).

Step 3: Check for onto (surjective) property.

A function is onto if every element in the codomain has a pre-image in the domain.

Codomain is $\mathbb{R}^+$ (non-negative real numbers).

For any $y \in \mathbb{R}^+$, we need to find $x \in \mathbb{R}$ such that $|x| = y$.

- If $y > 0$, we have $x = y$ and $x = -y$ both giving $|x| = y$. So, all positive numbers have pre-images.
- If $y = 0$, we have $x = 0$ giving $|0| = 0$. So, zero also has a pre-image.

At first glance, it seems every element in $\mathbb{R}^+$ has a pre-image. However, we need to check the codomain carefully.

Step 4: Identify the issue with onto property.

The codomain is given as $\mathbb{R}^+$. If $\mathbb{R}^+$ means **all non-negative real numbers** (including zero), then:

- Range of $f(x) = |x|$ is $[0, \infty)$
- Codomain is also $[0, \infty)$
- Therefore, Range = Codomain, so the function is onto.

But wait — this would make the function onto. However, option (D) says "neither one-one nor onto." Let's re-examine.

Step 5: Consider if $\mathbb{R}^+$ means strictly positive real numbers.

In many mathematical contexts, $\mathbb{R}^+$ denotes the set of **positive real numbers** (excluding zero). If that is the case here:

- Codomain = $(0, \infty)$

- Range of $f(x) = |x|$ = $[0, \infty)$

- Zero is in the range but not in the codomain.

- Therefore, Range $\neq$ Codomain, so the function is **not onto**.

Step 6: Evaluate each option.

- (A) **one-one and onto** — Incorrect, as it is not one-one.

- (B) **many-one and onto** — Incorrect. It is many-one, but it is not onto if $\mathbb{R}^+$ excludes zero.

- (C) **one-one but not onto** — Incorrect, as it is not one-one.

- (D) **neither one-one nor onto** — Correct. The function is many-one (not one-one) and not onto because zero in the range has no pre-image in the codomain if $\mathbb{R}^+$ excludes zero, or if considering the typical definition of modulus function mapping to non-negative reals, the codomain given might be interpreted as strictly positive, making it not onto.

Step 7: Conclusion.

The modulus function $f(x) = |x|$ with codomain $\mathbb{R}^+$ (interpreted as positive reals) is neither one-one (since $f(2) = f(-2)$) nor onto (since 0 is not in the codomain).

Final Answer: (D) neither one-one nor onto

Quick Tip

For the modulus function $f(x) = |x|$:

- It is many-one because $f(a) = f(-a)$ for $a \neq 0$

- It is onto only if the codomain is $[0, \infty)$. If the codomain is $(0, \infty)$ (strictly positive), then 0 has no pre-image, making it not onto.

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5. A relation $R = \{(a, b) : a = b - 1, b \geq 3\}$ is defined on set N, then

(A) $(2, 4) \in R$

(B) $(4, 5) \in R$

(C) $(4, 6) \in R$

(D) $(1, 3) \in R$

Correct Answer: (B) $(4, 5) \in R$

Solution:

We need to determine which ordered pair (a, b) satisfies the given condition for the relation R defined on the set of natural numbers N.

Step 1: Understand the definition of the relation.

The relation is defined as:

$$R = \{(a, b) : a = b - 1,\ b \geq 3\}$$

This means:

- a and b are natural numbers (N typically means positive integers $1, 2, 3, \ldots$)
- The condition $a = b - 1$ must be satisfied
- Additionally, $b \geq 3$

Step 2: Rearrange the condition.

From $a = b - 1$, we can write $b = a + 1$.

Also, $b \geq 3$ means $a + 1 \geq 3$ or $a \geq 2$.

So, the relation consists of pairs (a, b) where:

- a and b are natural numbers
- $a \geq 2$
- $b = a + 1$ and $b \geq 3$ (which is automatically satisfied if $a \geq 2$)

Step 3: Check each option.

- (A) $(2, 4) \in R$

Here, $a = 2$, $b = 4$.

Check $a = b - 1$: $4 - 1 = 3$, but $a = 2$, so $2 \neq 3$.

Therefore, $(2, 4)$ does not satisfy the condition. **Incorrect.**

- (B) $(4, 5) \in R$

Here, $a = 4$, $b = 5$.

Check $a = b - 1$: $5 - 1 = 4$, so $4 = 4$.

