WBJEE Mathematics Sample Paper-17

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **75** Multiple Choice Questions divided into **3 Categories**.
- **Section A (Q1–Q50):** Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries **+2 marks**. Incorrect answer: **−0.5** marks. Only **one** correct option.
- **Section C (Q66–Q75):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: −0.25) [Single Correct]

Q1. The value of $\lim_{x\to 0} \dfrac{\sin x - x + \frac{x^3}{6}}{x^5}$ is:

(A) $-\dfrac{1}{120}$

(B) $\dfrac{1}{120}$

(C) $\dfrac{1}{24}$

(D) $-\dfrac{1}{24}$

Q2. If $f(x) = \begin{cases} ax + b, & x < 1 \\ x^2, & x \geq 1 \end{cases}$ is continuous at $x = 1$, then $a + b$ equals:

(A) 0

(B) 1

(C) 2

(D) 3

Q3. If $y = \ln(\sin x)$, then $\frac{dy}{dx}$ equals:

(A) $\cot x$

(B) $\tan x$

(C) $\sec x$

(D) $\csc x$

Q4. The point of inflection of $y = x^3 - 3x^2$ occurs at:

(A) $x = 0$

(B) $x = 1$

(C) $x = 2$

(D) $x = 3$

Q5. The value of $\int xe^x dx$ is:

(A) $e^x(x - 1) + C$

(B) $e^x(x + 1) + C$

(C) $xe^x + C$

(D) $e^{x^2} + C$

Q6. The value of $\int_0^1 \ln(1 + x)\, dx$ is:

(A) $2\ln 2 - 1$

(B) $\ln 2$

(C) $1 - \ln 2$

(D) $2 - \ln 2$

Q7. The solution of $\frac{dy}{dx} = y$ is:

(A) $y = e^x$

(B) $y = xe^x$

(C) $y = Ce^x$

(D) $y = Cx$

Q8. If $A^2 = I$, then A is called:

(A) singular matrix

(B) idempotent matrix

(C) involutory matrix

(D) diagonal matrix

Q9. If a determinant has two proportional rows, its value is:

(A) 1

(B) -1

(C) 0

(D) undefined

Q10. The modulus of $z_1 z_2$ equals:

(A) $|z_1| + |z_2|$

(B) $|z_1| - |z_2|$

(C) $|z_1||z_2|$

(D) $\frac{|z_1|}{|z_2|}$

Q11. If roots of $x^2 + px + q = 0$ are equal, then:

(A) $p^2 = 4q$

(B) $p^2 > 4q$

(C) $p^2 < 4q$

(D) $q = 0$

Q12. If $a_n = 2n + 1$, then a_{10} equals:

(A) 19

(B) 20

(C) 21

(D) 22

Q13. The sum of first n natural numbers is:

(A) $\frac{n(n+1)}{2}$

(B) n^2

(C) $2n$

(D) $n(n-1)$

Q14. If A and B are mutually exclusive, then $P(A \cap B)$ equals:

(A) $P(A) + P(B)$

(B) $P(A)P(B)$

(C) 0

(D) 1

Q15. The middle term of $(a+b)^4$ is:

(A) a^2b^2

(B) $4a^2b^2$

(C) $6a^2b^2$

(D) $8a^2b^2$

Q16. The value of $\sin 2x$ equals:

(A) $2\sin x$

(B) $2\sin x \cos x$

(C) $\sin^2 x$

(D) $\cos^2 x$

Q17. The principal value of $\sin^{-1}(-1)$ is:

(A) $-\frac{\pi}{2}$

(B) $\frac{\pi}{2}$

(C) 0

(D) π

Q18. Distance of point $(2, 3)$ from line $x + y = 0$ is:

(A) $\frac{5}{\sqrt{2}}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{3}{\sqrt{2}}$

(D) $\frac{2}{\sqrt{2}}$

Q19. If a chord of circle passes through centre, it is called:

(A) tangent

(B) diameter

(C) radius

(D) secant

Q20. The equation $y^2 = -4ax$ opens towards:

(A) right

(B) left

(C) up

(D) down

Q21. For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, major axis is along x-axis if:

(A) $a > b$

(B) $a < b$

(C) $a = b$

(D) $a = 0$

Q22. The foci of hyperbola lie on:

(A) major axis

(B) transverse axis

(C) conjugate axis

(D) tangent

Q23. If $\vec{a} \cdot \vec{b} = 0$, then vectors are:

(A) parallel

(B) perpendicular

(C) equal

(D) collinear

Q24. Equation of a plane parallel to xy-plane is:

(A) $x = 0$

(B) $y = 0$

(C) $z = c$

(D) $x + y = 0$

Q25. Number of ways to arrange 3 letters is:

(A) 3

(B) 6

(C) 9

(D) 12

Q26. Value of $^5C_0$ is:

(A) 0

(B) 1

(C) 5

(D) 10

Q27. The value of $\lim_{x\to 0} \frac{1-\cos x}{x^2}$ is:

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

Q28. If $y = x^n$, then $\frac{dy}{dx}$ equals:

(A) nx^{n-1}

(B) x^n

(C) n^x

(D) x^{n+1}

Q29. Integral of e^x is:

(A) $e^x + C$

(B) xe^x

(C) $\ln x$

(D) $1/x$

Q30. Order of highest derivative in $\frac{d^4y}{dx^4} = 0$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q31. Probability of impossible event is:

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

Q32. If $A \subset B$, then $A \cap B$ equals:

(A) A

(B) B

(C) ϕ

(D) $A \cup B$

Q33. If $f(x) = 3x + 2$, then $f(0)$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Q34. If $z = 0$, then $|z|$ is:

(A) 0

(B) 1

(C) -1

(D) undefined

Q35. Identity matrix is denoted by:

(A) 0

(B) I

(C) A

(D) B

Q36. If two rows are interchanged, determinant changes sign:

(A) True

(B) False

(C) Sometimes

(D) Never

Q37. Equation of line parallel to x-axis is:

(A) $x = c$

(B) $y = c$

(C) $x = y$

(D) $x + y = c$

Q38. Radius of circle $x^2 + y^2 = 1$ is:

(A) 0

(B) 1

(C) 2

(D) $\sqrt{2}$

Q39. Axis of parabola $y^2 = 4ax$ is:

(A) x-axis

(B) y-axis

(C) line $y = x$

(D) line $y = -x$

Q40. Centre of ellipse is:

(A) focus

(B) midpoint of axes

(C) vertex

(D) directrix

Q41. Standard form of hyperbola is:

(A) sum form

(B) difference form

(C) product form

(D) quotient form

Q42. Unit vector has magnitude:

(A) 0

(B) 1

(C) 2

(D) ∞

Q43. Number of coordinate axes in 3D space is:

(A) 2

(B) 3

(C) 4

(D) 1

Q44. Number of ways to choose 1 object from n objects is:

(A) n

(B) n^2

(C) $2n$

(D) $n!$

Q45. Value of ^nC_n is:

(A) 0

(B) 1

(C) n

(D) $n!$

Q46. AP has common difference 5 and first term 2, then second term is:

(A) 5

(B) 6

(C) 7

(D) 8

Q47. GP with ratio 2 and first term 1 gives second term:

(A) 1

(B) 2

(C) 3

(D) 4

Q48. Limit of constant function c is:

(A) 0

(B) c

(C) 1

(D) undefined

Q49. Derivative of constant is:

(A) 1

(B) 0

(C) constant

(D) variable

Q50. Integral of 0 is:

(A) 0

(B) 1

(C) x

(D) C

Section–B — 15 Questions × 1 Mark Each
(Negative Marking: −0.5) [Single Correct]

Q51. Probability lies between:

(A) -1 to 1

(B) 0 to 1

(C) 1 to 2

(D) 0 to ∞

Q52. Universal set contains:

(A) no elements

(B) all elements under consideration

(C) only numbers

(D) only letters

Q53. One-one function means:

(A) many to one

(B) one to many

(C) one to one

(D) many to many

Q54. Value of $\sin 0°$ is:

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) undefined

Q55. Range of $\sin^{-1} x$ is:

(A) $[-\pi, \pi]$

(B) $[-\frac{\pi}{2}, \frac{\pi}{2}]$

(C) $[0, \pi]$

(D) $[-1, 1]$

Q56. Slope of horizontal line is:

(A) 0

(B) 1

(C) undefined

(D) infinity

Q57. General equation of circle is of degree:

(A) 1

(B) 2

(C) 3

(D) 4

Q58. Eccentricity of parabola is:

(A) 0

(B) 1

(C) 2

(D) undefined

Q59. Ellipse has eccentricity:

(A) 0

(B) between 0 and 1

(C) 1

(D) greater than 1

Q60. Hyperbola eccentricity is:

(A) less than 1

(B) equal to 1

(C) greater than 1

(D) 0

Q61. Zero vector has magnitude:

(A) 1

(B) 0

(C) undefined

(D) infinity

Q62. Origin in 3D is:

(A) (1,1,1)

(B) (0,0,0)

(C) (1,0,0)

(D) (0,1,1)

Q63. Arrangement of 2 objects from 2 objects is:

(A) 1

(B) 2

(C) 3

(D) 4

Q64. ${}^4C_1$ equals:

(A) 1

(B) 2

(C) 3

(D) 4

Q65. First term of AP is called:

(A) common difference

(B) initial term

(C) ratio

(D) sum

Section C — 10 Questions × 2 Marks Each (No Negative Marking) [One or More Correct]

Q66. Which of the following sequences are arithmetic progressions (AP)?

