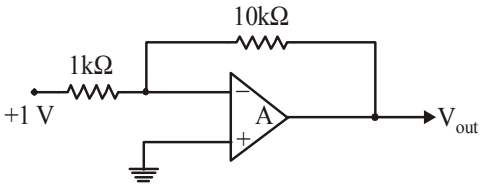
SOLVED PAPER VITEEE 2008

PART - I (PHYSICS)

1. Two beams of light will not give rise to an interference patern, if
   (a) they are coherent
   (b) they have the same wavelength
   (c) they are linearly polarized perpendicular to each other
   (d) they are not monochromatic
2. A slit of width 'a' is illuminated with a monochromatic light of wavelength λ from a distant source and the diffraction pattern is observed on a screen placed at a distance 'D' from the slit. To increase the width of the central maximum one should
   (a) decrease D
   (b) decrease a
   (c) decrease λ
   (d) the width cannot be changed
3. A thin film of soap solution (n = 1.4) lies on the top of a glass plate (n = 1.5). When visible light is incident almost normal to the plate, two adjacent reflection maxima are observed at two wavelengths 420 and 630 nm. The minimum thickness of the soap solution is
   (a) 420 nm (b) 450 nm
   (c) 630 nm (d) 1260 nm
4. If the speed of a wave doubles as it passes from shallow water into deeper water, its wavelength will be
   (a) unchanged (b) halved
   (c) doubled (d) quadrupled
5. A light whose frequency is equal to 6×10^{14} Hz is incident on a metal whose work function is 2eV. $\left[h = 6.63 \times 10^{-34}\ \text{Js}\right]$ $\left[1\text{eV} = 1.6 \times 10^{-19}\ \text{J}\right]$
   The maximum energy of the electrons emitted will be
   (a) 2.49 eV (b) 4.49 eV
   (c) 0.49 eV (d) 5.49 eV
6. An electron microscope is used to probe the atomic arrangements to a resolution of 5 Å. What should be the electric potential to which the electrons need to be accelerated ?
   (a) 2.5 V (b) 5 V
   (c) 2.5 kV (d) 5 kV
7. Which phenomenon best supports the theory that matter has a wave nature ?
   (a) Electron momentum
   (b) Electron diffraction
   (c) Photon momentum
   (d) Photon diffraction
8. The radioactivity of a certain material drops to $\frac{1}{16}$ of the initial value in 2 hours. The half life of this radionuclide is
   (a) 10 min (b) 20 min
   (c) 30 min (d) 40 min
9. An observer 'A' sees an asteroid with a radioactive element moving by at a speed = 0.3 c and measures the radioactivity decay time to be T_A. Another observer 'B' is moving with the asteroid and measures its decay time as T_B. Then T_A and T_B are related as below
   (a) $T_B < T_A$
   (b) $T_A = T_B$
   (c) $T_B > T_A$
   (d) Either (A) or (C) depending on whether the asteroid is approaching or moving away from A
10. ^{234}U has 92 protons and 234 nucleons total in its nucleus. It decays by emitting an alpha particle. After the decay it becomes
    (a) ^{232}U (b) ^{232}Pa
    (c) ^{230}Th (d) ^{230}Ra
11. K_α and K_β x-rays are emitted when there is a transition of electron between the levels
    (a) n = 2 to n = 1 and n = 3 to n = 1 respectively
    (b) n = 2 to n = 1 and n = 3 to n = 2 respectively
    (c) n = 3 to n = 2 and n = 4 to n = 2 respectively
    (d) n = 3 to n = 2 and n = 4 to n = 3 respectively

12. A certain radioactive material ${}_{Z}X^{A}$ starts emitting α and β particles successively such that the end product is ${}_{Z-3}Y^{A-8}$. The number of α and β particles emitted are
(a) 4 and 3 respectively
(b) 2 and 1 respectively
(c) 3 and 4 respectively
(d) 3 and 8 respectively

13. 

