

# UP Board Class 10 Mathematics Question Paper with Solutions(Memory Based)

Time Allowed :3 Hour	Maximum Marks :60	Total Questions :24
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## General Instructions

Read the following instructions very carefully and strictly follow them:

- Answers to this Paper must be written on the paper provided separately.
- You will not be allowed to write during the first 15 minutes
- This time is to be spent in reading the question paper.
- The time given at the head of this Paper is the time allowed for writing the answers,
- The paper has four Sections.
- Section A is compulsory - All questions in Section A must be answered.
- You must attempt one question from each of the Sections B, C and D and one other question from any Section of your choice.

1. Find the HCF and LCM of 96 and 404 by the prime factorisation method and verify that  $\text{HCF} \times \text{LCM} = \text{product of the two numbers}$ .

**Solution:**

**Concept:**

- Prime factorisation expresses numbers as a product of prime numbers.
- HCF (Highest Common Factor) = product of common primes with the smallest powers.
- LCM (Least Common Multiple) = product of all primes with the greatest powers.
- Verification property:

$$\text{HCF} \times \text{LCM} = \text{Product of the two numbers}$$

**Step 1: Prime factorisation of 96.**

$$96 = 2 \times 48 = 2^2 \times 24 = 2^3 \times 12 = 2^4 \times 6 = 2^5 \times 3$$

$$\therefore 96 = 2^5 \times 3$$

**Step 2: Prime factorisation of 404.**

$$404 = 2 \times 202 = 2^2 \times 101$$

$$\therefore 404 = 2^2 \times 101$$

**Step 3: Find the HCF.**

Common prime factor = 2

Smallest power of 2 in both numbers =  $2^2$

$$\text{HCF} = 2^2 = 4$$

**Step 4: Find the LCM.**

Take highest powers of all primes:

$$\text{LCM} = 2^5 \times 3 \times 101$$

$$= 32 \times 3 \times 101$$

$$= 96 \times 101 = 9696$$

**Step 5: Verification.**

$$\text{HCF} \times \text{LCM} = 4 \times 9696 = 38784$$

Product of the numbers:

$$96 \times 404 = 38784$$

$\therefore \text{HCF} \times \text{LCM} = \text{Product of the numbers (Verified)}$ .

#### Quick Tip

For two numbers: - HCF uses minimum powers of common primes. - LCM uses maximum powers of all primes. Always verify using:  $\text{HCF} \times \text{LCM} = a \times b$ .

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**2. Prove that  $\sqrt{3}$  (or  $\sqrt{5}$ ) is an irrational number.**

**Correct Answer:**  $\sqrt{3}$  and  $\sqrt{5}$  are irrational numbers.

**Solution: Concept:** A number is irrational if it cannot be written in the form  $\frac{p}{q}$ , where  $p, q$  are integers and  $q \neq 0$ . To prove a number is irrational, we commonly use the **proof by contradiction** method:

- Assume the number is rational.
- Write it in lowest terms.
- Show a contradiction (both numerator and denominator divisible by same number).

**Step 1:** Assume  $\sqrt{3}$  is rational.

Suppose  $\sqrt{3} = \frac{p}{q}$ , where  $p$  and  $q$  are coprime integers (no common factor).

**Step 2:** Square both sides.

$$3 = \frac{p^2}{q^2} \Rightarrow p^2 = 3q^2.$$

This means  $p^2$  is divisible by 3, so  $p$  must also be divisible by 3.

**Step 3:** Substitute  $p = 3k$ .

Let  $p = 3k$ . Then:

$$(3k)^2 = 3q^2 \Rightarrow 9k^2 = 3q^2 \Rightarrow q^2 = 3k^2.$$

Thus,  $q^2$  is divisible by 3, so  $q$  is also divisible by 3.

**Step 4:** Contradiction.

Both  $p$  and  $q$  are divisible by 3, contradicting the assumption that they are coprime. Hence, our assumption is false.

**Conclusion:** Therefore,  $\sqrt{3}$  is irrational.

**Similarly for  $\sqrt{5}$ :**

Assume  $\sqrt{5} = \frac{p}{q}$  in lowest terms.

$$5 = \frac{p^2}{q^2} \Rightarrow p^2 = 5q^2.$$

This implies  $p$  is divisible by 5. Let  $p = 5k$ . Then:

$$25k^2 = 5q^2 \Rightarrow q^2 = 5k^2.$$

So  $q$  is also divisible by 5 — again a contradiction. Hence,  $\sqrt{5}$  is irrational.

#### Quick Tip

To prove square roots of primes are irrational, assume a rational form and show both numerator and denominator become divisible by the same prime — creating a contradiction.

**3. Find the value of  $k$  for which the quadratic equation  $2x^2 + kx + 3 = 0$  has two equal real roots.**

**Correct Answer:**  $k = \pm 2\sqrt{6}$

**Solution: Concept:** A quadratic equation  $ax^2 + bx + c = 0$  has:

- Two equal real roots when the discriminant  $D = 0$
- Where  $D = b^2 - 4ac$

Thus, we use the discriminant condition to determine  $k$ .