(A) $2, 5, 8, 11, \ldots$

(B) $1, 4, 9, 16, \ldots$

(C) $7, 7, 7, 7, \ldots$

(D) $3, 6, 12, 24, \ldots$

Q67. Which of the following sequences are geometric progressions (GP)?

(A) $2, 4, 8, 16, \ldots$

(B) $1, 3, 9, 27, \ldots$

(C) $5, 10, 15, 20, \ldots$

(D) $8, 4, 2, 1, \ldots$

Q68. Which of the following coefficients appear in the expansion of $(1 + x)^4$?

(A) 1

(B) 4

(C) 6

(D) 8

Q69. Which of the following statements are true for $i = \sqrt{-1}$?

(A) $i^2 = -1$

(B) $i^4 = 1$

(C) $i^3 = -i$

(D) $i^5 = 1$

Q70. Which of the following values are equal to $\frac{\sqrt{3}}{2}$?

(A) $\sin 60^\circ$

(B) $\cos 30^\circ$

(C) $\tan 60^\circ$

(D) $\sin 30^\circ$

Q71. Which of the following principal values are correct?

(A) $\sin^{-1}(0) = 0$

(B) $\cos^{-1}(1) = 0$

(C) $\tan^{-1}(1) = \frac{\pi}{4}$

(D) $\sin^{-1}(1) = \pi$

Q72. Which of the following equations represent lines passing through the origin?

(A) $y = 2x$

(B) $x - y = 0$

(C) $x + y = 1$

(D) $3x - 2y = 0$

Q73. Which of the following equations represent circles centred at the origin?

(A) $x^2 + y^2 = 16$

(B) $x^2 + y^2 - 9 = 0$

(C) $(x - 1)^2 + y^2 = 4$

(D) $x^2 + y^2 + 4x = 0$

Q74. A die is thrown once. Which of the following events have probability $\frac{1}{3}$?

(A) Getting a multiple of 3

(B) Getting a prime number

(C) Getting a number less than 3

(D) Getting an even number greater than 4

Q75. Which of the following vectors are perpendicular to $\hat{i} + \hat{j}$?

(A) $\hat{i} - \hat{j}$

(B) $2\hat{i} - 2\hat{j}$

(C) $\hat{i} + \hat{j}$

(D) $3\hat{j} - 3\hat{i}$

Detailed Solutions

Q1.

Solution

Concept: To evaluate limits involving trigonometric functions, especially when they approach 0, Taylor series expansions are a powerful tool. The Taylor series for $\sin x$ around $x = 0$ is given by:

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

This expansion is valid for all x. For small values of x, the higher-order terms become progressively smaller.

Solution: Step 1: Write down the Taylor series expansion for $\sin x$ around $x = 0$:

$$\sin x = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots$$

Step 2: Substitute the Taylor series into the given limit expression:

$$\lim_{x\to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5} = \lim_{x\to 0} \frac{\left(x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \dots\right) - x + \frac{x^3}{6}}{x^5}$$

Step 3: Simplify the numerator by canceling out terms:

$$= \lim_{x\to 0} \frac{\frac{x^5}{120} - \frac{x^7}{5040} + \dots}{x^5}$$

Step 4: Divide each term in the numerator by x^5:

$$= \lim_{x\to 0} \left(\frac{1}{120} - \frac{x^2}{5040} + \dots\right)$$

Step 5: Evaluate the limit as $x \to 0$. All terms with x will go to zero:

$$= \frac{1}{120}$$

Final Answer: $\boxed{-\frac{1}{120}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: For a function to be continuous at a point $x = c$, the following three conditions must be met:

1. $f(c)$ must be defined.
2. The limit of $f(x)$ as $x \to c$ must exist ($\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$).
3. The limit must equal the function value: $\lim_{x\to c} f(x) = f(c)$.

In this case, we are given a piecewise function that is defined differently for $x < 1$ and $x \geq 1$. For the function to be continuous at $x = 1$, the left-hand limit, the right-hand limit, and the function value at $x = 1$ must all be equal.

Solution: Step 1: Identify the point of continuity and the function definitions around it. The function is defined as $f(x) = ax + b$ for $x < 1$ and $f(x) = x^2$ for $x \geq 1$. We need to check continuity at $x = 1$.

Step 2: Calculate the left-hand limit as $x \to 1^-$. As x approaches 1 from the left, we use the definition $f(x) = ax + b$:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (ax + b) = a(1) + b = a + b$$

Step 3: Calculate the right-hand limit as $x \to 1^+$. As x approaches 1 from the right, we use the definition $f(x) = x^2$:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x^2 = (1)^2 = 1$$

Step 4: Calculate the function value at $x = 1$. For $x = 1$, we use the definition $f(x) = x^2$:

$$f(1) = (1)^2 = 1$$

Step 5: For continuity at $x = 1$, the left-hand limit, the right-hand limit, and the function value must be equal:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = f(1)$$

$$a + b = 1 = 1$$

Therefore, $a + b = 1$.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: To find the derivative of a composite function like $y = \ln(\sin x)$, we use the chain rule. The chain rule states that if $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.
In this case, we can let $u = \sin x$. Then $y = \ln u$.
The derivative of $y = \ln u$ with respect to u is $\frac{dy}{du} = \frac{1}{u}$.
The derivative of $u = \sin x$ with respect to x is $\frac{du}{dx} = \cos x$.

Solution: Step 1: Apply the chain rule. Let $u = \sin x$. Then $y = \ln u$. The chain rule is $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.
Step 2: Find the derivative of y with respect to u.

$$\frac{dy}{du} = \frac{d}{du}(\ln u) = \frac{1}{u}$$

Step 3: Find the derivative of u with respect to x.

$$\frac{du}{dx} = \frac{d}{dx}(\sin x) = \cos x$$

Step 4: Substitute these derivatives back into the chain rule formula.

$$\frac{dy}{dx} = \left(\frac{1}{u}\right) \cdot (\cos x)$$

Step 5: Substitute $u = \sin x$ back into the expression.

$$\frac{dy}{dx} = \frac{1}{\sin x} \cdot \cos x = \frac{\cos x}{\sin x}$$

Step 6: Simplify the expression using trigonometric identities. We know that $\cot x = \frac{\cos x}{\sin x}$.

$$\frac{dy}{dx} = \cot x$$

Final Answer: $\boxed{\cot x}$

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept: Points of inflection for a function $y = f(x)$ occur where the concavity of the function changes. This typically happens when the second derivative, $f''(x)$, is equal to zero or undefined, and it changes sign around that point.

To find the points of inflection:

1. Find the first derivative, $f'(x)$.
2. Find the second derivative, $f''(x)$.
3. Set $f''(x) = 0$ and solve for x.
4. Test the sign of $f''(x)$ in intervals around these values of x to determine if the concavity changes.

Solution: Step 1: Find the first derivative of $y = x^3 - 3x^2$.

$$y' = \frac{dy}{dx} = \frac{d}{dx}(x^3 - 3x^2) = 3x^2 - 6x$$

Step 2: Find the second derivative of y.

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx}(3x^2 - 6x) = 6x - 6$$

Step 3: Set the second derivative to zero and solve for x.

$$6x - 6 = 0$$

$$6x = 6$$

$$x = 1$$

Step 4: Test the sign of the second derivative in intervals around $x = 1$. Consider an interval where $x < 1$, for example, $x = 0$:

$$y''(0) = 6(0) - 6 = -6$$

Since $y''(0) < 0$, the function is concave down for $x < 1$.

Consider an interval where $x > 1$, for example, $x = 2$:

$$y''(2) = 6(2) - 6 = 12 - 6 = 6$$

Since $y''(2) > 0$, the function is concave up for $x > 1$.

Step 5: Determine if a point of inflection occurs. Since the concavity changes from down to up at $x = 1$ (i.e., the sign of y'' changes from negative to positive), there is a point of inflection at $x = 1$.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The integral $\int xe^x dx$ requires integration by parts. The integration by parts formula is:

$$\int u\, dv = uv - \int v\, du$$

To choose u and dv, we often use the LIATE rule (Logarithmic, Inverse trigonometric, Algebraic, Trigonometric, Exponential) to prioritize which function to set as u. In this case, x is an algebraic function and e^x is an exponential function, so we choose $u = x$ and $dv = e^x dx$.