In the circuit shown above, an input of 1V is fed into the inverting input of an ideal Op-amp A. The output signal V_{out} will be
(a) +10 V (b) –10 V
(c) 0 V (d) infinity

14. When a solid with a band gap has a donor level just below its empty energy band, the solid is
(a) an insulator
(b) a conductor
(c) a p-type semiconductor
(d) an n-type semiconductor

15. A p-n junction has acceptor impurity concentration of 10^{17} cm^{-3} in the p-side and donor impurity concentration of 10^{16} cm^{-3} in the n-side. What is the contact potential at the junction (kT = thermal energy, intrinsic carrier concentration $n_i = 1.4\times10^{10}$ cm^{-3}) ?
(a) (kT/e) ln (4×10^{12})
(b) (kT/e) ln (2.5×10^{23})
(c) (kT/e) ln (10^{23})
(d) (kT/e) ln (10^{9})

16. A Zener diode has a contact potential of 1V in the absence of biasing. It undergoes Zener breakdown for an electric field of 10^6 V/m at the depletion region of p-n junction. If the width of the depletion region is 2.5 μm, what should be the reverse biased potential for the Zener breakdown to occur ?
(a) 3.5 V (b) 2.5 V
(c) 1.5 V (d) 0.5 V

17. In Colpitt oscillator the feedback network consists of
(a) two inductors and a capacitor
(b) two capacitors and an inductor
(c) three pairs of RC circuit
(d) three pairs of RL circuit

18. The reverse saturation of p-n diode
(a) depends on doping concentrations
(b) depends on diffusion lengths of carriers
(c) depends on the doping concentrations and diffusion lengths
(d) depends on the doping concentrations, diffusion length and device temperature

19. A radio station has two channels. One is AM at 1020 kHz and the other FM at 89.5 MHz. For good results you will use
(a) longer antenna for the AM channel and shorter for the FM
(b) shorter antenna for the AM channel and longer for the FM
(c) same length antenna will work for both
(d) information given is not enough to say which one to use for which

20. The communication using optical fibers is based on the principle of
(a) total internal reflection
(b) Brewster angle
(c) polarization
(d) resonance

21. In nature, the electric charge of any system is always equal to
(a) half integral multiple of the least amount of charge
(b) zero
(c) square of the least amount of charge
(d) integral multiple of the least amount of charge

22. The energy stored in the capacitor as shown in Fig. (a) is 4.5×10^{-6} J. If the battery is replaced by another capacitor of 900 pF as shown in Fig. (b), then the total energy of system is

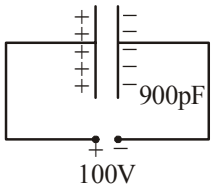


Fig. (a)

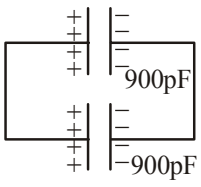


Fig. (b)

(a) 4.5×10^{-6} J (b) 2.25×10^{-6} J
(c) zero (d) 9×10^{-6} J

23. Equal amounts of a metal are converted into cylindrical wires of different lengths (L) and cross-sectional area (A). The wire with the maximum resistance is the one, which has
(a) length = L and area = A
(b) length = $\frac{L}{2}$ and area = 2A
(c) length = 2L and area = $\frac{A}{2}$
(d) all have the same resistance, as the amount of the metal is the same

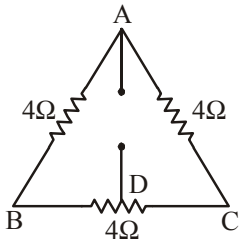
24. If the force exerted by an electric dipole on a charge q at a distance of 1m is F, the force at a point 2m away in the same direction will be
(a) $\frac{F}{2}$ (b) $\frac{F}{4}$
(c) $\frac{F}{6}$ (d) $\frac{F}{8}$

25. A solid sphere of radius R_1 and volume charge density $\rho = \frac{\rho_0}{r}$ is enclosed by a hollow sphere of radius R_2 with negative surface charge density σ, such that the total charge in the system is zero. ρ_0 is a positive constant and r is the distance from the centre of the sphere. The ratio $\frac{R_2}{R_1}$ is
(a) $\frac{\sigma}{\rho_0}$ (b) $\sqrt{2\sigma/\rho_0}$
(c) $\sqrt{\rho_0/(2\sigma)}$ (d) $\frac{\rho_0}{\sigma}$

26. A solid spherical conductor of radius R has a spherical cavity of radius a (a < R) at its centre. A charge + Q is kept at the centre. The charge at the inner surface, outer surface and at a position r (a < r < R) are respectively
(a) + Q, – Q, 0 (b) – Q, + Q, 0
(c) 0, – Q, 0 (d) + Q, 0, 0

