

WBJEE 2026 Mathematics

Question Paper

Conducted by West Bengal Joint Entrance Examinations Board



General Instructions

- (i) **Duration:** The total duration of the examination is 2 hours (120 minutes).
- (ii) **Total Marks:** The paper carries a maximum of 100 marks.
- (iii) **Structure:** The paper has 3 question categories:
 - **Category 1:** 50 questions for 1 mark each and negative marking of 0.25.
 - **Category 2:** 15 questions for 2 marks each and negative marking of 0.5.
 - **Category 3:** 10 questions for 2 marks each and no negative marking.
- (iv) **Compulsory Questions:** All 75 questions are compulsory

1. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that $x = 0$ is the only real root of $P'(x) = 0$. If $P(-1) < P(1)$, then in the interval $[-1, 1]$:

- (A) $P(-1)$ is the minimum but $P(1)$ is not the maximum of P
- (B) $P(-1)$ is not minimum but $P(1)$ is the maximum of P
- (C) neither $P(-1)$ is the minimum nor $P(1)$ is the maximum of P
- (D) $P(-1)$ is the minimum and $P(1)$ is the maximum of P

2. If α, β are the roots of the equation $x^2 - px + q = 0$ and $\alpha > 0, \beta > 0$, then $\alpha^{\frac{1}{4}} + \beta^{\frac{1}{4}} = (p + 6\sqrt{q} + 4q^{\frac{1}{4}}\sqrt{p + 2\sqrt{q}})^k$, where K is:

- (A) $\frac{3}{2}$
- (B) 2
- (C) $\frac{1}{3}$
- (D) $\frac{1}{4}$

3. If $\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{2r^2}\right) = a$, then $\tan a$ is equal to:

- (A) 1
- (B) 0
- (C) $\sqrt{3}$
- (D) $\frac{\pi}{4}$

4. Consider a function $f(x)$ which has exactly two roots at $x = a$. If $\lim_{x \rightarrow a} \left(\frac{\lambda f'(x)}{f(x)} - \frac{1}{x-a} \right) = m$ ($\neq 0$), then the value of λ is:

- (A) 2
- (B) $\frac{1}{2}$
- (C) 1
- (D) $\frac{1}{4}$

5. A vector given by $\vec{P} = f(t)\hat{i} + g(t)\hat{j} + \hat{k}$ moves in such a way that it is always parallel to the vector $\vec{Q} = -f''(t)\hat{i} + f'(t)\hat{j} + \hat{k}$. The magnitude of \vec{P} is:

- (A) a linear function of time
- (B) a quadratic function of time
- (C) a cubic function of time
- (D) constant

6. The expression $\sum_{K=1}^{32} (3K + 2) \left\{ \sum_{r=1}^{10} \left(\sin \frac{2r\pi}{11} - i \cos \frac{2r\pi}{11} \right) \right\}^K$ represents:

- (A) $48(1 + i)$
- (B) $48(1 - i)$
- (C) $-\frac{48}{11}(1 - i)$
- (D) $48(1 - i)$

7. θ elimination from the equations $x^2 + y^2 = \frac{x \cos 3\theta + y \sin 3\theta}{\cos^3 \theta} = \frac{y \cos 3\theta - x \sin 3\theta}{\sin^3 \theta}$ will be:

- (A) $4(x^4 + y^4) = 3x + 4y$
- (B) $(x^2 + y^2 + 2x)(x^2 + y^2 - x) = 2y^2$

(C) $(x^2 + y^2 - 2x)(x^2 + y^2 + x) = 9y$

(D) $x^{2/3} + y^{2/3} = 1$

8. t_n denotes the n th term of an A.P and $t_p = \frac{1}{q}$, $t_q = \frac{1}{p}$. Then which one of the following options is a root of the equation $(p + 2q - 3r)x^2 + (q + 2r - 3p)x + (r + 2p - 3q) = 0$?