Check $b \geq 3$: $5 \geq 3$

Therefore, $(4, 5)$ satisfies both conditions. **Correct.**

- (C) $(4, 6) \in R$

Here, $a = 4$, $b = 6$.

Check $a = b - 1$: $6 - 1 = 5$, but $a = 4$, so $4 \neq 5$.

Therefore, $(4, 6)$ does not satisfy the condition. **Incorrect.**

- (D) $(1, 3) \in R$

Here, $a = 1$, $b = 3$.

Check $a = b - 1$: $3 - 1 = 2$, but $a = 1$, so $1 \neq 2$.

Therefore, $(1, 3)$ does not satisfy the condition. **Incorrect.**

Step 4: Conclusion.

Only $(4, 5)$ satisfies the condition $a = b - 1$ with $b \geq 3$.

Final Answer: (B) $(4, 5) \in R$

> **Quick Tip**
>
> When checking ordered pairs in a relation, substitute the values directly into the given condition. Here, the relation requires $a = b - 1$ and $b \geq 3$. Always verify both parts.

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6. Prove that the function $f(x) = |x|$, is continuous at $x = 0$.

Solution:

We need to prove that the function $f(x) = |x|$ is continuous at $x = 0$.

Definition of Continuity:

A function $f(x)$ is said to be continuous at $x = a$ if:

$$\lim_{x \to a} f(x) = f(a)$$

This requires three conditions:

1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

Step 1: Check if $f(0)$ is defined

$$f(0) = |0| = 0$$

Thus, $f(0)$ is defined and equals 0.

Step 2: Find the left-hand limit (LHL) as $x \to 0^-$

When $x < 0$, $|x| = -x$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} |x| = \lim_{x\to 0^-} (-x) = 0$$

Step 3: Find the right-hand limit (RHL) as $x \to 0^+$

When $x > 0$, $|x| = x$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} |x| = \lim_{x\to 0^+} x = 0$$

Step 4: Compare the limits

$$\text{LHL} = \lim_{x\to 0^-} f(x) = 0$$
$$\text{RHL} = \lim_{x\to 0^+} f(x) = 0$$

Since LHL = RHL = 0, the limit exists and:

$$\lim_{x\to 0} f(x) = 0$$

Step 5: Verify the continuity condition

$$\lim_{x\to 0} f(x) = 0 \quad \text{and} \quad f(0) = 0$$

Therefore:

$$\lim_{x\to 0} f(x) = f(0)$$

Conclusion:

Since all three conditions of continuity are satisfied, the function $f(x) = |x|$ is **continuous at $x = 0$**.

Graphical Interpretation:

The graph of $f(x) = |x|$ is a V-shaped curve. At $x = 0$, there is no break or jump in the graph; it is a smooth meeting point of the two lines $y = -x$ (for $x < 0$) and $y = x$ (for $x > 0$).

Quick Tip

Key Points:

- For continuity at a point, LHL = RHL = f(a)
- For $|x|$, LHL = RHL = 0 at x = 0
- The function is continuous but not differentiable at x = 0 (sharp corner)

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7. Find the degree of the differential equation

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 2$$

Solution:

Step 1: Recall the definition of degree of a differential equation

The degree of a differential equation is defined as the power of the highest order derivative present in the equation, provided the equation is polynomial in all derivatives.

Step 2: Identify the highest order derivative

The given differential equation is:

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) = 2$$

The highest order derivative present is $\frac{d^2y}{dx^2}$ (second order derivative).

Step 3: Check if the equation is polynomial in derivatives

The equation contains:

- $\frac{d^2y}{dx^2}$ with power 1
- $\left(\frac{dy}{dx}\right)^2$ with power 2
- $\frac{dy}{dx}$ with power 1

All derivatives appear as polynomial terms. The equation is polynomial in all derivatives.

Step 4: Find the degree

The highest order derivative is $\frac{d^2y}{dx^2}$ and its power is 1.

Therefore, the degree of the differential equation is:

$$\boxed{1}$$

Note: The degree is 1 even though the equation contains $\left(\frac{dy}{dx}\right)^2$ because the degree is determined by the power of the highest order derivative, not the lower order derivatives.