27. A cylindrical capacitor has charge Q and length L. If both the charge and length of the capacitor are doubled, by keeping other parameters fixed, the energy stored in the capacitor
(a) remains same
(b) increases two times
(c) decreases two times
(d) increases four times

28. Three resistances of 4 Ω each are connected as shown in figure. If the point D divides the resistance into two equal halves, the resistance between point A and D will be



(a) 12 Ω (b) 6 Ω
(c) 3 Ω (d) $\frac{1}{3}$ Ω

29. The resistance of a metal increases with increasing temperature because
(a) the collisions of the conducting electrons with the electrons increase
(b) the collisions of the conducting electrons with the lattice consisting of the ions of the metal increase
(c) the number of conduction electrons decreases
(d) the number of conduction electrons increases

30. In the absence of applied potential, the electric current flowing through a metallic wire is zero because
(a) the electrons remain stationary
(b) the electrons are drifted in random direction with a speed of the order of 10^{-2} cm/s
(c) the electrons move in random direction with a speed of the order close to that of velocity of light
(d) electrons and ions move in opposite direction

31. A meter bridge is used to determine the resistance of an unknown wire by measuring the balance point length *l*. If the wire is replaced by another wire of same material but with double the length and half the thickness, the balancing point is expected to be
(a) $\frac{1}{8\ell}$ (b) $\frac{1}{4\ell}$
(c) 8ℓ (d) 16ℓ

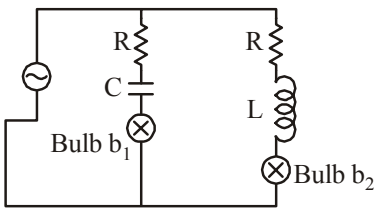
32. Identify the INCORRECT statement regarding a superconducting wire
(a) transport current flows through its surface
(b) transport current flows through the entire area of cross-section of the wire
(c) it exhibits zero electrical resistivity and expels applied magnetic field
(d) it is used to produce large magnetic field

33. A sample of HCl gas is placed in an electric field 3×10^4 NC^{-1}. The dipole moment of each HCl molecule is 6×10^{-30}cm. The maximum torque that can act on a molecule is
(a) $2\times10^{-34}\,C^2Nm^{-1}$ (b) $2\times10^{-34}\,Nm$
(c) $18\times10^{-26}\,Nm$ (d) $0.5\times10^{34}\,C^{-2}\,Nm^{-1}$

34. When a metallic plate swings between the poles of a magnet
(a) no effect on the plate
(b) eddy currents are set up inside the plate and the direction of the current is along the motion of the plate
(c) eddy currents are set up inside the plate and the direction of the current oppose the motion of the plate
(d) eddy currents are set up inside the plate

35. When an electrical appliance is switched on, it responds almost immediately, because
(a) the electrons in the connecting wires move with the speed of light
(b) the electrical signal is carried by electromagnetic waves moving with the speed of light
(c) the electrons move with the speed which is close to but less than speed of light
(d) the electrons are stagnant.

36. Two identical incandescent light bulbs are connected as shown in the Figure. When the circuit is an AC voltage source of frequency f, which of the following observations will be correct ?



(a) both bulbs will glow alternatively
(b) both bulbs will glow with same brightness provided frequency $f=\frac{1}{2\pi\sqrt{(1/LC)}}$
(c) bulb b_1 will light up initially and goes off, bulb b_2 will be ON constantly
(d) bulb b_1 will blink and bulb b_2 will be ON constantly

37. A transformer rated at 10 kW is used to connect a 5kV transmission line to a 240V circuit. The ratio of turns in the windings of the transformer is
(a) 5 (b) 20.8
(c) 104 (d) 40

38. Three solenoid coils of same dimension, same number of turns and same number of layers of winding are taken. Coil 1 with inductance L_1 was wound using a Mn wire of resistance $11\,\Omega$/m; Coil 2 with inductance L_2 was wound using the similar wire but the direction of winding was reversed in each layer; Coil 3 with inductance L_3 was wound using a superconducting wire. The self inductance of the coils L_1, L_2, L_3 are
(a) $L_1=L_2=L_3$ (b) $L_1=L_2;\ L_3=0$
(c) $L_1=L_3;\ L_2=0$ (d) $L_1>L_2>L_3$

