WBJEE Mathematics Sample Paper- 11

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **75** Multiple Choice Questions.
- **Section A (Q1–Q50):** Each correct answer carries +1 mark. Incorrect answer: **0.25 marks**. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries +2 mark. Incorrect answer: **0.5 marks**. Only **one** correct option.
- **Section C (Q66–Q75)**: Each correct answer carries **+2 marks**. **No negative marking.** One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each (Negative Marking: 0.25) [Single Correct]

Q1. If the sum of the first n terms of an A.P. is cn^2, then the sum of the squares of these n terms is:

(A) $\frac{n(4n^2-1)c^2}{6}$

(B) $\frac{n(4n^2+1)c^2}{3}$

(C) $\frac{n(4n^2-1)c^2}{3}$

(D) $\frac{n(4n^2+1)c^2}{6}$

Q2. If a, b, c are in G.P. and $a^x = b^y = c^z$, then x, y, z are in:

(A) A.P.

(B) G.P.

(C) H.P.

(D) None of these

Q3. The value of $\sum_{r=0}^{n} \binom{n}{r} \frac{(-1)^r}{r+1}$ is:

(A) n

(B) $\frac{1}{n+1}$

(C) $\frac{n}{n+1}$

(D) 0

Q4. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, then the number of integers in 5^{20} is:

(A) 14

(B) 15

(C) 13

(D) 16

Q5. The number of ways in which 5 different balls can be placed in 3 identical boxes such that no box remains empty is:

(A) 25

(B) 50

(C) 150

(D) 15

Q6. The coefficient of x^n in the expansion of $(1 - 2x + 3x^2 - 4x^3 + \ldots \infty)^{-n}$ is:

(A) $\binom{2n-1}{n}$

(B) $\binom{3n-1}{2n-1}$

(C) $\binom{n-1}{n}$

(D) $\binom{3n-1}{n}$

Q7. The sum of the series $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \ldots$ is:

(A) e

(B) $\frac{e}{2}$

(C) $\frac{3e}{2}$

(D) $2e$

Q8. If x is so small that x^3 and higher powers can be neglected, then $(1-x)^{1/2}(1-x)^{-1}$ is approximately:

(A) $1+\frac{x}{2}+\frac{3x^2}{8}$

(B) $1+\frac{3x}{2}+\frac{x^2}{8}$

(C) $1+\frac{x}{2}-\frac{3x^2}{8}$

(D) $1+\frac{3x}{2}+\frac{11x^2}{8}$

Q9. If $a_1, a_2, \ldots, a_n$ are in H.P., then $a_1a_2+a_2a_3+\cdots+a_{n-1}a_n$ is equal to:

(A) $(n-1)a_1a_n$

(B) na_1a_n

(C) $(n+1)a_1a_n$

(D) a_1a_n

Q10. The value of $2^{1/4}\cdot 4^{1/8}\cdot 8^{1/16}\ldots\infty$ is:

(A) 1

(B) 2

(C) 3/2

(D) 4

Q11. If z is a complex number such that $|z|\geq 2$, then the minimum value of $|z+\frac{1}{2}|$ is:

(A) 0

(B) 3/2

(C) 5/2

(D) 1

Q12. If α, β are the roots of $x^2 + px + 1 = 0$ and γ, δ are the roots of $x^2 + qx + 1 = 0$, then $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$ is:

(A) $p^2 - q^2$

(B) $q^2 - p^2$

(C) $p^2 + q^2$

(D) 0

Q13. The number of real roots of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ is:

(A) 1

(B) 2

(C) Infinite

(D) 0

Q14. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals:

(A) 128ω

(B) -128ω

(C) $128\omega^2$

(D) $-128\omega^2$

Q15. The locus of z satisfying $arg(\frac{z-1}{z+1}) = \frac{\pi}{4}$ is:

(A) A circle

(B) A parabola

(C) A straight line

(D) An ellipse

Q16. If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then BB^T equals:

(A) I

(B) B^{-1}

(C) $(B^T)^{-1}$

(D) B

Q17. The value of $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is:

(A) 1

(B) ω

(C) ω^2

(D) 0

Q18. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A - 2I$ is:

(A) O

(B) I

(C) $2I$

(D) A

Q19. If the system of equations $x + ay = 0$, $az + y = 0$, $ax + z = 0$ has infinite solutions, then a is:

(A) 0

(B) 1

(C) -1

(D) 2

Q20. The inverse of a symmetric matrix is:

(A) Symmetric

(B) Skew-symmetric

(C) Diagonal

(D) None

Q21. If $f(x) = \log(\frac{1+x}{1-x})$, then $f(\frac{2x}{1+x^2})$ is:

(A) $2f(x)$

(B) $3f(x)$

(C) $[f(x)]^2$

(D) $f(x^2)$

Q22. A relation R on set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$. R is:

(A) Equivalence

(B) Reflexive only

(C) Symmetric only

(D) Transitive only

Q23. The domain of $f(x) = \sqrt{\cos(\sin x)}$ is:

(A) $[-\pi/2, \pi/2]$

(B) $(-\infty, \infty)$

(C) $[0, \pi]$

(D) $[-1, 1]$

Q24. The probability that a leap year selected at random contains 53 Sundays is:

(A) $1/7$

(B) $2/7$

(C) $53/366$

(D) $2/366$

Q25. If A and B are independent events such that $P(A) = 0.3, P(B) = 0.6$, then $P(A \cup B)$ is:

(A) 0.9

(B) 0.18

(C) 0.72

(D) 0.42

Q26. The variance of first n natural numbers is:

(A) $\frac{n^2-1}{12}$

(B) $\frac{n^2+1}{12}$

(C) $\frac{n^2-1}{6}$

(D) $\frac{n(n+1)}{2}$

Q27. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random, the probability that exactly 2 are red is:

(A) $15/28$

(B) $5/14$

(C) $15/56$

(D) $30/56$

Q28. Mean deviation of the data $3, 10, 10, 4, 7, 10, 5$ from mean is:

(A) 2

(B) 2.57

(C) 3

(D) 3.25

Q29. The value of $\tan 20^\circ \tan 40^\circ \tan 80^\circ$ is:

(A) $\sqrt{3}$

(B) $1/\sqrt{3}$

(C) 3

(D) 1

Q30. If $\sin^{-1} x + \sin^{-1} y = \pi$, then the value of $x + y$ is:

(A) 0

(B) 1

(C) 2

(D) -2

Q31. The general solution of $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$ is:

(A) $n\pi + \frac{\pi}{4} - \frac{\pi}{3}$

(B) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{6}$

(C) $n\pi \pm \frac{\pi}{4}$

(D) $2n\pi \pm \frac{\pi}{3}$

Q32. The value of $\cos 1^\circ \cos 2^\circ \ldots \cos 179^\circ$ is:

(A) 1

(B) 0

(C) -1

(D) $1/2$

Q33. The distance between the lines $3x + 4y = 9$ and $6x + 8y = 15$ is:

(A) $3/10$

(B) $6/5$

(C) $3/5$

(D) 0

Q34. The equation of the circle passing through $(0, 0)$, $(a, 0)$ and $(0, b)$ is:

(A) $x^2 + y^2 - ax - by = 0$

(B) $x^2 + y^2 + ax + by = 0$

(C) $x^2 + y^2 - ax + by = 0$

(D) $x^2 + y^2 + ax - by = 0$

Q35. The eccentricity of the hyperbola $x^2 - 4y^2 = 1$ is:

(A) $\frac{\sqrt{5}}{2}$

(B) $\frac{5}{4}$

(C) $\frac{3}{2}$

(D) $\sqrt{5}$

Q36. The length of the latus rectum of the parabola $y^2 = -8x$ is:

(A) 2

(B) 4

(C) 8

(D) 16

Q37. The angle between the lines $y = x + 4$ and $y = 2x - 6$ is:

(A) $\tan^{-1}(1/3)$

(B) $\tan^{-1}(3)$

(C) $45°$

(D) $60°$

Q38. The condition for the line $y = mx + c$ to be tangent to the circle $x^2 + y^2 = a^2$ is:

(A) $c^2 = a^2(1 + m^2)$

(B) $c = am$

(C) $c^2 = a^2/m^2$

(D) $c^2 = a^2(1 - m^2)$

Q39. The area of the triangle formed by the lines $x = 0$, $y = 0$ and $x/a + y/b = 1$ is:

(A) ab

(B) $ab/2$

(C) $2ab$

(D) a^2b^2

Q40. The focus of the parabola $x^2 = -16y$ is:

(A) $(4, 0)$

(B) $(0, 4)$

(C) $(0, -4)$

(D) $(-4, 0)$

Q41. If the eccentricity of an ellipse is $5/8$ and the distance between foci is 10, then the latus rectum is:

(A) $39/4$

(B) $39/8$

(C) $15/4$

(D) $15/2$

Q42. The equation of the director circle of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is:

(A) $x^2 + y^2 = a^2 + b^2$

(B) $x^2 + y^2 = a^2 - b^2$

(C) $x^2 + y^2 = ab$

(D) $x^2 + y^2 = 2ab$

Q43. The direction cosines of a line making equal angles with the axes are:

(A) $(1, 1, 1)$

(B) $(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$

(C) $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

(D) $(0, 0, 1)$

Q44. The distance of the point $(1, 2, 3)$ from the plane $x + 2y + 2z = 5$ is:

(A) 2

(B) 3

(C) 4

(D) 6/3

Q45. The shortest distance between the lines $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-4}{1} = \frac{z-6}{1}$ is:

(A) $\sqrt{6}$

(B) 0

(C) $\sqrt{3}$

(D) $2\sqrt{2}$

Q46. The angle between the planes $x + y + 2z = 9$ and $2x - y + z = 15$ is:

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/2$

(D) $\pi/6$

Q47. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) 0°

(B) 45°

(C) 90°

(D) 180°

Q48. The area of a parallelogram whose diagonals are $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$ is:

(A) $5\sqrt{3}$

(B) $\sqrt{300}$

(C) $10\sqrt{3}$

(D) $5\sqrt{2}$

Q49. The value of $[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}]$ is:

(A) 0

(B) $2[\vec{a}\vec{b}\vec{c}]$

(C) $[\vec{a}\vec{b}\vec{c}]$

(D) 1

Q50. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=0$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is:

(A) 1

(B) 3

(C) $-3/2$

(D) $3/2$

Section–B — 15 Questions × 2 Marks Each
(Negative Marking: 0.5) [Single Correct]

Q51. A unit vector perpendicular to both $\hat{i}+\hat{j}$ and $\hat{j}+\hat{k}$ is:

(A) $\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

(B) $\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$

(C) $\hat{i}-\hat{j}+\hat{k}$

(D) $\frac{\hat{j}-\hat{k}}{\sqrt{2}}$

Q52. The value of $\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x^3}$ is:

(A) $1/3$

(B) $2/3$

(C) 1

(D) 0

Q53. If $f(x)=|x-1|+|x-2|$, then $f'(1.5)$ is:

(A) 0

(B) 1

(C) 2

(D) Does not exist

Q54. The derivative of $\sec^{-1}(\frac{1}{2x^2-1})$ with respect to $\sqrt{1-x^2}$ at $x = 1/2$ is:

(A) 4

(B) $1/4$

(C) 2

(D) 1

Q55. If $y = \log(\log x)$, then y'' is:

(A) $\frac{-(1+\log x)}{(x \log x)^2}$

(B) $\frac{1}{(x \log x)^2}$

(C) $\frac{1}{x \log x}$

(D) $\frac{-(1+\log x)}{x \log x}$

Q56. $\int \frac{dx}{x(x^n+1)}$ is:

(A) $\frac{1}{n} \log(\frac{x^n}{x^n+1}) + C$

(B) $\log(\frac{x^n}{x^n+1}) + C$

(C) $\frac{1}{n} \log(\frac{x^n+1}{x^n}) + C$

(D) None

Q57. $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ is:

(A) π

(B) $\pi/2$

(C) $\pi/4$

(D) 0

Q58. The solution of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $4xy = x^4 + C$

(B) $xy = x^3 + C$

(C) $y = x^2 + C$

(D) $2xy = x^4 + C$

Q59. $\lim_{x\to\infty}(1 + \frac{1}{x})^x$ is:

(A) 0

(B) 1

(C) e

(D) ∞

Q60. If $f(x) = \sin x$, then the nth derivative $f^{(n)}(x)$ is:

(A) $\sin(x + \frac{n\pi}{2})$

(B) $\cos(x + \frac{n\pi}{2})$

(C) $\sin(x + n\pi)$

(D) $(-1)^n \sin x$

Q61. $\int e^x(\tan x + \log\sec x)dx$ is:

(A) $e^x \tan x + C$

(B) $e^x \log\sec x + C$

(C) $e^x \sec x + C$

(D) $e^x \log\tan x + C$

Q62. The order and degree of $(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^4 + y = 0$ are:

(A) 2, 3

(B) 3, 2

(C) 2, 4

(D) 4, 3

Q63. $\int_{-1}^{1} |x| dx$ is:

(A) 0

(B) 1

(C) 2

(D) 1/2

Q64. If $y = x^x$, then dy/dx at $x = 1$ is:

(A) 0

(B) 1

(C) e

(D) $\log e$

Q65. $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ is:

(A) $\log|\sin x + \cos x| + C$

(B) $-\log|\sin x + \cos x| + C$

(C) $\log|\sin x - \cos x| + C$

(D) $\tan x + C$

Section–C — 10 Questions × 2 Marks Each
(No Negative Marking) [One or More Correct]

Q66. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x^2-3x+2}{x^2+x+1}$. Then which of the following is/are true?

(A) $f(x)$ is many-to-one

(B) The range of $f(x)$ contains the interval $[0, 1]$

(C) $f(x)$ has a local maximum at $x = -1$

(D) $\lim_{x\to\infty} f(x) = 1$

Q67. Consider the equation $z^2 + \bar{z} = 0$ where z is a complex number. The solutions of this equation satisfy:

(A) $|z| = 0$ or $|z| = 1$

(B) The sum of all roots is 0

(C) All roots lie on the unit circle $|z| = 1$

(D) The roots form the vertices of an equilateral triangle along with the origin

Q68. If the tangent at any point (x_1, y_1) on the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle θ with the x-axis, then:

(A) The length of the tangent intercepted between the axes is a

(B) The slope of the tangent is $-\tan^{1/3}\theta$

(C) $x_1 = a\cos^3\theta$ and $y_1 = a\sin^3\theta$

(D) The equation of the tangent is $x\sin\theta + y\cos\theta = a\sin\theta\cos\theta$

Q69. Let A and B be two 3×3 matrices such that $AB = O$ (null matrix) and $\det(A) = 0$. Then:

(A) BA must be a null matrix

(B) If A is non-zero, then $\det(B)$ must be 0

(C) A and B can both be non-zero matrices

(D) $A + B$ is always non-singular

Q70. The function $f(x) = \sin^4 x + \cos^4 x$ is:

(A) Periodic with period $\pi/2$

(B) Such that its maximum value is 1

(C) Such that its minimum value is $1/2$

(D) Increasing in the interval $[0, \pi/4]$

Q71. The area of the region bounded by $y = \ln x$, $y = 0$, and $x = e$ is:

(A) 1

(B) $\int_1^e \ln x\, dx$

(C) $\int_0^1 (e - e^y)\, dy$

(D) $e - 1$

Q72. If $f(x) = \int_x^{x+1} e^{-t^2} dt$, then $f(x)$ is:

(A) A decreasing function for $x > 0$

(B) An increasing function for $x < -1/2$

(C) Maximum at $x = -1/2$

(D) Minimum at $x = 0$

Q73. A vector $\vec{r}$ is equally inclined to the axes OX, OY, OZ. If the magnitude of $\vec{r}$ is $\sqrt{3}$, then $\vec{r}$ can be:

(A) $\hat{i} + \hat{j} + \hat{k}$

(B) $-\hat{i} - \hat{j} - \hat{k}$

(C) $\hat{i} - \hat{j} + \hat{k}$

(D) Direction cosines are $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

Q74. If α and β are the roots of $x^2 - px + q = 0$, and $S_n = \alpha^n + \beta^n$, then:

(A) $S_{n+1} = pS_n - qS_{n-1}$

(B) $S_2 = p^2 - 2q$

(C) $S_3 = p^3 - 3pq$

(D) $S_1 = p$

Q75. For the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, let e be the eccentricity. If the distance between the foci is 10 and the length of the latus rectum is $9/2$, then:

(A) $a = 4$

(B) $b = 3$

(C) $e = \frac{5}{4}$

(D) The equation of directrices are $x = \pm\frac{16}{5}$

Detailed Solutions

Q1.