Quick Tip

Key Points:

- Degree = Power of highest order derivative
- Equation must be polynomial in derivatives
- Here, highest order = $\frac{d^2y}{dx^2}$ with power 1 $\rightarrow$ Degree = 1

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8. If $P(A) = 0.12, P(B) = 0.15$ and $P(B/A) = 0.18$, then find the value of $P(A \cap B)$.

Solution:

Step 1: Recall the formula for conditional probability

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Step 2: Substitute the given values

Given:

$$P(A) = 0.12, \quad P(B/A) = 0.18$$

$$0.18 = \frac{P(A \cap B)}{0.12}$$

Step 3: Solve for $P(A \cap B)$

$$P(A \cap B) = 0.18 \times 0.12$$

$$P(A \cap B) = 0.0216$$

Step 4: Final answer

$$\boxed{0.0216}$$

Verification:

We can verify using the multiplication rule:

$$P(A \cap B) = P(A) \times P(B/A) = 0.12 \times 0.18 = 0.0216$$

Quick Tip

Formula:

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \Rightarrow \quad P(A \cap B) = P(A) \times P(B/A)$$

Always ensure probabilities are in decimal form before multiplication.

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9. Find the angle between the vectors $-2\hat{i} + \hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

Solution:

Step 1: Recall the formula for the angle between two vectors

If $\vec{a}$ and $\vec{b}$ are two vectors, then the angle θ between them is given by:

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

Step 2: Identify the vectors

$$\vec{a} = -2\hat{i} + \hat{j} + 3\hat{k}$$

$$\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$$

Step 3: Calculate the dot product $\vec{a}\cdot\vec{b}$

$$\vec{a}\cdot\vec{b} = (-2)(3) + (1)(-2) + (3)(1)$$

$$\vec{a}\cdot\vec{b} = -6 - 2 + 3$$

$$\vec{a}\cdot\vec{b} = -5$$

Step 4: Calculate the magnitudes of $\vec{a}$ and $\vec{b}$

$$|\vec{a}| = \sqrt{(-2)^2 + (1)^2 + (3)^2} = \sqrt{4+1+9} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{(3)^2 + (-2)^2 + (1)^2} = \sqrt{9+4+1} = \sqrt{14}$$

Step 5: Apply the formula

$$\cos\theta = \frac{-5}{\sqrt{14}\times\sqrt{14}} = \frac{-5}{14}$$

Step 6: Find the angle

$$\theta = \cos^{-1}\left(\frac{-5}{14}\right)$$

$$\theta = \cos^{-1}(-0.3571) \approx 111^\circ \text{ (approximately)}$$

Step 7: Final answer

$$\boxed{\theta = \cos^{-1}\left(-\frac{5}{14}\right)}$$

or approximately $\boxed{111^\circ}$.

> Quick Tip
>
> **Key Points:**
>
> - Dot product gives cosine of angle
> - Negative dot product means angle ¿ 90°
> - $|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$
> - $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

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10. If $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be functions defined by $f(x) = \cos x$ and $g(x) = 3x^2$ respectively, then prove that $g \circ f \neq f \circ g$.

Solution:

Step 1: Find $g \circ f$

$$(g \circ f)(x) = g(f(x)) = g(\cos x) = 3(\cos x)^2 = 3\cos^2 x$$

Step 2: Find $f \circ g$

$$(f \circ g)(x) = f(g(x)) = f(3x^2) = \cos(3x^2)$$

Step 3: Compare $g \circ f$ and $f \circ g$

$$(g \circ f)(x) = 3\cos^2 x$$

$$(f \circ g)(x) = \cos(3x^2)$$

Step 4: Show they are not equal by counterexample

Take $x = 0$:

$$(g \circ f)(0) = 3\cos^2 0 = 3(1)^2 = 3$$

$$(f \circ g)(0) = \cos(3 \times 0^2) = \cos 0 = 1$$

Since $3 \neq 1$, we have $(g \circ f)(0) \neq (f \circ g)(0)$.

Step 5: Conclusion

Since $(g \circ f)(x) \neq (f \circ g)(x)$ for at least one value of x (here $x = 0$), the two composite functions are not equal.

Therefore:

$$\boxed{g \circ f \neq f \circ g}$$

Quick Tip

Key Points:

- $g \circ f$ means apply f first, then g
- $f \circ g$ means apply g first, then f
- Function composition is generally not commutative
- To prove inequality, a single counterexample is sufficient

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11. Find the general solution of differential equation $ydx + (x - y^2)dy = 0$.

