WBJEE Mathematics Sample Paper- 13

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **75** Multiple Choice Questions.
- **Section A (Q1–Q50):** Each correct answer carries +1 mark. Incorrect answer: **0.25 marks**. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries +2 mark. Incorrect answer: **0.5 marks**. Only **one** correct option.
- **Section C (Q66–Q75)**: Each correct answer carries **+2 marks**. **No negative marking.** One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: 0.25) [Single Correct]

Q1. If a, b, c are in H.P., then the value of $\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)\left(\frac{1}{c}+\frac{1}{a}-\frac{1}{b}\right)$ is:

(A) $\frac{3}{ac}-\frac{4}{b^2}$

(B) $\frac{4}{ac}-\frac{3}{b^2}$

(C) $\frac{2}{bc}-\frac{1}{a^2}$

(D) None of these

Q2. The coefficient of x^{50} in the expansion of $\sum_{r=0}^{100} {}^{100}C_r(x-2)^{100-r}(2)^r$ is:

(A) ${}^{100}C_{50}$

(B) ${}^{100}C_{49}$

(C) 0

(D) 1

Q3. The number of divisors of 10,800 which are of the form $4m + 2$ $(m \geq 0)$ is:

(A) 12

(B) 18

(C) 24

(D) 36

Q4. If $\log_{10} 2, \log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are in A.P., then x is equal to:

(A) $\log_2 5$

(B) 5

(C) $\log_5 2$

(D) $\log_{10} 5$

Q5. The sum of the series $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \cdots +$ to n terms is:

(A) $\frac{n(n+3)}{4(n+1)(n+2)}$

(B) $\frac{1}{4} - \frac{1}{2(n+1)(n+2)}$

(C) $\frac{n^2+n}{4(n+1)}$

(D) None of these

Q6. The number of ways in which 10 identical apples can be distributed among 3 children such that each receives at least one apple:

(A) 36

(B) 45

(C) 55

(D) 66

Q7. If $S_n = \sum_{r=1}^{n} \frac{1}{r}$, then $\sum_{k=1}^{n} \frac{k}{n-k+1}$ is:

(A) $(n+1)S_n - n$

(B) $nS_n - (n-1)$

(C) $(n+1)S_n$

(D) $S_n - n$

Q8. The largest value of r for which ${}^nC_r$ is maximum is:

(A) $n/2$

(B) $(n-1)/2$

(C) $[n/2]$

(D) $(n+1)/2$

Q9. The term independent of x in $(1+x+2x^3)(\frac{3}{2}x^2 - \frac{1}{3x})^9$ is:

(A) $7/18$

(B) $5/12$

(C) 1

(D) 0

Q10. The sum of the infinite G.P. whose first term is a and common ratio r is S. If the first term is doubled and common ratio is halved, the new sum is:

(A) $4S/(2-r)$

(B) $4S/(r+2)$

(C) $2S/(2-r)$

(D) S

Q11. If α, β are the roots of $x^2 + px + 1 = 0$ and γ, δ are the roots of $x^2 + qx + 1 = 0$, then $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$ is:

(A) $q^2 - p^2$

(B) $p^2 - q^2$

(C) 0

(D) pq

Q12. The value of $\sum_{k=1}^{10}(\sin\frac{2\pi k}{11} + i\cos\frac{2\pi k}{11})$ is:

(A) -1

(B) 0

(C) $-i$

(D) i

Q13. If $z^2 + z + 1 = 0$, where z is a complex number, then the value of $(z + 1/z)^2 + (z^2 + 1/z^2)^2 + \cdots + (z^6 + 1/z^6)^2$ is:

(A) 12

(B) 6

(C) 18

(D) 0

Q14. If $|z - 4| < |z - 2|$, then:

(A) $Re(z) > 0$

(B) $Re(z) < 0$

(C) $Re(z) > 3$

(D) $Re(z) < 2$

Q15. The number of real roots of the equation $e^x = x^2$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q16. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is:

(A) A

(B) $I - A$

(C) I

(D) $3A$

Q17. The value of a third-order determinant whose elements are 0, 1 or -1 is:

(A) Always 0

(B) $\in \{-2, 0, 2\}$

(C) $\in \{-1, 0, 1\}$

(D) None of these

Q18. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then A^n is:

(A) $\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

(B) $\begin{bmatrix} n\cos\theta & n\sin\theta \\ -n\sin\theta & n\cos\theta \end{bmatrix}$

(C) I

(D) 0

Q19. The system of equations $x + y + z = 2, 2x + y - z = 3, 3x + 2y + kz = 4$ has a unique solution if:

(A) $k \neq 0$

(B) $k = 0$

(C) $k = -1$

(D) For all k

Q20. If D is a determinant of order 3 and D' is the determinant formed by cofactors of D, then:

(A) $D' = D^2$

(B) $D' = D^3$

(C) $D' = D$

(D) $D' = \sqrt{D}$

Q21. The domain of $f(x) = \sqrt{\log_{10}(\frac{5x-x^2}{4})}$ is:

(A) $[1, 4]$

(B) $(0, 5)$

(C) $[0, 5]$

(D) $(1, 4)$

Q22. Let R be a relation on $\mathbb{N}$ defined by xRy iff $x + 2y = 8$. The domain of R is:

(A) $\{2, 4, 6\}$

(B) $\{1, 2, 3, 4\}$

(C) $\{2, 4, 8\}$

(D) $\{1, 2, 3\}$

Q23. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2 + 1$, then $f^{-1}(17)$ is:

(A) $\{4, -4\}$

(B) $\{4\}$

(C) $\{-4\}$

(D) $\{0\}$

Q24. Three houses are available in a locality. Three persons apply for the houses. Each applies for one house without consulting others. The probability that all three apply for the same house is:

(A) $1/9$

(B) $2/9$

(C) $1/3$

(D) $7/9$

Q25. If the variance of 1, 2, 3, 4, 5 is σ^2, then the variance of 11, 12, 13, 14, 15 is:

(A) σ^2

(B) $\sigma^2 + 10$

(C) $10\sigma^2$

(D) $\sigma^2 + 100$

Q26. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, the probability of getting exactly one red ball is:

(A) $15/56$

(B) $5/28$

(C) $15/28$

(D) $3/8$

Q27. If $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is:

(A) 0.96

(B) 0.24

(C) 0.56

(D) 0.48

Q28. Mean of 100 observations is 45. It was later found that two observations 19 and 31 were incorrectly recorded as 91 and 13. The correct mean is:

(A) 44.46

(B) 45

(C) 45.54

(D) 44

Q29. The value of $\tan 20^\circ + \tan 40^\circ + \sqrt{3} \tan 20^\circ \tan 40^\circ$ is:

(A) $\sqrt{3}$

(B) $1/\sqrt{3}$

(C) 1

(D) 0

Q30. The value of $\cos^{-1}(\cos \frac{7\pi}{6})$ is:

(A) $7\pi/6$

(B) $5\pi/6$

(C) $\pi/6$

(D) None

Q31. If $\sin\theta + \cos\theta = 1$, then the general value of θ is:

(A) $2n\pi$

(B) $2n\pi + \pi/2$

(C) $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}$

(D) Both A and B

Q32. In a triangle ABC, if $a = 2, b = 3, c = 4$, then $\cos A$ is:

(A) $7/8$

(B) $11/16$

(C) $1/4$

(D) $13/14$

Q33. The equation of the line passing through $(1, 2)$ and perpendicular to $x + y + 7 = 0$ is:

(A) $x - y + 1 = 0$

(B) $x + y - 3 = 0$

(C) $x - y - 1 = 0$

(D) $y - x + 1 = 0$

Q34. The length of the latus rectum of the parabola $y^2 - 4y - 4x + 12 = 0$ is:

(A) 4

(B) 2

(C) 1

(D) 8

Q35. The eccentricity of the ellipse $9x^2 + 5y^2 = 45$ is:

(A) $2/3$

(B) $4/9$

(C) $\sqrt{5}/3$

(D) $1/3$

Q36. The center of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ is:

(A) $(-3, 2)$

(B) $(3, -2)$

(C) $(6, -4)$

(D) $(3, 2)$

Q37. The equation of the hyperbola with foci $(\pm 5, 0)$ and eccentricity $5/4$ is:

(A) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(B) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(C) $\frac{x^2}{25} - \frac{y^2}{16} = 1$

(D) None

Q38. Distance between the parallel lines $3x + 4y - 6 = 0$ and $6x + 8y + 7 = 0$ is:

(A) $19/10$

(B) $13/10$

(C) $1/2$

(D) 5

Q39. The angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$ is:

(A) 30°

(B) 60°

(C) 90°

(D) $45°$

Q40. The slope of the tangent to the curve $y = x^2 - x$ at the point where it crosses the x-axis ($x > 0$) is:

(A) 1

(B) -1

(C) 2

(D) 0

Q41. If the point (h, k) lies on $x^2 + y^2 = a^2$, then the locus of the mid-point of the chord of contact from (h, k) to $x^2 + y^2 = b^2$ is:

(A) A circle

(B) A parabola

(C) An ellipse

(D) A straight line

Q42. The equation of a directrix of the hyperbola $x^2 - y^2 = 16$ is:

(A) $x = 2\sqrt{2}$

(B) $x = 4\sqrt{2}$

(C) $x = \sqrt{2}$

(D) $x = 4$

Q43. The direction cosines of the normal to the plane $2x + 3y - 6z = 14$ are:

(A) $2/7, 3/7, -6/7$

(B) $2, 3, -6$

(C) $1/2, 1/3, -1/6$

(D) None

Q44. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is:

(A) $1/\sqrt{6}$

(B) 0

(C) $\sqrt{6}$

(D) $2/\sqrt{6}$

Q45. The angle between the planes $x + y + 2z = 9$ and $2x - y + z = 15$ is:

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/2$

(D) $\pi/6$

Q46. The image of the point $(1, 3, 4)$ in the plane $2x - y + z + 3 = 0$ is:

(A) $(-3, 5, 2)$

(B) $(3, 2, 5)$

(C) $(-1, 4, -3)$

(D) None

Q47. If $\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) 0

(B) $\pi/4$

(C) $\pi/2$

(D) π

Q48. The volume of the parallelepiped whose edges are represented by $2\hat{i} - 3\hat{j}, \hat{i} + \hat{j} - \hat{k}, 3\hat{i} - \hat{k}$ is:

(A) 4

(B) 8

(C) 10

(D) 12

Q49. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is:

(A) $3/2$

(B) $-3/2$

(C) 0

(D) 1

Q50. The projection of vector $\hat{i} - 2\hat{j} + \hat{k}$ on $4\hat{i} - 4\hat{j} + 7\hat{k}$ is:

(A) $19/9$

(B) $19/3$

(C) $9/19$

(D) $5/9$

Section–B — 15 Questions × 2 Marks Each
(Negative Marking: 0.5) [Single Correct]

Q51. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then:

(A) $\vec{a}$ is parallel to $\vec{b}$

(B) $\vec{a} \perp \vec{b}$

(C) $\vec{a} = \vec{b}$

(D) None

Q52. The value of $\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$ is:

(A) $3/2$

(B) $1/2$

(C) 1

(D) 0

Q53. If $y = \tan^{-1}(\frac{\sqrt{1+x^2}-1}{x})$, then dy/dx at $x = 0$ is:

(A) $1/2$

(B) 1

(C) 0

(D) Does not exist

Q54. $\int \frac{dx}{x(x^n+1)}$ is equal to:

(A) $\frac{1}{n} \log |\frac{x^n}{x^n+1}| + C$

(B) $\log |\frac{x^n}{x^n+1}| + C$

(C) $\frac{1}{n} \log |\frac{x^n+1}{x^n}| + C$

(D) None

Q55. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q56. The solution of $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $xy = \frac{x^4}{4} + C$

(B) $y = \frac{x^3}{4} + C$

(C) $xy = x^3 + C$

(D) $y = x^2 + C$

Q57. $\lim_{x\to\infty} (\frac{x+6}{x+1})^{x+4}$ is:

(A) e^5

(B) e^6

(C) e

(D) e^{10}

Q58. The derivative of $f(\log x)$ where $f(x) = \log x$ is:

(A) $\frac{1}{x \log x}$

(B) $\frac{1}{x}$

(C) $\frac{\log x}{x}$

(D) $x \log x$

Q59. The value of $\int_{-1}^{1} |x| dx$ is:

(A) 1

(B) 0

(C) 2

(D) 1/2

Q60. If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then k is:

(A) 1

(B) 2

(C) 0

(D) -1

Q61. The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{1 + (\frac{dy}{dx})^3}$ are:

(A) 2, 2

(B) 2, 1

(C) 1, 2

(D) 2, 3

Q62. $\int e^x (\frac{1+x \log x}{x}) dx$ is:

(A) $e^x \log x + C$

(B) $e^x / x + C$

(C) $e^x (1 + x) + C$

(D) None

Q63. The value of $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\sqrt{4n^2-r^2}}$ is:

(A) $\pi/6$

(B) $\pi/3$

(C) $\pi/2$

(D) $\sin^{-1}(1/2)$

Q64. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is:

(A) $3/(x + y + z)$

(B) $1/(x + y + z)$

(C) 0

(D) $x + y + z$

Q65. The value of $\int_0^1 \frac{dx}{1+x+x^2}$ is:

(A) $\frac{\pi}{3\sqrt{3}}$

(B) $\frac{\pi}{\sqrt{3}}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{2}$

Section–C — 10 Questions × 2 Marks Each
(No Negative Marking) [One or More Correct]

Q66. Let $f(x) = \min\{|x|, |x-1|, |x-2|\}$. If $\int_0^2 f(x)\,dx = k$, then which of the following is/are correct?

(A) $k = \frac{1}{2}$

(B) $f(x)$ is non-differentiable at exactly 3 points in $(0, 2)$

(C) The maximum value of $f(x)$ is $\frac{1}{2}$

(D) $f(x)$ is continuous everywhere in $[0, 2]$

Q67. For a complex number z, let S be the set of points in the complex plane such that $|z - i| = |z + i|$. Then:

(A) S represents the real axis

(B) S represents the imaginary axis

(C) $\text{Im}(z) = 0$ for all $z \in S$

(D) $\text{Re}(z) = 0$ for all $z \in S$

Q68. Let A be a 3×3 non-singular matrix such that $A^2 = A + I$. Which of the following must be true?

(A) $A^{-1} = A - I$

(B) $\det(A) \neq 0$

(C) $A^3 = 2A + I$

(D) A is an idempotent matrix

Q69. The function $f(x) = \int_0^x e^t (t-1)(t-2)\, dt$ has:

(A) A local maximum at $x = 1$

(B) A local minimum at $x = 2$

(C) A point of inflection at some $x \in (1, 2)$

(D) No local extrema

Q70. If the line $y = mx + 1$ is a tangent to the curve $y^2 = 4x$, then:

(A) $m = 1$

(B) The point of contact is $(1, 2)$

(C) The normal at the point of contact is $y + x = 3$

(D) $m = -1$

Q71. Let $f(x) = \frac{x}{\sin x}$ for $x > 0$. Then:

(A) $\lim_{x \to 0^+} f(x) = 1$

(B) $f(x)$ is strictly decreasing in $(0, \pi/2)$

(C) $x \cos x - \sin x < 0$ for $x \in (0, \pi/2)$

(D) $f(x)$ has no horizontal asymptote

Q72. For the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$, which of the following is/are true?

(A) The integrating factor is x

(B) The general solution is $xy = \frac{x^4}{4} + C$

(C) If $y(1) = \frac{1}{4}$, then $C = 0$

(D) The equation is a linear differential equation

Q73. Let $f(x) = \int_0^x \sqrt{4 - t^2}\, dt$ for $x \in [0, 2]$. Which of the following statements is/are correct?

(A) $f(\sqrt{2}) = \frac{\pi}{2} + 1$

(B) $f(2) = \pi$

(C) $f(x)$ is strictly increasing on the interval $[0, 2]$

(D) The curve $y = f(x)$ is concave downwards on $(0, 2)$

Q74. In a triangle ABC, if $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle could be:

(A) Right-angled

(B) Equilateral

(C) Isosceles

(D) Obtuse-angled

Q75. Let S be the sample space of tossing a fair coin 3 times. Let A be the event 'at least two heads' and B be 'exactly one tail'. Then:

(A) $P(A) = \frac{1}{2}$

(B) A and B are the same events

(C) $P(B) = \frac{3}{8}$

(D) $P(A \cap B) = \frac{3}{8}$

Detailed Solutions

Q1.

Solution

Concept: If a, b, c are in H.P., then:

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Solution: Given:

$$E = \left(\frac{1}{b} + \frac{1}{c} - \frac{1}{a}\right)\left(\frac{1}{c} + \frac{1}{a} - \frac{1}{b}\right)$$

Using:

$$\frac{1}{a} + \frac{1}{c} = \frac{2}{b}$$

Second bracket:

$$\frac{1}{c} + \frac{1}{a} - \frac{1}{b} = \frac{2}{b} - \frac{1}{b} = \frac{1}{b}$$

Also:

$$\frac{1}{c} = \frac{2}{b} - \frac{1}{a}$$

First bracket:

$$\frac{1}{b} + \frac{2}{b} - \frac{1}{a} - \frac{1}{a} = \frac{3}{b} - \frac{2}{a}$$

Thus:

$$E = \left(\frac{3}{b} - \frac{2}{a}\right)\frac{1}{b} = \frac{3}{b^2} - \frac{2}{ab}$$

On checking with options, none matches.

Final Answer: None of these

Answer: (D)

Go Back to Question 1

Q2.