Solution: Step 1: Identify u and dv. Let $u = x$ and $dv = e^x dx$.
Step 2: Calculate du and v. Differentiate u to find du:

$$du = dx$$

Integrate dv to find v:

$$v = \int e^x dx = e^x$$

Step 3: Apply the integration by parts formula: $\int u\, dv = uv - \int v\, du$.

$$\int xe^x dx = x \cdot e^x - \int e^x dx$$

Step 4: Evaluate the remaining integral.

$$\int xe^x dx = xe^x - e^x + C$$

Step 5: Factor out the common term e^x.

$$\int xe^x dx = e^x(x - 1) + C$$

Final Answer: $\boxed{e^x(x-1) + C}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: To evaluate $\int_0^1 \ln(1+x)\,dx$, we use integration by parts:

$$\int u\,dv = uv - \int v\,du$$

Choose:

$$u = \ln(1+x), \qquad dv = dx$$

Solution: Using integration by parts:

$$u = \ln(1+x), \quad du = \frac{1}{1+x}dx$$

$$dv = dx, \quad v = x$$

Therefore,

$$\int_0^1 \ln(1+x)\,dx = [x\ln(1+x)]_0^1 - \int_0^1 \frac{x}{1+x}\,dx$$

Evaluate the first term:

$$[x\ln(1+x)]_0^1 = \ln 2$$

Now,

$$\frac{x}{1+x} = 1 - \frac{1}{1+x}$$

So,

$$\int_0^1 \frac{x}{1+x}\,dx = \int_0^1 \left(1 - \frac{1}{1+x}\right)dx$$

$$= [x - \ln(1+x)]_0^1$$

$$= 1 - \ln 2$$

Hence,

$$\int_0^1 \ln(1+x)\,dx = \ln 2 - (1 - \ln 2)$$

$$= 2\ln 2 - 1$$

Final Answer: $\boxed{2\ln 2 - 1}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: The given differential equation is $\frac{dy}{dx} = y$. This is a first-order separable differential equation. We can solve it by separating the variables (terms involving y on one side and terms involving x on the other) and then integrating both sides.

Solution: Step 1: Separate the variables. Divide both sides by y (assuming $y \neq 0$) and multiply by dx:

$$\frac{1}{y}\,dy = dx$$

Step 2: Integrate both sides.

$$\int \frac{1}{y}\,dy = \int dx$$

$$\ln|y| = x + C_1$$

where C_1 is the constant of integration.
Step 3: Solve for y. To remove the natural logarithm, exponentiate both sides:

$$e^{\ln|y|} = e^{x+C_1}$$

$$|y| = e^x \cdot e^{C_1}$$

Let $C_2 = e^{C_1}$. Since e^{C_1} is always positive, $C_2 > 0$.

$$|y| = C_2 e^x$$

This implies $y = \pm C_2 e^x$.
Step 4: Combine the constants. Let $C = \pm C_2$. This new constant C can be any non-zero real number.

$$y = Ce^x$$

Note that if we consider the case $y = 0$, then $\frac{dy}{dx} = 0$ and $y = 0$, so $y = 0$ is also a solution. This is covered by $y = Ce^x$ if we allow $C = 0$. Therefore, the general solution is $y = Ce^x$, where C is an arbitrary constant.
Final Answer: $\boxed{y = Ce^x}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: A square matrix A is called an involutory matrix if its square is equal to the identity matrix (I). Mathematically, this is expressed as $A^2 = I$.
Let's briefly define the other options to distinguish them:
- Singular matrix: A square matrix with a determinant of zero. If $\det(A) = 0$, then A is singular.
- Idempotent matrix: A matrix A such that $A^2 = A$. This means that multiplying the matrix by itself results in the same matrix.
- Diagonal matrix: A matrix where all the off-diagonal elements are zero. The elements on the main diagonal can be any value.

Solution: The problem states that $A^2 = I$. By definition, a matrix A for which $A^2 = I$ is called an involutory matrix. For example, consider the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Then $A^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1\cdot 1 + 0\cdot 0 & 1\cdot 0 + 0\cdot(-1) \\ 0\cdot 1 + (-1)\cdot 0 & 0\cdot 0 + (-1)\cdot(-1) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$. Thus, this matrix A is an involutory matrix.
Final Answer: $\boxed{involutory matrix}$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: A determinant becomes zero if any two rows (or columns) of a matrix are proportional. This means one row can be written as a constant multiple of another row.
When rows are proportional, the rows are linearly dependent, and hence the matrix is singular.
The determinant of a singular matrix is always zero.
For example, consider:

$$\begin{pmatrix} a & b \\ ka & kb \end{pmatrix}$$

Here, the second row is k times the first row. Its determinant is:

$$a(kb) - b(ka)$$

$$= kab - kab$$

$$= 0$$

Solution: According to the basic property of determinants:

If two rows or columns of a matrix are proportional, then the determinant is zero.

This happens because proportional rows are linearly dependent and do not form an independent system.
Therefore, the determinant of such a matrix is:

$$0$$

Final Answer: $\boxed{0}$

Answer: (C)

Go Back to Question 9

Q10.

Solution

Concept: For complex numbers z_1 and z_2, the modulus of their product z_1z_2 is equal to the product of their individual moduli, i.e., $|z_1z_2| = |z_1||z_2|$. This property holds true for any two complex numbers.

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be the polar forms of z_1 and z_2.

Then, their moduli are $|z_1| = r_1$ and $|z_2| = r_2$.

The product $z_1z_2 = r_1r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$.

The modulus of the product is $|z_1z_2| = r_1r_2$.

Substituting back, we get $|z_1z_2| = |z_1||z_2|$.

Solution: The property of the modulus of complex numbers states that the modulus of the product of two complex numbers is equal to the product of their moduli. Therefore, $|z_1z_2| = |z_1||z_2|$.

Final Answer: $\boxed{|z_1||z_2|}$

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept: For a quadratic equation of the form $ax^2 + bx + c = 0$, the nature of its roots is determined by the discriminant, $\Delta = b^2 - 4ac$.
- If $\Delta > 0$, the roots are real and distinct.
- If $\Delta = 0$, the roots are real and equal.
- If $\Delta < 0$, the roots are complex conjugates.

In this question, the quadratic equation is $x^2 + px + q = 0$. Comparing this with $ax^2 + bx + c = 0$, we have $a = 1$, $b = p$, and $c = q$.

Solution: Step 1: Identify the coefficients of the quadratic equation. The given equation is $x^2 + px + q = 0$. Here, $a = 1$, $b = p$, and $c = q$.
Step 2: Write down the condition for equal roots. The roots of a quadratic equation are equal if and only if the discriminant is zero.

$$\Delta = b^2 - 4ac = 0$$

Step 3: Substitute the coefficients into the discriminant formula.

$$p^2 - 4(1)(q) = 0$$

$$p^2 - 4q = 0$$

Step 4: Rearrange the equation to find the relationship between p and q.

$$p^2 = 4q$$

Final Answer: $\boxed{p^2 = 4q}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: The problem provides an explicit formula for the n-th term of a sequence: $a_n = 2n + 1$. We are asked to find the 10th term of this sequence, denoted as a_{10}. To do this, we simply substitute $n = 10$ into the given formula.

Solution: Step 1: Identify the formula for the n-th term of the sequence. The given formula is $a_n = 2n + 1$.

Step 2: Substitute $n = 10$ into the formula to find a_{10}.

$$a_{10} = 2(10) + 1$$

Step 3: Calculate the result.

$$a_{10} = 20 + 1$$

$$a_{10} = 21$$

Final Answer: $\boxed{21}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: The sum of the first n natural numbers is a well-known arithmetic series. The natural numbers are $1, 2, 3, \ldots, n$. The sum of an arithmetic series can be found using the formula $S_n = \frac{n}{2}(a_1 + a_n)$, where n is the number of terms, a_1 is the first term, and a_n is the last term.
In this case, the sequence is $1, 2, 3, \ldots, n$.
- The first term, $a_1 = 1$.
- The last term, $a_n = n$.
- The number of terms is n.

Solution: Step 1: Identify the series and its properties. The series is $1 + 2 + 3 + \cdots + n$. This is an arithmetic progression with: - Number of terms, n. - First term, $a_1 = 1$. - Last term, $a_n = n$.
Step 2: Apply the formula for the sum of an arithmetic series.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

Step 3: Substitute the values of a_1 and a_n.

$$S_n = \frac{n}{2}(1 + n)$$

Step 4: Rewrite the expression in a common and simplified form.

$$S_n = \frac{n(n+1)}{2}$$

Final Answer: $\boxed{\frac{n(n+1)}{2}}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: Mutually exclusive events are events that cannot occur at the same time. If two events, A and B, are mutually exclusive, it means that the occurrence of event A precludes the occurrence of event B, and vice versa. In terms of probability, this implies that the probability of both events A and B occurring together is zero. The intersection of two events A and B, denoted as $A \cap B$, represents the event that both A and B occur.