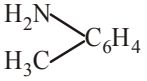
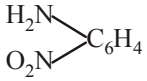
39. Light travels with a speed of 2×10^8 m/s in crown glass of refractive index 1.5. What is the speed of light in dense flint glass of refractive index 1.8 ?
(a) 1.33×10^8 m/s (b) 1.67×10^8 m/s
(c) 2.0×10^8 m/s (d) 3.0×10^8 m/s

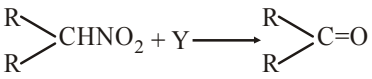
40. A parallel beam of fast moving electrons is incident normally on a narrow slit. A screen is placed at a large distance from the slit. If the speed of the electrons is increased, which of the following statement is correct ?
(a) diffraction pattern is not observed on the screen in the case of electrons
(b) the angular width of the central maximum of the diffraction pattern will increase
(c) the angular width of the central maximum will decrease
(d) the angular width of the central maximum will remain the same

PART - II (CHEMISTRY)

41. $CH_3CH_3 + HNO_3 \xrightarrow{675\ K}$
(a) $CH_3CH_2NO_2$
(b) $CH_3CH_2NO_2 + CH_3NO_2$
(c) $2CH_3NO_2$
(d) $CH_2 = CH_2$

42. When acetamide is hydrolysed by boiling with acid, the product obtained is :
(a) acetic acid (b) ethyl amine
(c) ethanol (d) acetamide

43. Which will not go for diazotization ?
(a) $C_6H_5NH_2$ (b) $C_6H_5CH_2NH_2$
(c)  (d) 

44. Secondary nitroalkanes can be converted into ketones by using Y. Identify the Y from the following

(a) Aqueous HCl (b) Aqueous NaOH
(c) $KMnO_4$ (d) CO

45. Alkyl cyanides undergo Stephen reduction to produce
(a) aldehyde (b) secondary amine
(c) primary amine (d) amide

46. The continuous phase contains the dispersed phase throughout, Example is
(a) Water in milk
(b) Fat in milk
(c) Water droplets in mist
(d) Oil in water

47. The number of hydrogen atoms present in 25.6 g of sucrose ($C_{12}H_{22}O_{11}$) which has a molar mass of 342.3 g is
(a) 22×10^{23} (b) 9.91×10^{23}
(c) 11×10^{23} (d) 44×10^{23}

48. Milk changes after digestion into :
(a) cellulose (b) fructose
(c) glucose (d) lactose

49. Which of the following sets consists only of essential amino acids ?
(a) Alanine, tyrosine, cystine
(b) Leucine, lysine, tryptophane
(c) Alanine, glutamine, lycine
(d) Leucine, proline, glycine

50. Which of the following is ketohexose ?
(a) Glucose (b) Sucrose
(c) Fructose (d) Ribose

51. The oxidation number of oxygen in KO_3, Na_2O_2 is
(a) 3, 2 (b) 1, 0
(c) 0, 1 (d) –0.33, –1

52. Reaction of PCl_3 and PhMgBr would give
(a) bromobenzene
(b) chlorobenzene
(c) triphenylphosphine
(d) dichlorobenzene

53. Which of the following is not a characteristic of transition elements ?
(a) Variable oxidation states
(b) Formation of colored compounds
(c) Formation of interstitial compounds
(d) Natural radioactivity

54. Cl - P - Cl bond angles in PCl_5 molecule are
(a) 120° and 90° (b) 60° and 90°
(c) 60°and 120° (d) 120° and 30°

55. The magnetic moment of a salt containing Zn^{2+} ion is
(a) 0 (b) 1.87
(c) 5.92 (d) 2

56. The number of formula units of calcium flouride CaF_2 present in 146.4 g of CaF_2 are (molar mass of CaF_2 is 78.08 g/mol)
(a) $1.129 \times 10^{24}\ CaF_2$ (b) $1.146 \times 10^{24}\ CaF_2$
(c) $7.808 \times 10^{24}\ CaF_2$ (d) $1.877 \times 10^{24}\ CaF_2$

57. The IUPAC name of the given compound $\left[Co(NH_3)_5Cl\right]Cl_2$ is
(a) pentaamino cobalt chloride chlorate
(b) cobalt pentaamine chloro chloride
(c) pentaamine chloro cobalt (III) chloride
(d) pentaamino cobalt (III) chlorate

