Vector Algebra JEE Main PYQ - 3

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Vector Algebra

1. (+4, -1)

Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to

a. 2

b. $\frac{39}{5}$

c. 9

d. $\frac{46}{5}$

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2. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2$ is equal to (+4, -1)

a. 3

b. 5

c. 1

d. 4

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3. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i} + \hat{j}$ and the plane determined by the vectors $\hat{i} - \hat{j}, \hat{i} + \hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ is (+4, -1)

a. $\frac{3\pi}{4}$

b. $\frac{2\pi}{3}$

c. $\frac{4\pi}{5}$

d. $\frac{5\pi}{6}$

4. Let $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 4\hat{i} + \hat{j} + 7\hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} + 4\hat{k}$ be 3 vectors. If a vector $\vec{p}$ satisfies $\vec{p}_x\vec{b} = \vec{c}_x\vec{b}$ and $\vec{p}_x\vec{a} = 0$ then $\vec{p}.(\hat{i} - \hat{j} - \hat{k})$ is equal to **(+4, -1)**

a. 32

b. 23

c. 16

d. 61

5. Let S be the set of all a∈ R for which the angle between the vectors $\vec{u} = a\left(\log_e b\right)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = \left(\log_e b\right)\hat{i} + 2\hat{j} + 2a\left(\log_e b\right)\hat{k}, (b > 1)$ is acute. Then S is equal to **(+4, -1)**

a. $(\infty, -\frac{4}{3})$

b. ϕ

c. $(-\frac{4}{3}, 0)$

d. $(\frac{12}{7}, \infty)$

6. A plane P is parallel to two lines whose direction rations are −2, 1, −3 and −1, 2, −2 and it contains the point (2, 2, −2). Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron OABC, where O is the origin and p = α + β + γ, then the ordered pair (V, p) is equal to : **(+4, -1)**

a. (48,-13)

b. (24,-13)

c. (48,11)

d. (24,-5)

7. Let the lines (+4, -1)
$L_1 : \vec{r} = \lambda(\hat{i} + 2\hat{j} + 3\hat{k}), \ \ \lambda \in R$
$L_2 : \vec{r} = (\hat{i} + 3\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} + 5\hat{k}), \ \ \mu \in R$
intersect at the point S. If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines L_1 and L_2 then the value of $a + b + d$ is equal to _______.

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8. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4$, $|\vec{b}| = 3$ and $\theta(\frac{\pi}{4}, \frac{\pi}{3})$. Then $|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})|^2 + 4(\vec{a}.\vec{b})^2$ is equal to ______. (+4, -1)

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9. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $a_i > 0$, $i = 1, 2, 3$ be a vector which makes equal angles with the coordinate axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i} + 4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}, \vec{b}$ and x-axis are co-planar, then projection of a vector $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is equal to : (+4, -1)

a. $\sqrt{7}$

b. $\sqrt{2}$

c. 2

d. 7

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10. Let $\lambda \in R, \vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$.If $((\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})) \times (\vec{a} - \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then $|\lambda(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|^2$ is equal to (+4, -1)

a. 136

b. 132

c. 140

d. 144

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11. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-zero vectors and $\hat{n}$ is a unit vector perpendicular to $\vec{c}$ such that $\vec{a}$=$\alpha\vec{b}$-$\hat{n}$, $\alpha \neq 0$ and $\vec{b}$.$\vec{c}$=12, then |$\vec{c} \times (\vec{a} \times \vec{b})$| is equal to: (+4, -1)

a. 9

b. 15

c. 6

d. 12

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12. **Let: $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be there vectors If $\vec{r}$ is a vector such that, $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, then $25|\vec{r}|^2$ is equal to** **(+4, -1)**

a. 560

b. 339

c. 449

d. 336

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13. Let p, q and r be three mutually perpendicular vectors of the same magnitude. If a vector x satisfies equation p x {(x - q) x p} + q x {(x - r) x r} + r x {(x - p) x r} = 0, then x is given by **(+4, -1)**

a. 1/2 (p + q - 2r)

b. 1/2 (p + q + r)

c. 1/3 (2p + q - r)

d. 1/3 (2p + q - r)

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14. If ( ) = 3 ^{2} + 15 + 5 then the approximate value of (3.02) is: **(+4, -1)**

a. (A) 47.66

b. (B) 57.66

c. (C) 67.66

d. (D) 77.66

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15. What is one of the square roots of 3 + 4 , where = $\sqrt{-1}$? **(+4, -1)**

a. (A) 2 +

b. (B) 2 −

c. (C) −2 +

d. (D) −3 −

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16. The relation in the set R of real numbers, defined as = {(,): a ≤ ^{2}} is: **(+4, -1)**

a. (A) is neither reflexive, nor transitive, but symmetric.

b. (B) is neither reflexive, nor symmetric, but transitive.

c. (C) is neither reflexive, nor symmetric, nor transitive.

d. (D) is reflexive, symmetric, and transitive.

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17. The value of lim $_0 \frac{\int_0^{2} \sec^2}{\sin}$ is **(+4, -1)**

a. (A) 3

b. (B) 2

c. (C) 1
Correct Answer

d. (D) 0

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18. The sum of all values of α, for which the points whose position vectors are: **(+4, -1)**

$$\mathbf{r_1} = \hat{i} - 2\hat{j} + 3\hat{k}, \quad \mathbf{r_2} = 2\hat{i} - 3\hat{j} + 4\hat{k}, \quad \mathbf{r_3} = (\alpha + 1)\hat{i} + 2\hat{k}, \quad \mathbf{r_4} = \hat{j} + 2\hat{k}$$

are coplanar, is equal to:

a. -2

b. 2

c. 6

d. 4

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19. Let $\vec{a}$ = $2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b}$ = $\hat{i}-2\hat{j}-2\hat{k}$, $\vec{c}$ = $-\hat{i}+4\hat{j}+3\hat{k}$ and $\vec{d}$ is a vector perpendicular to $\vec{b}$ and $\vec{c}$, $\vec{a}.\vec{d}$ = 18, then find $|\vec{a}x\vec{d}|^2$ **(+4, -1)**