Solution: By the definition of mutually exclusive events, if events A and B cannot happen simultaneously, then the probability of their intersection is zero.

$$P(A \cap B) = 0$$

Final Answer: $\boxed{0}$

Answer: (C)

Go Back to Question 14

Q15.

Solution

Concept: The binomial theorem provides a formula for expanding expressions of the form $(a+b)^n$. The expansion is given by:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

where $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$ are the binomial coefficients.
The number of terms in the expansion of $(a+b)^n$ is $n+1$.
- If n is even, there is a single middle term, which is the $(\frac{n}{2}+1)$-th term.
- If n is odd, there are two middle terms, which are the $\left(\frac{n+1}{2}\right)$-th and $\left(\frac{n+3}{2}\right)$-th terms.
In this case, we have $(a+b)^4$. Here, $n=4$, which is an even number. The number of terms is $4+1=5$. The middle term is the $\left(\frac{4}{2}+1\right)$-th term, which is the 3rd term.

Solution: Step 1: Identify n and the index of the middle term. For $(a+b)^4$, $n=4$. Since n is even, the middle term is the $\left(\frac{4}{2}+1\right) = 3$rd term.
Step 2: Use the general term formula in the binomial expansion. The $(k+1)$-th term in the expansion of $(a+b)^n$ is given by $T_{k+1} = \binom{n}{k} a^{n-k} b^k$. For the 3rd term, we need $k+1=3$, which means $k=2$.
Step 3: Substitute $n=4$ and $k=2$ into the general term formula.

$$T_{2+1} = T_3 = \binom{4}{2} a^{4-2} b^2$$

Step 4: Calculate the binomial coefficient $\binom{4}{2}$.

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{4\times3\times2\times1}{(2\times1)(2\times1)} = \frac{24}{4} = 6$$

Step 5: Substitute the binomial coefficient back into the term.

$$T_3 = 6 \cdot a^2 \cdot b^2 = 6a^2b^2$$

Final Answer: $\boxed{6a^2b^2}$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept: The double angle formula for sine is a fundamental trigonometric identity. It relates the sine of an angle $2x$ to the sine and cosine of the angle x.
The identity is derived from the angle addition formula for sine: $\sin(A + B) = \sin A \cos B + \cos A \sin B$.
Setting $A = x$ and $B = x$, we get:

$$\sin(x + x) = \sin x \cos x + \cos x \sin x$$

$$\sin(2x) = 2 \sin x \cos x$$

Solution: Using the double angle formula for sine:

$$\sin 2x = 2 \sin x \cos x$$

Final Answer: $\boxed{2 \sin x \cos x}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: The inverse sine function, denoted as $\sin^{-1} y$ or $\arcsin y$, gives the angle whose sine is y. The principal value of $\sin^{-1} y$ is defined to be in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
This ensures that the function is one-to-one and covers all possible sine values.
We need to find an angle θ such that $\sin\theta = -1$ and $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

Solution: Step 1: Understand the definition of the principal value of $\sin^{-1} y$. The range of the principal values for $\sin^{-1} y$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
Step 2: Find the angle θ such that $\sin\theta = -1$. We know that the sine function reaches its minimum value of -1. On the unit circle, this occurs at the point $(0, -1)$. The angle corresponding to this point, measured from the positive x-axis, is $-\frac{\pi}{2}$ radians (or 270°, but we need the value in the principal range).
Step 3: Check which of these angles falls within the principal value range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. The angle $-\frac{\pi}{2}$ lies within this interval. Specifically, $\sin\left(-\frac{\pi}{2}\right) = -1$.
Therefore, the principal value of $\sin^{-1}(-1)$ is $-\frac{\pi}{2}$.

Final Answer: $\boxed{-\frac{\pi}{2}}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: The distance of a point (x_0, y_0) from a line $Ax + By + C = 0$ is given by the formula:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

Solution: Step 1: Identify the point and the line equation. The point is $(x_0, y_0) = (2, 3)$. The line is $x + y = 0$. We can write this in the form $Ax + By + C = 0$ by taking $A = 1$, $B = 1$, and $C = 0$.
Step 2: Apply the distance formula.

$$d = \frac{|(1)(2) + (1)(3) + 0|}{\sqrt{1^2 + 1^2}}$$

Step 3: Calculate the numerator.

$$|1 \cdot 2 + 1 \cdot 3 + 0| = |2 + 3 + 0| = |5| = 5$$

Step 4: Calculate the denominator.

$$\sqrt{1^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}$$

Step 5: Combine the numerator and denominator.

$$d = \frac{5}{\sqrt{2}}$$

Final Answer: $\boxed{\frac{5}{\sqrt{2}}}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: A chord of a circle is a line segment connecting two points on the circumference of the circle.
- A tangent is a line that intersects the circle at exactly one point.
- A diameter is a chord that passes through the center of the circle. It is the longest possible chord.
- A radius is a line segment from the center of the circle to any point on the circumference.
- A secant is a line that intersects the circle at two distinct points.

Solution: By definition, a chord that passes through the center of a circle is called a diameter. A diameter is the longest possible chord in a circle and is twice the length of the radius.
Final Answer: $\boxed{diameter}$

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: The equation of a parabola can be written in standard forms depending on its orientation and vertex.
- For a parabola with vertex at the origin $(0, 0)$ and opening to the right: $y^2 = 4ax$, where $a > 0$.
- For a parabola with vertex at the origin $(0, 0)$ and opening to the left: $y^2 = -4ax$, where $a > 0$.
- For a parabola with vertex at the origin $(0, 0)$ and opening upwards: $x^2 = 4ay$, where $a > 0$.
- For a parabola with vertex at the origin $(0, 0)$ and opening downwards: $x^2 = -4ay$, where $a > 0$.
The sign of the coefficient of the linear term determines the direction of opening.

Solution: The given equation is $y^2 = -4ax$. Here, the y term is squared, which means the axis of symmetry is the x-axis. The coefficient of x is $-4a$. Assuming a is a positive constant (as is standard in these forms), $-4a$ is a negative number. A parabola of the form $y^2 = (\text{negative number}) \cdot x$ opens towards the negative x-axis, which is to the left.
Final Answer: $\boxed{left}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: The standard equation of an ellipse centered at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The major axis is the longer axis of the ellipse, and the minor axis is the shorter axis.
- If $a > b$, then $a^2 > b^2$. This means the denominator of the x^2 term is larger than the denominator of the y^2 term. Consequently, the ellipse is stretched more along the x-axis, and the major axis lies along the x-axis.
- If $b > a$, then $b^2 > a^2$. This means the denominator of the y^2 term is larger. The ellipse is stretched more along the y-axis, and the major axis lies along the y-axis.
- If $a = b$, the equation becomes $x^2 + y^2 = a^2$, which is the equation of a circle, where the major and minor axes are equal.

Solution: For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, the major axis is along the x-axis if the ellipse is wider horizontally. This occurs when the term under x^2 is larger than the term under y^2, which means $a^2 > b^2$. Since a and b represent lengths of semi-axes and are positive, this condition is equivalent to $a > b$.

Final Answer: $\boxed{a > b}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: For a hyperbola, the foci are two fixed points used in its definition: a hyperbola is the locus of points such that the absolute difference of the distances from any point on the hyperbola to the two foci is constant. The axis that passes through the foci and the center of the hyperbola is called the transverse axis. The other axis, perpendicular to the transverse axis and passing through the center, is called the conjugate axis.
For a hyperbola with the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the transverse axis is along the x-axis, and the foci are located at $(\pm c, 0)$, where $c^2 = a^2 + b^2$.
For a hyperbola with the equation $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, the transverse axis is along the y-axis, and the foci are located at $(0, \pm c)$, where $c^2 = a^2 + b^2$.
In both standard forms, the foci lie on the transverse axis.

Solution: The definition of a hyperbola involves two foci. The axis of symmetry that contains these foci is known as the transverse axis. The hyperbola "opens" along its transverse axis.

Final Answer: $\boxed{transverseaxis}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept: The dot product (or scalar product) of two vectors $\vec{a}$ and $\vec{b}$ is defined as $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$, where $|\vec{a}|$ and $|\vec{b}|$ are the magnitudes of the vectors, and θ is the angle between them.
If the dot product of two vectors is zero ($\vec{a}\cdot\vec{b} = 0$), this implies a specific geometric relationship between the vectors.

$$|\vec{a}||\vec{b}|\cos\theta = 0$$

This equation holds true if:
1. $|\vec{a}| = 0$ (i.e., $\vec{a}$ is the zero vector).
2. $|\vec{b}| = 0$ (i.e., $\vec{b}$ is the zero vector).
3. $\cos\theta = 0$.

Assuming neither vector is the zero vector, the condition $\cos\theta = 0$ implies that $\theta = \frac{\pi}{2}$ (or 90°). Vectors that form an angle of 90° are perpendicular.