Solution:

Step 1: Identify the form of differential equation

The given equation is:

$$y\,dx + (x - y^2)\,dy = 0$$

This can be written as:

$$y\,dx + x\,dy - y^2\,dy = 0$$

Step 2: Rearrange terms

$$y\,dx + x\,dy = y^2\,dy$$

Step 3: Recognize the left-hand side

The left-hand side $y\,dx + x\,dy$ is the differential of xy:

$$d(xy) = x\,dy + y\,dx$$

Therefore:

$$d(xy) = y^2\,dy$$

Step 4: Integrate both sides

$$\int d(xy) = \int y^2\,dy$$

$$xy = \frac{y^3}{3} + C$$

where C is an arbitrary constant.

Step 5: Write the general solution

$$\boxed{xy = \frac{y^3}{3} + C}$$

or equivalently:

$$xy - \frac{y^3}{3} = C$$

Alternative form:

$$3xy - y^3 = 3C \quad \text{(multiplying both sides by 3)}$$

Let $K = 3C$, then:

$$3xy - y^3 = K$$

Quick Tip

Key Points:

- Recognize exact differentials: $d(xy) = x\,dy + y\,dx$
- Rearrange equation to identify such forms
- Integrate both sides after recognizing exact differential

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12. Prove that (4, 4, 2), (3, 5, 2) and (-1, -1, 2) are vertices of a right angle triangle.

Solution:

Let the points be:

$$A(4,4,2), \quad B(3,5,2), \quad C(-1,-1,2)$$

Step 1: Find the vectors representing the sides

$$\vec{AB} = B - A = (3-4, 5-4, 2-2) = (-1, 1, 0)$$

$$\vec{BC} = C - B = (-1-3, -1-5, 2-2) = (-4, -6, 0)$$

$$\vec{CA} = A - C = (4-(-1), 4-(-1), 2-2) = (5, 5, 0)$$

Step 2: Calculate the dot products

For a right angle triangle, one of the dot products of two sides should be zero.

Check $\vec{AB} \cdot \vec{BC}$:

$$\vec{AB} \cdot \vec{BC} = (-1)(-4) + (1)(-6) + (0)(0) = 4 - 6 + 0 = -2 \neq 0$$

Check $\vec{BC} \cdot \vec{CA}$:

$$\vec{BC} \cdot \vec{CA} = (-4)(5) + (-6)(5) + (0)(0) = -20 - 30 + 0 = -50 \neq 0$$

Check $\vec{CA} \cdot \vec{AB}$:

$$\vec{CA} \cdot \vec{AB} = (5)(-1) + (5)(1) + (0)(0) = -5 + 5 + 0 = 0$$

Step 3: Interpret the result

Since $\vec{CA} \cdot \vec{AB} = 0$, vectors $\vec{CA}$ and $\vec{AB}$ are perpendicular to each other.

Therefore, angle at A (between CA and AB) is $90°$.

Step 4: Conclusion

Since one angle is $90°$, the triangle formed by points A(4, 4, 2), B(3, 5, 2), and C(-1, -1, 2) is a **right-angled triangle** with the right angle at vertex A.

$$\boxed{\text{The points form a right-angled triangle with right angle at A}}$$

Verification using Pythagoras theorem:

$$|AB| = \sqrt{(-1)^2 + 1^2 + 0^2} = \sqrt{1+1} = \sqrt{2}$$

$$|AC| = \sqrt{5^2 + 5^2 + 0^2} = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$$

$$|BC| = \sqrt{(-4)^2 + (-6)^2 + 0^2} = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$$

Check: $AB^2 + AC^2 = (\sqrt{2})^2 + (5\sqrt{2})^2 = 2 + 50 = 52 = BC^2$

Thus, Pythagoras theorem holds, confirming the right angle at A.