a. 720

b. 700

c. 360

d. 300

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20. Let $\overrightarrow{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}, \overrightarrow{b} = \hat{i} + \hat{k} and \overrightarrow{c} = \hat{i} + 2\hat{j} - 3\hat{k}$ be three given vectors If $\overrightarrow{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}\ \ and\ \ \vec{r} \cdot \vec{b} = 0$, then $|\vec{r}|$ is equal to : **(+4, -1)**

a. $\frac{11}{7}\sqrt{2}$

b. $\frac{11}{7}$

c. $\frac{\sqrt{914}}{7}$

d. $\frac{11}{5}\sqrt{2}$

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21. Let $\vec{u} = \hat{i} - \hat{j} - 2\hat{k}, \vec{v} = 2\hat{i} + \hat{j} - \hat{k}, \vec{v} \cdot \vec{w} = 2$ and $\vec{v} \times \vec{w} = \vec{u} + \lambda\vec{v}$. Then $\vec{u} \cdot \vec{w}$ is equal to **(+4, -1)**

a. 1

b. 2

c. $\frac{3}{2}$

d. $-\frac{2}{3}$

22. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{A\ \text{Area}\ (\triangle PQR)}{\text{Area}\ (\triangle ABC)}$ is equal to **(+4, -1)**

a. 4

b. 3

c. 2

d. $\frac{5}{2}$

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23. Let $\vec{a} = -\hat{i} - \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{i} - \hat{j}$. Then $\vec{a} - 6\vec{b}$ is equal to **(+4, -1)**

a. $3(\hat{i} - \hat{j} - \hat{k})$

b. $3(\hat{i} - \hat{j} + \hat{k})$

c. $3(\hat{i} + \hat{j} - \hat{k})$

d. $3(\hat{i} + \hat{j} + \hat{k})$

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24. Let $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b}$ be a vector such that $\vec{a} \times \vec{b} = 2\hat{i} - \hat{k}$ and $\vec{a} \cdot \vec{b} = 3$ Then the projection of $\vec{b}$ on the vector $\vec{a} - \vec{b}$ is :- **(+4, -1)**

a. $\frac{2}{\sqrt{21}}$

b. $2\sqrt{\frac{3}{7}}$

c. $\frac{2}{3}\sqrt{\frac{7}{3}}$

d. $\frac{2}{3}$

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25. The foot of perpendicular of the point $(2, 0, 5)$ on the line $\frac{x+1}{2} = \frac{y-1}{5} - \frac{z+1}{-1}$ is (α, β, γ). Then which of the following is NOT correct? **(+4, -1)**

a. $\frac{\gamma}{\alpha} = \frac{5}{8}$

b. $\frac{\beta}{\gamma} = -5$

c. $\frac{\alpha\beta}{\gamma} = \frac{4}{15}$

d. $\frac{\alpha}{\beta} = -8$

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26. If the foot of the perpendicular drawn from $(1, 9, 7)$ to the line passing through the point $(3, 2, 1)$ and parallel to the planes $x + 2y + z = 0$ and $3y - z = 3$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to **(+4, -1)**

a. -1

b. 1

c. 3

d. 5

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27. **Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$ If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2$ is equal to _____** **(+4, -1)**

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28. Let $\vec{v} = \alpha\hat{i} + 2\hat{j} - 3\hat{k}, \vec{w} = 2\alpha\hat{i} + \hat{j} - \hat{k}$ and $\vec{u}$ be a vector such that $|\vec{u}| = \alpha >$ If the minimum value of the scalar triple product $[\vec{u}\ \vec{v}\ \vec{w}]$ is $-\alpha\sqrt{3401}$, and $|\vec{u} \cdot \hat{i}|^2 = \frac{m}{n}$ where m and n are coprime natural numbers, then $m + n$ is equal to _____ **(+4, -1)**

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29. Let x and y be distinct integers where $1 \leq x \leq 25$ and $1 \leq y \leq 25$ Then, the number of ways of choosing x and y, such that $x + y$ is divisible by 5 , is ____ **(+4, -1)**

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30. **Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6}$ and $|\vec{a} \times \vec{b}| = \sqrt{48}$ Then $(\vec{a} \cdot \vec{b})^2$ is equal to** **(+4, -1)**

Answers

1. Answer: d

Explanation:

To solve the problem, we need to determine the projection of the vector $\vec{b} - 2\vec{a}$ on the vector $\vec{b} + \vec{a}$.

First, let's express the vectors given in the problem:

- $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$
- $\vec{b} = 3\hat{i} + 5\hat{j} + 4\hat{k}$

The cross product of these vectors is given by:

- $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$

The cross product formula in terms of vector components is:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & 5 & 4 \end{vmatrix}$$

Expanding the determinant, we get:

1. Coefficient of $\hat{i}$: $1 \cdot 4 - \beta \cdot 5 = 4 - 5\beta$
2. Coefficient of $\hat{j}$: $-(\alpha \cdot 4 - \beta \cdot 3) = -4\alpha + 3\beta$
3. Coefficient of $\hat{k}$: $\alpha \cdot 5 - 1 \cdot 3 = 5\alpha - 3$

Therefore, $\vec{a} \times \vec{b} = (4 - 5\beta)\hat{i} + (-4\alpha + 3\beta)\hat{j} + (5\alpha - 3)\hat{k}$, which is given to be:

(-1)\hat{i} + 9\hat{j} + 12\hat{k}

Equating components, we have the equations:

- $4 - 5\beta = -1 \rightarrow \beta = 1$
- $-4\alpha + 3(1) = 9 \rightarrow \alpha = -1.5$
- $5\alpha - 3 = 12 \rightarrow$ (already consistent with the second equation)

Now substituting α and β into $\vec{a}$, we find:

- $\vec{a} = -1.5\hat{i} + \hat{j} + 1\hat{k}$

Vectors required are:

- $\vec{b} - 2\vec{a} = (3 - 2(-1.5))\hat{i} + (5 - 2(1))\hat{j} + (4 - 2(1))\hat{k} = (3 + 3)\hat{i} + (5 - 2)\hat{j} + (4 - 2)\hat{k} = 6\hat{i} + 3\hat{j} + 2\hat{k}$
- $\vec{b} + \vec{a} = (3 - 1.5)\hat{i} + (5 + 1)\hat{j} + (4 + 1)\hat{k} = 1.5\hat{i} + 6\hat{j} + 5\hat{k}$

The projection of $\vec{u}$ on $\vec{v}$ is given by:

$\text{Projection of } \vec{u} \text{ on } \vec{v} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\vec{v}$

First, let's calculate $\vec{u} \cdot \vec{v}$:

- $\vec{u} \cdot \vec{v} = 6 \times 1.5 + 3 \times 6 + 2 \times 5 = 9 + 18 + 10 = 37$

Calculate $|\vec{v}|^2$:

- $|\vec{v}|^2 = (1.5)^2 + 6^2 + 5^2 = 2.25 + 36 + 25 = 63.25$

The projection magnitude is:

$\frac{37}{63.25} \times \sqrt{1.5^2 + 6^2 + 5^2} = \frac{46}{5}$. Thus the correct answer is $\frac{46}{5}$.

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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2. Answer: b

Explanation:

To solve the problem, we need to find the square of the magnitude of the vector $(\vec{b} \times \vec{a}) - \vec{b}$. Given that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$, let's proceed step by step:

1. Start with the expression whose magnitude we need to find: $(\vec{b} \times \vec{a}) - \vec{b}$
2. Calculate the magnitude squared, which is given by the dot product of the vector with itself: $\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2 = \left[(\vec{b} \times \vec{a}) - \vec{b}\right] \cdot \left[(\vec{b} \times \vec{a}) - \vec{b}\right]$
3. Expand the dot product: $= (\vec{b} \times \vec{a}) \cdot (\vec{b} \times \vec{a}) - 2(\vec{b} \times \vec{a}) \cdot \vec{b} + \vec{b} \cdot \vec{b}$
4. We know:
   - $|\vec{b} \times \vec{a}| = 2$
   - $\vec{b} \cdot (\vec{b} \times \vec{a}) = 0$ (This is because the dot product of a vector and a cross product of the same vector with another vector is zero.)
   - $\vec{b} \cdot \vec{b} = |\vec{b}|^2 = 1^2 = 1$
5. Substitute these values into the expression: $\left|(\vec{b} \times \vec{a}) - \vec{b}\right|^2 = 2^2 - 2 \times 0 + 1 = 4 + 1 = 5$

Therefore, the magnitude squared of the vector $(\vec{b} \times \vec{a}) - \vec{b}$ is **5**.

Concepts:

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3. **Answer: a**

Explanation:

If $\vec{n}_1$ is a vector normal to the plane determined by $\hat{i}$ and $\hat{i}+\hat{j}$ then

$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = k$

If $\vec{n}_2$ is a vector normal to the plane determined by $\hat{i}-\hat{j}$ and $\hat{i}+\hat{k}$ then

n2 = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$| = $-\hat{i}-\hat{j}+\hat{k}$

Vector $\vec{a}$ is parallel to $\vec{n}_1 \times \vec{n}_2$ i.e

$\vec{a}$ is parallel to $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ -1 & -1 & 1 \end{vmatrix} = \hat{i}-\hat{j}$

Given

$b = \hat{i} - 2\hat{j} + 2\hat{k}$

cosine of acute angle between

$a \ and \ b = |\frac{\vec{a}.\vec{b}}{|\vec{a}|.|\vec{b}|}| = \frac{1}{\sqrt{2}}$

Obtuse angle between

$\vec{a}$ and $\vec{b} = \frac{3\pi}{4}$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

4. **Answer: a**

Explanation:

The correct option is (A): 32

Concepts:

1. Vector Algebra:

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The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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5. **Answer: b**

Explanation:

$\vec{u} = a\left(\log_e b\right)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = \left(\log_e b\right)\hat{i} + 2\hat{j} + 2a\left(\log_e b\right)\hat{k}$

For acute angle, $\vec{u}\cdot\vec{v} > 0$

$\Rightarrow\ a\left(\log_e b\right)^2 - 12 + 6a\left(\log_e b\right) > 0$

∵ $b > 1$ so, $\log_e b = t \Rightarrow t > 0$ as $b > 1$

$at^2 + 6at - 12 > 0\ \ \forall t > 0$

$\Rightarrow a \in \phi$

Concepts:

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The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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6. Answer: b

Explanation:

Assume $\vec{a}_1 = (-2, 1, -3)$ and $\vec{a}_2 = (-1, 2, -2)$ Vector normal to plane

So, $\overline{n} = \vec{a}_1 \times \vec{a}_2 \Rightarrow \overline{n} = (4, -1, -3)$

$\text{Plane through } (2, 2, -2) \text{ and normal to } \overline{n}$

$(x–2, y–2, z+2) \cdot (4, -1, -3) = 0$

$\Rightarrow 4x–y–3z = 12$

$\Rightarrow \frac{x}{3} + \frac{y}{-12} + \frac{z}{-4} = 1$

$Intercepts\ \alpha, \beta, \gamma\ are\ 3, –12, –4$

$P = \alpha + \beta + \gamma = –13$

$V = \frac{1}{6} \times 3 \times 12 \times 4 = 24$

Concepts:

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The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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7. **Answer: 5 - 5**

Explanation:

As plane is parallel to both the lines we have d.r's of normal to the plane as $(7, –2, –1)$

from $\begin{vmatrix} \hat{i} & \hat{i} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix} = 7\hat{i} - \hat{j}(2) + \hat{k}(-1)$

The point of intersection of lines is $2\hat{i} + 4\hat{j} + 6\hat{k}$

So, the equation of plane is,

$7(x -2)–2(y -4)–1(z -6) = 0$

$7x -2y - z \ = 0$

$a \ + \ b \ + \ d \ = 7–2 + 0 = 5$

$a \ + \ b \ + \ d \ = 5$

So, the answer is 5.