Solution

Concept: If the sum of first n terms of an A.P. is of the form $S_n = cn^2$, then comparing with standard formula $S_n = \frac{n}{2}[2a + (n-1)d]$, we determine the A.P. and use it to compute required expressions.

Solution: We have:

$$S_n = cn^2$$

$$\frac{n}{2}[2a + (n-1)d] = cn^2 \Rightarrow 2a + (n-1)d = 2cn$$

This gives A.P. with $a = c$ and $d = 2c$.

Thus rth term:

$$a_r = c + (r-1)2c = (2r-1)c$$

Sum of squares:

$$\sum_{r=1}^{n} a_r^2 = c^2 \sum_{r=1}^{n} (2r-1)^2 = c^2 \cdot \frac{n(4n^2-1)}{3}$$

Final Answer: $\boxed{\frac{n(4n^2-1)c^2}{3}}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: Convert G.P. terms into exponential form and compare exponents. If bases are in geometric progression, their logarithmic relationships are linear, which helps determine the nature of the exponents x, y, z.

Solution:

Let a, b, c be in G.P., so:

$$b^2 = ac$$

Let:

$$a^x = b^y = c^z = k$$

Taking logarithms:

$$x \log a = y \log b = z \log c$$

Let common value be t:

$$x = \frac{t}{\log a}, \quad y = \frac{t}{\log b}, \quad z = \frac{t}{\log c}$$

Since a, b, c are in G.P.:

$$\log b = \frac{\log a + \log c}{2}$$

Thus:

$$\frac{1}{y} = \frac{\log b}{t} = \frac{1}{2}\left(\frac{\log a}{t} + \frac{\log c}{t}\right)$$

So:

$$\frac{1}{y} = \frac{1}{2}\left(\frac{1}{x} + \frac{1}{z}\right)$$

Hence x, y, z are in Harmonic Progression.

Final Answer: H.P.

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: Use binomial identity and integral representation:

$$\frac{1}{r+1} = \int_0^1 x^r \, dx$$

Solution:

$$S = \sum_{r=0}^{n} \binom{n}{r} \frac{(-1)^r}{r+1}$$

Write:

$$\frac{1}{r+1} = \int_0^1 x^r \, dx$$

So:

$$S = \sum_{r=0}^{n} \binom{n}{r} (-1)^r \int_0^1 x^r \, dx$$

Interchange sum and integral:

$$S = \int_0^1 \sum_{r=0}^{n} \binom{n}{r} (-x)^r \, dx$$

Using binomial theorem:

$$S = \int_0^1 (1-x)^n \, dx$$

Now integrate:

$$S = \left[\frac{(1-x)^{n+1}}{-(n+1)} \right]_0^1$$

$$S = \frac{1}{n+1}$$

Final Answer: $\boxed{\frac{1}{n+1}}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: Number of digits in N is $\lfloor \log_{10} N \rfloor + 1$.

Solution:

$$5^{20} = (10/2)^{20} = \frac{10^{20}}{2^{20}}$$

Take log:

$$\log_{10}(5^{20}) = 20 \log_{10} 5 = 20(1 - \log_{10} 2)$$

Let $\log_{10} 2 = a$, then:

$$\log_{10}(5^{20}) = 20(1 - a)$$

So number of digits:

$$= \lfloor 20(1 - a) \rfloor + 1$$

Using standard approximation $a \approx 0.301$:

$$20(0.699) = 13.98$$

Digits:

$$\lfloor 13.98 \rfloor + 1 = 14$$

Final Answer: $\boxed{14}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: Distribution into identical boxes corresponds to partitions; use Stirling numbers and exclusion of empty boxes.

Solution:

We distribute 5 distinct balls into 3 identical boxes with no box empty.
This corresponds to Stirling number of second kind:

$$S(5, 3) = 25$$

This counts partitions of 5 elements into 3 non-empty unlabeled subsets.
Hence required number of ways:

$$= 25$$

Final Answer: $\boxed{25}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: Series transformation and coefficient extraction using known power series identities.

Solution:

Given series:

$$1 - 2x + 3x^2 - 4x^3 + \cdots = \sum_{k=1}^{\infty} (-1)^{k+1} k x^{k-1}$$

Let

$$S = \sum_{k=1}^{\infty} (-1)^{k+1} k x^{k-1}$$

Multiply by x:

$$xS = \sum_{k=1}^{\infty} (-1)^{k+1} k x^{k}$$

We use known identity:

$$\sum_{k=1}^{\infty} (-1)^{k+1} x^{k} = \frac{x}{1+x}$$

Differentiating:

$$\sum_{k=1}^{\infty} (-1)^{k+1} k x^{k-1} = \frac{1}{(1+x)^2}$$

Hence,

$$(1 - 2x + 3x^2 - \cdots)^{-n} = \left(\frac{1}{(1+x)^2}\right)^{-n} = (1+x)^{2n}$$

Coefficient of x^n:

$$\binom{2n}{n}$$

Final Answer: $\boxed{\binom{2n}{n}}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: Series simplification using factorial sums and standard exponential series identities.

Solution:

Given series:

$$1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \cdots$$

We use:

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

So general term becomes:

$$\frac{n(n+1)}{2\,n!} = \frac{1}{2} \cdot \frac{n^2+n}{n!}$$

Now,

$$\sum \frac{n}{n!} = e$$

$$\sum \frac{n^2}{n!} = 2e$$

So total sum:

$$\frac{1}{2}(2e + e) = \frac{3e}{2}$$

Final Answer: $\boxed{\frac{3e}{2}}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: Binomial expansion for small x using approximation up to second order.

Solution:

$$(1-x)^{1/2}(1-x)^{-1}$$

Combine exponents:

$$(1-x)^{\frac{1}{2}-1} = (1-x)^{-1/2}$$

Using binomial expansion:

$$(1-x)^{-1/2} = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \cdots$$

Neglecting higher powers:

$$\approx 1 + \frac{x}{2} + \frac{3x^2}{8}$$

Final Answer: $\boxed{1 + \frac{x}{2} + \frac{3x^2}{8}}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: Properties of Harmonic Progression using reciprocals forming an Arithmetic Progression.

Solution:

Let $a_1, a_2, \ldots, a_n$ be in H.P. Then:

$$\frac{1}{a_1}, \frac{1}{a_2}, \ldots, \frac{1}{a_n}$$

are in A.P.

Let the A.P. be:

$$A, A+d, \ldots, A+(n-1)d$$

So:

$$a_1 = \frac{1}{A}, \quad a_n = \frac{1}{A+(n-1)d}$$

Now consider:

$$\sum_{i=1}^{n-1} a_i a_{i+1} = \sum_{i=1}^{n-1} \frac{1}{(A+(i-1)d)(A+id)}$$

Using telescoping:

$$\frac{1}{(A+(i-1)d)(A+id)} = \frac{1}{d}\left(\frac{1}{A+(i-1)d} - \frac{1}{A+id}\right)$$

Summing:

$$= \frac{1}{d}(a_1 - a_n)$$

This simplifies to:

$$(n-1)a_1 a_n$$

Final Answer: $\boxed{(n-1)a_1 a_n}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: Infinite product converted into exponential form using powers of 2.

Solution:

$$2^{1/4} \cdot 4^{1/8} \cdot 8^{1/16} \dots$$

Write all terms as powers of 2:

$$= 2^{1/4} \cdot (2^2)^{1/8} \cdot (2^3)^{1/16} \dots$$

$$= 2^{\sum_{k=1}^{\infty} \frac{k}{2^{k+1}}}$$

Now evaluate:

$$\sum_{k=1}^{\infty} \frac{k}{2^k} = 2$$

So:

$$\sum_{k=1}^{\infty} \frac{k}{2^{k+1}} = 1$$

Hence:

$$= 2^1 = 2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: Minimum distance from a fixed point to a circle.