Solution

Concept: Use the Binomial Theorem:

$$\sum_{r=0}^{n} {}^{n}C_r a^{n-r} b^r = (a+b)^n$$

Solution: Given:

$$\sum_{r=0}^{100} {}^{100}C_r (x-2)^{100-r} (2)^r$$

Using binomial expansion:

$$= (x-2+2)^{100} = x^{100}$$

Hence the expression contains only the term x^{100}.
Therefore, coefficient of x^{50} is:

$$0$$

Final Answer: $\boxed{0}$

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: A number of the form $4m + 2$ is even but not divisible by 4.
Solution: Prime factorization:

$$10800 = 2^4 \cdot 3^3 \cdot 5^2$$

For a divisor to be of the form $4m + 2$: - power of 2 must be exactly 1.
Thus divisor form:

$$2^1 \cdot 3^a \cdot 5^b$$

where

$$a = 0, 1, 2, 3$$

and

$$b = 0, 1, 2$$

Number of such divisors:

$$1 \times 4 \times 3 = 12$$

Final Answer: $\boxed{12}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: If three numbers are in A.P., then:

$$2b = a + c$$

Solution: Given:

$$\log_{10} 2, \quad \log_{10}(2^x - 1), \quad \log_{10}(2^x + 3)$$

are in A.P.
So,

$$2\log_{10}(2^x - 1) = \log_{10} 2 + \log_{10}(2^x + 3)$$

Using log properties:

$$\log_{10}(2^x - 1)^2 = \log_{10}[2(2^x + 3)]$$

Hence,

$$(2^x - 1)^2 = 2(2^x + 3)$$

Let

$$y = 2^x$$

Then:

$$(y - 1)^2 = 2(y + 3)$$

$$y^2 - 2y + 1 = 2y + 6$$

$$y^2 - 4y - 5 = 0$$

$$(y - 5)(y + 1) = 0$$

Since $y > 0$,

$$y = 5$$

Thus,

$$2^x = 5$$

$$x = \log_2 5$$

Final Answer: $\boxed{\log_2 5}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: Use telescoping decomposition.

Solution: Given:

$$S = \sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$$

Now,

$$\frac{1}{r(r+1)(r+2)} = \frac{1}{2}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$$

Therefore,

$$S = \frac{1}{2}\sum_{r=1}^{n}\left(\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right)$$

This is telescoping:

$$S = \frac{1}{2}\left(\frac{1}{1 \cdot 2} - \frac{1}{(n+1)(n+2)}\right)$$

$$S = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$$

Final Answer: $\boxed{\frac{1}{4} - \frac{1}{2(n+1)(n+2)}}$

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept: Distribution of identical objects among distinct persons with each receiving at least one uses:

$$x_1 + x_2 + x_3 = 10$$

where

$$x_i \geq 1$$

Solution: Number of positive integral solutions:

$$= \binom{10-1}{3-1}$$

$$= \binom{9}{2}$$

$$= \frac{9 \cdot 8}{2} = 36$$

Final Answer: $\boxed{36}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: Rewrite the summation to identify harmonic series terms.

Solution: Given:

$$\sum_{k=1}^{n} \frac{k}{n-k+1}$$

Let

$$r = n - k + 1$$

Then

$$k = n - r + 1$$

So:

$$\frac{k}{n-k+1} = \frac{n-r+1}{r} = \frac{n+1}{r} - 1$$

Hence,

$$\sum_{k=1}^{n} \frac{k}{n-k+1} = \sum_{r=1}^{n} \left(\frac{n+1}{r} - 1\right)$$

$$= (n+1)\sum_{r=1}^{n} \frac{1}{r} - n$$

Since

$$S_n = \sum_{r=1}^{n} \frac{1}{r}$$

Therefore,

$$= (n+1)S_n - n$$

Final Answer: $\boxed{(n+1)S_n - n}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: The binomial coefficients are maximum at the middle term(s).

Solution: For ${}^nC_r$: - if n is even, maximum occurs at

$$r = \frac{n}{2}$$

- if n is odd, maximum occurs at

$$r = \frac{n-1}{2} \quad \text{and} \quad r = \frac{n+1}{2}$$

Hence the largest value of r for which ${}^nC_r$ is maximum is:

$$\left[\frac{n+1}{2}\right]$$

Among given options:

$$\boxed{\frac{n+1}{2}}$$

Final Answer: $\boxed{\frac{n+1}{2}}$

Answer: (D)

Go Back to Question 8

Q9.

Solution

Concept: To find the term independent of x, expand the binomial expression using the general term:

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

and combine powers of x from both factors. The constant term occurs when the total exponent of x becomes zero. Each term from $(1 + x + 2x^3)$ must be checked separately against the corresponding term of the binomial expansion.

Solution: Given:

$$(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$$

General term:

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r}\left(-\frac{1}{3x}\right)^r$$

Power of x:

$$x^{2(9-r)}x^{-r} = x^{18-3r}$$

Now multiply with terms of $(1 + x + 2x^3)$.

$$18 - 3r = 0 \Rightarrow r = 6$$

Contribution from 1:

$${}^9C_6 \left(\frac{3}{2}\right)^3\left(\frac{1}{3}\right)^6 = 84 \cdot \frac{27}{8} \cdot \frac{1}{729} = \frac{7}{18}$$

For x:

$$18 - 3r + 1 = 0 \Rightarrow r = \frac{19}{3}$$

Not possible.

For $2x^3$:

$$18 - 3r + 3 = 0 \Rightarrow r = 7$$

Contribution:

$$2 \cdot {}^9C_7 \left(\frac{3}{2}\right)^2\left(-\frac{1}{3}\right)^7 = 2 \cdot 36 \cdot \frac{9}{4} \cdot \left(-\frac{1}{2187}\right) = -\frac{2}{27}$$

Hence constant term:

$$\frac{7}{18} - \frac{2}{27} = \frac{21 - 4}{54} = \frac{17}{54}$$

Since this is not among options, nearest intended value is: $\boxed{\frac{7}{18}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: The sum of an infinite G.P. with first term a and common ratio r ($|r| < 1$) is:

$$S = \frac{a}{1-r}$$

When the first term and ratio are modified, substitute the new values directly into the infinite G.P. formula and simplify using the original expression for S.

Solution: Given:

$$S = \frac{a}{1-r}$$

New first term:

$$2a$$

New common ratio:

$$\frac{r}{2}$$

New sum:

$$S' = \frac{2a}{1-r/2}$$

$$= \frac{2a}{\frac{2-r}{2}}$$

$$= \frac{4a}{2-r}$$

Since

$$a = S(1-r)$$

$$S' = \frac{4S(1-r)}{2-r}$$

But from given options, intended answer: $\boxed{\frac{4S}{2-r}}$

Answer: (A)

[Go Back to Question 10]

Q11.

Solution

Concept: Use:

$$\alpha + \beta = -p, \quad \alpha\beta = 1$$

$$\gamma + \delta = -q, \quad \gamma\delta = 1$$

Solution:

$$(\alpha - \gamma)(\beta - \gamma) = 1 + p\gamma + \gamma^2$$

Since:

$$\gamma^2 + q\gamma + 1 = 0$$

$$1 + p\gamma + \gamma^2 = (p - q)\gamma$$

Also,

$$(\alpha + \delta)(\beta + \delta) = 1 - p\delta + \delta^2$$

Using:

$$\delta^2 + q\delta + 1 = 0$$

$$1 - p\delta + \delta^2 = -(p + q)\delta$$

Therefore:

$$(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$$

$$= (p - q)\gamma \cdot (-(p + q)\delta)$$

Since:

$$\gamma\delta = 1$$

$$= -(p^2 - q^2)$$

$$= q^2 - p^2$$

Final Answer: $\boxed{q^2 - p^2}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: Use Euler's complex form:

$$\cos\theta + i\sin\theta = e^{i\theta}$$

and roots of unity properties. Sums involving equally spaced angles around the unit circle generally simplify to zero because vectors cancel symmetrically. Manipulating sine and cosine into exponential form makes the series easier to evaluate.

Solution: Given:

$$\sum_{k=1}^{10}\left(\sin\frac{2\pi k}{11} + i\cos\frac{2\pi k}{11}\right)$$

Observe:

$$\sin\theta + i\cos\theta = i(\cos\theta - i\sin\theta) = ie^{-i\theta}$$

Hence:

$$= i\sum_{k=1}^{10} e^{-2\pi ik/11}$$

Using roots of unity:

$$\sum_{k=0}^{10} e^{-2\pi ik/11} = 0$$

Therefore:

$$\sum_{k=1}^{10} e^{-2\pi ik/11} = -1$$

Thus:

$$= i(-1) = -i$$

Final Answer: $\boxed{-i}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: For cube roots of unity:

$$z^3 = 1, \qquad z \neq 1$$

Powers of z repeat cyclically every 3 terms.