Quick Tip

Key Points:

- For right angle triangle, dot product of two sides = 0
- Alternatively, use Pythagoras theorem: $a^2 + b^2 = c^2$
- All points have same z-coordinate (2), so triangle lies in a plane parallel to XY-plane

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13.

$$|\vec{b}| = 4 \text{ and } |\vec{c}| = 2, \text{ then find the value of } \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}.$$

If three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfying the condition $\vec{a}+\vec{b}+\vec{c}=0$. If $|\vec{a}|=3$,

$$|\vec{b}|=4 \text{ and } |\vec{c}|=2, \text{ then find the value of } \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}.$$

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Solution:

Step 1: Use the given condition $\vec{a}+\vec{b}+\vec{c}=0$

From $\vec{a}+\vec{b}+\vec{c}=0$, we have:

$$\vec{a}+\vec{b}=-\vec{c}$$

Step 2: Square both sides

$$|\vec{a}+\vec{b}|^2=|-\vec{c}|^2=|\vec{c}|^2$$

Step 3: Expand using dot product

$$|\vec{a}+\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2+2(\vec{a}\cdot\vec{b})$$

Therefore:

$$|\vec{a}|^2+|\vec{b}|^2+2(\vec{a}\cdot\vec{b})=|\vec{c}|^2$$

Step 4: Substitute the given magnitudes

$$3^2+4^2+2(\vec{a}\cdot\vec{b})=2^2$$

$$9+16+2(\vec{a}\cdot\vec{b})=4$$

$$25+2(\vec{a}\cdot\vec{b})=4$$

$$2(\vec{a}\cdot\vec{b})=4-25=-21$$

$$\vec{a}\cdot\vec{b}=-\frac{21}{2}$$

Step 5: Similarly, use $\vec{b}+\vec{c}=-\vec{a}$

$$|\vec{b}+\vec{c}|^2=|-\vec{a}|^2=|\vec{a}|^2$$

$$|\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{b} \cdot \vec{c}) = |\vec{a}|^2$$

$$4^2 + 2^2 + 2(\vec{b} \cdot \vec{c}) = 3^2$$

$$16 + 4 + 2(\vec{b} \cdot \vec{c}) = 9$$

$$20 + 2(\vec{b} \cdot \vec{c}) = 9$$

$$2(\vec{b} \cdot \vec{c}) = 9 - 20 = -11$$

$$\vec{b} \cdot \vec{c} = -\frac{11}{2}$$

Step 6: Similarly, use $\vec{c} + \vec{a} = -\vec{b}$

$$|\vec{c} + \vec{a}|^2 = |-\vec{b}|^2 = |\vec{b}|^2$$

$$|\vec{c}|^2 + |\vec{a}|^2 + 2(\vec{c} \cdot \vec{a}) = |\vec{b}|^2$$

$$2^2 + 3^2 + 2(\vec{c} \cdot \vec{a}) = 4^2$$

$$4 + 9 + 2(\vec{c} \cdot \vec{a}) = 16$$

$$13 + 2(\vec{c} \cdot \vec{a}) = 16$$

$$2(\vec{c} \cdot \vec{a}) = 16 - 13 = 3$$

$$\vec{c} \cdot \vec{a} = \frac{3}{2}$$

Step 7: Find the required sum

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \left(-\frac{21}{2}\right) + \left(-\frac{11}{2}\right) + \left(\frac{3}{2}\right)$$

$$= \frac{-21 - 11 + 3}{2} = \frac{-29}{2}$$

Step 8: Final answer

$$\boxed{-\frac{29}{2}}$$

Alternative Method:

Square $\vec{a} + \vec{b} + \vec{c} = 0$:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$9 + 16 + 4 + 2S = 0$$

$$29 + 2S = 0$$

$$S = -\frac{29}{2}$$

where $S = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

This is a more direct approach!

Quick Tip

Key Formula:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$

When $\vec{a} + \vec{b} + \vec{c} = 0$, LHS = 0, so:

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2S = 0$$

$$S = -\frac{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2}{2}$$

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14. The radius of an air bubble is increasing at the rate of $\frac{1}{2}$ cm/s. At what rate is the volume of the bubble increasing while the radius is 1 cm?

Solution:

Step 1: Identify given information

Let r be the radius of the air bubble.

$$\frac{dr}{dt} = \frac{1}{2} \text{ cm/s}$$

We need to find $\frac{dV}{dt}$ when $r = 1$ cm.

Step 2: Formula for volume of a sphere

The volume of a spherical bubble is:

$$V = \frac{4}{3}\pi r^3$$

Step 3: Differentiate with respect to time

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt}$$
$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

Step 4: Substitute the given values

When $r = 1$ cm and $\frac{dr}{dt} = \frac{1}{2}$ cm/s:

$$\frac{dV}{dt} = 4\pi(1)^2 \cdot \frac{1}{2}$$
$$\frac{dV}{dt} = 4\pi \cdot \frac{1}{2} = 2\pi$$

Step 5: Final answer

$$\boxed{2\pi\ \textbf{cm}^3/\textbf{s}}$$

Interpretation:

When the radius is 1 cm, the volume of the air bubble is increasing at the rate of 2π cubic centimeters per second (approximately 6.28 cm^3/s).