**Step 1:** Identify coefficients.

Given equation:

$$2x^2 + kx + 3 = 0$$

So,

$$a = 2, \quad b = k, \quad c = 3.$$

**Step 2:** Use the condition for equal roots.

For equal real roots:

$$D = b^2 - 4ac = 0.$$

**Step 3:** Substitute values.

$$k^2 - 4(2)(3) = 0$$

$$k^2 - 24 = 0.$$

**Step 4:** Solve for  $k$ .

$$k^2 = 24$$

$$k = \pm\sqrt{24} = \pm 2\sqrt{6}.$$

**Conclusion:**

The quadratic equation has equal real roots when:

$$k = \pm 2\sqrt{6}.$$

#### Quick Tip

For equal roots in a quadratic equation, always set the discriminant  $b^2 - 4ac = 0$ . This is the fastest method in exams.

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**4. Solve the pair of linear equations  $2x + 3y = 11$  and  $2x - 4y = -24$  using the substitution or elimination method.**

**Correct Answer:**  $x = -1, y = \frac{13}{3}$

**Solution: Concept:** A pair of linear equations can be solved using:

- Substitution method
- Elimination method

Here, we use the **elimination method** to remove one variable.

**Step 1:** Write the given equations.

$$(1) \quad 2x + 3y = 11$$

$$(2) \quad 2x - 4y = -24$$

**Step 2:** Eliminate  $x$  by subtraction.

Subtract equation (2) from equation (1):

$$(2x + 3y) - (2x - 4y) = 11 - (-24)$$

$$2x + 3y - 2x + 4y = 35$$

$$7y = 35$$

**Step 3:** Solve for  $y$ .

$$y = \frac{35}{7} = 5.$$

**Step 4:** Substitute  $y = 5$  into equation (1).

$$2x + 3(5) = 11$$

$$2x + 15 = 11$$

$$2x = -4$$

$$x = -2.$$

**Conclusion:**

The solution of the pair of linear equations is:

$$x = -2, \quad y = 5.$$

**Note:** The computed values satisfy both equations.

#### Quick Tip

If the coefficients of one variable are already equal, subtract the equations directly to eliminate that variable quickly.

**5.** If  $\tan(A + B) = \sqrt{3}$  and  $\tan(A - B) = \frac{1}{\sqrt{3}}$ , find the values of  $A$  and  $B$ .

**Correct Answer:**  $A = 45^\circ$ ,  $B = 15^\circ$

**Solution: Concept:** We use standard tangent values:

- $\tan 60^\circ = \sqrt{3}$
- $\tan 30^\circ = \frac{1}{\sqrt{3}}$

Thus, we convert the trigonometric equations into angle equations and solve simultaneously.

**Step 1:** Use known tangent values.

$$\tan(A + B) = \sqrt{3} \Rightarrow A + B = 60^\circ \quad (\text{principal value})$$

$$\tan(A - B) = \frac{1}{\sqrt{3}} \Rightarrow A - B = 30^\circ$$

**Step 2:** Solve the system of equations.

Add both equations:

$$(A + B) + (A - B) = 60^\circ + 30^\circ$$

$$2A = 90^\circ \Rightarrow A = 45^\circ.$$

**Step 3:** Substitute  $A$  back.

$$A + B = 60^\circ$$
$$45^\circ + B = 60^\circ \Rightarrow B = 15^\circ.$$

**Conclusion:**

The required angles are:

$$A = 45^\circ, \quad B = 15^\circ.$$

#### Quick Tip

Memorize key tangent values:  $\tan 30^\circ = \frac{1}{\sqrt{3}}$ ,  $\tan 45^\circ = 1$ ,  $\tan 60^\circ = \sqrt{3}$ . They help solve trig equations quickly.

**6. Prove the identity:**  $\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2 \sec A$ .

**Correct Answer:** LHS = RHS =  $2 \sec A$  (Identity proved)

**Solution: Concept:** To prove trigonometric identities:

- Start from the more complicated side (usually LHS).
- Rationalize or combine terms using algebra.
- Use identities like  $\sin^2 A + \cos^2 A = 1$  and  $\sec A = \frac{1}{\cos A}$ .

**Step 1:** Start with LHS.

$$\text{LHS} = \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}.$$

**Step 2:** Rationalize the first term.

Multiply numerator and denominator by  $1 - \sin A$ :

$$\frac{\cos A}{1 + \sin A} \cdot \frac{1 - \sin A}{1 - \sin A} = \frac{\cos A(1 - \sin A)}{1 - \sin^2 A}.$$

**Step 3:** Use identity  $1 - \sin^2 A = \cos^2 A$ .

$$= \frac{\cos A(1 - \sin A)}{\cos^2 A} = \frac{1 - \sin A}{\cos A}.$$

**Step 4:** Substitute back into LHS.

$$\text{LHS} = \frac{1 - \sin A}{\cos A} + \frac{1 + \sin A}{\cos A}.$$

**Step 5:** Combine the fractions.

$$\text{LHS} = \frac{(1 - \sin A) + (1 + \sin A)}{\cos A} = \frac{2}{\cos A}.$$

**Step 6:** Convert to secant.

$$\frac{2}{\cos A} = 2 \sec A.$$

**Conclusion:**

$$\text{LHS} = \text{RHS} = 2 \sec A.$$

Hence, the identity is proved.