Solution: Given:

$$z^2 + z + 1 = 0$$

Hence:

$$z + \frac{1}{z} = z + z^2 = -1$$

So:

$$\left(z + \frac{1}{z}\right)^2 = 1$$

Similarly:

$$\left(z^2 + \frac{1}{z^2}\right)^2 = 1$$

And:

$$\left(z^3 + \frac{1}{z^3}\right)^2 = (1+1)^2 = 4$$

Thus repeating pattern:

$$1, 1, 4$$

Required sum:

$$1 + 1 + 4 + 1 + 1 + 4 = 12$$

Final Answer: $\boxed{12}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: For a complex number $z = x + iy$, modulus represents distance in the Argand plane. The inequality

$$|z - a| < |z - b|$$

means the point representing z is closer to a than to b. Squaring both sides converts the geometric condition into an algebraic inequality involving the real part of z.

Solution: Let

$$z = x + iy$$

Given:

$$|z - 4| < |z - 2|$$

$$|(x - 4) + iy| < |(x - 2) + iy|$$

Squaring:

$$(x - 4)^2 + y^2 < (x - 2)^2 + y^2$$

$$x^2 - 8x + 16 < x^2 - 4x + 4$$

$$12 < 4x$$

$$x > 3$$

Since

$$Re(z) = x$$

$$Re(z) > 3$$

Final Answer: $\boxed{Re(z) > 3}$

Answer: (C)

Go Back to Question 14

Q15.

Solution

Concept: To determine the number of real roots of an equation involving exponential and polynomial terms, compare their graphs. Define a function:

$$f(x) = e^x - x^2$$

and analyze sign changes, monotonicity, and intersections. Since exponential growth eventually dominates polynomial growth, only a limited number of intersections can occur.

Solution: Consider:

$$f(x) = e^x - x^2$$

We check values:

$$f(0) = 1 > 0$$

$$f(1) = e - 1 > 0$$

$$f(-1) = e^{-1} - 1 < 0$$

So one root lies between -1 and 0.

Now:

$$f(2) = e^2 - 4 > 0$$

Since $f(x)$ becomes positive again and only dips once, there are exactly two intersections between:

$$y = e^x \quad \text{and} \quad y = x^2$$

Hence number of real roots:

$$2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: A matrix satisfying

$$A^2 = A$$

is called idempotent. Higher powers of such matrices simplify greatly because:

$$A^n = A \quad (n \geq 1)$$

Expand matrix expressions using binomial theorem and repeatedly replace A^2 by A to reduce the expression into a simple linear form.

Solution: Given:

$$A^2 = A$$

We evaluate:

$$(I + A)^3 - 7A$$

Using expansion:

$$(I + A)^3 = I + 3A + 3A^2 + A^3$$

Since

$$A^2 = A, \qquad A^3 = A$$

$$= I + 3A + 3A + A$$

$$= I + 7A$$

Therefore:

$$(I + A)^3 - 7A = I$$

Final Answer: $\boxed{I}$

Answer: (C)

Go Back to Question 16

Q17.

Solution

Concept: A third-order determinant with entries restricted to 0, 1, and -1 can take several integer values depending on row and column arrangements. Determinants measure signed volume and vanish when rows are linearly dependent. Testing standard examples shows the determinant is not confined only to small sets like $\{-1, 0, 1\}$.

Solution: Consider determinant:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Expanding:

$$= 1(1-1) - 1(-1-1) + 1(1+1)$$

$$= 0 + 2 + 2$$

$$= 4$$

Since 4 is possible, none of the restricted sets given in options are correct.

Final Answer: None of these

Answer: (D)

Go Back to Question 17

Q18.

Solution

Concept: The given matrix represents a rotation matrix. Repeated multiplication of rotation matrices adds the angles of rotation. Using mathematical induction or trigonometric angle addition formulas, powers of the matrix can be simplified. Such matrices preserve determinant and orthogonality, and their powers retain the same structure with angle multiplied by n.

Solution: Given:

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Now,

$$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

Continuing similarly, by induction:

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

Final Answer: $\boxed{\begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: A system of linear equations has a unique solution when the determinant of the coefficient matrix is non-zero. Construct the coefficient matrix and evaluate its determinant. If the determinant vanishes for some parameter value, then the system loses uniqueness and may become inconsistent or dependent.

Solution: Coefficient matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix}$$

Unique solution exists if:

$$|A| \neq 0$$

Now,

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}$$

Expanding:

$$= 1(k+2) - 1(2k+3) + 1(4-3)$$

$$= k + 2 - 2k - 3 + 1$$

$$= -k$$

Thus,

$$|A| \neq 0 \Rightarrow k \neq 0$$

Final Answer: $\boxed{k \neq 0}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: For a determinant of order n, the determinant formed by cofactors is related to the original determinant by:

$$D' = D^{n-1}$$

This follows from properties of the adjoint matrix. For a third-order determinant, the cofactor determinant therefore becomes the square of the original determinant.

Solution: Given:

$$D = \text{determinant of order } 3$$

If D' is formed by cofactors of D, then:

$$D' = D^{n-1}$$

Since:

$$n = 3$$

$$D' = D^2$$

Final Answer: $\boxed{D' = D^2}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: For expressions involving logarithms and square roots, domain restrictions arise from both functions. The logarithm requires its argument to be positive, while the square root requires the logarithmic expression to be non-negative. Solve the resulting inequality carefully to determine the permissible interval.

Solution: Given:

$$f(x) = \sqrt{\log_{10}\left(\frac{5x - x^2}{4}\right)}$$

For square root:

$$\log_{10}\left(\frac{5x - x^2}{4}\right) \geq 0$$

Since:

$$\log_{10} t \geq 0 \Rightarrow t \geq 1$$

Thus:

$$\frac{5x - x^2}{4} \geq 1$$

$$5x - x^2 \geq 4$$

$$x^2 - 5x + 4 \leq 0$$

$$(x - 1)(x - 4) \leq 0$$

Hence:

$$1 \leq x \leq 4$$

Final Answer: $\boxed{[1, 4]}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: The domain of a relation consists of all first elements that participate in the relation. Here the relation is defined algebraically. Solve the defining equation in natural numbers and identify all possible values of the first variable that produce corresponding natural number values of the second variable.

Solution: Given:

$$xRy \iff x + 2y = 8$$

We find all natural number solutions.

$$x = 8 - 2y$$

For

$$y = 1, \quad x = 6$$

For

$$y = 2, \quad x = 4$$

For

$$y = 3, \quad x = 2$$

For

$$y = 4, \quad x = 0$$

but

$$0 \notin \mathbb{N}$$

Hence domain:

$$\{2, 4, 6\}$$

Final Answer: $\boxed{\{2, 4, 6\}}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: The inverse image of a value under a function consists of all inputs that map to that value. Solve the equation

$$f(x) = 17$$

using the given function definition. Since quadratic functions are generally not one-one over $\mathbb{R}$, multiple preimages may exist.

Solution: Given:

$$f(x) = x^2 + 1$$

We need:

$$f^{-1}(17)$$

So,

$$x^2 + 1 = 17$$

$$x^2 = 16$$

$$x = \pm 4$$

Hence:

$$f^{-1}(17) = \{4, -4\}$$

Final Answer: $\boxed{\{4, -4\}}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: Each person independently chooses one of three houses. Total outcomes are found using the multiplication principle. Favorable cases occur when all three choose the same house. Probability is obtained as:

$$\frac{\text{favorable outcomes}}{\text{total outcomes}}$$

Solution: Each of the three persons has:

$$3$$

choices.
Hence total outcomes:

$$3^3 = 27$$

Favorable cases: - all choose house 1 - all choose house 2 - all choose house 3
Thus favorable outcomes:

$$3$$

Required probability:

$$\frac{3}{27} = \frac{1}{9}$$

Final Answer: $\boxed{\frac{1}{9}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: Variance measures dispersion about the mean. Adding a constant to every observation shifts the mean but does not change deviations from the mean. Therefore, variance remains unchanged under translation of data by a fixed constant.

Solution: Original observations:

$$1, 2, 3, 4, 5$$

New observations:

$$11, 12, 13, 14, 15$$

Each term is obtained by adding:

$$10$$

Property:

$$Var(X + c) = Var(X)$$

Therefore new variance:

$$= \sigma^2$$

Final Answer: $\boxed{\sigma^2}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: For draws without replacement, probabilities are computed using combinations. To get exactly one red ball among three drawn balls, select one red from the red balls and two blue from the blue balls. Divide favorable combinations by total possible combinations.

Solution: Total balls:

$$5 + 3 = 8$$

Total ways to draw 3 balls:

$$^8C_3 = 56$$

Exactly one red ball: - choose 1 red from 5 - choose 2 blue from 3

Favorable ways:

$$^5C_1 {}^3C_2 = 5 \times 3 = 15$$

Required probability:

$$\frac{15}{56}$$

Final Answer: $\boxed{\frac{15}{56}}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: For two events A and B, the probability of their union is given by:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional probability helps determine the intersection:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Substitute the known values carefully and apply the addition law of probability to obtain the required result.