58. When SCN^- is added to an aqueous solution containing $Fe(NO_3)_3$,the complex ion produced is
(a) $\left[Fe(OH_2)_2(SCN^-)\right]^{2+}$
(b) $\left[Fe(OH_2)_5(SCN^-)\right]^{2+}$
(c) $\left[Fe(OH_2)_8(SCN^-)\right]^{2+}$
(d) $\left[Fe(OH_2)(SCN^-)\right]^{6+}$

59. Hair dyes contain
(a) copper nitrate (b) gold chloride
(c) silver nitrate (d) copper sulphate

60. Schottky defects occurs mainly in electrovalent compounds where
(a) positive ions and negative ions are of different size
(b) positive ions and negative ions are of same size
(c) positive ions are small and negative ions are big
(d) positive ions are big and negative ions are small

61. The number of unpaired electrons calculated in $\left\{Co(NH_3)_6\right\}^{3+}$ and $\left\{Co(F_6)\right\}^{3-}$ are
(a) 4 and 4 (b) 0 and 2
(c) 2 and 4 (d) 0 and 4

62. The standard free energy change of a reaction is $\Delta G^\circ = -115 kJ$ at 298 K. Calculate the equilibrium constant k_p in log k_p $\left(R = 8.314\ Jk^{-1}\ mol^{-1}\right)$.
(a) 20.16 (b) 2.303
(c) 2.016 (d) 13.83

63. If an endothermic reaction occurs spontaneously at constant temperature T and P, then which of the following is true ?
(a) $\Delta G > 0$ (b) $\Delta H < 0$
(c) $\Delta S > 0$ (d) $\Delta S < 0$

64. If a plot of $\log_{10}C$ versus t gives a straight line for a given reaction, then the reaction is
(a) zero order (b) first order
(c) second order (d) third order

65. A spontaneous process is one in which the system suffers :
(a) no energy change
(b) a lowering of free energy
(c) a lowering of entropy
(d) an increase in internal energy

66. The half life period of a first order reaction is 1 min 40 secs. Calculate its rate constant.
(a) $6.93 \times 10^{-3}\ min^{-1}$ (b) $6.93 \times 10^{-3}\ sec^{-1}$
(c) 6.93×10^{-3} sec (d) 6.93×10^{3} sec

67. The molar conductivities of KCl, NaCl and KNO_3 are 152, 128 and 111 S $cm^2\ mol^{-1}$ respectively. What is the molar conductivity of $NaNO_3$?
(a) 101 S $cm^2\ mol^{-1}$ (b) 87 S $cm^2\ mol^{-1}$
(c) –101 S $cm^2\ mol^{-1}$ (d) –391 S $cm^2\ mol^{-1}$

68. The electrochemical cell stops working after sometime because :
(a) electrode potential of both the electrodes becomes zero
(b) electrode potential of both the electrodes becomes equal
(c) one of the electrodes is eaten away
(d) the cell reaction gets reversed

69. The amount of electricity required to produce one mole of copper from copper sulphate solution will be
(a) 1 Faraday (b) 2.33 Faraday
(c) 2 Faraday (d) 1.33 Faraday

70. Dipping iron article into a strongly alkaline solution of sodium phosphate
(a) does not affect the article
(b) forms $Fe_2O_3 . xH_2O$ on the surface
(c) forms iron phosphate film
(d) forms ferric hydroxide

71. Hydroboration oxidation of 4-methyl-octene would give
(a) 4-methyl octanol
(b) 2-methyl decane
(c) 4-methyl heptanol
(d) 4-methyl-2-octanone

72. When ethyl alcohol is heated with conc. H_2SO_4, the product obtained is :
(a) $CH_3COOC_2H_5$ (b) C_2H_2
(c) C_2H_6 (d) C_2H_4

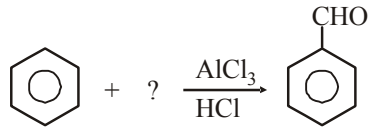
73. Anisole is the product obtained from phenol by the reaction known as
(a) coupling (b) etherification
(c) oxidation (d) esterification

74. Ethylene glycol gives oxalic acid on oxidation with
(a) acidified $K_2Cr_2O_7$
(b) acidified $KMnO_4$
(c) alkaline $KMnO_4$
(d) periodic acid

75. Diamond is hard because
(a) all the four valence electrons are bonded to each carbon atoms by covalent bonds
(b) it is a giant molecule
(c) it is made up of carbon atoms
(d) it cannot be burnt

76. A wittig reaction with an aldehyde gives
(a) ketone compound
(b) a long chain fatty acid
(c) olefin compound
(d) epoxide

77. Cannizaro reaction is given by
(a) HCHO (b) CH_3COCH_3
(c) CH_3CHO (d) CH_3CH_2OH

78. 