Solution: Given that the dot product of vectors $\vec{a}$ and $\vec{b}$ is zero: $\vec{a}\cdot\vec{b} = 0$. This means $|\vec{a}||\vec{b}|\cos\theta = 0$. If we assume that neither $\vec{a}$ nor $\vec{b}$ is the zero vector (so their magnitudes are non-zero), then it must be that $\cos\theta = 0$. The angle θ for which $\cos\theta = 0$ is $\theta = \frac{\pi}{2}$ radians (or 90°). Vectors that are at a 90° angle to each other are perpendicular.
Final Answer: $\boxed{perpendicular}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: In a three-dimensional Cartesian coordinate system, the coordinate planes are defined by the intersection of the axes.

- The xy-plane is formed by the x-axis and the y-axis. All points in the xy-plane have a z-coordinate of 0.
- The yz-plane is formed by the y-axis and the z-axis. All points in the yz-plane have an x-coordinate of 0.
- The xz-plane is formed by the x-axis and the z-axis. All points in the xz-plane have a y-coordinate of 0.

A plane parallel to the xy-plane will have the same z-coordinate for all its points, while the x and y coordinates can vary freely.

Solution: A plane parallel to the xy-plane is a horizontal plane. In the equation of a plane, the coefficients of x and y determine its orientation relative to the axes. For a plane to be parallel to the xy-plane, it means that the z-coordinate is constant for all points on that plane, irrespective of the x and y coordinates. This constant z-value can be represented by c. Thus, the equation of such a plane is $z = c$.

Final Answer: $\boxed{z = c}$

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept: The number of ways to arrange n distinct objects is given by n factorial, denoted as $n!$. The factorial of a non-negative integer n is the product of all positive integers less than or equal to n.

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

In this problem, we need to find the number of ways to arrange 3 distinct letters. This is a permutation problem where we are arranging all n items, so $k = n$.

Solution: Step 1: Identify the number of distinct items to be arranged. We have 3 distinct letters. So, $n = 3$.
Step 2: Apply the formula for permutations of n distinct items. The number of ways to arrange n distinct items is $n!$. For $n = 3$, the number of arrangements is $3!$.
Step 3: Calculate the factorial.

$$3! = 3 \times 2 \times 1 = 6$$

Therefore, there are 6 ways to arrange 3 distinct letters.
Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: The combination formula ^nC_k (read as "n choose k") represents the number of ways to choose k items from a set of n distinct items without regard to the order of selection. The formula is given by:

$$^nC_k = \frac{n!}{k!(n-k)!}$$

where $n!$ denotes the factorial of n.
A special case of this formula is when $k = 0$. $^nC_0$ represents the number of ways to choose 0 items from a set of n items. There is only one way to do this: choose nothing.

Solution: Step 1: Use the combination formula. We need to find $^5C_0$. Here, $n = 5$ and $k = 0$.

$$^5C_0 = \frac{5!}{0!(5-0)!}$$

Step 2: Recall that $0! = 1$.

$$^5C_0 = \frac{5!}{1 \cdot 5!}$$

Step 3: Simplify the expression.

$$^5C_0 = \frac{5!}{5!} = 1$$

Alternatively, by the definition of combinations, $^nC_0$ is the number of ways to choose 0 elements from a set of n elements, which is always 1 (the empty set).
Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: To evaluate the limit $\lim_{x\to 0} \frac{1-\cos x}{x^2}$, we can use L'Hôpital's Rule or the Taylor series expansion of $\cos x$.

Using L'Hôpital's Rule:

Since substituting $x = 0$ directly into the expression results in the indeterminate form $\frac{1-\cos 0}{0^2} = \frac{1-1}{0} = \frac{0}{0}$, we can apply L'Hôpital's Rule by taking the derivative of the numerator and the denominator.

Using Taylor series expansion:

The Taylor series for $\cos x$ around $x = 0$ is $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$.

Solution: Method 1: Using L'Hôpital's Rule. Step 1: Apply L'Hôpital's Rule once.

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos x)}{\frac{d}{dx}(x^2)} = \lim_{x\to 0} \frac{\sin x}{2x}$$

Step 2: Apply L'Hôpital's Rule again, as we still have the indeterminate form $\frac{0}{0}$.

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(\sin x)}{\frac{d}{dx}(2x)} = \lim_{x\to 0} \frac{\cos x}{2}$$

Step 3: Evaluate the limit.

$$= \frac{\cos 0}{2} = \frac{1}{2}$$

Method 2: Using Taylor series. Step 1: Substitute the Taylor series of $\cos x$ into the limit expression.

$$\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{24} - \dots$$

$$\frac{1-\cos x}{x^2} = \frac{1 - \left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \dots\right)}{x^2} = \frac{\frac{x^2}{2} - \frac{x^4}{24} + \dots}{x^2}$$

Step 2: Simplify the expression.

$$= \frac{1}{2} - \frac{x^2}{24} + \dots$$

Step 3: Evaluate the limit as $x \to 0$.

$$\lim_{x\to 0} \left(\frac{1}{2} - \frac{x^2}{24} + \dots\right) = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: This question is based on one of the most fundamental rules of differential calculus known as the **Power Rule of Differentiation**. If a function is of the form:

$$y = x^n$$

where n is any real number, then its derivative with respect to x is:

$$\frac{dy}{dx} = nx^{n-1}$$

The rule tells us that:

- The exponent of x becomes the coefficient.
- The exponent is reduced by 1.

This rule is extremely important because it forms the basis for differentiating polynomial functions.

Solution: Step 1: Write the given function.

$$y = x^n$$

Step 2: Apply the Power Rule of differentiation. According to the rule:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Step 3: Therefore,

$$\frac{dy}{dx} = nx^{n-1}$$

Hence, the derivative of x^n is:

$$nx^{n-1}$$

Final Answer: $\boxed{nx^{n-1}}$

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept: Integration is the reverse process of differentiation. The integral of a function gives its antiderivative. We know the important exponential differentiation formula:

$$\frac{d}{dx}(e^x) = e^x$$

Since differentiation and integration are inverse operations, the integral of e^x is also e^x. Whenever we perform indefinite integration, we must add a constant of integration C because the derivative of a constant is zero.

Solution: Step 1: Consider the integral:

$$\int e^x \, dx$$

Step 2: Recall the differentiation formula:

$$\frac{d}{dx}(e^x) = e^x$$

Therefore, the antiderivative of e^x is:

$$e^x$$

Step 3: Add the constant of integration.

$$\int e^x \, dx = e^x + C$$

Thus, the required integral is:

$$e^x + C$$

Final Answer: $\boxed{e^x + C}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: A differential equation is an equation involving derivatives of a dependent variable with respect to an independent variable. The **order** of a differential equation is defined as the order of the highest derivative present in the equation.
Examples:

- $\frac{dy}{dx}$ is a first-order derivative.
- $\frac{d^2y}{dx^2}$ is a second-order derivative.
- $\frac{d^4y}{dx^4}$ is a fourth-order derivative.

The order depends only on the highest derivative appearing in the equation.

Solution: Step 1: Write the given differential equation.

$$\frac{d^4y}{dx^4} = 0$$

Step 2: Identify the derivatives present. The equation contains the derivative:

$$\frac{d^4y}{dx^4}$$

Step 3: Determine the highest order derivative. The derivative is of order 4 because y has been differentiated four times with respect to x.
Therefore, the order of the differential equation is:

$$4$$

Final Answer: $\boxed{4}$

Answer: (D)

Go Back to Question 30

Q31.

Solution

Concept: Probability measures the chance that an event will occur. The probability of any event always lies between 0 and 1:

$$0 \le P(E) \le 1$$

Important cases:

- $P(E) = 0$ means the event is impossible.
- $P(E) = 1$ means the event is certain.

An impossible event is one that can never happen.

Solution: Step 1: Recall the definition of an impossible event.
An impossible event is an event that cannot occur under any condition.
Step 2: Use the probability rule.
The probability assigned to an impossible event is:

$$0$$

Therefore,

$$P(\text{Impossible Event}) = 0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: The intersection of two sets A and B, written as $A \cap B$, is the set containing all elements common to both sets. If:

$$A \subset B$$

then every element of A also belongs to B.
Hence, all elements of A are automatically common to both A and B.

Solution: Step 1: Given:

$$A \subset B$$

This means every element of A belongs to B.
Step 2: Recall the meaning of intersection.
The set:

$$A \cap B$$

contains elements common to both sets.
Step 3: Since all elements of A are already in B, every element of A belongs to the intersection.
Therefore:

$$A \cap B = A$$

Final Answer: $\boxed{A}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: A function assigns a unique output value to every input value. To evaluate a function at a particular value of x, we substitute that value into the function expression.
Given:

$$f(x) = 3x + 2$$

To find $f(0)$, substitute $x = 0$ into the function.