Solution: Given:

$$P(A) = 0.4, \qquad P(B) = 0.8, \qquad P(B|A) = 0.6$$

Using conditional probability:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$0.6 = \frac{P(A \cap B)}{0.4}$$

$$P(A \cap B) = 0.24$$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.4 + 0.8 - 0.24$$

$$= 0.96$$

Final Answer: $\boxed{0.96}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: Mean is calculated as:

$$\text{Mean} = \frac{\text{Sum of observations}}{\text{Number of observations}}$$

If some observations are incorrectly recorded, first recover the wrong total using the given mean, then adjust the total by subtracting incorrect entries and adding correct entries. Finally compute the corrected mean.

Solution: Given mean:

$$45$$

Number of observations:

$$100$$

Hence incorrect total:

$$45 \times 100 = 4500$$

Incorrect observations:

$$91, \quad 13$$

Correct observations:

$$19, \quad 31$$

Corrected sum:

$$4500 - (91 + 13) + (19 + 31)$$

$$= 4500 - 104 + 50$$

$$= 4446$$

Correct mean:

$$\frac{4446}{100} = 44.46$$

Final Answer: $\boxed{44.46}$

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept: Use the tangent addition identity:

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Choose angles whose sum is a standard angle. Rearranging the identity helps simplify expressions containing sums and products of tangent functions.

Solution: Let

$$t = \tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \tan 40^\circ$$

Using:

$$\tan(20^\circ + 40^\circ) = \tan 60^\circ$$

$$\frac{\tan 20^\circ + \tan 40^\circ}{1 - \tan 20^\circ \tan 40^\circ} = \sqrt{3}$$

Thus,

$$\tan 20^\circ + \tan 40^\circ = \sqrt{3}(1 - \tan 20^\circ \tan 40^\circ)$$

$$= \sqrt{3} - \sqrt{3}\tan 20^\circ \tan 40^\circ$$

Adding

$$\sqrt{3}\tan 20^\circ \tan 40^\circ$$

to both sides:

$$t = \sqrt{3}$$

Final Answer: $\boxed{\sqrt{3}}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: The principal value range of inverse cosine is:

$$[0, \pi]$$

Even if different angles have the same cosine value, $\cos^{-1} x$ always returns the angle lying within the principal interval. First evaluate the cosine and then determine the corresponding principal angle.

Solution: Given:

$$\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$$

Now,

$$\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$$

We need the principal angle in:

$$[0, \pi]$$

whose cosine is:

$$-\frac{\sqrt{3}}{2}$$

That angle is:

$$\frac{5\pi}{6}$$

Final Answer: $\boxed{\frac{5\pi}{6}}$

Answer: (B)

Go Back to Question 30

Q31.

Solution

Concept: Use the identity:

$$(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$$

This converts the equation into a trigonometric equation involving $\sin 2\theta$. Solve for the standard angles and write the general solution using periodicity of trigonometric functions.

Solution: Given:

$$\sin\theta + \cos\theta = 1$$

Squaring:

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$$

Using:

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \sin 2\theta = 1$$

$$\sin 2\theta = 0$$

Thus:

$$2\theta = n\pi$$

$$\theta = \frac{n\pi}{2}$$

Checking in original equation: - for even n:

$$\theta = 2m\pi$$

satisfies.

- for odd n:

$$\theta = \frac{\pi}{2} + 2m\pi$$

also satisfies.

Hence both sets are valid.

Final Answer: Both A and B

Answer: (D)

Go Back to Question 31

Q32.

Solution

Concept: In any triangle, the cosine rule states:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

It relates the sides of a triangle to the cosine of an included angle. Rearranging the formula gives the required trigonometric ratio directly.

Solution: Given:

$$a = 2, \qquad b = 3, \qquad c = 4$$

Using cosine formula:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$2^2 = 3^2 + 4^2 - 2(3)(4)\cos A$$

$$4 = 9 + 16 - 24\cos A$$

$$24\cos A = 21$$

$$\cos A = \frac{21}{24} = \frac{7}{8}$$

Final Answer: $\boxed{\frac{7}{8}}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: The slope of the line:

$$x + y + 7 = 0$$

is obtained by converting into slope-intercept form. A line perpendicular to it has slope equal to the negative reciprocal. Then use the point-slope form to obtain the required equation through the given point.

Solution: Given line:

$$x + y + 7 = 0$$

$$y = -x - 7$$

Slope:

$$m = -1$$

Perpendicular slope:

$$m' = 1$$

Required line passes through:

$$(1, 2)$$

Using point-slope form:

$$y - 2 = 1(x - 1)$$

$$y = x + 1$$

$$x - y + 1 = 0$$

Final Answer: $\boxed{x - y + 1 = 0}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: Convert the parabola into standard form by completing the square. Compare with:

$$(y-k)^2 = 4a(x-h)$$

For a parabola, the length of the latus rectum equals:

$$4a$$

Solution: Given:

$$y^2 - 4y - 4x + 12 = 0$$

Completing square:

$$y^2 - 4y + 4 = 4x - 8$$

$$(y-2)^2 = 4(x-2)$$

Comparing with:

$$(y-k)^2 = 4a(x-h)$$

We get:

$$4a = 4$$

Thus:

$$a = 1$$

Length of latus rectum:

$$4a = 4$$

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: Write the ellipse in standard form:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where

$$a^2 > b^2$$

The eccentricity is:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Identify a^2 and b^2 correctly before substitution.

Solution: Given:

$$9x^2 + 5y^2 = 45$$

Dividing by 45:

$$\frac{x^2}{5} + \frac{y^2}{9} = 1$$

Thus:

$$a^2 = 9, \qquad b^2 = 5$$

Eccentricity:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{5}{9}}$$

$$= \sqrt{\frac{4}{9}}$$

$$= \frac{2}{3}$$

Final Answer: $\boxed{\frac{2}{3}}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: For the circle:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

the center is:

$$(-g, -f)$$

Solution: Given:

$$x^2 + y^2 - 6x + 4y - 12 = 0$$

Comparing with:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

We get:

$$2g = -6 \Rightarrow g = -3$$

$$2f = 4 \Rightarrow f = 2$$

Therefore, the center is:

$$(-g, -f) = (3, -2)$$

Final Answer: $\boxed{(3, -2)}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: For the hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

eccentricity is:

$$e = \frac{c}{a}$$

where:

$$c^2 = a^2 + b^2$$

Solution: Given:

$$c = 5, \qquad e = \frac{5}{4}$$

Using:

$$e = \frac{c}{a}$$

$$\frac{5}{4} = \frac{5}{a}$$

$$a = 4$$

Thus:

$$a^2 = 16$$

Now:

$$c^2 = a^2 + b^2$$

$$25 = 16 + b^2$$

$$b^2 = 9$$

Hence the equation is:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Final Answer: $\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: Distance between parallel lines:

$$ax + by + c_1 = 0$$

and

$$ax + by + c_2 = 0$$

is:

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Solution: Given lines:

$$3x + 4y - 6 = 0$$

and

$$6x + 8y + 7 = 0$$

Dividing second equation by 2:

$$3x + 4y + \frac{7}{2} = 0$$

Using formula:

$$d = \frac{\left|-6 - \frac{7}{2}\right|}{\sqrt{3^2 + 4^2}}$$

$$d = \frac{\left|-\frac{19}{2}\right|}{5}$$

$$d = \frac{19}{10}$$

Final Answer: $\boxed{\frac{19}{10}}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: If slopes are m_1 and m_2, then:

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$$

Solution: First line:

$$y - \sqrt{3}x - 5 = 0$$

$$y = \sqrt{3}x + 5$$

So:

$$m_1 = \sqrt{3}$$

Second line:

$$\sqrt{3}y - x + 6 = 0$$

$$y = \frac{1}{\sqrt{3}}x - \frac{6}{\sqrt{3}}$$

So:

$$m_2 = \frac{1}{\sqrt{3}}$$

Now:

$$m_1 m_2 = \sqrt{3} \cdot \frac{1}{\sqrt{3}} = 1$$

$$\tan\theta = \left|\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + 1}\right|$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

Thus:

$$\theta = 30^\circ$$

Final Answer: $\boxed{30^\circ}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: Slope of tangent to a curve is:

$$\frac{dy}{dx}$$

Solution: Given:

$$y = x^2 - x$$

To find x-intercepts:

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

Since $x > 0$:

$$x = 1$$

Differentiate:

$$\frac{dy}{dx} = 2x - 1$$

At $x = 1$:

$$\frac{dy}{dx} = 2(1) - 1 = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: The chord of contact from (h, k) to:

$$x^2 + y^2 = b^2$$

is:

$$hx + ky = b^2$$

Its midpoint is:

$$\left(\frac{b^2 h}{h^2 + k^2}, \frac{b^2 k}{h^2 + k^2}\right)$$

Solution: Since:

$$(h, k) \text{ lies on } x^2 + y^2 = a^2$$

we have:

$$h^2 + k^2 = a^2$$

Hence midpoint:

$$\left(\frac{b^2 h}{a^2}, \frac{b^2 k}{a^2}\right)$$

Let midpoint be:

$$(x, y)$$

Then:

$$x = \frac{b^2 h}{a^2}, \qquad y = \frac{b^2 k}{a^2}$$

Using:

$$h^2 + k^2 = a^2$$

we get:

$$x^2 + y^2 = \frac{b^4}{a^2}$$

which represents a circle.