(A) t_{pq}

(B) t_p

(C) t_q

(D) t_{p+q}

9. Consider the sequence of numbers $\{1, 2, 3, \dots, 13\}$. A person chooses three numbers at random from the sequence. The probability that the chosen three numbers form an A.P is:

(A) $\frac{21}{157}$

(B) $\frac{18}{143}$

(C) $\frac{29}{180}$

(D) $\frac{24}{163}$

10. If $f(x) = \frac{1+x}{1-x}$ and A is a matrix such that $A^3 = 0$, then $f(A) =$

(A) $I + 2A + 2A^2$

(B) $I + 2A + A^2$

(C) $I - 2A + A^2$

(D) $I + A + A^2$

11. Which of the following statements is always true?

(A) If $f(x)$ is decreasing, then $\frac{1}{f(x)}$ is increasing

(B) If $f(x)$ is decreasing, then $\frac{1}{f(x)}$ is also decreasing

(C) If both f and g are positive functions such that f is decreasing and g is increasing, then $\frac{f}{g}$ is a decreasing function

(D) If both f and g are positive functions such that f is increasing and g is decreasing, then $\frac{f}{g}$ is a decreasing function

12. If $0 < \alpha < \beta < \gamma < \frac{\pi}{2}$ then the equation $\frac{1}{x-\sin\alpha} + \frac{1}{x-\sin\beta} + \frac{1}{x-\sin\gamma} = 0$ has:

- (A) real and unequal roots
 - (B) imaginary roots
 - (C) real and equal roots
 - (D) rational roots
-

13. On the set \mathbb{R} of real numbers the relation ρ , defined by $x\rho y$ ($x, y \in \mathbb{R}$) iff:

- (A) $|x - y| < 2$ is reflexive but neither symmetric nor transitive
 - (B) $|x| \geq y$ is reflexive and transitive but not symmetric
 - (C) $x > |y|$ is transitive but neither reflexive nor symmetric
 - (D) $x - y < 2$ is reflexive and symmetric but not transitive
-

14. If $\int \frac{\csc^2 x - 2010}{\cos^{2010} x} dx = -\frac{f(x)}{(g(x))^{2010}} + c$, where $f(\frac{\pi}{4}) = 1$, then the number of solutions of the equation $\frac{f(x)}{g(x)} = \{x\}$ in $[0, 2\pi]$ is/are (where $\{\cdot\}$ represents fractional part function):

- (A) 3
 - (B) 1
 - (C) 0
 - (D) 2
-

15. If the locus of mid point of any normal chord of the parabola $y^2 = 4x$ is $x - \lambda = \frac{\mu}{y^2} + \frac{y^2}{\nu}$, $\lambda, \mu, \nu \in \mathbb{N}$, then $(\lambda + \mu + \nu)$ equals to:

- (A) 8
 - (B) 16
 - (C) 10
 - (D) 17
-

16. The true set of values of 'K' for which $\sin^{-1}\left(\frac{1}{1+\sin^2 x}\right) = \frac{K\pi}{6}$ may have a solution is:

- (A) $[\frac{1}{6}, \frac{1}{2}]$
 - (B) $[\frac{1}{4}, \frac{1}{2}]$
 - (C) $[2, 4]$
 - (D) $[1, 3]$
-

17. A mapping is selected at random from all mappings $f : A \rightarrow A$ where set $A = \{1, 2, 3, \dots, n\}$. If the probability that the mapping is injective is $\frac{3}{32}$, then the value of n is:

- (A) 8
 - (B) 14
 - (C) 3
 - (D) 4
-

18. Let $A = [a, \infty)$ denotes the domain, then $f : [a, \infty) \rightarrow B$ which is defined by $f(x) = 2x^3 - 3x^2 + 6$ will have an inverse for the smallest real value of 'a' if:

- (A) $a = 0, B = [6, \infty)$
 - (B) $a = 2, B = [10, \infty)$
 - (C) $a = 1, B = [5, \infty)$
 - (D) $a = -1, B = [5, \infty)$
-

19. If $a = \lim_{n \rightarrow \infty} \cos^{2n} x, (x = n\pi)$ and $b = \lim_{n \rightarrow \infty} \cos^{2n} x, (x \neq m\pi)$, then numerical value of the area of the triangle whose vertices are $(a, b), (-2, 1)$ and $(2, 1)$ is:

- (A) 2
 - (B) 4
 - (C) 1
 - (D) $\frac{1}{2}$
-

20. The position vectors of two adjacent sides of a rectangle $OACB$ are \vec{a} and \vec{b} respectively, where O is the origin. If $16|\vec{a} \times \vec{b}| = 3(|\vec{a}| + |\vec{b}|)^2$ and θ be the acute angle between the diagonals OC and AB , then the value of $\tan(\frac{\theta}{2})$ is:

- (A) $\frac{1}{3}$
 - (B) $\frac{1}{\sqrt{3}}$
 - (C) $\sqrt{3}$
 - (D) 1
-

21. The point of intersection of $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$ and $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$, where $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{k}$ is:

- (A) $3\hat{i} + 2\hat{j} + \hat{k}$
 (B) $\hat{i} - \hat{j} - \hat{k}$
 (C) $4\hat{i} + 2\hat{j} - \hat{k}$
 (D) $3\hat{i} + \hat{j} - \hat{k}$
-

22. Let a_1, a_2, a_3, \dots are in G.P such that $n > m$, $a_n > a_m$ and $a_1 + a_n = 66$, $a_2 \cdot a_{n-1} = 128$. If $\sum_{r=1}^n a_r = 126$, then n is:

- (A) 11
 (B) 8
 (C) 6
 (D) 64
-

23. The minimum length of intercept on any tangent to the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ cut by the circle $x^2 + y^2 = 25$ is:

- (A) 6
 (B) 9
 (C) 11
 (D) 8
-

24. Intercepts of the plane $\vec{r} \cdot \vec{n} = d$ ($\neq 0$) on the coordinate axes respectively are:

- (A) $\frac{\hat{i} \cdot \vec{n}}{d}, \frac{\hat{j} \cdot \vec{n}}{d}, \frac{\hat{k} \cdot \vec{n}}{d}$
 (B) $\left| \frac{\hat{i} \cdot \vec{n}}{d} \right|, \left| \frac{\hat{j} \cdot \vec{n}}{d} \right|, \left| \frac{\hat{k} \cdot \vec{n}}{d} \right|$
 (C) $\frac{d}{\hat{i} \cdot \vec{n}}, \frac{d}{\hat{j} \cdot \vec{n}}, \frac{d}{\hat{k} \cdot \vec{n}}$
 (D) $\frac{d}{\hat{i} \cdot \vec{n}}, \frac{d}{\hat{j} \cdot \vec{n}}, \frac{d}{\hat{k} \cdot \vec{n}}$
-

25. The general solution of the equation $\sin^{100} x - \cos^{100} x = 1$ is:

- (A) $\{2n\pi + \frac{\pi}{3} : n \in I\}$
 (B) $\{n\pi \pm \frac{\pi}{2} : n \in I\}$
 (C) $\{n\pi + \frac{\pi}{4} : n \in I\}$
 (D) $\{2m\pi - \frac{\pi}{3} : n \in I\}$

26. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$ then the value of $\begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$ is equal to:

- (A) 64
(B) 0
(C) 14
(D) 16
-

27. Number of elements in the range set of $f(x) = \left[\frac{x}{15} \right] \left[-\frac{15}{x} \right]$, for all $x \in (0, 90)$; (where $[\cdot]$ denotes the greatest integer function) is:

- (A) 8
(B) 7
(C) 6
(D) 5
-

28. Let 10 Bags B_1, B_2, \dots, B_{10} which contain 21, 22, ..., 30 different articles respectively. Then the total number of ways to bring out 10 articles from a Bag is:

- (A) ${}^{31}C_{20} + {}^{21}C_{10}$
(B) ${}^{31}C_{20} - {}^{21}C_{10}$
(C) ${}^{30}C_{20} - {}^{20}C_{10}$
(D) ${}^{30}C_{20} + {}^{20}C_{10}$
-

29. Let domain and range of $f(x)$ and $g(x)$ is $[0, \infty)$. If $f(x)$ is an increasing function, $g(x)$ is a decreasing function, $h(x) = f\{g(x)\}$, $h(0) = 0$ and $p(x) = h(x^3 - 2x^2 + 2x) - h(4)$ then for all $x \in (0, 2)$:

- (A) $p(x) = -3$
(B) $p(x) = 0$
(C) $0 < p(x) < -h(4)$
(D) $0 \leq p(x) \leq -h(4)$
-

30. Consider the following ellipse: $\frac{x^2}{f(K^2+2K+5)} + \frac{y^2}{f(K+11)} = 1$, where $f(x)$ is a positive decreasing function. Then the value (values) of K for which the major axis coincides with x -axis is:

- (A) $K = -5$
 - (B) $K \in (-3, 2)$
 - (C) $K \in (-7, -5)$
 - (D) $K = 2$
-

31. The solution of the differential equation $2x^2y \frac{dy}{dx} = \tan(x^2y^2) - 2xy^2$, given $y(1) = \sqrt{\frac{\pi}{2}}$ is:

- (A) $\sin(x^2y^2) = e^{x-1}$
 - (B) $\sin(x^2y^2) = e^{2(x-1)}$
 - (C) $\cos\left(\frac{\pi}{2} + x^2y^2\right) + x = 0$
 - (D) $\sin(x^2y^2) = 1$
-

32. The value of the integral $\int \frac{(\sqrt[3]{x+\sqrt{2-x^2}})(\sqrt[6]{1-x\sqrt{2-x^2}})}{\sqrt[3]{1-x^2}} dx$ for $x \in (0, 1)$ is:

- (A) $2^{\frac{1}{12}}x + c$
 - (B) $2^{\frac{3}{4}}x + c$
 - (C) $2^{\frac{1}{3}}x + c$
 - (D) $2^{\frac{1}{6}}x + c$
-

33. Consider the function $y = f(x)$ defined implicitly by the equation $y^3 - 3y + x = 0$ on the interval $(-\infty, -2) \cup (2, \infty)$. The area of the region bounded by the curve $y = f(x)$, the x -axis and the lines $x = a, x = b$, where $-\infty < a < b < -2$ is:

- (A) $\int_a^b \frac{x dx}{3((f(x))^2-1)} - b f(b) + a f(a)$
 - (B) $-\int_a^b \frac{x dx}{3((f(x))^2-1)} - b f(b) + a f(a)$
 - (C) $\int_a^b \frac{x dx}{3((f(x))^2-1)} + b f(b) - a f(a)$
 - (D) $-\int_a^b \frac{x dx}{3((f(x))^2-1)} + b f(b) - a f(a)$
-

34. The total number of polynomials of the form $x^3 + ax^2 + bx + c$ which are divisible by $x^2 + 1$, where $a, b, c \in \{1, 2, 3, \dots, 10\}$ is:

- (A) 120
- (B) 45

- (C) 10
(D) 15
-

35. The term independent of x in the expansion of $\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1} - \frac{x-1}{x-x^{\frac{1}{2}}}\right)^{15}$ is equal to:

- (A) 5105
(B) 5005
(C) 1365
(D) 105
-

36. For a real number y , consider $[y]$ denotes the greatest integer less than or equal to y . If

$f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$, then:

- (A) $f'(x)$ exists for all x
(B) $f'(x)$ does not exist
(C) $f'(1) = \frac{\pi}{4}$
(D) $f'(1) = -\frac{\pi}{4}$
-

37. If $\int_0^1 \left(\sum_{r=1}^{2013} \frac{x}{x^2+r^2}\right) \left(\prod_{r=1}^{2013} (x^2+r^2)\right) dx = \frac{1}{2} \left[\left(\prod_{r=1}^{2013} (1+r^2)\right) - K\right]$, then K is:

- (A) $\frac{2013(2014)(4027)}{6}$
(B) $(2013)^{2013}$
(C) $(2013)!$
(D) $((2013)!)^2$
-

38. The least positive value of 'a' for which the equation $\int_0^x (t^2 - 8t + 13)dt = x \sin \frac{a}{x}$ has a solution is:

- (A) 3π
(B) 4π
(C) π
(D) 2π
-

39. Let all the points on the curve $x^2 + y^2 - 10x = 0$ are reflected about the line $y = x + 3$. If the locus of the reflected points is in the form $x^2 + y^2 + gx + f + c = 0$, then the value of $(g + f + c)$ is:

- (A) 38
 - (B) -28
 - (C) 28
 - (D) -38
-

40. The equation $|x + 1|^{\log_{x+1}(3+2x-x^2)} = (x - 3)|x|$ has:

- (A) no solution
 - (B) two solutions
 - (C) unique solution
 - (D) infinite no. of solutions
-

41. If the domain of $f(x)$ is $(0, 1)$, then the domain of $y = f(e^x) + f(\ln|x|)$ is:

- (A) $(-1, -\frac{1}{e})$
 - (B) $(\frac{1}{e}, 1)$
 - (C) $(-e, -1)$
 - (D) $(-e, -1) \cup (1, e)$
-

42. The number of 3-digit numbers of the form xyz with $x < y, z < y$ and $x \neq 0$ is:

- (A) 284
 - (B) 240
 - (C) 44
 - (D) 270
-

43. Suppose A is denoted the set of all numbers between 1 and 700 which are divisible by 3 and let B is denoted the set of all numbers between 1 and 300 which are divisible by 7. If $C = \{(a, b) | a \in A, b \in B, a \neq b \text{ and } a + b = \text{even number}\}$, then order of C is:

- (A) 4879
- (B) 4789

- (C) 6789
(D) 9876

44. Let us define the power of a matrix A as the maximum $m \in \mathbb{Z}^+$ such that $A^m = I$. For two matrices A and B if $A^5 = I$ and $ABA^{-1} = B^2$, then the power of the matrix B is between:

- (A) 20 and 24
(B) 28 and 32
(C) 36 and 40
(D) 4 and 8

45. If for two real numbers a, b with $|a| \leq 1$ and $|b| \leq 1$, $\frac{1}{3} + \frac{\sin^{-1} a + \sin^{-1} b}{4} + \frac{(\sin^{-1} a + \sin^{-1} b)^2}{16} + \frac{(\sin^{-1} a + \sin^{-1} b)^3}{64} + \dots = \frac{2(8-3\pi)}{3(16+3\pi)}$, then the value of $\sin^{-1}(a\sqrt{1-b^2} + b\sqrt{1-a^2})$ is:

- (A) $\frac{2(32+15\pi)}{3\pi-8}$
(B) $-\frac{\pi}{4}$
(C) $-\frac{3\pi}{4}$
(D) $\frac{1}{3} + \frac{\pi}{4}$

46. Let $\det A = \begin{vmatrix} l & m & n \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$. If $(l-m)^2 + (p-q)^2 = 9$, $(m-n)^2 + (q-r)^2 = 16$, $(n-l)^2 + (r-p)^2 = 25$, then the value of $(\det A)^2$ is:

- (A) 169
(B) 144
(C) 121
(D) 100

47. Let $f : (0, 1) \rightarrow (0, 1)$ be a bijective differentiable function such that $f'(x) \neq 0 \forall x \in (0, 1)$ and $f\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$. Suppose for all x ,

$$\lim_{t \rightarrow x} \frac{\int_0^t \sqrt{1-(f(s))^2} ds - \int_0^x \sqrt{1-(f(s))^2} ds}{f(t) - f(x)} = f(x)$$

Then the value of $f\left(\frac{1}{4}\right)$ belongs to:

- (A) $\{\sqrt{7}, \sqrt{6}\}$
(B) $\left\{\frac{\sqrt{7}}{2}, \frac{\sqrt{15}}{2}\right\}$
(C) $\left\{\frac{\sqrt{7}}{4}, \frac{\sqrt{15}}{4}\right\}$
(D) $\left\{\frac{\sqrt{7}}{3}, \frac{\sqrt{15}}{3}\right\}$
-

48. If 'a' is an integer lying in $[-5, 30]$, then the probability that the graph of $y = x^2 + 2(a + 4)x - 5a + 64$ lies above the x-axis is:

- (A) $\frac{1}{6}$
(B) $\frac{7}{36}$
(C) $\frac{2}{9}$
(D) $\frac{3}{5}$
-

49. Consider a square $ABCD$ of diagonal length $2a$. The square is folded along the diagonal AC so that the plane of $\triangle ABC$ is perpendicular to the plane of $\triangle ADC$. In this case the shortest distance between AB and CD is:

- (A) $\frac{2a}{\sqrt{3}}$
(B) $\frac{a}{2\sqrt{3}}$
(C) $\frac{a}{\sqrt{3}}$
(D) $\frac{\sqrt{3}a}{2}$
-

50. If $\int \frac{(1-x^2)dx}{\sqrt{x}\sqrt{(1+x^2)^3}} = \alpha \frac{x^\beta}{(1+x^2)^\gamma} + C$, $\alpha, \beta, \gamma \in \mathbb{R}$ and C is constant of integration, then $\alpha : \beta : \gamma$ will be:

- (A) $4 : 1 : 1$
(B) $2 : 2 : \frac{1}{2}$
(C) $\frac{1}{6} : 2 : \frac{1}{2}$
(D) $1 : 2 : \frac{1}{2}$
-

51. Let $\vec{a} = (x, y, z)$ be the vector with $|\vec{a}| = 2\sqrt{3}$, which makes equal angles with the vector $\vec{b} = (y, -2z, 3x)$ and $\vec{c} = (2z, 3x, -y)$ and is perpendicular to the vector $\vec{d} = (1, -1, 2)$. If the angle between \vec{a} and the unit vector \hat{j} is obtuse, then \vec{a} is:

- (A) $(2, -2, -2)$
-

- (B) $(-2, -2, 2)$
(C) $(-2, 2, -2)$
(D) $(2, -2, 2)$
-

52. Let A_1, A_2, \dots, A_6 are six sets, each with four elements and B_1, B_2, \dots, B_n are n sets, each with two elements. Let $S = A_1 \cup A_2 \cup \dots \cup A_6 = B_1 \cup B_2 \cup \dots \cup B_n$. Given that each element of S belongs to exactly four of the A 's and to exactly three of the B 's. Then n is:

- (A) 12
(B) 24
(C) 6
(D) 9
-

53. A figure is bounded by the curves $y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 1$. The point at which a tangent should be drawn to the curve $y = x^2 + 1$ for it to cut off a trapezium of the greatest area from the figure is:

- (A) $(1, 2)$
(B) $(-1, 2)$
(C) $(\frac{1}{2}, \frac{5}{4})$
(D) $(0, 1)$
-

54. The ends A, B of a straight line segment of constant length c slide upon the fixed rectangular axes OX, OY respectively. If the rectangle $OAPB$ is completed, then the locus of the foot of the perpendicular drawn from P to AB is:

- (A) $x^2 + y^2 = c^2$
(B) $x^{2/3} + y^{2/3} = c^{2/3}$
(C) $\sqrt{x} + \sqrt{y} = \sqrt{c}$
(D) $xy = c^2$
-

55. Let 1 lies between the roots of the equation $y^2 - my + 1 = 0$ and $[x]$ denotes the greatest integer function. Then the value of $\left[\left(\frac{4|x|}{x^2+16} \right)^m \right]$ is:

- (A) 5
(B) 4
-

- (C) 0
(D) 1
-

56. Let $f(x)$ be a twice differentiable function in $[1, 3]$ and $f(1) = f(3)$. Further if $|f''(x)| \leq 2$, then for all x in $[1, 3]$:

- (A) $|f'(x)| \geq 4$
(B) $|f'(x)| \leq -1$
(C) $|f'(x)| > 2$
(D) $|f'(x)| < 4$
-

57. The quantities a_1, a_2, a_3, \dots form an infinite decreasing G.P. If $a_1 = 1$, then the common ratio of the progression for which the expression $6a_5 - 16a_4 - 3a_3 + 12a_2$ is at a maximum is:

- (A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $\frac{1}{3}$
(D) $-\frac{1}{4}$
-

58. If f be a real valued function defined for all real numbers x such that for some fixed $a > 0$, it satisfies $f(x+a) = \frac{1}{2} + \sqrt{f(x) - (f(x))^2} \forall x$, then $f(x)$ is periodic with period:

- (A) a
(B) $4a$
(C) $\frac{a}{2}$
(D) $2a$
-

59. Four natural numbers selected at random are multiplied together, then the probability that the digit in the unit's place in the product be 1, 3, 7 or 9 is:

- (A) $\frac{16}{625}$
(B) $\frac{18}{625}$
(C) $\frac{4}{625}$
(D) $\frac{5}{625}$
-

60. Let $f(x)$ be a real valued function which is monotonic and differentiable. Then for any reals a and b , $\int_{f(a)}^{f(b)} 2x\{b - f^{-1}(x)\}dx =$

- (A) $\int_a^b (f^2(x) - f^2(a))dx$
 - (B) $\int_a^b (f(x) - f(a))^2 dx$
 - (C) $\int_a^b (bf^2(x) - af^2(a))dx$
 - (D) $bf^2(b) + f^{-1}(a)$
-

61. Tangent at a point P_1 (other than $(0,0)$) on the curve $y = x^3$ meets the curve again at P_2 . The tangent at P_2 meets the curve at P_3 and so on. Then the abscissae of $P_1, P_2, P_3, \dots, P_n$ form:

- (A) an A.P with common difference 1
 - (B) an H.P with common difference $\frac{1}{2}$
 - (C) a G.P with common ratio 2
 - (D) a G.P with common ratio (-2)
-

62. The equation $x^3 + 5x^2 + px + q = 0$ and $x^3 + 7x^2 + px + r = 0$ have two roots in common. If the third root of each equation is represented by x_1 and x_2 respectively, then GCD of x_1, x_2 will be:

- (A) 3
 - (B) 1
 - (C) p
 - (D) 2
-

63. Let a, b, c be non-zero real numbers, such that $\int_0^1 (1 + \cos^8 x)(ax^2 + bx + c)dx = \int_0^2 (1 + \cos^8 x)(ax^2 + bx + c)dx$. Then $ax^2 + bx + c = 0$ has:

- (A) no solution in $(0, 2)$
 - (B) at least one root in $(1, 2)$
 - (C) two imaginary roots
 - (D) two roots in $(0, 2)$
-

64. Let Z_1, Z_2 be the roots of the equation $Z^2 + pZ + q = 0$, where the coefficients p and q may be complex numbers and also let A, B represent Z_1, Z_2 respectively in the complex plane. If $\angle AOB = \alpha \neq 0$ and $OA = OB$, where O is the origin, then the value of $\frac{p^2}{q} \sec^2 \frac{\alpha}{2}$ will be:

- (A) $\frac{1}{4}$
 (B) $\frac{3}{4}$
 (C) 4
 (D) 1
-

65. Let $g(x) = ax + b$, where $a < 0$ and g is defined from $[1, 3]$ onto $[0, 2]$. Then the value of $\cot(\cos^{-1}(|\sin x| + |\cos x|) + \sin^{-1}(-|\cos x| - |\sin x|))$ is equal to:

- (A) $g(2) + g(3)$
 (B) $g(2)$
 (C) $g(3)$
 (D) $g(1) + g(2)$
-

66. If $\sum_{r=0}^{2n} a_r(x-2)^r = \sum_{r=0}^{2n} b_r(x-3)^r$ and $a_k = 1 \forall k \geq 1$, then the value of $\frac{b_n}{{}_{2n+1}C_{n+1}}$ is:

- (A) $\frac{1}{2}$
 (B) 2
 (C) $\frac{1}{4}$
 (D) 1
-

67. If $f(x)$ is differentiable for all $x \in \mathbb{R}$ and satisfies the relation $x = \lim_{n \rightarrow \infty} \frac{[1^2(f(x))^x] + [2^2(f(x))^x] + \dots + [n^2(f(x))^x]}{n^3}$, where $[\cdot]$ denotes the greatest integer function, then $f'(x)$ is equal to:

- (A) $\frac{1}{3x^2} \ln x$
 (B) $3x^{1/x}(1 - \ln 3x)$
 (C) $(3x)^{\frac{1}{x}} \left[\frac{1 - \ln 3x}{x^2} \right]$
 (D) $(3x)^{\frac{1}{x}} \frac{(\ln 3x + 1)}{x^2}$
-

68. If a differentiable function satisfies

$$(x - y)f(x + y) - (x + y)f(x - y) = 2(x^2y - y^3), \quad \forall x, y \in \mathbb{R}$$

and $f(1) = 2$, then:

- (A) $f(x)$ must be a polynomial function
 (B) $f(3) = 13$
-

(C) $f(3) = 12$

(D) $f(0) = 0$

69. Let $f(x) > 0$ for all $x \in \mathbb{R}$ and $f(x)$ is bounded. If $\lim_{n \rightarrow \infty} \sum_{r=1}^n a^{r-1} \int_{(r-1)a}^{ra} \frac{f(x)dx}{f(x)+f(2ra-a-x)} = \frac{3}{5}$, where $0 < a < 1$, then the value(s) of a is/are:

(A) $\frac{5}{11}$

(B) $\frac{7}{11}$

(C) $\frac{6}{11}$

(D) $\frac{1}{11}$

70. Consider the curve $x = 1 - 3t^2, y = t - 3t^3$. The tangent to the curve at the point is inclined at an angle ϕ to OX and the tangent at $P(-2, 2)$ meets the curve again at Q . Then:

(A) the curve is symmetrical about x -axis

(B) the curve is symmetrical about y -axis

(C) $3t = \tan \phi + \sec \phi$

(D) tangents at P and Q are at right angle

71. If $f(x) = x(1331x^2 - 3630x + 3300)$, then for $a = \cos^2(\tan^{-1}(\sin(\cot^{-1} 3)))$:

(A) $f(a + 1) = 2331$

(B) $f'(a) = 11$

(C) $\lim_{x \rightarrow a} f(x) = 1000$

(D) $\int_0^a (f(x) - 1000)dx = \frac{2500}{11}$

72. Let $\vec{r} = \sin x(\vec{a} \times \vec{b}) + \cos y(\vec{b} \times \vec{c}) + 2(\vec{c} \times \vec{a})$, where \vec{a}, \vec{b} and \vec{c} are three non-coplanar vectors. It is given that \vec{r} is perpendicular to $(\vec{a} + \vec{b} + \vec{c})$. Then the possible value(s) of $(x^2 + y^2)$ is/are:

(A) $\frac{5\pi^2}{4}$

(B) $\frac{35\pi^2}{4}$

(C) $\frac{37\pi^2}{4}$

(D) $\frac{\pi^2}{4}$

73. The parabola $y = 4 - x^2$ has vertex P . It intersects the x -axis at A and B . If the parabola is translated from its initial position to a new position by moving its vertex along the line $y = x + 4$, so that it intersects the x -axis at B and C , then the abscissa of C will be:

- (A) 12
 - (B) 8
 - (C) 6
 - (D) $\frac{7}{3}$
-

74. If $A_1, A_2, A_3, \dots, A_{1006}$ be independent events such that $P(A_i) = \frac{1}{2^i}$ ($i = 1, 2, \dots, 1006$) and the probability that none of the events occurs be $\frac{\alpha!}{2^{\alpha(\beta!)^2}}$, then:

- (A) β is of the form $4k + 2, k \in I$
 - (B) $\alpha = 2\beta$
 - (C) β is of the form $4k + 1, k \in I$
 - (D) β is a prime number
-

75. If $(4^{\sec^2 \alpha})x^2 + 2x + (\beta^2 - \beta + \frac{1}{2}) = 0$ has real roots, then the value/values of $(\cos \alpha + \cos^{-1} \beta)$ is/are:

- (A) $1 + \frac{\pi}{3}$ if $\alpha = 2n\pi$
 - (B) $-1 - \frac{\pi}{3}$ if $\alpha = (2n + 1)\pi$
 - (C) $-1 + \frac{\pi}{3}$ if $\alpha = (2n + 1)\pi$
 - (D) $-1 + \frac{\pi}{3}$ if $\alpha = 2n\pi$
-