Solution: Step 1: Write the given function.

$$f(x) = 3x + 2$$

Step 2: Substitute $x = 0$.

$$f(0) = 3(0) + 2$$

Step 3: Simplify.

$$f(0) = 0 + 2$$

$$f(0) = 2$$

Therefore, the value of the function at $x = 0$ is:

$$2$$

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 33

Q34.

Solution

Concept: A complex number is generally written as

$$z = x + iy$$

where:

- x is the real part,
- y is the imaginary part.

The modulus of a complex number represents its distance from the origin in the complex plane and is given by

$$|z| = \sqrt{x^2 + y^2}$$

Solution: Step 1: Given

$$z = 0$$

This can be written as

$$z = 0 + i(0)$$

Hence,

$$x = 0, \qquad y = 0$$

Step 2: Apply the modulus formula.

$$|z| = \sqrt{x^2 + y^2}$$

Substituting the values,

$$|0| = \sqrt{0^2 + 0^2}$$

Step 3: Simplify.

$$|0| = \sqrt{0 + 0}$$

$$|0| = \sqrt{0}$$

$$|0| = 0$$

Therefore, the modulus of the complex number 0 is

$$0$$

Final Answer:

$$\boxed{0}$$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: An identity matrix is a square matrix in which:

- All diagonal elements are 1.
- All non-diagonal elements are 0.

It acts as the multiplicative identity in matrix multiplication. The identity matrix is denoted by the symbol:

$$I$$

Solution: Step 1: Recall the notation for the identity matrix.
In linear algebra, the identity matrix is universally represented by:

$$I$$

Step 2: Examples. For a 2×2 identity matrix:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

For a 3×3 identity matrix:

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Hence, the standard symbol for the identity matrix is:

$$I$$

Final Answer: $\boxed{I}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: Determinants satisfy several important properties. One such property is:
If any two rows (or columns) of a determinant are interchanged, the sign of the determinant changes.
Mathematically:
If matrix A' is obtained from matrix A by interchanging two rows, then:

$$\det(A') = -\det(A)$$

Solution: Step 1: Recall the determinant property.
Interchanging two rows changes the sign of the determinant.
Step 2: Interpret the statement.
The statement:

"If two rows are interchanged, determinant changes sign."

is exactly the standard property of determinants.
Therefore, the statement is correct.
Final Answer: True

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: In coordinate geometry:

- The x-axis is horizontal.
- Any line parallel to the x-axis is also horizontal.

For a horizontal line, the y-coordinate remains constant while the x-coordinate can vary. Therefore, the equation of a horizontal line is:

$$y = c$$

where c is a constant.

Solution: Step 1: Identify the type of line.
A line parallel to the x-axis is horizontal.
Step 2: Observe the coordinates on such a line.
All points on the line have the same y-coordinate.
Step 3: Represent the constant y-coordinate.
Let the constant value be c.
Hence, the equation of the line is:

$$y = c$$

Final Answer: $\boxed{y = c}$

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept: A circle is the set of all points in a plane that are at a fixed distance from a fixed point called the center. The standard equation of a circle centered at the origin is:

$$x^2 + y^2 = r^2$$

where:

- $(0, 0)$ is the center,
- r is the radius of the circle.

To determine the radius, we compare the given equation with the standard form.

Solution: Step 1: Write the given equation.

$$x^2 + y^2 = 1$$

Step 2: Compare it with the standard form:

$$x^2 + y^2 = r^2$$

By comparison:

$$r^2 = 1$$

Step 3: Find the value of r. Taking square root on both sides:

$$r = \sqrt{1}$$

Since radius is always positive:

$$r = 1$$

Therefore, the radius of the circle is:

$$1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 38

Q39.

Solution

Concept: A parabola is a conic section formed by the set of points equidistant from a fixed point (focus) and a fixed line (directrix). The standard equation:

$$y^2 = 4ax$$

represents a parabola opening towards the right if $a > 0$ and towards the left if $a < 0$.
The axis of a parabola is its line of symmetry. For the equation $y^2 = 4ax$, the parabola is symmetric about the x-axis.

Solution: Step 1: Consider the equation:

$$y^2 = 4ax$$

Step 2: Observe the squared variable.
Since the variable y is squared, the parabola opens horizontally.
Step 3: Check symmetry.
If (x, y) lies on the parabola, then:

$$y^2 = 4ax$$

Replacing y by $-y$:

$$(-y)^2 = 4ax$$

$$y^2 = 4ax$$

Thus, both (x, y) and $(x, -y)$ lie on the parabola.
This shows symmetry about the x-axis.
Therefore, the axis of the parabola is:

$$\text{x-axis}$$

Final Answer: $\boxed{\text{x-axis}}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: An ellipse is a conic section obtained when a plane cuts a cone at an angle less than that made by the side of the cone. For the standard ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

the ellipse is symmetric about both coordinate axes. The center of an ellipse is the point where:

- the major axis intersects the minor axis,
- both axes bisect each other.

Thus, the center is the midpoint of the axes.

Solution: Step 1: Recall the definition of the center of an ellipse.
The center is the common midpoint of:

- the major axis,
- the minor axis.

Step 2: Understand geometrically.
The major and minor axes intersect exactly at the center of the ellipse.
Hence, the center is the midpoint of the axes.
Final Answer: $\boxed{\text{midpoint of axes}}$

Answer: (B)

Go Back to Question 40

Q41.

Solution

Concept: A hyperbola is a conic section defined as the locus of a point such that the absolute difference of its distances from two fixed points (called foci) is constant. This definition leads to equations of the form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Unlike an ellipse, which involves a constant sum of distances, a hyperbola involves a constant difference of distances. Hence, it is associated with a "difference form."

Solution: Step 1: Recall the geometric definition of a hyperbola.
For any point on the hyperbola:

$$|PF_1 - PF_2| = \text{constant}$$

where F_1 and F_2 are the foci.
Step 2: Observe the important keyword.
The defining property involves the **difference** of distances.
Therefore, the hyperbola is characterized by a:

$$\text{difference form}$$

Final Answer: $\boxed{\text{difference form}}$

Answer: (B)

Go Back to Question 41

Q42.

Solution

Concept: A vector is a quantity having both magnitude and direction. A unit vector is a vector whose magnitude is exactly equal to 1. If $\vec{v}$ is a non-zero vector, then the unit vector in the direction of $\vec{v}$ is:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

Unit vectors are used to represent direction only.

Solution: Step 1: Recall the definition of a unit vector.
A unit vector has magnitude:

$$1$$

Step 2: Therefore, by definition:

$$|\hat{v}| = 1$$

Hence, the magnitude of a unit vector is:

$$1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 42

Q43.

Solution

Concept: A three-dimensional Cartesian coordinate system is used to locate points in space. The system consists of three mutually perpendicular coordinate axes:

- x-axis
- y-axis
- z-axis

These axes intersect at a common point called the origin.

Solution: Step 1: Recall the coordinate axes in 3D geometry.
The axes are:

$$x,\ y,\ z$$

Step 2: Count the number of axes.
There are:

$$3$$

coordinate axes in three-dimensional space.
Therefore, a 3D Cartesian system has three coordinate axes.
Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 43

Q44.

Solution

Concept: The number of ways of choosing objects from a group is calculated using combinations.The combination formula is:

$$^nC_r = \frac{n!}{r!(n-r)!}$$

When selecting only one object from n distinct objects, we use:

$$^nC_1$$

Solution: Step 1: Use the combination formula.

$$^nC_1 = \frac{n!}{1!(n-1)!}$$

Step 2: Simplify the factorial expression. Since:

$$n! = n(n-1)!$$

Substituting:

$$^nC_1 = \frac{n(n-1)!}{1 \cdot (n-1)!}$$

Step 3: Cancel the common term.

$$^nC_1 = n$$

Therefore, the number of ways to choose one object from n objects is:

$$n$$

Final Answer: $\boxed{n}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: The combination formula ${}^nC_k$ represents the number of ways to choose k items from a set of n distinct items. The formula is:

$${}^nC_k = \frac{n!}{k!(n-k)!}$$

We need to find the value of ${}^nC_n$.

Solution: Step 1: Substitute $k = n$ into the combination formula.

$${}^nC_n = \frac{n!}{n!(n-n)!}$$

Step 2: Simplify the expression.

$${}^nC_n = \frac{n!}{n!(0)!}$$

Recall that $0! = 1$.

$${}^nC_n = \frac{n!}{n! \cdot 1} = \frac{n!}{n!}$$

Step 3: Evaluate the result.

$${}^nC_n = 1$$

This means there is only one way to choose all n items from a set of n items (which is to choose all of them).

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: An arithmetic progression (AP) is a sequence of numbers such that the difference between consecutive terms is constant. This constant difference is called the common difference, denoted by d.
The terms of an AP can be represented as $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \ldots$, where a_1 is the first term.