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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8. Answer: 576 - 576

Explanation:

$$|(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})|^2+4(\vec{a}.\vec{b})^2$$
$$=|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}|^2+4(\vec{a}\cdot\vec{b})^2$$
$$=|2(\vec{a}\times\vec{b})|^2+4(\vec{a}\cdot\vec{b})^2$$
$$=4(\vec{a}\times\vec{b})^2+(\vec{a}\cdot\vec{b})^2$$
$$=4|\vec{a}|^2|\vec{b}|^2$$
$$=4x16x9$$
$$=576$$

So, the correct answer is 576.

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

9. Answer: b

Explanation:

$cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

$\Rightarrow cos^2\alpha = \frac{1}{3}$

$\Rightarrow cos\ \alpha = \frac{1}{\sqrt{3}}$

$\vec{a} = \frac{\lambda}{3}(\hat{i} + \hat{j} + \hat{k}), \ \ \lambda > 0$

$\frac{\lambda}{\sqrt{3}} \frac{(\hat{i}+\hat{j}+\hat{k})\cdot(3\hat{i}+4\hat{j})}{\sqrt{3^2+4^2}} = 7$

$\Rightarrow \frac{\lambda}{\sqrt{3}}(3 + 4) = 7 \times 5$

$\therefore \lambda = 5\sqrt{3}$

$\vec{a} = 5(\hat{i} + \hat{j} + \hat{k})$

Let

$\vec{b} = p\hat{i} + q\hat{j} + r\hat{k}$

$\vec{a} \cdot \vec{b} = 0$ and $[\vec{a}\vec{b}\hat{i}] = 0$

$\Rightarrow \ p + q + r \ = 0$ ...(i)

And

$\begin{vmatrix} p & q & r \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{vmatrix} = 0$

$\Rightarrow \vec{b} = -2r\hat{i} + r\hat{j} + r\hat{k}$

$\Rightarrow \vec{b} = r(-2\hat{i} + \hat{j} + \hat{k})$

Now

$|\vec{a}| = |\vec{b}|$

$5\sqrt{3} = |r|\sqrt{b}$

$|r| = 5\sqrt{2}$

Projection of $\vec{b}$ on $3\hat{i} + 4\hat{j}$

= $|\frac{\vec{b}\cdot(3\hat{i}+4\hat{j})}{\sqrt{3^2+4^2}}|$

= $|r| \frac{(-6+4)}{5}$

= $| - \frac{2}{5}|$

Projection = $\frac{2}{5} \times \frac{5}{\sqrt{2}}$

Projection = $\sqrt{2}$

So, the correct option is (b): $\sqrt{2}$

Concepts:

1. Vector Algebra:

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The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

10. **Answer: c**

Explanation:

Let $\mathbf{a} = \lambda i + 2j - 3k$
and $\mathbf{b} = i - \lambda j + 2k$. We are given the equation:

$$((\mathbf{a}+\mathbf{b}) \times (\mathbf{a}\times\mathbf{b}))\,((\mathbf{a}-\mathbf{b})) = 8i - 40j - 24k$$

Now, solve for the value of λ:

$$(\mathbf{a}-\mathbf{b}) \times (\mathbf{a}\times\mathbf{b}) = 8i - 40j - 24k$$

$$8(\mathbf{a}\times\mathbf{b}) = 8i - 40j - 24k$$

Now, calculate $\mathbf{a}\times\mathbf{b}$: $\mathbf{a}\times\mathbf{b} = \left|i \quad j \quad k\ \lambda \quad 2 \quad -3\ 1 \quad -\lambda \quad 2\right| = (4-3\lambda)i - (2\lambda+3)j + (-\lambda^2-2)k$

Since $\mathbf{a}\times\mathbf{b} = 8i - 40j - 24k$, we solve for $\lambda = 1$. Thus, we have:

$$\mathbf{a}+\mathbf{b} = 2i + 3j - 5k$$

Then, $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b}) = \left| i \quad j \quad k\, 2 \quad 3 \quad -5\, 1 \quad -1 \quad 2 \right| = 2i+10j+6k$

Hence, the required answer is $4+100+36=140$.

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

11. Answer: d

Explanation:

The correct option is (D) : 12

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

12. Answer: b

Explanation:

We are given the following vectors:

$$\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \mathbf{b} = \hat{i} - \hat{j} + 2\hat{k}, \quad \mathbf{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}.$$

Step 1: We know that $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ and $\mathbf{r} \cdot \mathbf{a} = 0$. From the condition $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$, we have:

$$\mathbf{r} - \mathbf{c} = \lambda \mathbf{b} \quad (\text{where } \lambda \text{ is a constant}).$$

Step 2: Next, substitute the expression for $\mathbf{r}$:

$$\mathbf{r} = \mathbf{c} + \lambda \mathbf{b}.$$

Substituting the values of $\mathbf{c}$ and $\mathbf{b}$, we get:

$$\mathbf{r} = 5\hat{i} - 3\hat{j} + 3\hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k}).$$

Thus, the vector $\mathbf{r}$ is:

$$\mathbf{r} = (5+\lambda)\hat{i} + (-3-\lambda)\hat{j} + (3+2\lambda)\hat{k}.$$

Step 3: Using the condition $\mathbf{r} \cdot \mathbf{a} = 0$, we calculate the dot product:

$$\mathbf{r} \cdot \mathbf{a} = (5+\lambda)(1) + (-3-\lambda)(2) + (3+2\lambda)(3).$$

Simplifying:

$$\mathbf{r} \cdot \mathbf{a} = 5 + \lambda - 6 - 2\lambda + 9 + 6\lambda = 8 + 5\lambda = 0.$$

Thus, solving for λ, we get:

$$5\lambda = -8 \quad \Rightarrow \quad \lambda = -\frac{8}{5}.$$

Step 4: Substitute $\lambda = -\frac{8}{5}$ into the expression for $\mathbf{r}$:

$$\mathbf{r} = \left(5 - \frac{8}{5}\right)\hat{i} + \left(-3 + \frac{8}{5}\right)\hat{j} + \left(3 - \frac{16}{5}\right)\hat{k}.$$