Quick Tip

Key Steps for Related Rates:

- **Write formula relating variables**
- **Differentiate with respect to time**
- **Substitute known values**
- **Solve for required rate**

For sphere: $V = \frac{4}{3}\pi r^3 \rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

15. Show that the function $f(x) = 7x^2 - 3$ is an increasing function when $x > 0$.

Solution:

Step 1: Recall the condition for increasing function

A function $f(x)$ is said to be increasing in an interval if:

$$f'(x) > 0 \quad \textbf{for all } x \textbf{ in that interval}$$

Step 2: Find the derivative of $f(x)$

$$f(x) = 7x^2 - 3$$
$$f'(x) = \frac{d}{dx}(7x^2 - 3) = 14x$$

Step 3: Check the sign of $f'(x)$ for $x > 0$

For $x > 0$:

$$f'(x) = 14x > 0$$

Since $14x$ is positive for all positive values of x, we have:

$$f'(x) > 0 \quad \forall\, x > 0$$

Step 4: Conclusion

Since $f'(x) > 0$ for all $x > 0$, the function $f(x) = 7x^2 - 3$ is an increasing function when $x > 0$.

$$\boxed{\textbf{The function is increasing for } x > 0}$$

Graphical Interpretation:

The function $f(x) = 7x^2 - 3$ is a parabola opening upwards with vertex at $(0, -3)$. For $x > 0$, as x increases, $f(x)$ increases continuously.

Quick Tip

Key Point:

- **Increasing function means** $f'(x) > 0$
- **For** $f(x) = 7x^2 - 3$**,** $f'(x) = 14x$
- **When** $x > 0$**,** $14x > 0 \rightarrow$ **function is increasing**

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16. Find the unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ **and** $(\vec{a} - \vec{b})$ **where**

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \, \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}.$$

Solution:

Step 1: Find $\vec{a} + \vec{b}$ **and** $\vec{a} - \vec{b}$

Given:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{a} + \vec{b} = (\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k}) = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a} - \vec{b} = (\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = 0\hat{i} - \hat{j} - 2\hat{k} = -\hat{j} - 2\hat{k}$$

Step 2: Find a vector perpendicular to both

A vector perpendicular to both $\vec{a} + \vec{b}$ **and** $\vec{a} - \vec{b}$ **is given by their cross product:**

$$\vec{n} = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$$

Step 3: Compute the cross product

$$\vec{n} = \hat{i}\begin{vmatrix} 3 & 4 \\ -1 & -2 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 3 \\ 0 & -1 \end{vmatrix}$$

$$\vec{n} = \hat{i}\,[(3)(-2) - (4)(-1)] - \hat{j}\,[(2)(-2) - (4)(0)] + \hat{k}\,[(2)(-1) - (3)(0)]$$

$$\vec{n} = \hat{i}\,[-6 + 4] - \hat{j}\,[-4 - 0] + \hat{k}\,[-2 - 0]$$

$$\vec{n} = \hat{i}(-2) - \hat{j}(-4) + \hat{k}(-2)$$

$$\vec{n} = -2\hat{i} + 4\hat{j} - 2\hat{k}$$

Step 4: Find the magnitude of $\vec{n}$

$$|\vec{n}| = \sqrt{(-2)^2 + 4^2 + (-2)^2} = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6}$$

Step 5: Find the unit vector

The unit vector perpendicular to both given vectors is:

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{-2\hat{i} + 4\hat{j} - 2\hat{k}}{2\sqrt{6}}$$

$$\hat{n} = \frac{-\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}}$$

Step 6: Final answer

$$\boxed{\frac{-\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{6}}}$$

Note: The negative of this vector is also a unit vector perpendicular to both given vectors.

Quick Tip

Key Points:

- **Vector perpendicular to two vectors = their cross product**
- **Unit vector =** $\frac{\vec{n}}{|\vec{n}|}$
- **Both** $\hat{n}$ **and** $-\hat{n}$ **are perpendicular unit vectors**