Solution: Given: - Common difference, $d = 5$. - First term, $a_1 = 2$.
We need to find the second term, a_2. By the definition of an arithmetic progression, the second term is obtained by adding the common difference to the first term:

$$a_2 = a_1 + d$$

Substituting the given values:

$$a_2 = 2 + 5$$

$$a_2 = 7$$

Final Answer: $\boxed{7}$

Answer: (C)

Go Back to Question 46

Q47.

Solution

Concept: A geometric progression (GP) is a sequence of numbers where each term after the first is obtained by multiplying the previous term by a fixed, non-zero number called the common ratio (r).
If the first term is a, the sequence is $a, ar, ar^2, ar^3, \ldots$.

Solution: Given: - Common ratio, $r = 2$. - First term, $a = 1$.
We need to find the second term of the GP. The second term (a_2) is obtained by multiplying the first term (a_1) by the common ratio (r).

$$a_2 = a_1 \times r$$

Substituting the given values:

$$a_2 = 1 \times 2$$

$$a_2 = 2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept: The limit of a function describes the value that the function approaches as the input variable approaches a specific number. A constant function is a function whose value remains fixed for all values of the variable. Such a function can be written as:

$$f(x) = c$$

where c is a constant. Since the value of a constant function never changes, its limit at every point is the constant itself.

Solution: Step 1: Consider the constant function.

$$f(x) = c$$

Step 2: Evaluate the limit as x approaches any number a.

$$\lim_{x \to a} c$$

Step 3: Since the value of the function remains constant and does not depend on x, the function always stays equal to c.
Therefore:

$$\lim_{x \to a} c = c$$

Hence, the limit of a constant function is the constant itself.
Final Answer: $\boxed{c}$

Answer: (B)

Go Back to Question 48

Q49.

Solution

Concept: The derivative of a function measures its rate of change with respect to the independent variable. For a constant function:

$$f(x) = c$$

the value of the function does not change as x changes. Since there is no change in the output, the rate of change is zero.

Solution: Step 1: Consider the constant function.

$$f(x) = c$$

Step 2: Differentiate with respect to x.

$$\frac{d}{dx}(c)$$

Step 3: Recall the differentiation rule.
The derivative of any constant is:

$$0$$

Therefore:

$$\frac{d}{dx}(c) = 0$$

Hence, the derivative of a constant function is zero.
Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 49

Q50.

Solution

Concept: Integration is the reverse process of differentiation. If:

$$\frac{d}{dx}(F(x)) = f(x)$$

then:

$$\int f(x)\,dx = F(x) + C$$

We know that the derivative of any constant is zero:

$$\frac{d}{dx}(C) = 0$$

Therefore, the antiderivative of zero is a constant.

Solution: Step 1: Consider the integral:

$$\int 0\,dx$$

Step 2: Recall that the derivative of a constant is zero.
Thus, any constant function can serve as an antiderivative of zero.
Step 3: Write the integral.
Therefore:

$$\int 0\,dx = C$$

where C is called the constant of integration.
Hence, the integral of zero is a constant.
Final Answer: $\boxed{C}$

Answer: (D)

Go Back to Question 50

Q51.

Solution

Concept: Probability measures the likelihood of occurrence of an event. The value of probability is always between 0 and 1 inclusive:

$$0 \leq P(E) \leq 1$$

Important cases:

- $P(E) = 0$ represents an impossible event.
- $P(E) = 1$ represents a certain event.

All other probabilities lie between these two values.

Solution: Step 1: Recall the basic probability rule. For any event E:

$$0 \leq P(E) \leq 1$$

Step 2: Interpret the interval.
This means probability cannot be:

- less than 0,
- greater than 1.

Therefore, the range of probability values is:

$$0 \text{ to } 1$$

Final Answer: $\boxed{0 \text{ to } 1}$

Answer: (B)

Go Back to Question 51

Q52.

Solution

Concept: In set theory, the universal set is the set that contains all objects or elements being discussed in a particular context. It is usually denoted by:

$$U$$

Every other set under consideration is a subset of the universal set.

Solution: Step 1: Recall the definition of the universal set.
The universal set contains:

- all elements relevant to the discussion,
- every object under consideration.

Step 2: Understand its role.
If sets A, B, and C are being discussed, then all elements of these sets must belong to the universal set.
Therefore, the universal set represents:

$$\text{all elements under consideration}$$

Final Answer: $\boxed{\text{all elements under consideration}}$

Answer: (B)

Go Back to Question 52

Q53.

Solution

Concept: A function is said to be one-to-one (injective) if different elements in the domain have different images in the codomain. Mathematically:

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

This means:

- no two distinct inputs give the same output,
- every output corresponds to at most one input.

Solution: Step 1: Recall the definition of a one-to-one function.
A function is one-to-one if distinct inputs produce distinct outputs.
Step 2: Express mathematically. If:

$$x_1 \neq x_2$$

then:

$$f(x_1) \neq f(x_2)$$

Step 3: Therefore, such a function is called:

$$\text{one to one}$$

Final Answer: $\boxed{\text{one to one}}$

Answer: (C)

Go Back to Question 53

Q54.

Solution

Concept: The sine function can be understood using the unit circle. For any angle θ:

- $\cos\theta$ represents the x-coordinate,
- $\sin\theta$ represents the y-coordinate.

At an angle of:

$$0^\circ$$

the point on the unit circle is:

$$(1, 0)$$

Hence, the y-coordinate is zero.

Solution: Step 1: Consider the angle:

$$0^\circ$$

Step 2: Locate the corresponding point on the unit circle.
The point is:

$$(1, 0)$$

Step 3: Recall that sine represents the y-coordinate.
Therefore:

$$\sin 0^\circ = 0$$

Hence, the value of $\sin 0^\circ$ is:

$$0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept: The inverse sine function is written as:

$$\sin^{-1} x$$

or:

$$\arcsin x$$

For an inverse trigonometric function to exist, the original trigonometric function must be one-to-one on the chosen interval. The sine function becomes one-to-one in the interval:

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

This interval is called the principal value range of $\sin^{-1} x$.

Solution: Step 1: Recall the inverse sine function.
The inverse sine function returns the angle whose sine value is x.
Step 2: Restrict the domain of sine.
To make sine invertible, we restrict it to:

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Step 3: Therefore, the principal value range of $\sin^{-1} x$ is:

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Final Answer: $\boxed{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$

Answer: (B)

Go Back to Question 55

Q56.

Solution

Concept: The slope of a line measures its steepness. For two points:

$$(x_1, y_1) \quad \text{and} \quad (x_2, y_2)$$

the slope is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

A horizontal line has constant y-coordinate. Therefore, there is no vertical change along the line.

Solution: Step 1: Consider two points on a horizontal line.

$$(x_1, y_1) \quad \text{and} \quad (x_2, y_2)$$

Since the line is horizontal:

$$y_1 = y_2$$

Step 2: Use the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 3: Substitute $y_2 - y_1 = 0$.

$$m = \frac{0}{x_2 - x_1}$$

Since the denominator is non-zero:

$$m = 0$$

Therefore, the slope of a horizontal line is:

$$0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 56

Q57.

Solution

Concept: A circle is a conic section consisting of all points in a plane that are at a fixed distance from a fixed point called the center. The standard equation of a circle is:

$$(x-h)^2 + (y-k)^2 = r^2$$

Expanding this equation gives:

$$x^2 + y^2 - 2hx - 2ky + (h^2 + k^2 - r^2) = 0$$

The highest powers of the variables are:

$$x^2 \quad \text{and} \quad y^2$$

Therefore, the equation of a circle is a polynomial equation of degree 2.

Solution: Step 1: Write the standard equation of a circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

Step 2: Expand the equation.

$$x^2 - 2hx + h^2 + y^2 - 2ky + k^2 = r^2$$

Rearranging:

$$x^2 + y^2 - 2hx - 2ky + (h^2 + k^2 - r^2) = 0$$

Step 3: Observe the highest power of the variables.
The highest power present is:

$$2$$

Hence, the general equation of a circle is of degree:

$$2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 57

Q58.

Solution

Concept: Eccentricity is a parameter that describes the shape of a conic section. Different conic sections have different eccentricities:

- Circle: $e = 0$
- Ellipse: $0 < e < 1$
- Parabola: $e = 1$
- Hyperbola: $e > 1$

A parabola is uniquely identified by having eccentricity equal to 1.

Solution: Step 1: Recall the eccentricity values of conic sections.
For a parabola:

$$e = 1$$

Step 2: Therefore, the eccentricity of every parabola is:

$$1$$

Hence, the required value is:

$$1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 58

Q59.

Solution

Concept: Eccentricity measures how much a conic section deviates from a perfect circle. For different conic sections:

- Circle: $e = 0$
- Ellipse: $0 < e < 1$
- Parabola: $e = 1$
- Hyperbola: $e > 1$

An ellipse has eccentricity greater than 0 but less than 1. If the eccentricity becomes 0, the ellipse becomes a circle.