Identify the reactant
(a) H_2O (b) HCHO
(c) CO (d) CH_3CHO

79. Maleic acid and Fumaric acids are
(a) Position Isomers (b) Geometric Isomers
(c) Enentiomers (d) Functional Isomers

80. The gas evolved on heating alkali formate with soda-lime is
(a) CO (b) CO_2
(c) Hydrogen (d) water vapor

PART - III (MATHEMATICS)

81. If $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b},\vec{b}$ and $\vec{c}$ being non-parallel. If θ_1 is the angle between $\vec{a}$ and $\vec{b}$ and θ_2 is the angle between $\vec{a}$ and $\vec{c}$, then

(a) $\theta_1=\frac{\pi}{6},\theta_2=\frac{\pi}{3}$ (b) $\theta_1=\frac{\pi}{3},\theta_2=\frac{\pi}{6}$

(c) $\theta_1=\frac{\pi}{2},\theta_2=\frac{\pi}{3}$ (d) $\theta_1=\frac{\pi}{3},\theta_2=\frac{\pi}{2}$

82. The equation $\vec{r}^2-2\vec{r}.\vec{c}+h=0,|\vec{c}|>\sqrt{h}$, represents
(a) circle (b) ellipse
(c) cone (d) sphere

83. The simplified expression of $\sin(\tan^{-1}x)$, for any real number x is given by

(a) $\frac{1}{\sqrt{1+x^2}}$ (b) $\frac{x}{\sqrt{1+x^2}}$

(c) $-\frac{1}{\sqrt{1+x^2}}$ (d) $-\frac{x}{\sqrt{1+x^2}}$

84. If $\left|\frac{z-25}{z-1}\right|=5$, the value of $|z|$
(a) 3 (b) 4
(c) 5 (d) 6

85. Argument of the complex number $\left(\frac{-1-3i}{2+i}\right)$ is
(a) 45° (b) 135°
(c) 225° (d) 240°

86. In a triangle ABC, the sides b and c are the roots of the equation $x^2-61x+820=0$ and $A=\tan^{-1}\left(\frac{4}{3}\right)$, then a^2 is equal to
(a) 1098 (b) 1096
(c) 1097 (d) 1095

87. The shortest distance between the straight lines through the points $A_1=(6,2,2)$ and $A_2=(-4,0,-1)$, in the directions of $(1,-2,2)$ and $(3,-2,-2)$ is
(a) 6 (b) 8
(c) 12 (d) 9

88. The center and radius of the sphere $x^2+y^2+z^2+3x-4z+1=0$ are

(a) $\left(-\frac{3}{2},0,-2\right);\frac{\sqrt{21}}{2}$

(b) $\left(\frac{3}{2},0,2\right);\sqrt{21}$

(c) $\left(-\frac{3}{2},0,2\right);\frac{\sqrt{21}}{2}$

(d) $\left(-\frac{3}{2},2,0\right);\frac{21}{2}$

89. Let A and B are two fixed points in a plane then locus of another point C on the same plane such that CA + CB = constant, (> AB) is
(a) circle (b) ellipse
(c) parabola (d) hyperbola

90. The directrix of the parabola $y^2+4x+3=0$ is

(a) $x-\frac{4}{3}=0$ (b) $x+\frac{1}{4}=0$

(c) $x-\frac{3}{4}=0$ (d) $x-\frac{1}{4}=0$

91. If g (x) is a polynomial satisfying
$g(x)\,g(y)=g(x)+g(y)+g(xy)-2$
for all real x and y and g (2) = 5 then $\lim_{x\to3} g(x)$ is
(a) 9 (b) 10
(c) 25 (d) 20

92. The value of f(0) so that $\frac{(-e^x + 2^x)}{x}$ may be continuous at x = 0 is

(a) $\log\left(\frac{1}{2}\right)$ (b) 0

(c) 4 (d) $-1 + \log 2$

93. Let [ ] denote the greatest integer function and $f(x) = [\tan^2 x]$. Then

(a) $\lim_{x \to 0} f(x)$ does not exist

(b) f(x) is continuous at x = 0

(c) f(x) is not differentiable at x = 0

(d) f(x) = 1

94. A spherical balloon is expanding. If the radius is increasing at the rate of 2 centimeters per minute, the rate at which the volume increases (in cubic centimeters per minute) when the radius is 5 centimetres is