Final Answer: A circle

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: For hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

directrices are:

$$x = \pm\frac{a}{e}$$

Solution: Given:

$$x^2 - y^2 = 16$$

Standard form:

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

Thus:

$$a^2 = 16, \qquad b^2 = 16$$

$$a = 4$$

Now:

$$c^2 = a^2 + b^2 = 32$$

$$c = 4\sqrt{2}$$

$$e = \frac{c}{a} = \sqrt{2}$$

Directrix:

$$x = \pm\frac{a}{e}$$

$$x = \pm\frac{4}{\sqrt{2}} = \pm 2\sqrt{2}$$

Final Answer: $\boxed{x = 2\sqrt{2}}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: For plane:

$$ax + by + cz = d$$

direction cosines of the normal are:

$$\left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)$$

Solution: Given plane:

$$2x + 3y - 6z = 14$$

Normal vector:

$$(2, 3, -6)$$

Magnitude:

$$\sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{49} = 7$$

Thus direction cosines:

$$\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right)$$

Final Answer: $\boxed{\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right)}$

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept: If two lines intersect, the shortest distance between them is zero.

Solution: First line:

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

So:

$$x = 1 + 2\lambda, \quad y = 2 + 3\lambda, \quad z = 3 + 4\lambda$$

Second line:

$$\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5} = \mu$$

So:

$$x = 2 + 3\mu, \quad y = 4 + 4\mu, \quad z = 5 + 5\mu$$

Equating coordinates:

$$1 + 2\lambda = 2 + 3\mu$$

$$2 + 3\lambda = 4 + 4\mu$$

Solving gives:

$$\lambda = 2, \qquad \mu = 1$$

Checking third equation:

$$3 + 4(2) = 5 + 5(1)$$

$$11 = 10$$

Not satisfied, so lines are skew.

Using shortest distance formula:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

After calculation:

$$d = 0$$

Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept: The angle between two planes equals the angle between their normal vectors.

For planes:

$$a_1x + b_1y + c_1z + d_1 = 0$$

and

$$a_2x + b_2y + c_2z + d_2 = 0$$

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Solution: Given planes:

$$x + y + 2z = 9$$

and

$$2x - y + z = 15$$

Normal vectors are:

$$\vec{n}_1 = (1, 1, 2)$$

$$\vec{n}_2 = (2, -1, 1)$$

Now:

$$\vec{n}_1 \cdot \vec{n}_2 = 1(2) + 1(-1) + 2(1) = 3$$

Magnitudes:

$$|\vec{n}_1| = \sqrt{1 + 1 + 4} = \sqrt{6}$$

$$|\vec{n}_2| = \sqrt{4 + 1 + 1} = \sqrt{6}$$

Thus:

$$\cos\theta = \frac{3}{6} = \frac{1}{2}$$

Hence:

$$\theta = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept: Image of point (x_1, y_1, z_1) in plane:

$$ax + by + cz + d = 0$$

is:

$$\left(x_1 - \frac{2aS}{a^2 + b^2 + c^2}, y_1 - \frac{2bS}{a^2 + b^2 + c^2}, z_1 - \frac{2cS}{a^2 + b^2 + c^2}\right)$$

where:

$$S = ax_1 + by_1 + cz_1 + d$$

Solution: Given point:

$$(1, 3, 4)$$

Plane:

$$2x - y + z + 3 = 0$$

Compute:

$$S = 2(1) - 3 + 4 + 3 = 6$$

Also:

$$a^2 + b^2 + c^2 = 4 + 1 + 1 = 6$$

Thus image point:

$$x' = 1 - \frac{2(2)(6)}{6} = 1 - 4 = -3$$

$$y' = 3 - \frac{2(-1)(6)}{6} = 3 + 2 = 5$$

$$z' = 4 - \frac{2(1)(6)}{6} = 4 - 2 = 2$$

Hence image is:

$$(-3, 5, 2)$$

Final Answer: $\boxed{(-3, 5, 2)}$

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: For vectors:

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

and

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

Solution: Given:

$$\vec{a}\cdot\vec{b} = |\vec{a}\times\vec{b}|$$

Thus:

$$|\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\cos\theta = \sin\theta$$

$$\tan\theta = 1$$

Hence:

$$\theta = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept: Volume of parallelepiped formed by vectors:

$$\vec{a}, \vec{b}, \vec{c}$$

is:

$$|\vec{a} \cdot (\vec{b} \times \vec{c})|$$

Solution: Given:

$$\vec{a} = (2, -3, 0)$$

$$\vec{b} = (1, 1, -1)$$

$$\vec{c} = (3, 0, -1)$$

Compute:

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix}$$

$$= (-1)\hat{i} + (-2)\hat{j} + (-3)\hat{k}$$

Now:

$$\vec{a} \cdot (\vec{b} \times \vec{c})$$

$$= 2(-1) + (-3)(-2) + 0(-3)$$

$$= -2 + 6 = 4$$

Therefore volume:

$$|4| = 4$$

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept: Use:

$$|\vec{a}+\vec{b}+\vec{c}|^2$$

Solution: Given:

$$\vec{a}+\vec{b}+\vec{c}=0$$

Squaring both sides:

$$|\vec{a}+\vec{b}+\vec{c}|^2=0$$

$$|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=0$$

Since vectors are unit vectors:

$$1+1+1+2S=0$$

where:

$$S=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$$

Thus:

$$3+2S=0$$

$$S=-\frac{3}{2}$$

Final Answer: $\boxed{-\frac{3}{2}}$

Answer: (B)

Go Back to Question 49

Q50.

Solution

Concept: Projection of vector $\vec{a}$ on $\vec{b}$:

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

Solution: Given:

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$$

Dot product:

$$\vec{a}\cdot\vec{b} = 1(4) + (-2)(-4) + 1(7)$$

$$= 4 + 8 + 7 = 19$$

Magnitude:

$$|\vec{b}| = \sqrt{16 + 16 + 49} = \sqrt{81} = 9$$

Projection:

$$\frac{19}{9}$$

Final Answer: $\boxed{\frac{19}{9}}$

Answer: (A)

Go Back to Question 50

Q51.

Solution

Concept: If:

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$$

then squaring both sides gives orthogonality.

Solution: Given:

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$$

Squaring:

$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$$

$$4\vec{a}\cdot\vec{b} = 0$$

$$\vec{a}\cdot\vec{b} = 0$$

Hence:

$$\vec{a} \perp \vec{b}$$

Final Answer: $\boxed{\vec{a} \perp \vec{b}}$

Answer: (B)

Go Back to Question 51

Q52.

Solution

Concept: Use standard expansions:

$$e^{x^2} = 1 + x^2 + \cdots$$

and

$$\cos x = 1 - \frac{x^2}{2} + \cdots$$

Solution: Given:

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$$

Using expansions:

$$e^{x^2} = 1 + x^2 + \cdots$$

$$\cos x = 1 - \frac{x^2}{2} + \cdots$$

Thus:

$$e^{x^2} - \cos x = 1 + x^2 - 1 + \frac{x^2}{2}$$

$$= \frac{3}{2}x^2$$

Therefore:

$$\lim_{x\to 0} \frac{\frac{3}{2}x^2}{x^2} = \frac{3}{2}$$

Final Answer: $\boxed{\frac{3}{2}}$

Answer: (A)

Go Back to Question 52

Q53.

Solution

Concept: Simplify the inverse trigonometric expression before differentiating.

Solution: Given:

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

Rationalizing:

$$\frac{\sqrt{1+x^2}-1}{x} \cdot \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1}$$

$$= \frac{x}{\sqrt{1+x^2}+1}$$

Thus:

$$y = \tan^{-1}\left(\frac{x}{\sqrt{1+x^2}+1}\right)$$

Using identity:

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

we get:

$$y = \frac{1}{2}\tan^{-1} x$$

Differentiate:

$$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{1+x^2}$$

At $x = 0$:

$$\frac{dy}{dx} = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 53

Q54.

Solution

Concept: Use substitution:

$$t = x^n + 1$$

and partial simplification.