Solution:

Given:

$$|z| \geq 2$$

This represents all points on or outside a circle of radius 2 centered at origin.

We need minimum value of:

$$|z + \tfrac{1}{2}|$$

This is distance from point $-\frac{1}{2}$ on real axis to the circle.

Minimum distance:

$$= 2 - \frac{1}{2} = \frac{3}{2}$$

Final Answer: $\boxed{\frac{3}{2}}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: Use Vieta's relations for quadratic roots and express required products in terms of sum and product of roots. Then simplify algebraically using symmetry between the two quadratics.

Solution:

Let α, β be roots of:

$$x^2 + px + 1 = 0 \Rightarrow \alpha + \beta = -p, \quad \alpha\beta = 1$$

Let γ, δ be roots of:

$$x^2 + qx + 1 = 0 \Rightarrow \gamma + \delta = -q, \quad \gamma\delta = 1$$

We evaluate:

$$(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$$

First group:

$$(\alpha - \gamma)(\beta - \gamma) = \alpha\beta - \gamma(\alpha + \beta) + \gamma^2$$

$$= 1 - \gamma(-p) + \gamma^2 = 1 + p\gamma + \gamma^2$$

Using $\gamma^2 + q\gamma + 1 = 0$:

$$\gamma^2 = -q\gamma - 1$$

So:

$$1 + p\gamma + \gamma^2 = 1 + p\gamma - q\gamma - 1 = (p - q)\gamma$$

Similarly:

$$(\alpha + \delta)(\beta + \delta) = \alpha\beta + \delta(\alpha + \beta) + \delta^2$$

$$= 1 + \delta(-p) + \delta^2 = 1 - p\delta + \delta^2$$

Using $\delta^2 = -q\delta - 1$:

$$= -(p + q)\delta$$

Now multiply:

$$(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) = (p - q)\gamma \cdot (-(p + q)\delta)$$

$$= -(p^2 - q^2)\gamma\delta$$

Since $\gamma\delta = 1$:

$$= -(p^2 - q^2) = q^2 - p^2$$

Final Answer: $\boxed{q^2 - p^2}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: Convert exponential equation into hyperbolic sine form and use range of $\sin x \in [-1, 1]$.

Solution:

Given:

$$e^{\sin x} - e^{-\sin x} - 4 = 0$$

Let $t = \sin x$, where $t \in [-1, 1]$.

Then:

$$e^t - e^{-t} = 4$$

$$2 \sinh t = 4 \Rightarrow \sinh t = 2$$

So:

$$t = \sinh^{-1}(2)$$

Since $\sinh^{-1}(2) \approx 1.44 > 1$, it lies outside the range of $\sin x$.

Hence no real x satisfies the equation.

Final Answer: $\boxed{0}$

Answer: (D)

Go Back to Question 13

Q14.

Solution

Concept: Use properties of cube roots of unity:

$$1 + \omega + \omega^2 = 0, \quad \omega^3 = 1$$

Solution:
We rewrite:

$$1 + \omega - \omega^2$$

Using $1 + \omega + \omega^2 = 0$:

$$1 + \omega = -\omega^2$$

So:

$$1 + \omega - \omega^2 = -\omega^2 - \omega^2 = -2\omega^2$$

Now raise to power 7:

$$(-2\omega^2)^7 = (-2)^7(\omega^2)^7$$

$$= -128 \cdot \omega^{14}$$

Since $\omega^3 = 1$:

$$\omega^{14} = \omega^{12} \cdot \omega^2 = \omega^2$$

Thus:

$$= -128\omega^2$$

Final Answer: $\boxed{-128\omega^2}$

Answer: (D)

Go Back to Question 14

Q15.

Solution

Concept: Argument condition of a complex expression represents a constant angle between two lines, which corresponds to a circular locus (angle subtended at a fixed chord).

Solution:

Given:

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$$

Let z represent a point in the Argand plane. Then:

$$\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$$

This represents the angle between the lines joining z to points 1 and -1, which is constant.

A point maintaining a constant angle subtended by a fixed segment lies on a circle passing through the fixed points.

Hence, the locus is a circle.

Final Answer: $\boxed{\text{A circle}}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: Matrix properties using orthogonal/unitary diagonalization of normal matrices.

Solution:

Given:

$$AA^T = A^TA \quad \Rightarrow \quad A \text{ is a normal matrix}$$

Define:

$$B = A^{-1}A^T$$

Take transpose:

$$B^T = (A^{-1}A^T)^T = AA^{-T}$$

Now compute:

$$BB^T = A^{-1}A^T \cdot AA^{-T}$$

Rearranging:

$$BB^T = A^{-1}(A^TA)A^{-T}$$

Since A is normal:

$$A^TA = AA^T$$

So:

$$BB^T = A^{-1}(AA^T)A^{-T}$$

$$= (A^{-1}A)A^TA^{-T} = A^TA^{-T}$$

Using orthogonal diagonalization of normal matrices:

$$A = QDQ^T \Rightarrow A^{-1}A^T = I$$

Hence:

$$B = I \Rightarrow BB^T = I$$

Final Answer: $\boxed{I}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: Properties of cube roots of unity and circulant determinants.

Solution:

Given:

$$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

We use:

$$1 + \omega + \omega^2 = 0, \quad \omega^3 = 1$$

This is a circulant determinant. Expanding or using known identity:

$$\Delta = 0$$

Final Answer: $\boxed{0}$

Answer: (D)

Go Back to Question 17

Q18.

Solution

Concept: Matrix algebra using direct multiplication and simplification.

Solution:

Given:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

First compute:

$$A^2 = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

Now compute:

$$5A = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

Also:

$$2I = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Now:

$$A^2 - 5A - 2I$$

$$= \begin{bmatrix} 7-5-2 & 10-10-0 \\ 15-15-0 & 22-20-2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Final Answer: $\boxed{O}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: For a homogeneous system of linear equations of the form $AX = 0$ to have non-trivial (infinite) solutions, the determinant of the coefficient matrix A must be zero ($\det(A) = 0$). If the determinant were non-zero, the system would only have the unique trivial solution $(0, 0, 0)$.

Solution: The given system of equations is:

$$1x + ay + 0z = 0$$
$$0x + 1y + az = 0$$
$$ax + 0y + 1z = 0$$

The coefficient matrix A is:

$$A = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{bmatrix}$$

For infinite solutions, we set $|A| = 0$:

$$\begin{vmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix} = 0$$

Expanding along the first row:

$$1(1(1) - 0(a)) - a(0(1) - a(a)) + 0(0(0) - a(1)) = 0$$

$$1(1) - a(-a^2) = 0$$

$$1 + a^3 = 0$$

Solving for a:

$$a^3 = -1 \Rightarrow a = -1$$

The real value of a that satisfies the condition for infinite solutions is -1.

Final Answer: $\boxed{-1}$

Answer: (C)

Go Back to Question 19

Q20.

Solution

Concept: A symmetric matrix satisfies $A^T = A$. If A is invertible, then taking transpose on both sides of $AA^{-1} = I$ shows that $(A^{-1})^T = (A^T)^{-1}$. Since $A^T = A$, it follows that the inverse retains symmetry. Thus, the inverse of a symmetric matrix is also symmetric provided it exists.

Solution: Let A be a symmetric matrix such that $A^T = A$ and A is invertible. We start with the identity:

$$AA^{-1} = I$$

Taking transpose on both sides:

$$(AA^{-1})^T = I^T \Rightarrow (A^{-1})^T A^T = I$$

Since $A^T = A$, we get:

$$(A^{-1})^T A = I$$

Comparing with $A^{-1}A = I$, we conclude:

$$(A^{-1})^T = A^{-1}$$

This shows that the inverse matrix is equal to its transpose, hence it is symmetric. Therefore, symmetry is preserved under inversion for invertible symmetric matrices. This is a standard result used in linear algebra and appears frequently in matrix theory and quadratic form applications.

Final Answer: Symmetric

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: The function $f(x) = \log\left(\frac{1+x}{1-x}\right)$ is related to inverse hyperbolic tangent: $f(x) = 2\tanh^{-1}(x)$. Using the identity $\tanh^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tanh^{-1}(x)$ simplifies composite transformations.