(a) 10π (b) 100π

(c) 200π (d) 50π

95. The length of the parabola $y^2 = 12x$ cut off by the latus-rectum is

(a) $6\left(\sqrt{2} + \log(1+\sqrt{2})\right)$

(b) $3\left(\sqrt{2} + \log(1+\sqrt{2})\right)$

(c) $6\left(\sqrt{2} - \log(1+\sqrt{2})\right)$

(d) $3\left(\sqrt{2} - \log(1+\sqrt{2})\right)$

96. If $I = \int \frac{x^5}{\sqrt{1+x^3}} dx$, then I is equal to

(a) $\frac{2}{9}(1+x^3)^{\frac{5}{2}} + \frac{2}{3}(1+x^3)^{\frac{3}{2}} + C$

(b) $\log\left|\sqrt{x} + \sqrt{1+x^3}\right| + C$

(c) $\log\left|\sqrt{x} - \sqrt{1+x^3}\right| + C$

(d) $\frac{2}{9}(1+x^3)^{\frac{3}{2}} - \frac{2}{3}(1+x^3)^{\frac{1}{2}} + C$

97. Area enclosed by the curve $\pi\left[4\left(x - \sqrt{2}\right)^2 + y^2\right] = 8$ is

(a) π (b) 2

(c) 3π (d) 4

98. The value of $\int_0^a \sqrt{\frac{a-x}{x}} dx$ is

(a) $\frac{a}{2}$ (b) $\frac{a}{4}$

(c) $\frac{\pi a}{2}$ (d) $\frac{\pi a}{4}$

99. Let y be the number of people in a village at time t. Assume that the rate of change of the population is proportional to the number of people in the village at any time and further assume that the population never increases in time. Then the population of the village at any fixed time t is given by

(a) $y = e^{kt} + c$, for some constants $c \leq 0$ and $k \geq 0$

(b) $y = ce^{kt}$, for some constants $c \geq 0$ and $k \leq 0$

(c) $y = e^{ct} + k$, for some constants $c \leq 0$ and $k \geq 0$

(d) $y = ke^{ct}$, for some constants $c \geq 0$ and $k \leq 0$

100. The differential equation of all straight lines touching the circle $x^2 + y^2 = a^2$ is

(a) $\left(y - \frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$

(b) $\left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1 + \left(\frac{dy}{dx}\right)^2\right]$

(c) $\left(y - x\frac{dy}{dx}\right) = a^2\left[1 + \left(\frac{dy}{dx}\right)\right]$

(d) $\left(y - \frac{dy}{dx}\right) = a^2\left[1 - \frac{dy}{dx}\right]$

101. The differential equation $\left|\frac{dy}{dx}\right| + |y| + 3 = 0$ admits

(a) infinite number of solutions

(b) no solution

(c) a unique solution

(d) many solutions

102. Solution of the differential equation
$xdy - ydx - \sqrt{x^2 + y^2}dx = 0$ is

(a) $y - \sqrt{x^2 + y^2} = Cx^2$

(b) $y + \sqrt{x^2 + y^2} = Cx^2$

(c) $x + \sqrt{x^2 + y^2} = Cy^2$

(d) $x - \sqrt{x^2 + y^2} = Cy^2$

103. Let P, Q, R and S be statements and suppose that $P \to Q \to R \to P$. if $\sim S \to R$, then

(a) $S \to \sim Q$ (b) $\sim Q \to S$

(c) $\sim S \to \sim Q$ (d) $Q \to \sim S$

104. In how many number of ways can 10 students be divided into three teams, one containing four students and the other three?