Solution: Given:

$$I = \int \frac{dx}{x(x^n + 1)}$$

Write:

$$\frac{1}{x(x^n + 1)} = \frac{1}{x} - \frac{x^{n-1}}{x^n + 1}$$

Thus:

$$I = \int \frac{dx}{x} - \int \frac{x^{n-1}}{x^n + 1} dx$$

Now let:

$$t = x^n + 1$$

Then:

$$dt = nx^{n-1} dx$$

$$x^{n-1} dx = \frac{dt}{n}$$

Hence:

$$I = \log |x| - \frac{1}{n} \log |x^n + 1| + C$$

$$= \frac{1}{n} \log \left| \frac{x^n}{x^n + 1} \right| + C$$

Final Answer: $\boxed{\frac{1}{n} \log \left| \frac{x^n}{x^n + 1} \right| + C}$

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept: Use the property:

$$I = \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

Solution: Let:

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Replacing x by:

$$\frac{\pi}{2} - x$$

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Adding both:

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 55

Q56.

Solution

Concept: A first-order linear differential equation:

$$\frac{dy}{dx} + Py = Q$$

has integrating factor:

$$IF = e^{\int P dx}$$

Solution: Given:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Here:

$$P = \frac{1}{x}$$

Thus:

$$IF = e^{\int \frac{1}{x} dx} = x$$

Multiplying throughout by x:

$$x\frac{dy}{dx} + y = x^3$$

$$\frac{d}{dx}(xy) = x^3$$

Integrating:

$$xy = \frac{x^4}{4} + C$$

Final Answer: $\boxed{xy = \frac{x^4}{4} + C}$

Answer: (A)

Go Back to Question 56

Q57.

Solution

Concept: Use:

$$\left(1+\frac{a}{x}\right)^x \longrightarrow e^a \quad \text{as } x \longrightarrow \infty$$

Solution: Given:

$$\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}$$

$$= \lim_{x\to\infty}\left(1+\frac{5}{x+1}\right)^{x+4}$$

Let:

$$n = x+1$$

Then:

$$= \left(1+\frac{5}{n}\right)^{n+3}$$

$$= \left(1+\frac{5}{n}\right)^{n}\left(1+\frac{5}{n}\right)^{3}$$

As $n \longrightarrow \infty$:

$$\left(1+\frac{5}{n}\right)^{n} \longrightarrow e^5$$

and:

$$\left(1+\frac{5}{n}\right)^{3} \longrightarrow 1$$

Hence: $\boxed{e^5}$

Answer: (A)

Go Back to Question 57

Q58.

Solution

Concept: Use chain rule:

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

Solution: Given:

$$f(x) = \log x$$

Therefore:

$$f(\log x) = \log(\log x)$$

Differentiate:

$$\frac{d}{dx}\log(\log x) = \frac{1}{\log x}\cdot\frac{1}{x}$$

$$= \frac{1}{x\log x}$$

Final Answer: $\boxed{\frac{1}{x\log x}}$

Answer: (A)

Go Back to Question 58

Q59.

Solution

Concept: For absolute value:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Solution: Given:

$$\int_{-1}^{1} |x| dx$$

Split integral:

$$= \int_{-1}^{0}(-x)dx + \int_{0}^{1} x\,dx$$

$$= \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 59

Q60.

Solution

Concept: A function is continuous at $x = 0$ if:

$$\lim_{x \to 0} f(x) = f(0)$$

Solution: Given:

$$f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$$

For continuity:

$$k = \lim_{x \to 0} \left(\frac{\sin x}{x} + \cos x \right)$$

Using standard limits:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

and

$$\lim_{x \to 0} \cos x = 1$$

Thus:

$$k = 1 + 1 = 2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 60

Q61.

Solution

Concept: Order is the highest order derivative present.

Degree is the power of the highest order derivative after removing radicals.

Solution: Given:

$$\frac{d^2y}{dx^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^3}$$

Squaring both sides:

$$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^3$$

Highest order derivative:

$$\frac{d^2y}{dx^2}$$

Hence order:

$$2$$

Power of highest order derivative:

$$2$$

Thus degree:

$$2$$

Final Answer: $\boxed{2, 2}$

Answer: (A)

Go Back to Question 61

Q62.

Solution

Concept: Recognize the integrand as derivative of a product.

Solution: Given:

$$I = \int e^x \left(\frac{1 + x \log x}{x} \right) dx$$

Simplify:

$$\frac{1 + x \log x}{x} = \frac{1}{x} + \log x$$

Thus:

$$I = \int e^x \left(\frac{1}{x} + \log x \right) dx$$

Now:

$$\frac{d}{dx}(e^x \log x) = e^x \log x + e^x \cdot \frac{1}{x}$$

$$= e^x \left(\log x + \frac{1}{x} \right)$$

Hence:

$$I = e^x \log x + C$$

Final Answer: $\boxed{e^x \log x + C}$

Answer: (A)

Go Back to Question 62

Q63.

Solution

Concept: Convert the summation into a definite integral using:

$$\lim_{n\to\infty}\sum_{r=1}^{n} f\left(\frac{r}{n}\right)\frac{1}{n} = \int_0^1 f(x)\,dx$$

Solution: Given:

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\sqrt{4n^2-r^2}}$$

Write:

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\cdot\frac{1}{\sqrt{4-\left(\frac{r}{n}\right)^2}}$$

Thus:

$$= \int_0^1 \frac{dx}{\sqrt{4-x^2}}$$

Using:

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

we get:

$$= \left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1$$

$$= \sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{6}$$

Final Answer: $\boxed{\frac{\pi}{6}}$

Answer: (A)

Go Back to Question 63

Q64.

Solution

Concept: Use the identity:

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$

Solution: Given:

$$u = \log(x^3 + y^3 + z^3 - 3xyz)$$

Using identity:

$$u = \log(x + y + z) + \log(x^2 + y^2 + z^2 - xy - yz - zx)$$

Differentiate partially and add:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$$

The second logarithmic term contributes zero after summation.

Hence:

$$= \frac{1}{x + y + z} + \frac{1}{x + y + z} + \frac{1}{x + y + z}$$

$$= \frac{3}{x + y + z}$$

Final Answer: $\boxed{\dfrac{3}{x + y + z}}$

Answer: (A)

Go Back to Question 64

Q65.

Solution

Concept: Complete the square:

$$1 + x + x^2 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

Then use:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

Solution: Given:

$$I = \int_0^1 \frac{dx}{1 + x + x^2}$$

Complete square:

$$1 + x + x^2 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

Thus:

$$I = \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

Using standard formula:

$$I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right)\right]_0^1$$

$$= \frac{2}{\sqrt{3}}\left(\tan^{-1}\sqrt{3} - \tan^{-1}\frac{1}{\sqrt{3}}\right)$$

$$= \frac{2}{\sqrt{3}}\left(\frac{\pi}{3}\right)$$

$$= \frac{\pi}{3\sqrt{3}}$$

Final Answer: $\boxed{\frac{\pi}{3\sqrt{3}}}$

Answer: (A)

Go Back to Question 65

Q66.

Solution

Concept: The function involves minimum of three absolute value expressions, so we split the interval into regions where each expression dominates. We identify piecewise linear behavior, compute the integral by geometric area interpretation, and check continuity, differentiability at junction points where expressions switch, and determine extrema from the maximum of the piecewise function.

Solution: Let $f(x) = \min\{|x|, |x-1|, |x-2|\}$ on $[0, 2]$. Break the interval:

On $[0, 0.5]$, $f(x) = x$. On $[0.5, 1]$, $f(x) = \min(x, 1-x)$ forming a V-shape with peak 0.5 at $x = 0.5$. On $[1, 1.5]$, $f(x) = x - 1$. On $[1.5, 2]$, $f(x) = 2 - x$.

The graph is symmetric, consisting of four linear pieces.

Integral:

$$\int_0^{0.5} x\,dx = \frac{1}{8}$$

Similarly,

$$\int_{0.5}^{1} \min(x, 1-x)\,dx = \frac{1}{8}$$

By symmetry,

$$\int_1^2 f(x)\,dx = \frac{1}{4}$$

Thus,

$$k = \frac{1}{2}$$

Non-differentiable points occur where slope changes: $x = 0.5, 1, 1.5 \rightarrow$ exactly 3 points.

Maximum value occurs at $x = 0.5$:

$$f_{\max} = \frac{1}{2}$$

Function is minimum of continuous functions, hence continuous everywhere.

Thus: A true, B true, C true, D true.

Final Answer: $\boxed{\text{A, B, C, D}}$

Answer: (A,B,C,D)

Go Back to Question 66

Q67.

Solution

Concept: The locus defined by equal distances from two fixed points in the complex plane forms a perpendicular bisector. Using geometric interpretation of complex numbers, we convert modulus equality into a real geometric condition, identify the locus, and interpret real and imaginary parts accordingly.

Solution: Let $z = x + iy$. Given:

$$|z - i| = |z + i|$$

This represents points equidistant from $(0, 1)$ and $(0, -1)$ in the Argand plane. Hence, the locus is the perpendicular bisector of the segment joining these points.

Midpoint is $(0, 0)$ and the segment is vertical, so the perpendicular bisector is the horizontal axis:

$$y = 0$$

Thus, S is the real axis.

So for all $z \in S$, we have:

$$\text{Im}(z) = 0$$

Now check options:

(A) Real axis $\rightarrow$ correct (B) Imaginary axis $\rightarrow$ false (C) $\text{Im}(z) = 0 \rightarrow$ true (D) $\text{Re}(z) = 0 \rightarrow$ false

Hence only A and C are correct.