Simplifying:

$$\mathbf{r} = \frac{17}{5}\hat{i} - \frac{7}{5}\hat{j} + \frac{1}{5}\hat{k}.$$

Step 5: Now, calculate $|\mathbf{r}|^2$:

$$|\mathbf{r}|^2 = \left(\frac{17}{5}\right)^2 + \left(\frac{-7}{5}\right)^2 + \left(\frac{1}{5}\right)^2 = \frac{289}{25} + \frac{49}{25} + \frac{1}{25} = \frac{339}{25}.$$

Thus:

$$25|\mathbf{r}|^2 = 25 \times \frac{339}{25} = 339.$$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

13. **Answer: b**

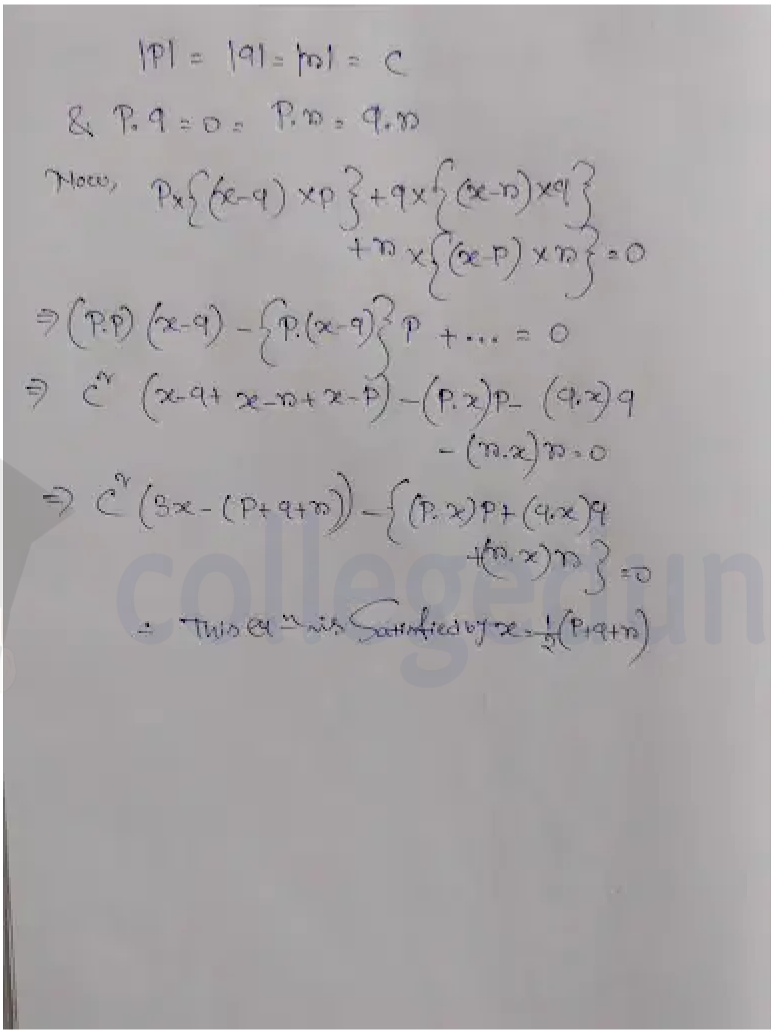
Explanation:

Correct option (b) 1/2 (p + q + r)

Let p, q and r be three mutually perpendicular vectors of the same magnitude.

If a vector x satisfies equation p x {(x - q) x p} + q x {(x - r) x r} + r x {(x - p) x r} = 0,

then x is given by



$|p| = |q| = |r| = c$

& $p.q = 0 = p.r = q.r$

Now, $p \times \{(x-q) \times p\} + q \times \{(x-r) \times q\} + r \times \{(x-p) \times r\} = 0$

$\Rightarrow (p.p)(x-q) - \{p.(x-q)\}p + \dots = 0$

$\Rightarrow c^2 (x-q+x-r+x-p) - (p.x)p - (q.x)q - (r.x)r = 0$

$\Rightarrow c^2 (3x-(p+q+r)) - \{(p.x)p + (q.x)q + (r.x)r\} = 0$

$\therefore$ This eqn is satisfied by $x = \frac{1}{2}(p+q+r)$

---

14. Answer: d

Explanation:

Explanation:

Let   = 3 and Δ   = 0.02 then we have (3.02) = (+ Δ) = 3(+ Δ)^2 + 15(+ Δ) + 5Now,
Δ   =   (+ Δ) − ( ) (+ Δ) = ( ) + Δ   ≈   ( ) +   ′( )Δ   (As   = Δ)

$(3.02) \approx (3^2 + 15 + 5) + (6 + 15)\Delta = [3(3)^2 + 15(3) + 5] + [6(3) + 15](0.02)$ [As $= 3, \Delta = 0.02$] $= (27 + 45 + 5) + (18 + 150(0.02) = 77 + (33)(0.02) = 77.66$Thus, the approximate value of $(3.02) = 77.66$.Hence, the correct option is (D).

---

15. Answer: a

Explanation:

Explanation:
Let $\sqrt{(3 + 4\ )} = \sqrt{\ } + \sqrt{\ }(\sqrt{3 + 4\ })^2 = (\sqrt{\ } + \sqrt{\ })^2$ $3 + 4 = - + 2\sqrt{\ }$Comparing real & imaginary part, we get, $3 = -$ and $2\sqrt{\ } = 4$ $= 4$We know, $(\ +\ )^2 = (\ -\ )^2 + 4$ $(\ +\ )^2 = 9 + 16$ $+ = \pm 5$ $(\ \neq -5;$ As $\sqrt{x}$ is real part) $= 4, y = 1$ $\sqrt{\ } = \pm 2, \sqrt{\ } = \pm 1$ Square root of $3 + 4$ is $2 +$ .Hence, the correct option is (A).

---

16. Answer: c

Explanation:

Explanation:
Given: $= \{(\ ,\ ): \leq\ ^2\}(\frac{1}{2}, \frac{1}{2})$ Because, $\frac{1}{2} > (\frac{1}{2})^2$ R is not reflexive.$(1, 4)$ as $1 < 4$But, 4 is not less than 1^2. $(4, 1)$ is not symmetric.$(3, 2), (2, 1.5)$ [Because $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25]3 > (1.5)^2 = 2.25$ $(3, 1.5)$ is not transitive. is neither reflexive, nor symmetric, nor transitive.Hence, the correct option is (C).

