

# WBJEE 2025 Mathematics Question Paper

Time Allowed :3 Hour	Maximum Marks :100	Total Questions :75
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## General Instructions

Read the following instructions very carefully and strictly follow them:

- All questions are of objective type having four answer options for each.
- Category-1: Carries 1 mark each and only one option is correct. In case of incorrect answer or any combination of more than one answer, 4 mark will be deducted.
- Category-2: Carries 2 marks each and only one option is correct. In case of incorrect answer or any combination of more than one answer,  $\frac{1}{2}$  mark will be deducted.
- Category-3: (a) One or more option(s) is/are correct; (b) Marking all correct option(s) only will yield 2 (two) marks; (c) For any combination of answers containing one or more incorrect options, the said answer will be treated as wrong, yielding a zero mark even if one or more of the chosen option(s) is/are correct; (d) For partially correct answers, i.e., when all right options are not marked and also no incorrect options are marked, marks awarded  $2 \times$  (no of correct options marked) + total no of the correct option(s); (e) Not attempting the question will fetch zero mark.
- Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C or D.
- Use only Black/Blue ink ball point pen to mark the answer by filling up of the respective bubbles completely.
- Do not put any mark other than where required in specified places on the OMR Sheet.
- Write question booklet number and your Roll Number carefully in the specified locations of the OMR Sheet. Also fill appropriate bubbles.
- Write your name (in block letter), name of the examination center and put your signature (as it appeared in the Admit Card) in appropriate boxes in the OMR Sheet.
- The OMR Sheet is liable to become invalid if there is any mistake in filling the correct bubbles for Question Booklet number/Roll Number or if there is any discrepancy in the name /signature of the candidate, name of the examination center. The OMR Sheet may also become invalid due to folding or putting stray marks on it or any damage made to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be the sole responsibility of the candidate.

1. The number of reflexive relations on a set  $A$  of  $n$  elements is equal to:

- (A)  $2^{n^2}$
  - (B)  $n^2$
  - (C)  $2^{n(n-1)}$
  - (D)  $n^2 - n$
- 

2. If  $\cos^{-1} \alpha + \cos^{-1} \beta + \cos^{-1} \gamma = 3\pi$ , then  $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$  is equal to:

- (A) 0
  - (B) 1
  - (C) 6
  - (D) 12
- 

3. An  $n \times n$  matrix is formed using 0, 1 and  $-1$  as its elements. The number of such matrices which are skew symmetric is:

- (A)  $\frac{n(n-1)}{2}$
  - (B)  $(n-1)^2$
  - (C)  $2^{\frac{n(n-1)}{2}}$
  - (D)  $3^{\frac{n(n-1)}{2}}$
- 

4. If  $a, b, c$  are positive real numbers each distinct from unity, then the value of the determinant

$$\begin{vmatrix} 1 & \log_a b & \log_a c \\ \log_b a & 1 & \log_b c \\ \log_c a & \log_c b & 1 \end{vmatrix} \text{ is:}$$

- (A) 0
  - (B) 1
  - (C)  $\log_e(abc)$
  - (D)  $\log_a e \cdot \log_b e \cdot \log_c e$
- 

5. Let  $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ . If  $|A|^2 = 25$ , then  $|\alpha|$  equals to:

- (A)  $5^2$
  - (B) 1
  - (C)  $\frac{1}{5}$
  - (D) 5
- 

6. The set of points of discontinuity of the function  $f(x) = x - [x]$ ,  $x \in \mathbb{R}$  is:

- (A)  $\mathbb{Q}$
  - (B)  $\mathbb{R}$
  - (C)  $\mathbb{N}$
  - (D)  $\mathbb{Z}$
- 

7. If  $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2, & x > 1 \end{cases}$ ,  $x \in \mathbb{R}$ , is everywhere differentiable, then:

- (A)  $a = 3, b = 5$
  - (B)  $a = 0, b = 5$
  - (C)  $a = 0, b = 3$
  - (D)  $a = b = 3$
- 

8. A function  $f : \mathbb{R} \rightarrow \mathbb{R}$  satisfies

$$f\left(\frac{x+y}{3}\right) = \frac{f(x) + f(y) + f(0)}{3} \quad \text{for all } x, y \in \mathbb{R}.$$

If  $f''$  is differentiable at  $x = 0$ , then  $f$  is:

- (A) linear
  - (B) quadratic
  - (C) cubic
  - (D) biquadratic
- 

9. The value of the integral

$$\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx \text{ is:}$$

- (A)  $\frac{1}{2}$
  - (B)  $\frac{3}{2}$
  - (C) 2
  - (D) 1
- 

10. The value of

$$\int_0^{1.5} [x^2] dx \text{ is equal to:}$$

- (A) 2
  - (B)  $2 - \sqrt{2}$
  - (C)  $2 + \sqrt{2}$
  - (D)  $\sqrt{2}$
-

**11. The function  $f(x) = 2x^3 - 3x^2 - 12x + 4$ ,  $x \in \mathbb{R}$  has:**

- (A) two points of local maximum.
  - (B) two points of local minimum.
  - (C) one local maximum and one local minimum.
  - (D) neither maximum nor minimum.
- 

**12. For what value of  $a$ , the sum of the squares of the roots of the equation**

$$x^2 - (a - 2)x - a + 1 = 0$$

**will have the least value?**

- (A) 2
  - (B) 0
  - (C) 3
  - (D) 1
- 

**13. Let  $p(x)$  be a real polynomial of least degree which has a local maximum at  $x = 1$  and a local minimum at  $x = 3$ . If  $p(1) = 6$  and  $p(3) = 2$ , then  $p'(0)$  is equal to:**

- (A) 8
  - (B) 9
  - (C) 3
  - (D) 6
- 

**14. If**

$$x = \int_0^y \frac{1}{\sqrt{1+9t^2}} dt \quad \text{and} \quad \frac{d^2y}{dx^2} = ay,$$

**then  $a$  is equal to:**

- (A) 3
  - (B) 6
  - (C) 9
  - (D) 1
- 

**15. The value of**

$$\int_{-1}^1 \frac{x^3 + |x| + 1}{x^2 + 2|x| + 1} dx \text{ is equal to:}$$

- (A)  $\log 2$
  - (B)  $2 \log 2$
  - (C)  $\frac{1}{2} \log 2$
  - (D)  $4 \log 2$
-

**16. A function  $f$  is defined by  $f(x) = 2 + (x - 1)^{2/3}$  on  $[0, 2]$ . Which of the following statements is incorrect?**

- (A)  $f$  is not derivable in  $(0, 2)$ .
  - (B)  $f$  is continuous in  $[0, 2]$ .
  - (C)  $f(0) = f(2)$ .
  - (D) Rolle's theorem is applicable on  $[0, 2]$ .
- 

**17. Let  $f(x)$  be a second degree polynomial. If  $f(1) = f(-1)$  and  $p, q, r$  are in A.P., then  $f'(p), f'(q), f'(r)$  are:**

- (A) in A.P.
  - (B) in G.P.
  - (C) in H.P.
  - (D) neither in A.P. nor G.P. nor H.P.
- 

**18. Let  $\vec{a}, \vec{b}, \vec{c}$  be vectors of equal magnitude such that the angle between  $\vec{a}$  and  $\vec{b}$  is  $\alpha$ , between  $\vec{b}$  and  $\vec{c}$  is  $\beta$ , and between  $\vec{c}$  and  $\vec{a}$  is  $\gamma$ . Then the minimum value of  $\cos \alpha + \cos \beta + \cos \gamma$  is:**

- (A)  $\frac{1}{2}$
  - (B)  $-\frac{1}{2}$
  - (C)  $\frac{3}{2}$
  - (D)  $-\frac{3}{2}$
- 

**19. If  $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar vectors and  $\lambda$  is a real number, then the vectors**

$$\vec{a} + 2\vec{b} + 3\vec{c}, \quad \lambda\vec{b} + 4\vec{c}, \quad (2\lambda - 1)\vec{c}$$

**are non-coplanar for:**

- (A) no value of  $\lambda$ .
  - (B) all except one value of  $\lambda$ .
  - (C) all except two values of  $\lambda$ .
  - (D) all values of  $\lambda$ .
- 

**20. The straight line**

$$\frac{x - 3}{3} = \frac{y - 2}{1} = \frac{z - 1}{0}$$

**is:**

- (A) parallel to the x-axis.
  - (B) parallel to the y-axis.
  - (C) parallel to the z-axis.
  - (D) perpendicular to the z-axis.
-