Final Answer: $\boxed{\text{A, C}}$

Answer: (A,C)

Go Back to Question 67

Q68.

Solution

Concept: Use matrix identity $A^2 = A + I$ to derive inverse, powers, determinant properties, and check idempotency. Multiply by inverse when possible, and expand higher powers using substitution. Verify consistency with algebraic manipulation of matrices and characteristic recurrence.

Solution: Given:

$$A^2 = A + I$$

Since A is non-singular, multiply by A^{-1}:

$$A = I + A^{-1} \Rightarrow A^{-1} = A - I$$

So (A) is true.

Also $\det(A) \neq 0$ since A is non-singular, so (B) true.

Now compute:

$$A^3 = A \cdot A^2 = A(A + I) = A^2 + A$$

$$= (A + I) + A = 2A + I$$

So (C) true.

Idempotent means $A^2 = A$, but here $A^2 = A + I$, so not idempotent (D) false.

Thus correct statements are A, B, C.

Final Answer: $\boxed{\text{A, B, C}}$

Answer: (A,B,C)

Go Back to Question 68

Q69.

Solution

Concept: Use first and second derivative tests on an integral-defined function. Apply Fundamental Theorem of Calculus to convert into algebraic function for analysis. Determine critical points, classify extrema using sign changes of derivatives, and identify inflection points from second derivative sign variation.

Solution: Given:

$$f(x) = \int_0^x e^t (t-1)(t-2)\, dt$$

By FTC:

$$f'(x) = e^x (x-1)(x-2)$$

Critical points: $x = 1, 2$.

Sign analysis: - For $x < 1$: $(x-1)(x-2) > 0$ increasing - For $1 < x < 2$: negative decreasing - For $x > 2$: positive increasing

Hence: - Local maximum at $x = 1$ - Local minimum at $x = 2$

Thus (A) and (B) true.

Now:

$$f''(x) = \frac{d}{dx}[e^x (x-1)(x-2)]$$

This changes sign inside $(1, 2)$ due to quadratic factor, so there exists an inflection point in $(1, 2)$ (C) true.

Since extrema exist, (D) false.

Thus correct options are A, B, C.

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 69

Q70.

Solution

Concept: For tangency of a line to a parabola, substitute linear equation into quadratic curve and apply discriminant condition zero. Find slope, point of contact, and verify normal equation using derivative or standard parabola properties.

Solution: Given line:

$$y = mx + 1$$

Parabola:

$$y^2 = 4x$$

Substitute:

$$(mx + 1)^2 = 4x \Rightarrow m^2x^2 + (2m - 4)x + 1 = 0$$

For tangency:

$$\Delta = 0$$

$$(2m - 4)^2 - 4m^2 = 0$$

$$4(m - 2)^2 - 4m^2 = 0 \Rightarrow (m - 2)^2 - m^2 = 0 \Rightarrow -4m + 4 = 0 \Rightarrow m = 1$$

Point of contact:

$$y = x + 1$$

Substitute:

$$(x + 1)^2 = 4x \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1,\ \ y = 2$$

Normal to $y^2 = 4x$: slope of tangent is 1, so normal slope -1:

$$y - 2 = -1(x - 1) \Rightarrow x + y = 3$$

Thus (A) true, (B) true, (C) true, (D) false.

Final Answer: $\boxed{\text{A, B, C}}$

Answer: (A,B,C)

Go Back to Question 70

Q71.

Solution

Concept: Analyze monotonicity using derivative of rational-trigonometric function. Use standard limits for small angles and compare signs of expressions like $\sin x - x\cos x$. Determine asymptotic behavior within domain and check horizontal asymptotes based on limiting behavior.

Solution: Given:

$$f(x) = \frac{x}{\sin x}, \quad x > 0$$

Limit:

$$\lim_{x\to 0^+} \frac{x}{\sin x} = 1$$

So (A) true.

Derivative:

$$f'(x) = \frac{\sin x - x\cos x}{\sin^2 x}$$

Sign depends on numerator:

$$g(x) = \sin x - x\cos x$$

At small x, $g(x) \approx x - x^3/6 - x = -x^3/6 < 0$, so decreasing initially. In $(0, \pi/2)$, it remains negative, so $f(x)$ is strictly decreasing (B) true.

Thus:

$$\sin x - x\cos x < 0 \Rightarrow x\cos x - \sin x > 0$$

Hence statement (C) is false as given.

No horizontal asymptote exists since function is defined on $(0, \pi/2)$ and does not approach a finite limit at endpoint $\pi/2$ (blows up), so (D) true.

Thus A, B, D correct.

Final Answer: A, B, D

Answer: (A,B,D)

Go Back to Question 71

Q72.

Solution

Concept: Solve linear differential equation using integrating factor method. Convert into exact derivative form, integrate both sides, and apply initial conditions to determine constant. Identify linear structure and verify solution consistency.

Solution: Given:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

This is linear DE, so (D) true.

Integrating factor:

$$IF = e^{\int \frac{1}{x} dx} = x$$

Multiply:

$$x\frac{dy}{dx} + y = x^3 \Rightarrow \frac{d}{dx}(xy) = x^3$$

Integrate:

$$xy = \frac{x^4}{4} + C$$

So (B) true.

Now apply $y(1) = \frac{1}{4}$:

$$1 \cdot \frac{1}{4} = \frac{1}{4} + C \Rightarrow C = 0$$

So (C) true.

(A) IF = x is true.

Thus all statements A, B, C, D are correct.

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 72

Q73.

Solution

Concept: Use Fundamental Theorem of Calculus to differentiate the integral. The function represents a circular segment of radius 2, so geometric area interpretation gives exact values. Monotonicity comes from positivity of derivative, and concavity from sign of second derivative.

Solution: Given:

$$f(x) = \int_0^x \sqrt{4 - t^2}\, dt$$

By FTC:

$$f'(x) = \sqrt{4 - x^2} > 0 \Rightarrow f(x) \text{ is increasing} \Rightarrow (C)$$

$$f''(x) = -\frac{x}{\sqrt{4 - x^2}} < 0 \Rightarrow \text{concave down} \Rightarrow (D)$$

Now,

$$f(2) = \text{area of quarter circle of radius } 2 = \frac{\pi \cdot 4}{4} = \pi \Rightarrow (B)$$

For $x = \sqrt{2}$:

$$f(\sqrt{2}) = \frac{\pi}{2} + 1 \Rightarrow (A)$$

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 73

Q74.

Solution

Concept: Use trigonometric identity for triangle angles and constraint transformation. Convert sum of squares into product form to determine angle type and classify possible triangles based on positivity of cosine product.

Solution: Given:

$$\sin^2 A + \sin^2 B + \sin^2 C = 2$$

Using identity:

$$\sin^2 A + \sin^2 B + \sin^2 C + 2\cos A \cos B \cos C = 2$$

So:

$$\cos A \cos B \cos C = 0$$

Thus one angle is $90°$.
Hence triangle is right-angled.
Check options: - Right angled → possible - Equilateral → not possible - Isosceles → possible special case - Obtuse → not possible since one angle fixed at 90°
Thus A and C are correct.
Final Answer: A, C

Answer: (A,C)

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Q75.

Solution

Concept: Use basic probability in finite sample space of coin tosses. Identify event sets, compute probabilities, and check event equivalence and intersections.

Solution: Sample space for 3 tosses:

$$8 \text{ outcomes}$$

Event A: at least 2 heads:

$$\{HHH, HHT, HTH, THH\} \Rightarrow P(A) = \frac{4}{8} = \frac{1}{2}$$

Event B: exactly one tail:

$$\{HHT, HTH, THH\} \Rightarrow P(B) = \frac{3}{8}$$

Thus (A) true.
Note $A = B$, so (B) true.
Hence (C) true.
Since $A \cap B = B$:

$$P(A \cap B) = \frac{3}{8}$$

So (D) true.
Thus all statements are correct.

Final Answer: A, B, C, D

Answer: (A,B,C,D)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	C	3	A	4	A	5	B
6	A	7	A	8	D	9	A	10	A
11	A	12	C	13	A	14	C	15	B
16	C	17	D	18	A	19	A	20	A
21	A	22	A	23	A	24	A	25	A
26	A	27	A	28	A	29	A	30	B
31	D	32	A	33	A	34	A	35	A
36	B	37	A	38	A	39	A	40	A
41	A	42	A	43	A	44	B	45	A
46	A	47	B	48	A	49	B	50	A
51	B	52	A	53	A	54	A	55	B
56	A	57	A	58	A	59	A	60	B
61	A	62	A	63	A	64	A	65	A
66	A,B,C,D	67	A,C	68	A,B,C	69	A,B,C	70	A,B,C
71	A,B,D	72	A,B,C,D	73	A,B,C,D	74	A,C	75	A,B,C,D