---

17. Answer: c

Explanation:

Explanation:

Given:The expression $\lim_{0} \frac{\int_0^2 \sec^2}{\sin}$We have to evaluate the given expression.Here, we'll use fundamental integrals of trigonometric functions.

$\lim_{0} \frac{\int_0^2 \sec^2}{\sin}$ $[\int \sec^2 = \tan\ ]$ $\lim_{0} \frac{[\tan\ ]^2}{\sin} = \lim_{0} \frac{\tan^2}{\sin}$

= lim 0 $\frac{\tan\ ^2}{\frac{2}{2}}$ [Dividing by ^{2} in Numerator & denominator] $\frac{\lim\ _0 (\frac{\tan\ ^2}{2})}{\sin}$ = $\frac{1}{1}$ = 1[Using limit of trigonometric functions]Hence, the correct option is (C).

---

18. Answer: b

Explanation:

For four points to be coplanar, the volume of the tetrahedron they form must be zero. This is determined using the scalar triple product of the vectors formed by three of these points.

Vectors formed:

$$\mathbf{AB} = (2\hat{i} - 3\hat{j} + 4\hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k}) = \hat{i} - \hat{j} + \hat{k}$$

$$\mathbf{AC} = ((\alpha + 1)\hat{i} + 2\hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k}) = \alpha\hat{i} + 2\hat{j} - \hat{k}$$

$$\mathbf{AD} = (\hat{j} + 2\hat{k}) - (\hat{i} - 2\hat{j} + 3\hat{k}) = -\hat{i} + 3\hat{j} - \hat{k}$$

The determinant of the matrix formed by these vectors must be zero:

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ -1 & 3 & -1 \end{vmatrix} = 0$$

Expanding along the first row:

$$1 \times \begin{vmatrix} 2 & -1 \\ 3 & -1 \end{vmatrix} - (-1) \times \begin{vmatrix} \alpha & -1 \\ -1 & -1 \end{vmatrix} + 1 \times \begin{vmatrix} \alpha & 2 \\ -1 & 3 \end{vmatrix} = 0$$

Computing determinants:

$$1(2 \times (-1) - (-1) \times 3) + 1(\alpha \times (-1) - (-1) \times (-1)) + 1(\alpha \times 3 - 2 \times (-1)) = 0$$

$$1(-2 + 3) + 1(-\alpha - 1) + 1(3\alpha + 2) = 0$$

$$1 + (-\alpha - 1) + (3\alpha + 2) = 0$$

$$1 - 1 + 2 + 3\alpha - \alpha = 0$$

$$2 + 2\alpha = 0$$

$$2\alpha = -2$$

$$\alpha = -1$$

Since the sum of all values of α is $\mathbf{2}$, the final answer is:

Final Answer: (2) $\mathbf{2}$.

Concepts:

1. Vector Algebra:

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The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

19. **Answer: a**

Explanation:

$\vec{d} = \lambda(\vec{b} \times \vec{c}) = \lambda(2\hat{i} - \hat{j} + 2\hat{k})$
$\vec{a}.\vec{d} = 18$
$\Rightarrow \lambda = 2$
Therefore, $|\vec{a} \times \vec{d}|^2 = \vec{a}^2\vec{d}^2 - (\vec{a}.\vec{d})^2$
$\Rightarrow |\vec{a} \times \vec{d}|^2 = 29 \times 36 - 324 = 1044 - 324 = 720$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

20. **Answer: a**

Explanation:

The given vectors are:

$$\vec{a} = 2\hat{i} - 7\hat{j} + 5\hat{k}, \quad \vec{b} = \hat{i} + \hat{k}, \quad \vec{c} = \hat{i} + 2\hat{j} - 3\hat{k}.$$

Step 1: Use the Condition for $\vec{r}$
From $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$, we can write:

$$(\vec{r} - \vec{c}) \times \vec{a} = 0.$$

This implies that $\vec{r} - \vec{c}$ is parallel to $\vec{a}$, so:

$$\vec{r} = \vec{c} + \lambda\vec{a},$$

where λ is a scalar.
Step 2: Use the Dot Product Condition
From $\vec{r} \cdot \vec{b} = 0$, substitute $\vec{r} = \vec{c} + \lambda\vec{a}$:

$$(\vec{c} + \lambda\vec{a}) \cdot \vec{b} = 0.$$

Expanding:

$$\vec{c} \cdot \vec{b} + \lambda(\vec{a} \cdot \vec{b}) = 0.$$

Calculate $\vec{c} \cdot \vec{b}$:

$$\vec{c} \cdot \vec{b} = (\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (\hat{i} + \hat{k}) = 1 + 0 - 3 = -2.$$

Calculate $\vec{a} \cdot \vec{b}$:

$$\vec{a} \cdot \vec{b} = (2\hat{i} - 7\hat{j} + 5\hat{k}) \cdot (\hat{i} + \hat{k}) = 2 + 0 + 5 = 7.$$

Substitute into the equation:

$$-2 + \lambda(7) = 0.$$

Solve for λ:

$$\lambda = \frac{2}{7}.$$

Step 3: Find $\vec{r}$
Substitute $\lambda = \frac{2}{7}$ into $\vec{r} = \vec{c} + \lambda\vec{a}$:

$$\vec{r} = (\hat{i} + 2\hat{j} - 3\hat{k}) + \frac{2}{7}(2\hat{i} - 7\hat{j} + 5\hat{k}).$$

Expand:

$$\vec{r} = \hat{i} + 2\hat{j} - 3\hat{k} + \frac{4}{7}\hat{i} - 2\hat{j} + \frac{10}{7}\hat{k}.$$

Combine terms:

$$\vec{r} = \left(1 + \frac{4}{7}\right)\hat{i} + (2 - 2)\,\hat{j} + \left(-3 + \frac{10}{7}\right)\hat{k}.$$

Simplify:

$$\vec{r} = \frac{11}{7}\hat{i} + 0\hat{j} + \frac{-11}{7}\hat{k}$$

.