21. If  $E$  and  $F$  are two independent events with  $P(E) = 0.3$  and  $P(E \cup F) = 0.5$ , then  $P(E/F) - P(F/E)$  equals:

- (A)  $\frac{2}{7}$
  - (B)  $\frac{3}{35}$
  - (C)  $\frac{1}{70}$
  - (D)  $\frac{1}{7}$
- 

22. If  $g(f(x)) = |\sin x|$  and  $f(g(x)) = (\sin \sqrt{x})^2$ , then:

- (A)  $f(x) = \sin^2 x, g(x) = \sqrt{x}$
  - (B)  $f(x) = \sin x, g(x) = |x|$
  - (C)  $f(x) = x^2, g(x) = \sin \sqrt{x}$
  - (D)  $f(x) = |x|, g(x) = \sin x$
- 

23. If  ${}^9P_5 + 5 \cdot {}^9P_4 = {}^{10}P_r$ , then the value of  $r$  is:

- (A) 4
  - (B) 8
  - (C) 5
  - (D) 7
- 

24. The value of the expression

$${}^{47}C_4 + \sum_{j=1}^5 {}^{52-j}C_3$$

is:

- (A)  ${}^{52}C_3$
  - (B)  ${}^{51}C_4$
  - (C)  ${}^{52}C_4$
  - (D)  ${}^{51}C_3$
- 

25. The sum of the first four terms of an arithmetic progression is 56. The sum of the last four terms is 112. If its first term is 11, then the number of terms is:

- (A) 10
  - (B) 11
  - (C) 12
  - (D) 13
- 

26. If the sum of  $n$  terms of an A.P. is  $3n^2 + 5n$  and its  $m$ -th term is 164, then the value of  $m$  is:

- (A) 26
  - (B) 27
  - (C) 28
  - (D) 29
- 

**27. If the sum of the squares of the roots of the equation**

$$x^2 - (a - 2)x - (a + 1) = 0$$

**is least for an appropriate real parameter  $a$ , then the value of  $a$  will be:**

- (A) 3
  - (B) 2
  - (C) 1
  - (D) 0
- 

**28. If for a matrix  $A$ ,  $|A| = 6$  and**

$$\text{adj } A = \begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & 1 \\ -1 & k & 0 \end{bmatrix},$$

**then  $k$  is equal to:**

- (A)  $-1$
  - (B) 1
  - (C) 2
  - (D) 0
- 

**29. Let  $\phi(x) = f(x) + f(2a - x)$ ,  $x \in [0, 2a]$ , and  $f''(x) > 0$  for all  $x \in [0, a]$ . Then  $\phi(x)$  is:**

- (A) increasing on  $[0, a]$ .
  - (B) decreasing on  $[0, a]$ .
  - (C) increasing on  $[0, 2a]$ .
  - (D) decreasing on  $[0, 2a]$ .
- 

**30. The value of the integral**

$$\int_0^{\pi/2} \log \left( \frac{4 + 3 \sin x}{4 + 3 \cos x} \right) dx \text{ is:}$$

- (A) 2
  - (B)  $\frac{3}{4}$
  - (C) 0
  - (D)  $-2$
-

31. If  $z_1, z_2$  are complex numbers such that  $\frac{2z_1}{3z_2}$  is a purely imaginary number, then the value of

$$\left| \frac{z_1 - z_2}{z_1 + z_2} \right|$$

is:

- (A) 1
  - (B) 2
  - (C) 3
  - (D) 4
- 

32. The line parallel to the x-axis passing through the intersection of the lines

$$ax + 2by + 3b = 0 \quad \text{and} \quad bx - 2ay - 3a = 0$$

where  $(a, b) \neq (0, 0)$ , is:

- (A) above x-axis at a distance  $\frac{3}{2}$  from it.
  - (B) above x-axis at a distance  $\frac{3}{3}$  from it.
  - (C) below x-axis at a distance  $\frac{3}{2}$  from it.
  - (D) below x-axis at a distance  $\frac{3}{3}$  from it.
- 

33. Consider three points  $P(\cos \alpha, \sin \beta)$ ,  $Q(\sin \alpha, \cos \beta)$  and  $R(0, 0)$ , where  $0 < \alpha, \beta < \frac{\pi}{4}$ . Then:

- (A)  $P$  lies on the line segment  $RQ$ .
  - (B)  $Q$  lies on the line segment  $PR$ .
  - (C)  $R$  lies on the line segment  $PQ$ .
  - (D)  $P, Q, R$  are non-collinear.
- 

34. If the matrix

$$\begin{pmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{pmatrix}$$

is orthogonal, then the values of  $a, b, c$  are:

- (A)  $a = \pm \frac{1}{\sqrt{3}}, b = \pm \frac{1}{\sqrt{6}}, c = \pm \frac{1}{\sqrt{2}}$
  - (B)  $a = \pm \frac{1}{\sqrt{2}}, b = \pm \frac{1}{\sqrt{6}}, c = \pm \frac{1}{\sqrt{3}}$
  - (C)  $a = \pm \frac{1}{\sqrt{2}}, b = \pm \frac{1}{\sqrt{6}}, c = \pm \frac{1}{\sqrt{3}}$
  - (D)  $a = \pm \frac{1}{\sqrt{3}}, b = \pm \frac{1}{\sqrt{6}}, c = \pm \frac{1}{\sqrt{3}}$
-

35. Suppose  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + qx + r = 0$  (with  $r \neq 0$ ) and they are in A.P. Then the rank of the matrix

$$\begin{pmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{pmatrix}$$

is:

- (A) 3
  - (B) 2
  - (C) 0
  - (D) 1
- 

36. If  $\text{adj } B = A$ ,  $|P| = |Q| = 1$ , then

$$\text{adj}(Q^{-1}BP^{-1}) = ?$$

- (A)  $PQ$
  - (B)  $QAP$
  - (C)  $PAQ$
  - (D)  $PA^{-1}Q$
- 

37. Let  $f(x) = |1 - 2x|$ , then

- (A)  $f(x)$  is continuous but not differentiable at  $x = \frac{1}{2}$ .
  - (B)  $f(x)$  is differentiable but not continuous at  $x = \frac{1}{2}$ .
  - (C)  $f(x)$  is both continuous and differentiable at  $x = \frac{1}{2}$ .
  - (D)  $f(x)$  is neither differentiable nor continuous at  $x = \frac{1}{2}$ .
- 

38. If  $(1 + x - 2x^2)^6 = 1 + a_1x + a_2x^2 + \dots + a_{12}x^{12}$ , then the value of  $a_2 + a_4 + a_6 + \dots + a_{12}$  is:

- (A) 21
  - (B) 31
  - (C) 32
  - (D) 64
- 

39. Let  $\omega (\neq 1)$  be a cube root of unity. Then the minimum value of the set

$$\{|a + b\omega + c\omega^2|^2 : a, b, c \text{ are distinct non-zero integers}\}$$

equals:

- (A) 15
- (B) 5
- (C) 3
- (D) 4

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40. The expression  $2^{4n} - 15n - 1$ , where  $n \in \mathbb{N}$ , is divisible by:

- (A) 125
  - (B) 225
  - (C) 325
  - (D) 425
- 

41. Let  $\vec{a}, \vec{b}, \vec{c}$  be unit vectors. Suppose  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$  and the angle between  $\vec{b}$  and  $\vec{c}$  is  $\frac{\pi}{6}$ . Then  $\vec{a}$  is:

- (A)  $\vec{b} \times \vec{c}$
  - (B)  $\vec{c} \times \vec{b}$
  - (C)  $\vec{b} + \vec{c}$
  - (D)  $\pm 2(\vec{b} \times \vec{c})$
- 

42. If  $\vec{a} = 3\hat{i} - \hat{k}$ ,  $|\vec{b}| = \sqrt{5}$  and  $\vec{a} \cdot \vec{b} = 3$ , then the area of the parallelogram for which  $\vec{a}$  and  $\vec{b}$  are adjacent sides is:

- (A)  $\sqrt{17}$
  - (B)  $\sqrt{14}$
  - (C)  $\sqrt{7}$
  - (D)  $\sqrt{41}$
- 

43. If  $\theta$  is the angle between two vectors  $\vec{a}$  and  $\vec{b}$  such that  $|\vec{a}| = 7$ ,  $|\vec{b}| = 1$  and

$$|\vec{a} \times \vec{b}|^2 = k^2 - (\vec{a} - \vec{b})^2,$$

then the values of  $k$  and  $\theta$  are:

- (A)  $k = 1, \theta = 45^\circ$
  - (B)  $k = 7, \theta = 60^\circ$
  - (C)  $k = 49, \theta = 90^\circ$
  - (D)  $k = 7$  and  $\theta$  arbitrary
- 

44. If  $f$  is the inverse function of  $g$  and  $g'(x) = \frac{1}{1+x^n}$ , then the value of  $f'(x)$  is:

- (A)  $1 + (f(x))^n$
  - (B)  $1 - (f(x))^n$
  - (C)  $\{1 + f(x)\}^n$
  - (D)  $(f(x))^n$
- 

45. Let

$$f_n(x) = \tan \frac{x}{2} (1 + \sec x)(1 + \sec 2x) \cdots (1 + \sec 2^{n-1}x),$$

then:

- (A)  $f_3\left(\frac{\pi}{16}\right) = 1$
  - (B)  $f_4\left(\frac{\pi}{16}\right) = 1$
  - (C)  $f_5\left(\frac{\pi}{16}\right) = 1$
  - (D)  $f_2\left(\frac{\pi}{16}\right) = 1$
- 

46. Evaluate

$$\lim_{x \rightarrow 0} \frac{\tan(\lfloor -\pi^2 \rfloor x^2) - x^2 \tan(\lfloor -\pi^2 \rfloor)}{\sin^2 x}$$

- (A) 0
  - (B)  $\tan 10 - 10$
  - (C)  $\tan 9 - 9$
  - (D) 1
- 

47. If  $x = -1$  and  $x = 2$  are extreme points of

$$f(x) = \alpha \log |x| + \beta x^2 + x \quad (x \neq 0),$$

then:

- (A)  $\alpha = -6, \beta = \frac{1}{2}$
  - (B)  $\alpha = -6, \beta = -\frac{1}{2}$
  - (C)  $\alpha = 2, \beta = -\frac{1}{2}$
  - (D)  $\alpha = 2, \beta = \frac{1}{2}$
- 

48. The line  $y - \sqrt{3}x + 3 = 0$  cuts the parabola  $y^2 = x + 2$  at the points  $P$  and  $Q$ . If the coordinates of the point  $X$  are  $(\sqrt{3}, 0)$ , then the value of  $XP \cdot XQ$  is:

- (A)  $\frac{4(2+\sqrt{3})}{3}$
  - (B)  $\frac{4(2-\sqrt{3})}{2}$
  - (C)  $\frac{5(2+\sqrt{3})}{3}$
  - (D)  $\frac{5(2-\sqrt{3})}{3}$
- 

49. Let  $f(x)$  be continuous on  $[0, 5]$  and differentiable in  $(0, 5)$ . If  $f(0) = 0$  and  $|f'(x)| \leq \frac{1}{5}$  for all  $x \in (0, 5)$ , then  $\forall x \in [0, 5]$ :

- (A)  $|f(x)| \leq 1$
  - (B)  $|f(x)| \leq \frac{1}{5}$
  - (C)  $f(x) = \frac{x}{5}$
  - (D)  $|f(x)| \geq 1$
-

**50.** Let  $f$  be a function which is differentiable for all real  $x$ . If  $f(2) = -4$  and  $f'(x) \geq 6$  for all  $x \in [2, 4]$ , then:

- (A)  $f(4) < 8$
  - (B)  $f(4) \geq 12$
  - (C)  $f(4) \geq 8$
  - (D)  $f(4) < 12$
- 

**51.** Let  $a_n$  denote the term independent of  $x$  in the expansion of

$$\left[ x + \frac{\sin(1/n)}{x^2} \right]^{3n},$$

then

$$\lim_{n \rightarrow \infty} \frac{(a_n)n!}{{}^{3n}P_n}$$

equals:

- (A) 0
  - (B) 1
  - (C)  $e$
  - (D)  $\frac{e}{\sqrt{3}}$
- 

**52.** The maximum number of common normals of  $y^2 = 4ax$  and  $x^2 = 4by$  is:

- (A) 3
  - (B) 4
  - (C) 5
  - (D) 6
- 

**53.** If  $|z_1| = |z_2| = |z_3| = 1$  and  $z_1 + z_2 + z_3 = 0$ , then the area of the triangle whose vertices are  $z_1, z_2, z_3$  is:

- (A)  $\frac{3\sqrt{3}}{4}$
  - (B)  $\frac{\sqrt{3}}{4}$
  - (C) 1
  - (D) 2
- 

**54.** The number of solutions of

$$\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$$

is:

- (A) 0
- (B) 1

- (C) 2  
(D) 4
- 

**55. If  $a, b, c$  are in A.P. and the equations**

$$(b - c)x^2 + (c - a)x + (a - b) = 0$$

$$2(c + a)x^2 + (b + c)x = 0$$

**have a common root, then:**

- (A)  $a^2, b^2, c^2$  are in A.P.  
(B)  $a^2, c^2, b^2$  are in A.P.  
(C)  $c^2, a^2, b^2$  are in A.P.  
(D)  $a^2, b^2, c^2$  are in G.P.
- 

**56. If  $f(x)$  and  $g(x)$  are polynomials such that**

$$\phi(x) = f(x^3) + xg(x^3)$$

**is divisible by  $x^2 + x + 1$ , then:**

- (A)  $\phi(x)$  divisible by  $x - 1$   
(B) none divisible by  $x - 1$   
(C)  $g(x)$  divisible by  $x - 1$ ,  $f(x)$  not  
(D)  $f(x)$  divisible by  $x - 1$ ,  $g(x)$  not
- 

**57. Let**

$$f(\theta) = \begin{vmatrix} 1 & \cos \theta & -1 \\ -\sin \theta & 1 & -\cos \theta \\ -1 & \sin \theta & 1 \end{vmatrix}.$$

**Suppose  $A$  and  $B$  are respectively maximum and minimum values of  $f(\theta)$ . Then**

**$(A, B)$  is:**

- (A)  $(2, 1)$   
(B)  $(2, 0)$   
(C)  $(\sqrt{2}, 1)$   
(D)  $(2, \frac{1}{\sqrt{2}})$
- 

**58. Let  $f(x) = |x - \alpha| + |x - \beta|$ , where  $\alpha, \beta$  are roots of  $x^2 - 3x + 2 = 0$ . Then the number of points in  $[\alpha, \beta]$  at which  $f$  is not differentiable is:**

- (A) 2  
(B) 0  
(C) 1  
(D) infinite

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**59.** Let  $x - y = 0$  and  $x + y = 1$  be two perpendicular diameters of a circle of radius  $R$ . The circle will pass through the origin if  $R$  equals:

- (A)  $\frac{1}{2}$
  - (B)  $\frac{1}{\sqrt{2}}$
  - (C)  $\frac{1}{\sqrt{3}}$
  - (D)  $\frac{1}{3}$
- 

**60.** If  $f(x) = \frac{3x-4}{2x-3}$ , then  $f(f(f(x)))$  will be:

- (A)  $x$
  - (B)  $2x$
  - (C)  $\frac{2x-3}{3x-4}$
  - (D)  $\frac{3x-4}{2x-3}$
- 

**61.** If  $\cos(\theta + \phi) = \frac{3}{5}$  and  $\sin(\theta - \phi) = \frac{5}{13}$ ,  $0 < \theta, \phi < \frac{\pi}{4}$ , then  $\cot(2\theta)$  equals:

- (A)  $\frac{16}{63}$
  - (B)  $\frac{63}{16}$
  - (C)  $\frac{3}{13}$
  - (D)  $\frac{13}{3}$
- 

**62.** The probability that a non-leap year selected at random will have 53 Sundays or 53 Saturdays is:

- (A)  $\frac{1}{7}$
  - (B)  $\frac{2}{7}$
  - (C) 1
  - (D)  $\frac{2}{365}$
- 

**63.** Let  $u + v + w = 3$ ,  $u, v, w \in \mathbb{R}$  and  $f(x) = ux^2 + vx + w$  be such that

$$f(x + y) = f(x) + f(y) + xy, \quad \forall x, y \in \mathbb{R}.$$

Then  $f(1)$  equals:

- (A)  $\frac{5}{2}$
  - (B)  $\frac{1}{2}$
  - (C)  $\frac{1}{\sqrt{2}}$
  - (D) 3
-

64. Let  $f(x) = \max\{x + [x], x - [x]\}$ , where  $[x]$  is the greatest integer  $\leq x$ . Then

$$\int_{-3}^3 f(x) dx$$

has the value:

- (A)  $\frac{51}{2}$
  - (B)  $\frac{21}{2}$
  - (C) 1
  - (D) 0
- 

65. The number of common tangents to the circles

$$x^2 + y^2 - 4x - 6y - 12 = 0, \quad x^2 + y^2 + 6x + 18y + 26 = 0$$

is:

- (A) 2
  - (B) 3
  - (C) 4
  - (D) 5
- 

66. The solution set of the equation

$$x \in \left(0, \frac{\pi}{2}\right), \quad \tan(\pi \tan x) = \cot(\pi \cot x)$$

is:

- (A)  $\{0\}$
  - (B)  $\left\{\frac{\pi}{4}\right\}$
  - (C)  $\emptyset$
  - (D)  $\left\{\frac{\pi}{6}\right\}$
- 

67. If  $P$  is a non-singular matrix of order  $5 \times 5$  and the sum of the elements of each row is 1, then the sum of the elements of each row in  $P^{-1}$  is:

- (A) 0
  - (B) 1
  - (C)  $\frac{1}{8}$
  - (D) 8
- 

68. If  $0 \leq a, b \leq 3$  and the equation

$$x^2 + 4 + 3 \cos(ax + b) = 2x$$

has real solutions, then the value(s) of  $(a + b)$  is/are:

- (A)  $\frac{\pi}{4}$
  - (B)  $\frac{\pi}{2}$
  - (C)  $\pi$
  - (D)  $2\pi$
- 

**69. If the equation**

$$\sin^2 x - (p + 2) \sin x - (p + 3) = 0$$

**has a solution, then  $p$  must lie in:**

- (A)  $[-3, -2]$
  - (B)  $(-3, -2)$
  - (C)  $(2, 3)$
  - (D)  $[-5, -3]$
- 

**70. If**

$$f(x) = \int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt, \quad g(x) = \int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt,$$

**then the value of  $f(x) + g(x)$  is:**

- (A)  $\pi$
  - (B)  $\frac{\pi}{4}$
  - (C)  $\frac{\pi}{2}$
  - (D) depends on  $x$
- 

**71. Three numbers are chosen at random without replacement from  $\{1, 2, \dots, 10\}$ . The probability that the minimum of the chosen numbers is 3 or the maximum is 7 is:**

- (A)  $\frac{5}{40}$
  - (B)  $\frac{3}{40}$
  - (C)  $\frac{11}{40}$
  - (D)  $\frac{9}{40}$
- 

**72. The population  $p(t)$  of a certain mouse species follows**

$$\frac{dp}{dt} = 0.5p - 450.$$

If  $p(0) = 850$ , then the time at which population becomes zero is:

- (A)  $\log 9$
  - (B)  $\frac{1}{2} \log 18$
  - (C)  $\log 18$
  - (D)  $2 \log 18$
-

**73. The value of**

$$\int_{-100}^{100} \frac{x + x^3 + x^5}{1 + x^2 + x^4 + x^6} dx$$

**is:**

- (A) 100
  - (B) 1000
  - (C) 0
  - (D) 10
- 

**74. Let  $f(x) = x^3$ ,  $x \in [-1, 1]$ . Then which of the following are correct?**

- (A)  $f'$  has a minimum at  $x = 0$ .
  - (B)  $f'$  has the maximum at  $x = 1$ .
  - (C)  $f'$  is continuous on  $[-1, 1]$ .
  - (D)  $f'$  is bounded on  $[-1, 1]$ .
- 

**75. Let  $f : [0, 1] \rightarrow \mathbb{R}$  and  $g : [0, 1] \rightarrow \mathbb{R}$  be defined as:**

$$f(x) = \begin{cases} 1, & x \text{ rational} \\ 0, & x \text{ irrational} \end{cases} \quad g(x) = \begin{cases} 0, & x \text{ rational} \\ 1, & x \text{ irrational} \end{cases}$$

**Then:**

- (A)  $f$  and  $g$  are continuous at  $x = \frac{1}{2}$ .
  - (B)  $f + g$  is continuous at  $x = \frac{2}{3}$  but  $f, g$  are discontinuous there.
  - (C)  $f(x), g(x) > 0$  for some  $x \in (0, 1)$ .
  - (D)  $f + g$  is not differentiable at  $x = \frac{3}{4}$ .
-