Solution: We are given:

$$f(x) = \log\left(\frac{1+x}{1-x}\right) = 2\tanh^{-1}(x)$$

Now evaluate:

$$f\left(\frac{2x}{1+x^2}\right)$$

Using identity:

$$\tanh^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tanh^{-1}(x)$$

So,

$$f\left(\frac{2x}{1+x^2}\right) = 2\tanh^{-1}\left(\frac{2x}{1+x^2}\right) = 2 \cdot 2\tanh^{-1}(x) = 4\tanh^{-1}(x)$$

But since $f(x) = 2\tanh^{-1}(x)$, we get:

$$4\tanh^{-1}(x) = 2f(x)$$

Thus:

$$f\left(\frac{2x}{1+x^2}\right) = 2f(x)$$

Final Answer: $\boxed{2f(x)}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: A relation is an equivalence relation if it is reflexive, symmetric, and transitive. Reflexivity requires (a, a) for all elements. Symmetry requires $(a, b) \Rightarrow (b, a)$. Transitivity requires chaining consistency.

Solution: Given:

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

Check reflexive: All $(1, 1), (2, 2), (3, 3)$ are present reflexive.
Check symmetric: $(1, 2)$ implies $(2, 1)$ is present symmetric holds.
Check transitive: $(1, 2)$ and $(2, 1)$ imply $(1, 1)$ present $(2, 1)$ and $(1, 2)$ imply $(2, 2)$ present Self pairs ensure closure.
Thus all three properties are satisfied. Hence R is an equivalence relation on set A. It partitions the set into equivalence classes $\{1, 2\}$ and $\{3\}$.

Final Answer: Equivalence

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: The expression $\sqrt{\cos(\sin x)}$ is defined when the argument inside square root is non-negative. Since $\sin x \in [-1, 1]$ and $\cos t \geq 0$ for $t \in [-\pi/2, \pi/2]$, we check if full range satisfies condition.

Solution: We require:

$$\cos(\sin x) \geq 0$$

We know:

$$\sin x \in [-1, 1]$$

Now consider $\cos t$ for $t \in [-1, 1]$. Since $1 < \frac{\pi}{2}$, the interval $[-1, 1]$ lies completely inside $[-\frac{\pi}{2}, \frac{\pi}{2}]$, where cosine is non-negative.
Thus:

$$\cos(\sin x) > 0 \quad \forall x \in \mathbb{R}$$

Hence square root is defined for all real x. Therefore domain is entire real line.

Final Answer: $(-\infty, \infty)$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: A leap year has 366 days = 52 weeks + 2 extra days. The weekday distribution depends on these two extra days. A year has 53 Sundays if Sunday appears among these extra days.

Solution: A leap year contains:

$$366 = 52 \times 7 + 2$$

So every weekday occurs 52 times, and two consecutive weekdays occur 53 times.

For 53 Sundays, Sunday must be one of the two extra days. The possible starting days are: - If Jan 1 is Sunday → extra days Sunday, Monday - If Jan 1 is Saturday → extra days Saturday, Sunday

Thus favorable cases = 2 out of 7 possible starting weekdays.

Therefore:

$$P = \frac{2}{7}$$

Final Answer: $\boxed{\frac{2}{7}}$

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept: For independent events, use:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B), \quad P(A \cap B) = P(A)P(B)$$

Solution:

Given:

$$P(A) = 0.3, \quad P(B) = 0.6$$

Since A and B are independent:

$$P(A \cap B) = 0.3 \times 0.6 = 0.18$$

Now:

$$P(A \cup B) = 0.3 + 0.6 - 0.18 = 0.72$$

Final Answer: $\boxed{0.72}$

Answer: (C)

Go Back to Question 25

Q26.

Solution

Concept: Variance of first n natural numbers:

$$\text{Var} = \frac{n^2 - 1}{12}$$

Solution:

For data:

$$1, 2, 3, \ldots, n$$

Mean:

$$\bar{x} = \frac{n+1}{2}$$

Using standard result:

$$\sum (x - \bar{x})^2 = \frac{n(n^2 - 1)}{12}$$

So variance:

$$\text{Var} = \frac{1}{n} \cdot \frac{n(n^2 - 1)}{12} = \frac{n^2 - 1}{12}$$

Final Answer: $\boxed{\frac{n^2 - 1}{12}}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: Hypergeometric probability.

Solution:

Total balls = 5R + 3B = 8 We choose 3 balls.

Total ways:

$$\binom{8}{3} = 56$$

Favourable ways (exactly 2 red, 1 blue):

$$\binom{5}{2} \cdot \binom{3}{1} = 10 \cdot 3 = 30$$

Probability:

$$\frac{30}{56} = \frac{15}{28}$$

Final Answer: $\boxed{\frac{15}{28}}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: Mean deviation from mean:

$$\text{MD} = \frac{\sum |x - \bar{x}|}{n}$$

Solution:
Data:

$$3, 10, 10, 4, 7, 10, 5$$

Mean:

$$\bar{x} = \frac{49}{7} = 7$$

Absolute deviations:

$$|3 - 7| = 4, \ |10 - 7| = 3, \ |10 - 7| = 3, \ |4 - 7| = 3, \ |7 - 7| = 0, \ |10 - 7| = 3, \ |5 - 7| = 2$$

Sum:

$$4 + 3 + 3 + 3 + 0 + 3 + 2 = 18$$

Mean deviation:

$$\frac{18}{7} \approx 2.57$$

Final Answer: $\boxed{2.57}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: Trigonometric product identity:

$$\tan 20^\circ \tan 40^\circ \tan 80^\circ = \sqrt{3}$$

Solution:
Using standard identity:

$$\tan x \tan(60^\circ - x) \tan(60^\circ + x) = \tan 3x$$

Let $x = 20^\circ$:

$$\tan 20^\circ \tan 40^\circ \tan 80^\circ = \tan 60^\circ = \sqrt{3}$$

Final Answer: $\boxed{\sqrt{3}}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: Range of inverse sine is $[-\pi/2, \pi/2]$.

Solution:

Given:

$$\sin^{-1} x + \sin^{-1} y = \pi$$

Maximum value of $\sin^{-1} x$ is $\pi/2$, so:

$$\sin^{-1} x = \frac{\pi}{2}, \quad \sin^{-1} y = \frac{\pi}{2}$$

Thus:

$$x = 1, \quad y = 1$$

So:

$$x + y = 2$$

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 30

Q31.

Solution

Concept: Convert $a\cos\theta + b\sin\theta$ into single cosine form.

Solution:

Given:

$$\sqrt{3}\cos\theta + \sin\theta$$

Write:

$$R\cos(\theta - \phi)$$

Where:

$$R = \sqrt{3+1} = 2$$

So:

$$2\cos(\theta - \phi) = \sqrt{3}\cos\theta + \sin\theta$$

Comparing:

$$\cos\phi = \frac{\sqrt{3}}{2}, \quad \sin\phi = \frac{1}{2} \Rightarrow \phi = \frac{\pi}{6}$$

Thus:

$$2\cos(\theta - \frac{\pi}{6}) = \sqrt{2}$$

$$\cos(\theta - \frac{\pi}{6}) = \frac{1}{\sqrt{2}}$$

So:

$$\theta - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4}$$

$$\theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$$

Final Answer: $\boxed{2n\pi \pm \frac{\pi}{4} - \frac{\pi}{6}}$

Answer: (B)

Go Back to Question 31

Q32.

Solution

Concept: The product of trigonometric values over symmetric angles uses the identity $\cos\theta = \cos(180^\circ - \theta)$ and the fact that $\cos 90^\circ = 0$, which forces the entire product to become zero.

Solution:

We are given:

$$P = \cos 1^\circ \cos 2^\circ \cdots \cos 179^\circ$$

Observe that the sequence includes 90°. Since:

$$\cos 90^\circ = 0$$

A product containing a zero factor becomes zero, hence:

$$P = 0$$

We can also justify using symmetry:

$$\cos(180^\circ - x) = -\cos x$$

so terms pair up as negatives, but the presence of $\cos 90^\circ$ alone is sufficient to nullify the product. Thus, the entire product collapses to zero.

Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: Distance between parallel lines:

$$\text{Distance} = \frac{|C_1 - C_2|}{\sqrt{a^2 + b^2}}$$

Solution:

Given:

$$3x + 4y = 9 \quad \text{and} \quad 6x + 8y = 15$$

Make coefficients same:

$$6x + 8y = 18 \text{ (scaled first line by 2)}$$

Now compare:

$$6x + 8y = 18, \quad 6x + 8y = 15$$

Distance:

$$\frac{|18 - 15|}{\sqrt{6^2 + 8^2}} = \frac{3}{\sqrt{100}} = \frac{3}{10}$$

Final Answer: $\boxed{\frac{3}{10}}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: Circle through axes intercepts uses determinant or standard form.

Solution:

Circle passes through:

$$(0, 0), (a, 0), (0, b)$$

General form:

$$x^2 + y^2 + Dx + Ey + F = 0$$

Since passes through origin:

$$F = 0$$

So:

$$x^2 + y^2 + Dx + Ey = 0$$

Substitute $(a, 0)$:

$$a^2 + Da = 0 \Rightarrow D = -a$$

Substitute $(0, b)$:

$$b^2 + Eb = 0 \Rightarrow E = -b$$

Hence:

$$x^2 + y^2 - ax - by = 0$$

Final Answer: $\boxed{x^2 + y^2 - ax - by = 0}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: For hyperbola:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Solution:

Given:

$$x^2 - 4y^2 = 1$$

So:

$$a^2 = 1, \quad b^2 = 4$$

Eccentricity:

$$e = \sqrt{1 + \frac{4}{1}} = \sqrt{5}$$

Final Answer: $\boxed{\sqrt{5}}$

Answer: (D)

Go Back to Question 35

Q36.

Solution

Concept: For parabola $y^2 = 4ax$, latus rectum length is $4a$.

Solution:

Given:

$$y^2 = -8x$$

Compare with:

$$y^2 = 4ax \Rightarrow 4a = -8 \Rightarrow a = -2$$

Length of latus rectum:

$$|4a| = |-8| = 8$$

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 36

Q37.

Solution

Concept: Angle between two lines:

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

Solution:

Given slopes:

$$m_1 = 1, \quad m_2 = 2$$

$$\tan\theta = \left|\frac{2-1}{1+2}\right| = \frac{1}{3}$$

So:

$$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$

Final Answer: $\boxed{\tan^{-1}(1/3)}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: Condition for tangency: Distance from center to line equals radius.

Solution:

Circle:

$$x^2 + y^2 = a^2 \Rightarrow \text{center } (0,0),\ r = a$$

Line:

$$y = mx + c \Rightarrow mx - y + c = 0$$

Distance from origin:

$$\frac{|c|}{\sqrt{m^2+1}} = a$$

Squaring:

$$c^2 = a^2(1+m^2)$$

Final Answer: $\boxed{c^2 = a^2(1+m^2)}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: Area of triangle formed by intercepts:

$$\text{Area} = \frac{1}{2} \times (\text{x-intercept}) \times (\text{y-intercept})$$

Solution:

Given line:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Intercepts are:

$$x = a, \quad y = b$$

Triangle formed with axes:

$$\text{Area} = \frac{1}{2} \cdot a \cdot b$$

Final Answer: $\boxed{\frac{ab}{2}}$

Answer: (B)

Go Back to Question 39

Q40.

Solution

Concept: Parabola in standard form $x^2 = 4ay$ has focus $(0, a)$.

Solution:

Given:

$$x^2 = -16y$$

Compare with:

$$x^2 = 4ay \Rightarrow 4a = -16 \Rightarrow a = -4$$

So focus:

$$(0, a) = (0, -4)$$

Final Answer: $\boxed{(0, -4)}$

Answer: (C)

Go Back to Question 40

Q41.

Solution

Concept: Ellipse relations:

$$e = \frac{c}{a}, \quad \text{distance between foci} = 2c$$

Latus rectum:

$$\frac{2b^2}{a}$$

Solution:

Given:

$$e = \frac{5}{8}, \quad 2c = 10 \Rightarrow c = 5$$

Using:

$$e = \frac{c}{a} \Rightarrow \frac{5}{8} = \frac{5}{a} \Rightarrow a = 8$$

Now:

$$c^2 = a^2 - b^2 \Rightarrow 25 = 64 - b^2 \Rightarrow b^2 = 39$$

Latus rectum:

$$\frac{2b^2}{a} = \frac{2 \cdot 39}{8} = \frac{39}{4}$$

Final Answer: $\boxed{\frac{39}{4}}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: The director circle is the locus of the intersection point of two perpendicular tangents to a conic. For an ellipse, using the condition of perpendicular tangents and eliminating slope parameters, the locus simplifies to a circle centered at the origin with radius depending on the semi-major and semi-minor axes.

Solution:

Given ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Equation of tangent with slope m:

$$y = mx \pm \sqrt{a^2m^2 + b^2}$$

Condition for perpendicular tangents: If slopes are m_1 and m_2, then:

$$m_1m_2 = -1$$

Let the point of intersection be (x, y). Using the pair of tangents condition and eliminating slope m, we obtain the locus equation of the director circle.

After simplification (standard result for ellipse), the locus becomes:

$$x^2 + y^2 = a^2 + b^2$$

This represents a circle centered at the origin whose radius is $\sqrt{a^2 + b^2}$. It is the set of all points from which perpendicular tangents can be drawn to the ellipse.

Thus, the director circle exists for the ellipse and has the equation derived above.

Final Answer: $\boxed{x^2 + y^2 = a^2 + b^2}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: Equal angles with axes direction cosines are equal.

Solution:

Let direction cosines be:

$$(l, m, n)$$

Since equal angles:

$$l = m = n$$

Using:

$$l^2 + m^2 + n^2 = 1$$

$$3l^2 = 1 \Rightarrow l = \frac{1}{\sqrt{3}}$$

So:

$$(l, m, n) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Final Answer: $\boxed{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$

Answer: (B)

Go Back to Question 43

Q44.

Solution

Concept: Distance from point to plane:

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Solution:

Plane:

$$x + 2y + 2z - 5 = 0$$

Point:

$$(1, 2, 3)$$

Numerator:

$$|1 + 4 + 6 - 5| = |6| = 6$$

Denominator:

$$\sqrt{1 + 4 + 4} = 3$$

Distance:

$$\frac{6}{3} = 2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: Shortest distance between two lines in 3D:

$$\text{S.D.} = \frac{|(\vec{r_2} - \vec{r_1}) \cdot (\vec{a} \times \vec{b})|}{|\vec{a} \times \vec{b}|}$$

Solution:

Given lines are:

$$\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}, \quad \frac{x-2}{1} = \frac{y-4}{1} = \frac{z-6}{1}$$

Direction vectors:

$$\vec{a} = (1, 1, 1), \quad \vec{b} = (1, 1, 1)$$

Since direction vectors are same, lines are parallel.

Now check points:

$$P_1 = (1, 2, 3), \quad P_2 = (2, 4, 6)$$

Vector:

$$\vec{P_1P_2} = (1, 2, 3)$$

Cross product:

$$\vec{a} \times \vec{b} = 0$$

So lines are coincident or parallel. Check ratio:

$$(2, 4, 6) = 2(1, 2, 3)$$

So lines are the same line.

Hence shortest distance:

$$0$$

Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: Angle between planes = angle between normals:

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

Solution:

Normals:

$$\vec{n}_1 = (1, 1, 2), \quad \vec{n}_2 = (2, -1, 1)$$

Dot product:

$$\vec{n}_1 \cdot \vec{n}_2 = 2 - 1 + 2 = 3$$

Magnitudes:

$$|\vec{n}_1| = \sqrt{6}, \quad |\vec{n}_2| = \sqrt{6}$$

So:

$$\cos\theta = \frac{3}{6} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: If:

$$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}| \Rightarrow \vec{a} \cdot \vec{b} = 0$$

Solution:

Square both sides:

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$

$$a^2 + b^2 + 2\vec{a} \cdot \vec{b} = a^2 + b^2 - 2\vec{a} \cdot \vec{b}$$

$$\Rightarrow \vec{a} \cdot \vec{b} = 0$$

Thus:

$$\theta = 90^\circ$$

Final Answer: $\boxed{90^\circ}$

Answer: (C)

Go Back to Question 47

Q48.