Step 4: Find $|\vec{r}|$

The magnitude of $\vec{r}$ is:

$$|\vec{r}| = \sqrt{\left(\frac{11}{7}\right)^2 + 0^2 + \left(\frac{-11}{7}\right)^2}.$$

Simplify:

$$|\vec{r}| = \sqrt{\frac{121}{49} + \frac{121}{49}} = \sqrt{\frac{242}{49}} = \frac{\sqrt{242}}{7}.$$

Simplify further:

$$|\vec{r}| = \frac{11}{7}\sqrt{2}.$$

Conclusion:

The magnitude of $\vec{r}$ is $\frac{11}{7}\sqrt{2}$.

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

21. **Answer: a**

Explanation:

$\overrightarrow{u} = (1, -1, -2), \overrightarrow{v} = (2, 1, -1), \overrightarrow{v} \cdot \overrightarrow{w} = 2$
$\overrightarrow{v} \times \overrightarrow{w} = \overrightarrow{u} + \lambda \overrightarrow{v} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$
Taking dot with $\overrightarrow{w}$ in (1)
$\overrightarrow{w} \cdot (\overrightarrow{v} \times \overrightarrow{w}) = \overrightarrow{u} \cdot \overrightarrow{w} + \lambda \overrightarrow{v} \cdot \overrightarrow{w}$
$\Rightarrow 0 = \overrightarrow{u} \cdot \overrightarrow{w} + 2\lambda$
Taking dot with $\vec{v}$ in (1)
$\overrightarrow{v} \cdot (\overrightarrow{v} \times \overrightarrow{w}) = \overrightarrow{u} \cdot \overrightarrow{v} + \lambda \overrightarrow{v} \cdot \overrightarrow{v}$
$\Rightarrow 0 = (2 - 1 + 2) + \lambda \cdot (6)$
$\lambda = -\frac{1}{2}$
$\Rightarrow \overrightarrow{u} \cdot \overrightarrow{w} = -2\lambda = 1$

Concepts:

1. **Vector Algebra:**

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

22. **Answer: b**

Explanation:

Let P is $\overrightarrow{0}$, Q is $\vec{q}$ and R is $\vec{r}$
A is $\frac{2\vec{q}+\vec{r}}{3}$, B is $\frac{2\vec{r}}{3}$ and C is $\frac{\vec{q}}{3}$
Area of $\triangle PQR$ is $= \frac{1}{2}|\vec{q} \times \vec{r}|$
Area of $\triangle ABC$ is $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$
$\overrightarrow{AB} = \frac{\vec{r}-2\vec{q}}{3}, \overrightarrow{AC} = \frac{-\vec{r}-\vec{q}}{3}$
Area of $\triangle ABC = \frac{1}{6}|\vec{q} \times r|$ $\frac{\text{Area}(\triangle PQR)}{\text{Area}(\triangle ABC)} = 3$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

23. Answer: d

Explanation:

Given the vector equations and properties, solving for b involves using the dot product and cross product information provided. Once b is calculated, $a - 6b$ is found by direct subtraction and scalar multiplication. The solution involves algebraic manipulation of vector components and verifying each option to match the calculated result.

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

24. Answer: a

Explanation:

Projection of $\vec{b}$ on vector $\vec{a}-\vec{b}$ is

$= \frac{\vec{b}\cdot(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}$

$= \frac{\vec{a}\cdot\vec{b}-|b|^2}{\sqrt{a^2+b^2-2a\cdot b}} = \frac{3-b^2}{\sqrt{6+b^2-6}} = \frac{3-b^2}{b}$

$|\vec{a}\times\vec{b}|^2 = 5$

$a^2b^2-(a\cdot b)^2 = 5$

$6b^2 = 14 \Rightarrow b^2 = \frac{7}{3}$

$\therefore \frac{3-b^2}{b} = \frac{3-\frac{7}{3}}{\sqrt{\frac{7}{3}}} = 2\times\sqrt{21}$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

25. **Answer: b**

Explanation:

To find the foot of the perpendicular, apply the formula for the point-line distance and the projection of a point onto a line using vector operations. Calculate $\mathbf{PA} \cdot \mathbf{b} = 0$ where $\mathbf{P}$ is the point on the line closest to $\mathbf{A}$ and $\mathbf{b}$ is the direction vector of the line. This provides the specific coordinates α, β, γ of the point $\mathbf{P}$, and subsequently, the ratios of these coordinates are compared against the options to identify the incorrect statement.

Concepts:

1. **Vector Algebra:**

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

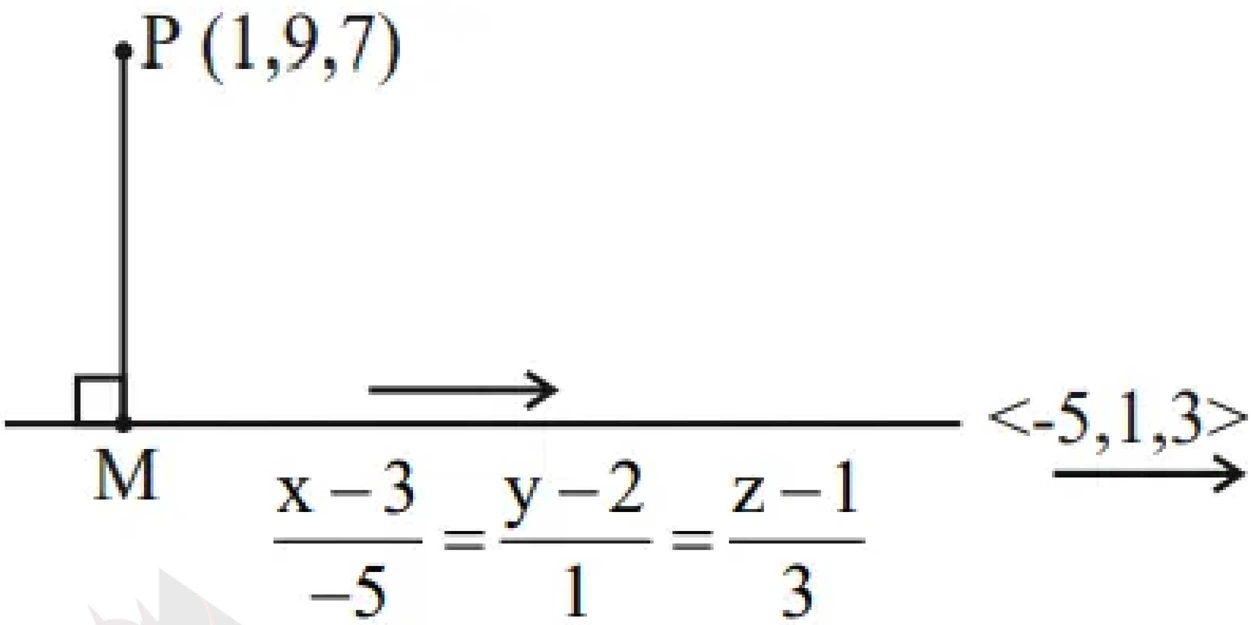
Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