#### Quick Tip

When expressions contain  $1 \pm \sin A$  or  $1 \pm \cos A$ , try rationalization using conjugates to simplify quickly.

**7. A 1.5 m tall boy is standing at some distance from a 30 m tall building; find the distance he walked towards the building if the angle of elevation of the top increases from  $30^\circ$  to  $60^\circ$ .**

**Correct Answer:**  $10\sqrt{3}$  m (approximately 17.32 m)

**Solution: Concept:** In height and distance problems:

- Use  $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$ .
- Effective height = building height - boy's height.

**Step 1: Find effective height.**

Building height = 30 m, boy's height = 1.5 m

$$\text{Effective height} = 30 - 1.5 = 28.5 \text{ m.}$$

**Step 2: Let initial distance be  $x$ .**

When angle of elevation =  $30^\circ$ :

$$\begin{aligned}\tan 30^\circ &= \frac{28.5}{x} \\ \frac{1}{\sqrt{3}} &= \frac{28.5}{x} \Rightarrow x = 28.5\sqrt{3}.\end{aligned}$$

**Step 3: Let new distance be  $y$ .**

When angle becomes  $60^\circ$ :

$$\begin{aligned}\tan 60^\circ &= \frac{28.5}{y} \\ \sqrt{3} &= \frac{28.5}{y} \Rightarrow y = \frac{28.5}{\sqrt{3}}.\end{aligned}$$

**Step 4: Distance walked towards building.**

$$\text{Distance walked} = x - y = 28.5\sqrt{3} - \frac{28.5}{\sqrt{3}}.$$

Take common factor:

$$= 28.5 \left( \frac{3-1}{\sqrt{3}} \right) = 28.5 \cdot \frac{2}{\sqrt{3}} = \frac{57}{\sqrt{3}}.$$

Rationalizing:

$$= \frac{57\sqrt{3}}{3} = 19\sqrt{3}.$$

**Conclusion:**

The boy walked:

$$19\sqrt{3} \text{ m} \approx 32.9 \text{ m}.$$

### Quick Tip

Always subtract observer height from object height in elevation problems to avoid common mistakes.

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**8. Prove that the lengths of tangents drawn from an external point to a circle are equal.**

**Correct Answer:** Tangents drawn from an external point to a circle are equal in length.

**Solution: Concept:** We use properties of tangents and triangles:

- A tangent is perpendicular to the radius at the point of contact.
- If two right triangles have equal hypotenuse and one equal side, they are congruent (RHS congruence).

**Step 1: Construct the figure.**

Let a circle have center  $O$ , and let  $P$  be an external point. Draw tangents  $PA$  and  $PB$  touching the circle at points  $A$  and  $B$ .

**Step 2: Join radii to points of contact.**

Join  $OA$  and  $OB$ . Since radius is perpendicular to tangent at the point of contact:

$$OA \perp PA, \quad OB \perp PB.$$

Thus,  $\triangle OAP$  and  $\triangle OBP$  are right triangles.

**Step 3: Compare the triangles.**

In triangles  $\triangle OAP$  and  $\triangle OBP$ :

- $OA = OB$  (radii of the same circle)
- $OP = OP$  (common side)
- $\angle OAP = \angle OBP = 90^\circ$

**Step 4: Apply RHS congruence.**

By RHS congruence rule:

$$\triangle OAP \cong \triangle OBP.$$

**Step 5: Corresponding sides are equal.**

Therefore, corresponding tangent lengths are equal:

$$PA = PB.$$

**Conclusion:**

The lengths of tangents drawn from an external point to a circle are equal.

#### Quick Tip

In circle geometry proofs, always connect the center to tangent points — it reveals right triangles and makes congruence proofs easier.

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**9. A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm; find the height of the cylinder.**

**Correct Answer:** 2.744 cm (approx.)

**Solution: Concept:** When a solid is melted and recast into another shape:

- Volume remains constant.

Formulas used:

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3, \quad \text{Volume of cylinder} = \pi R^2 h.$$

**Step 1: Equate the volumes.**

Let height of cylinder =  $h$ . Given:

$$r = 4.2 \text{ cm}, \quad R = 6 \text{ cm}.$$

$$\frac{4}{3}\pi(4.2)^3 = \pi(6)^2 h.$$

**Step 2: Cancel  $\pi$  and simplify.**

$$\frac{4}{3}(4.2)^3 = 36h.$$

**Step 3: Evaluate  $(4.2)^3$ .**

$$4.2^3 = 74.088.$$

$$\frac{4}{3} \times 74.088 = 98.784.$$

**Step 4: Solve for  $h$ .**

$$98.784 = 36h \Rightarrow h = \frac{98.784}{36}.$$

$$h \approx 2.744 \text{ cm}.$$

**Conclusion:**

The height of the cylinder is approximately:

$2.74 \text{ cm}$ .

**Quick Tip**

In recasting problems, always equate volumes and cancel  $\pi$  early to simplify calculations.

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