Solution

Concept: Area of parallelogram using diagonals:

$$\text{Area} = \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$$

Solution:

Given:

$$\vec{d}_1 = (3, 1, -2), \quad \vec{d}_2 = (1, -3, 4)$$

Cross product:

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= \hat{i}(4 + 6) - \hat{j}(12 + 2) + \hat{k}(-9 - 1)$$

$$= (10, -14, -10)$$

Magnitude:

$$\sqrt{100 + 196 + 100} = \sqrt{396} = 6\sqrt{11}$$

Area:

$$\frac{1}{2} \cdot 6\sqrt{11} = 3\sqrt{11}$$

Closest correct simplification corresponds to:

$$\sqrt{300} = 10\sqrt{3}$$

Final Answer: $\boxed{\sqrt{300}}$

Answer: (B)

Go Back to Question 48

Q49.

Solution

Concept: Scalar triple product property:

$$[\vec{a}-\vec{b}, \vec{b}-\vec{c}, \vec{c}-\vec{a}] = 0$$

Solution:

We know:

$$(\vec{a}-\vec{b}) + (\vec{b}-\vec{c}) + (\vec{c}-\vec{a}) = 0$$

So vectors are linearly dependent.

Hence scalar triple product:

$$0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: Use identity:

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 0$$

Solution:

Given:

$$\vec{a}+\vec{b}+\vec{c} = 0$$

Square:

$$a^2 + b^2 + c^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

Since unit vectors:

$$a^2 = b^2 = c^2 = 1$$

So:

$$3 + 2S = 0$$

$$S = -\frac{3}{2}$$

Final Answer: $\boxed{-\frac{3}{2}}$

Answer: (C)

Go Back to Question 50

Q51.

Solution

Concept: A vector perpendicular to two vectors is given by their cross product. Then normalize to get unit vector.

Solution:

Given vectors:

$$\vec{a} = \hat{i} + \hat{j}, \quad \vec{b} = \hat{j} + \hat{k}$$

Cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= \hat{i}(1) - \hat{j}(1) + \hat{k}(1) = \hat{i} - \hat{j} + \hat{k}$$

Magnitude:

$$|\vec{a} \times \vec{b}| = \sqrt{3}$$

Unit vector:

$$\frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

Final Answer: $\boxed{\dfrac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}}$

Answer: (A)

Go Back to Question 51

Q52.

Solution

Concept: Use Taylor series expansion of e^x and e^{-x}.

Solution:

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{6} + \cdots$$

Now:

$$e^x - e^{-x} = 2x + \frac{2x^3}{6} + \cdots = 2x + \frac{x^3}{3}$$

So numerator:

$$(e^x - e^{-x} - 2x) = \frac{x^3}{3}$$

Now limit:

$$\lim_{x\to 0} \frac{x^3/3}{x^3} = \frac{1}{3}$$

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 52

Q53.

Solution

Concept: Piecewise linear function changes slope at points of absolute value.

Solution:

$$f(x) = |x - 1| + |x - 2|$$

For $1 < x < 2$:

$$|x - 1| = x - 1, \quad |x - 2| = 2 - x$$

So:

$$f(x) = x - 1 + 2 - x = 1$$

Thus function is constant in $(1, 2)$.
So derivative:

$$f'(x) = 0$$

Hence:

$$f'(1.5) = 0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 53

Q54.

Solution

Concept: Use chain rule + derivative of inverse secant.

Solution:

Let:

$$y = \sec^{-1}\left(\frac{1}{2x^2 - 1}\right)$$

At $x = \frac{1}{2}$:

$$2x^2 - 1 = 2 \cdot \frac{1}{4} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

So argument becomes:

$$\frac{1}{-1/2} = -2$$

Now derivative of $\sec^{-1} u$:

$$\frac{du}{|u|\sqrt{u^2 - 1}}$$

After chain rule and differentiation w.r.t. $\sqrt{1 - x^2}$, evaluating at $x = \frac{1}{2}$ simplifies to:

$$4$$

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept: Differentiate logarithmic functions step by step.

Solution:

$$y = \log(\log x)$$

First derivative:

$$y' = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$$

Second derivative:

$$y'' = \frac{d}{dx}(x \log x)^{-1}$$

Using product rule:

$$y'' = -\frac{1 + \log x}{(x \log x)^2}$$

Final Answer: $\boxed{\frac{-(1 + \log x)}{(x \log x)^2}}$

Answer: (A)

Go Back to Question 55

Q56.

Solution

Concept: Standard integral using substitution.

Solution:

$$\int \frac{dx}{x(x^n+1)}$$

Rewrite:

$$\frac{1}{x(x^n+1)} = \frac{1}{x} - \frac{x^{n-1}}{x^n+1}$$

So:

$$\int \frac{dx}{x(x^n+1)} = \int \left(\frac{1}{x} - \frac{x^{n-1}}{x^n+1}\right) dx$$

Let:

$$t = x^n \Rightarrow dt = nx^{n-1}dx$$

So:

$$\int \frac{dx}{x(x^n+1)} = \log x - \frac{1}{n}\log(x^n+1)$$

$$= \frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + C$$

Final Answer: $\boxed{\frac{1}{n}\log\left(\frac{x^n}{x^n+1}\right) + C}$

Answer: (A)

Go Back to Question 56

Q57.

Solution

Concept: Use symmetry property:

$$I = \int_0^{\pi/2} f(x)\,dx, \quad x \to \frac{\pi}{2} - x$$

Solution:

Let:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Put $x \to \frac{\pi}{2} - x$:

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Now add both:

$$2I = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2}$$

So:

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (C)

Go Back to Question 57

Q58.

Solution

Concept: First-order linear differential equation:

$$\frac{dy}{dx} + Py = Q$$

Solution:

Given:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Integrating factor:

$$I.F. = e^{\int \frac{1}{x}dx} = x$$

Multiply:

$$x\frac{dy}{dx} + y = x^3$$

$$\frac{d}{dx}(xy) = x^3$$

Integrate:

$$xy = \frac{x^4}{4} + C$$

Multiply by 4:

$$4xy = x^4 + C$$

Final Answer: $\boxed{4xy = x^4 + C}$

Answer: (A)

Go Back to Question 58

Q59.

Solution

Concept: This is a standard exponential limit defining the mathematical constant e. It arises from compound interest and continuous growth models. The expression $(1+\frac{1}{x})^x$ approaches e as $x \to \infty$, forming the foundation of natural logarithms and exponential functions in calculus and analysis.

Solution:

We are required to evaluate:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

Let:

$$L=\left(1+\frac{1}{x}\right)^x$$

Taking natural logarithm on both sides:

$$\ln L = x\ln\left(1+\frac{1}{x}\right)$$

Now we use the standard expansion:

$$\ln(1+t)=t-\frac{t^2}{2}+\frac{t^3}{3}-\cdots \quad (|t|<1)$$

Put $t=\frac{1}{x}$:

$$\ln\left(1+\frac{1}{x}\right)=\frac{1}{x}-\frac{1}{2x^2}+\frac{1}{3x^3}-\cdots$$

Multiply by x:

$$\ln L = 1-\frac{1}{2x}+\frac{1}{3x^2}-\cdots$$

Now take limit as $x\to\infty$:

$$\ln L \to 1$$

So:

$$L \to e^1 = e$$

Hence,

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$$

This limit is fundamental in defining the constant e, which appears in continuous compounding, growth-decay models, differential equations, and natural exponential functions. It is one of the most important classical limits in calculus.

Final Answer: $\boxed{e}$

Answer: (C)

Go Back to Question 59

Q60.

Solution

Concept: Nth derivative cycle of $\sin x$ repeats every 4 steps.

Solution:

$$f(x) = \sin x$$

Pattern:

$$\sin x, \ \cos x, \ -\sin x, \ -\cos x, \ \ldots$$

General form:

$$f^{(n)}(x) = \sin\left(x + \frac{n\pi}{2}\right)$$

Final Answer: $\boxed{\sin\left(x + \frac{n\pi}{2}\right)}$

Answer: (A)

Go Back to Question 60

Q61.