---

26. **Answer: d**

Explanation:

The correct answer is (D) : 5



Direction ratio of line $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 0 & 3 & -1 \end{vmatrix}$

$= \hat{i}(-5) - \hat{j}(-1) + \hat{k}(3)$

$= -5\hat{i} + \hat{j} + 3\hat{k}$

$M(-5\lambda + 3, \lambda + 2, 3\lambda + 1)$

$\overrightarrow{\text{PM}} \perp (-5\hat{i} + \hat{j} + 3\hat{k})$

$-5(-5\lambda + 2) + (\lambda - 7) + 3(3\lambda - 6) = 0$

$\Rightarrow 25\lambda + \lambda + 9\lambda - 10 - 7 - 18 = 0$

$\Rightarrow \lambda = 1$

Point $M = (-2, 3, 4) = (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 5$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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27. Answer: 3 - 3

Explanation:

$$2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$$
$$\vec{a} \times (2\vec{b} + 3\vec{c}) = 0$$
$$\vec{a} = \lambda(2\vec{b} + 3\vec{c})$$
$$|\vec{a}|^2 = \lambda^2|2\vec{b} + 3\vec{c}|^2$$
$$|\vec{a}|^2 = \lambda^2(4|\vec{b}|^2 + 9|\vec{c}|^2 + 12\vec{b}.\vec{c})$$
$$31 = 31\lambda^2 \Rightarrow \lambda = \pm 1$$
$$\vec{a} = \pm(2\vec{b} + 3\vec{c})$$
$$\frac{|\vec{a}\times\vec{c}|}{|\vec{a}.\vec{b}|} = \frac{2|\vec{b}\times\vec{c}|}{2\vec{b}.\vec{b}+3\vec{c}.\vec{b}}$$
$$|\vec{b} \times \vec{c}| = |\vec{b}|^2|\vec{c}|^2 - (\vec{b}.\vec{c})^2 = \frac{3}{4}$$
$$\frac{|\vec{a}\times\vec{c}|}{|\vec{a}.\vec{b}|} = \frac{2\times\frac{\sqrt{3}}{2}}{2.\frac{1}{4}-\frac{3}{2}} = -\sqrt{3}$$
$$(\frac{\vec{a}\times\vec{c}}{\vec{a}.\vec{b}})^2 = 3$$

So, the correct answer is 3.

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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28. Answer: 3501 - 3501

Explanation:

The correct answer is 3501

$[\vec{u}\vec{v}\vec{w}] = \vec{u}.(\vec{v} \times \vec{w})$

$min.(|u||\vec{v} \times \vec{w}|cos\theta) = -\alpha\sqrt{3401}$

$\Rightarrow cos\theta = -1$

$|u| = \alpha(Given)$

$|\vec{v} \times \vec{w}| = \sqrt{3401}$

$\vec{v} \times \vec{w} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -3 \\ 2\alpha & 1 & -1 \end{vmatrix}$

$\vec{v} \times \vec{w} = \hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k}$

$|\vec{v} \times \vec{w}| = \sqrt{1 + 25\alpha^2 + 9\alpha^2} = \sqrt{3401}$

$34\alpha^2 = 3400$

$\alpha^2 = 100$

$\alpha = 10(as\ \alpha > 0)$

$so\ \vec{u} = \lambda(\hat{i} - 5\alpha\hat{j} - 3\alpha\hat{k})$

$\vec{u} = \sqrt{\lambda^2 + 25\alpha^2\lambda^2 + 9\alpha^2\lambda}$

$\alpha^2 = \lambda^2(1 + 25\alpha^2 + 9\alpha^2)$

$100 = \lambda^2(1 + 34 \times 100)$

$\lambda^2 = \frac{100}{3401} = \frac{m}{n}$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.

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29. **Answer: 120 - 120**

Explanation:

The correct answer is 120
x+y=5λ
Cases :

x	y	Number of ways
5λ	5λ	20
5λ+1	5λ+4	25
5λ+2	5λ+3	25
5λ+3	5λ+2	25
5λ+4	5λ+1	25

∴ Total number of ways are 120

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- "a" is a real number
- "b" is an imaginary number

The Complex Number consists of a symbol "i" which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

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30. **Answer: 36 - 36**

Explanation:

The correct answer is 36.

$|\vec{a}| = \sqrt{14}, |\vec{b}| = \sqrt{6} \quad |\vec{a} \times \vec{b}| = \sqrt{48}$

$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 \times |\vec{b}|^2$

$\Rightarrow (\vec{a} \cdot \vec{b})^2 = 84 - 48 = 36$

Concepts:

1. Vector Algebra:

A vector is an object which has both magnitudes and direction. It is usually represented by an arrow which shows the direction(→) and its length shows the magnitude. The arrow which indicates the vector has an arrowhead and its opposite end is the tail. It is denoted as

The magnitude of the vector is represented as |V|. Two vectors are said to be equal if they have equal magnitudes and equal direction.

Vector Algebra Operations:

Arithmetic operations such as addition, subtraction, multiplication on vectors. However, in the case of multiplication, vectors have two terminologies, such as dot product and cross product.