(a) 400 (b) 700

(c) 1050 (d) 2100

105. If R be a relation defined as a R b iff $|a - b| > 0$, then the relation is

(a) reflexive (b) symmetric

(c) transitive

(d) symmetric and transitive

106. Let S be a finite set containing n elements. Then the total number of commutative binary operation on S is

(a) $n^{\left[\frac{n(n+1}{2}\right]}$ (b) $n^{\left[\frac{n(n-1}{2}\right]}$

(c) $n^{(n^2)}$ (d) $2^{(n^2)}$

107. A manufacturer of cotter pins knows that 5% of his product is defective. He sells pins in boxes of 100 and guarantees that not more than one pin will be defective in a box. In order to find the probability that a box will fail to meet the guaranteed quality, the probability distribution one has to employ is

(a) Binomial (b) Poisson

(c) Normal (d) Exponential

108. The probability that a certain kind of component will survive a given shock test is $\frac{3}{4}$. The probability that exactly 2 of the next 4 components tested survive is

(a) $\frac{9}{41}$ (b) $\frac{25}{128}$

(c) $\frac{1}{5}$ (d) $\frac{27}{128}$

109. Mean and standard deviation of marks obtained in some particular subject by four classes are given below. Report the class with best performance

(a) 80, 18 (b) 75, 5

(c) 80, 21 (d) 76, 7

110. A random variable X follows binomial distribution with mean α and variance β. Then

(a) $0 < \alpha < \beta$ (b) $0 < \beta < \alpha$

(c) $\alpha < 0 < \beta$ (d) $\beta < 0 < \alpha$

111. The system of equations
$x + y + z = 0$
$2x + 3y + z = 0$
$x + 2y = 0$
has

(a) a unique solution; $x = 0, y = 0, z = 0$

(b) infinite solutions

(c) no solution

(d) finite number of non-zero solutions

112. $\begin{bmatrix} 0 & a \\ b & 0 \end{bmatrix}^4 = 1$, then

(a) $a = 1 = 2b$ (b) $a = b$

(c) $a = b^2$ (d) $ab = 1$

113. If $D = \text{diag}(d_1, d_2, ..., d_n)$ where $d_1 \neq 0$, for $i = 1, 2, ..., n$, then D^{-1} is equal to

(a) D^T

(b) D

(c) Adj (D)

(d) $\text{diag}(d_1^{-1}, d_2^{-1}, ...d_n^{-1})$

114. If x, y, z are different from zero and

$$\Delta = \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$$ then the value of the expression $\frac{a}{x} + \frac{b}{y} + \frac{c}{z}$ is

(a) 0 (b) –1

(c) 1 (d) 2

115. Probability of getting positive integral roots of the equation $x^2 - n = 0$ for the integer n, $1 \leq n \leq 40$ is

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$

(c) $\frac{3}{20}$ (d) $\frac{1}{20}$

116. The number of real roots of the equation $x^4 + \sqrt{x^4 + 20} = 22$ is

(a) 4 (b) 2

(c) 0 (d) 1

117. Let α, β be the roots of the equation $x^2 - ax + b = 0$ and $A_n = \alpha^n + \beta^n$. Then $A_{n+1} - aA_n + bA_{n-1}$ is equal to

(a) $-a$ (b) b

(c) 0 (d) $a - b$

118. If the sides of a right-angle triangle form an A.P., the 'Sin' of the acute angles are

(a) $\left(\frac{3}{5}, \frac{4}{5}\right)$

(b) $\left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

(c) $\left(\sqrt{\frac{\sqrt{5}-1}{2}}, \sqrt{\frac{\sqrt{5}-1}{2}}\right)$

(d) $\left(\sqrt{\frac{\sqrt{3}-1}{2}}, \sqrt{\frac{\sqrt{3}-1}{2}}\right)$

119. The plane through the point (–1, –1, –1) and containing the line of intersection of the planes $\vec{r}.(\hat{i} + 3\hat{j} - \hat{k}) = 0$ and $\vec{r}.(\hat{j} + 2\hat{k}) = 0$ is

(a) $\vec{r}.(\hat{i} + 2\hat{j} - 3\hat{k}) = 0$

(b) $\vec{r}.(\hat{i} + 4\hat{j} + \hat{k}) = 0$

(c) $\vec{r}.(\hat{i} + 5\hat{j} - 5\hat{k}) = 0$

(d) $\vec{r}.(\hat{i} + \hat{j} + 3\hat{k}) = 0$

120. $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} + 3\hat{k}$ are one of the sides and medians respectively, of a triangle through the same vertex, then area of the triangle is

(a) $\frac{1}{2}\sqrt{83}$ (b) $\sqrt{83}$

(c) $\frac{1}{2}\sqrt{85}$ (d) $\sqrt{86}$