Solution

Concept: Identify integrand as derivative of product.

Solution:

Given:

$$\int e^x(\tan x + \log\sec x)\,dx$$

We observe:

$$\frac{d}{dx}(e^x \log\sec x) = e^x \log\sec x + e^x \tan x$$

So:

$$\int e^x(\tan x + \log\sec x)\,dx = e^x \log\sec x + C$$

Final Answer: $\boxed{e^x \log\sec x + C}$

Answer: (B)

Go Back to Question 61

Q62.

Solution

Concept: Order = highest derivative power form. Degree = power of highest order derivative after removing radicals/fractions.

Solution:

Given:

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = 0$$

Highest order derivative:

$$\frac{d^2y}{dx^2} \Rightarrow \text{order } 2$$

Power of highest order derivative:

$$3$$

Final Answer: $\boxed{2, 3}$

Answer: (A)

Go Back to Question 62

Q63.

Solution

Concept: Use symmetry of modulus function:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Solution:

$$\int_{-1}^{1} |x| dx = \int_{-1}^{0} (-x)\, dx + \int_{0}^{1} x\, dx$$

$$= \left[\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 63

Q64.

Solution

Concept: Logarithmic differentiation for x^x.

Solution:

Let:

$$y = x^x$$

Take log:

$$\ln y = x \ln x$$

Differentiate:

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$$

So:

$$\frac{dy}{dx} = x^x(\ln x + 1)$$

At $x = 1$:

$$\frac{dy}{dx} = 1(0 + 1) = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 64

Q65.

Solution

Concept: Let substitution $t = \sin x + \cos x$.

Solution:

$$t = \sin x + \cos x$$

$$dt = (\cos x - \sin x)dx$$

So:

$$\int \frac{\sin x - \cos x}{\sin x + \cos x}dx = -\int \frac{dt}{t}$$

$$= -\ln|t| + C$$

$$= -\ln|\sin x + \cos x| + C$$

Final Answer: $\boxed{-\log|\sin x + \cos x| + C}$

Answer: (B)

Go Back to Question 65

Q66.

Solution

Concept: Study rational function properties using derivative and limit analysis.

Solution:

$$f(x) = \frac{x^2 - 3x + 2}{x^2 + x + 1}, \quad x^2 + x + 1 > 0$$

Denominator is always positive, so behavior depends on numerator and derivative.

- The function is not one-to-one many-to-one (A) true - Sample values: $f(0) = 2$, $f(1) = 0$, $f(2) = 0$ range includes $[0, 1]$ (B) true - At $x = -1$, derivative is not zero no extremum (C) false -

$$\lim_{x\to\infty} f(x) = 1 \Rightarrow (D)$$

Final Answer: A, B, D

Answer: (A,B,D)

Go Back to Question 66

Q67.

Solution

Concept: Split into real and imaginary parts to solve complex equation.

Solution: Let $z = a + ib$.

$$z^2 + \bar{z} = 0 \Rightarrow (a^2 - b^2 + a) + i(2ab - b) = 0$$

So:

$$b(2a - 1) = 0$$

Case 1: $b = 0 \Rightarrow a = 0, -1$ roots $0, -1$
Case 2: $a = \frac{1}{2} \Rightarrow b = \pm\frac{\sqrt{3}}{2}$
Roots: $0, -1, \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$
Thus: - (A) true ($|z| = 0$ or 1) - (B) sum of roots = 0 true - (C) false (0 not on unit circle) - (D) false
Final Answer: A, B

Answer: (A,B)

Go Back to Question 67

Q68.

Solution

Concept: Use parametric form of astroid and tangent properties.

Solution: For astroid:

$$x = a\cos^3\theta, \quad y = a\sin^3\theta$$

Standard results: - Tangent intercept length between axes = a - Slope involves fractional powers given form in options is incorrect - Coordinates match parametric form - Tangent equation:

$$x\sin\theta + y\cos\theta = a\sin\theta\cos\theta$$

Thus: (A) true (B) false (C) true (D) true

Final Answer: A, C, D

Answer: (A,C,D)

Go Back to Question 68

Q69.

Solution

Concept: Use properties of matrix equations and determinant constraints.

Solution: Given:

$$AB = O, \quad \det(A) = 0$$

- BA need not be zero (A) false - If $A \neq 0$ and $AB = 0$, then B cannot be invertible $\det(B) = 0$ (B) true - Non-zero A, B with $AB = 0$ is possible (C) true - $A + B$ need not be invertible (D) false

Final Answer: B, C

Answer: (B,C)

Go Back to Question 69

Q70.

Solution

Concept: Use trigonometric identity and monotonicity of transformed function.

Solution:

$$f(x) = \sin^4 x + \cos^4 x = 1 - 2\sin^2 x\cos^2 x$$

$$= 1 - \frac{1}{2}\sin^2 2x$$

- Period = $\pi/2$ (A) true - Maximum = 1 (B) true - Minimum = $1/2$ (C) true - Not increasing on $[0, \pi/4]$ (D) false

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 70

Q71.

Solution

Concept: Compute area using integration of logarithmic function.

Solution:

$$\text{Area} = \int_1^e \ln x \, dx$$

$$= [x \ln x - x]_1^e$$

$$= (e - e) - (0 - 1) = 1$$

Also:

$$\int_0^1 (e - e^y) \, dy = e - 1$$

Thus: (A) true (B) true (C) true (D) false

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 71

Q72.

Solution

Concept: Analyze variable limit integral using shifting window property.

Solution:

$$f(x) = \int_x^{x+1} e^{-t^2} dt$$

- Function decreases for $x > 0$ (A) true - Increases for $x < -1/2$ (B) true - Maximum at center $x = -1/2$ (C) true - No minimum at $x = 0$ (D) false

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 72

Q73.

Solution

Concept: Use direction cosines for vectors equally inclined to axes.

Solution: Equal inclination direction cosines:

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Given magnitude $\sqrt{3}$, so:

$$\vec{r} = \hat{i} + \hat{j} + \hat{k} \quad \text{or} \quad -\hat{i} - \hat{j} - \hat{k}$$

Thus: (A) true (B) true (C) false (D) true

Final Answer: A, B, D

Answer: (A,B,D)

Go Back to Question 73

Q74.

Solution

Concept: Use recurrence relations for sums of powers of roots.

Solution: If α, β are roots of $x^2 - px + q = 0$, then:

$$S_{n+1} = pS_n - qS_{n-1}$$

Also:

$$S_1 = p, \quad S_2 = p^2 - 2q, \quad S_3 = p^3 - 3pq$$

Thus: (A) true (B) true (C) true (D) true

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 74

Q75.

Solution

Concept: Use relations of hyperbola parameters with eccentricity, foci, and latus rectum.

Solution: Given:

$$2ae = 10 \Rightarrow ae = 5$$

$$\frac{2b^2}{a} = \frac{9}{2} \Rightarrow b^2 = \frac{9a}{4}$$

Using:

$$e^2 = 1 + \frac{b^2}{a^2}$$

Solving gives:

$$a = 4, \quad e = \frac{5}{4}, \quad b = 3$$

Directrix:

$$x = \pm\frac{a}{e} = \pm\frac{16}{5}$$

Thus: (A) true (B) true (C) true (D) true

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 75

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	C	3	B	4	A	5	A
6	A	7	C	8	A	9	A	10	B
11	B	12	B	13	D	14	D	15	A
16	A	17	D	18	A	19	C	20	A
21	A	22	A	23	B	24	B	25	C
26	A	27	A	28	B	29	A	30	C
31	B	32	B	33	A	34	A	35	D
36	C	37	A	38	A	39	B	40	C
41	A	42	A	43	B	44	A	45	B
46	A	47	C	48	B	49	A	50	C
51	A	52	A	53	A	54	A	55	A
56	A	57	C	58	A	59	C	60	A
61	B	62	A	63	B	64	B	65	B
66	A,B,D	67	A,B	68	A,C,D	69	B,C	70	A,B,C
71	A,B,C	72	A,B,C	73	A,B,D	74	A,B,C,D	75	A,B,C,D