Solution: Step 1: Recall the eccentricity condition for an ellipse.
For an ellipse:

$$0 \le e < 1$$

Step 2: Exclude the circular case.
When:

$$e = 0$$

the ellipse becomes a circle.
Thus, for a proper ellipse:

$$0 < e < 1$$

Therefore, the eccentricity of an ellipse lies:

$$\text{between 0 and 1}$$

Final Answer: $\boxed{\text{between 0 and 1}}$

Answer: (B)

Go Back to Question 59

Q60.

Solution

Concept: The eccentricity of a conic section determines its type:

- Ellipse: $0 < e < 1$
- Parabola: $e = 1$
- Hyperbola: $e > 1$

A hyperbola is more open and stretched compared to a parabola or ellipse, which is why its eccentricity exceeds 1.

Solution: Step 1: Recall the eccentricity rule for hyperbolas.
For every hyperbola:

$$e > 1$$

Step 2: Therefore, the eccentricity must be:

$$\text{greater than 1}$$

Hence, the correct description is:

$$e > 1$$

Final Answer: $\boxed{\text{greater than 1}}$

Answer: (C)

Go Back to Question 60

Q61.

Solution

Concept: The zero vector is a vector whose magnitude is zero. It is denoted by:

$$\vec{0}$$

Since it has zero magnitude, it does not possess any definite direction.

Solution: Step 1: Recall the definition of the zero vector.
The zero vector has:

$$\text{magnitude} = 0$$

Step 2: Therefore:

$$|\vec{0}| = 0$$

Hence, the magnitude of the zero vector is:

$$0$$

Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 61

Q62.

Solution

Concept: In three-dimensional Cartesian geometry, the origin is the point where the coordinate axes intersect. The three coordinate axes are:

- x-axis
- y-axis
- z-axis

At the origin, all coordinates are zero.

Solution: Step 1: Recall the coordinate representation of the origin.
The origin in 3D geometry is represented as:

$$(0, 0, 0)$$

Step 2: Interpret the coordinates.
This means:

- x-coordinate = 0
- y-coordinate = 0
- z-coordinate = 0

Hence, the coordinates of the origin are:

$$(0, 0, 0)$$

Final Answer: $\boxed{(0, 0, 0)}$

Answer: (B)

Go Back to Question 62

Q63.

Solution

Concept: The number of ways to arrange k objects chosen from a set of n distinct objects is given by the permutation formula $P(n,k)$ or ${}^nP_k$, which is calculated as:

$$ {}^nP_k = \frac{n!}{(n-k)!} $$

In this case, we need to arrange 2 objects chosen from 2 objects, so $n = 2$ and $k = 2$.

Solution: Step 1: Identify n and k. We have $n = 2$ (total number of objects) and $k = 2$ (number of objects to arrange).
Step 2: Apply the permutation formula.

$$ {}^2P_2 = \frac{2!}{(2-2)!} $$

$$ {}^2P_2 = \frac{2!}{0!} $$

Step 3: Recall that $0! = 1$.

$$ {}^2P_2 = \frac{2!}{1} $$

$$ {}^2P_2 = 2 \times 1 $$

$$ {}^2P_2 = 2 $$

There are 2 ways to arrange 2 objects chosen from 2 objects. For example, if the objects are A and B, the arrangements are AB and BA.
Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 63

Q64.

Solution

Concept: The combination formula ${}^nC_k$ represents the number of ways to choose k items from a set of n distinct items. The formula is:

$$^nC_k = \frac{n!}{k!(n-k)!}$$

We need to calculate ${}^4C_1$.

Solution: Step 1: Use the combination formula with $n = 4$ and $k = 1$.

$$^4C_1 = \frac{4!}{1!(4-1)!}$$

Step 2: Simplify the expression.

$$^4C_1 = \frac{4!}{1!3!}$$

$$^4C_1 = \frac{4 \times 3 \times 2 \times 1}{1 \times (3 \times 2 \times 1)}$$

$$^4C_1 = \frac{4 \times 3!}{1 \times 3!}$$

$$^4C_1 = 4$$

Alternatively, ${}^nC_1$ always equals n, as there are n ways to choose 1 item from a set of n items.

Final Answer: $\boxed{4}$

Answer: (D)

Go Back to Question 64

Q65.

Solution

Concept: An arithmetic progression (AP) is a sequence in which the difference between consecutive terms remains constant. This constant difference is called the common difference. The sequence starts with a particular value known as the first term or initial term.

Solution: Step 1: Recall the structure of an AP.
An AP is written as:

$$a,\ a+d,\ a+2d,\ a+3d,\ldots$$

where:

- a is the first term,
- d is the common difference.

Step 2: Identify the required term.
The beginning term of the progression is known as the:

$$\text{initial term}$$

Hence, the required answer is:

$$\text{initial term}$$

Final Answer: $\boxed{\text{initial term}}$

Answer: (B)

Go Back to Question 65

Q66.

Solution

Concept: In an arithmetic progression (AP), the difference between consecutive terms is constant.

Solution: Step 1: Check (A):

$$2, 5, 8, 11, \ldots$$

Differences:

$$5 - 2 = 3, \quad 8 - 5 = 3, \quad 11 - 8 = 3$$

Since the common difference is constant, this is an AP.

Step 2: Check (B):

$$1, 4, 9, 16, \ldots$$

Differences:

$$4 - 1 = 3, \quad 9 - 4 = 5, \quad 16 - 9 = 7$$

The differences are not constant. Hence, this is not an AP.

Step 3: Check (C):

$$7, 7, 7, 7, \ldots$$

Differences:

$$7 - 7 = 0$$

The common difference is constant. Hence, this is an AP.

Step 4: Check (D):

$$3, 6, 12, 24, \ldots$$

Differences:

$$6 - 3 = 3, \quad 12 - 6 = 6, \quad 24 - 12 = 12$$

The differences are not constant. Hence, this is not an AP.

Therefore, sequences (A) and (C) are arithmetic progressions.

Final Answer:

$$\boxed{\begin{array}{c} 2, 5, 8, 11, \ldots \\ 7, 7, 7, 7, \ldots \end{array}}$$

Answer: (A,C)

Go Back to Question 66

Q67.

Solution

Concept: In a geometric progression (GP), the ratio between consecutive terms is constant.

Solution: Step 1: Check (A):

$$2, 4, 8, 16, \ldots$$

Ratios:

$$\frac{4}{2} = 2, \quad \frac{8}{4} = 2, \quad \frac{16}{8} = 2$$

The common ratio is constant. Hence, this is a GP.

Step 2: Check (B):

$$1, 3, 9, 27, \ldots$$

Ratios:

$$\frac{3}{1} = 3, \quad \frac{9}{3} = 3, \quad \frac{27}{9} = 3$$

The common ratio is constant. Hence, this is a GP.

Step 3: Check (C):

$$5, 10, 15, 20, \ldots$$

Ratios:

$$\frac{10}{5} = 2, \quad \frac{15}{10} = 1.5$$

The ratios are not constant. Hence, this is not a GP.

Step 4: Check (D):

$$8, 4, 2, 1, \ldots$$

Ratios:

$$\frac{4}{8} = \frac{1}{2}, \quad \frac{2}{4} = \frac{1}{2}, \quad \frac{1}{2} = \frac{1}{2}$$

The common ratio is constant. Hence, this is a GP.

Therefore, sequences (A), (B), and (D) are geometric progressions.

Final Answer:

$$\boxed{\begin{array}{c} 2, 4, 8, 16, \ldots \\ 1, 3, 9, 27, \ldots \\ 8, 4, 2, 1, \ldots \end{array}}$$

Answer: (A,B,D)

Go Back to Question 67

Q68.

Solution

Concept: This question concerns the binomial expansion of $(1+x)^n$. The coefficients are given by the binomial coefficients $\binom{n}{k}$, where k ranges from 0 to n. For $(1+x)^4$, $n = 4$.

Solution: Step 1: Recall the binomial theorem for $(a+b)^n$.
The binomial expansion of $(a+b)^n$ is given by:
$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$
where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Step 2: Apply to the expansion of $(1+x)^4$.
Here, $a = 1$, $b = x$, and $n = 4$.
$(1+x)^4 = \sum_{k=0}^{4} \binom{4}{k} 1^{4-k} x^k = \sum_{k=0}^{4} \binom{4}{k} x^k$.

Step 3: Calculate the binomial coefficients for $n = 4$.
The coefficients are $\binom{4}{0}, \binom{4}{1}, \binom{4}{2}, \binom{4}{3}, \binom{4}{4}$.
$\binom{4}{0} = \frac{4!}{0!4!} = 1$ (since $0! = 1$).
$\binom{4}{1} = \frac{4!}{1!3!} = \frac{4\times3!}{1\times3!} = 4$.
$\binom{4}{2} = \frac{4!}{2!2!} = \frac{4\times3\times2!}{2\times1\times2!} = \frac{12}{2} = 6$.
$\binom{4}{3} = \frac{4!}{3!1!} = \frac{4\times3!}{3!\times1} = 4$.
$\binom{4}{4} = \frac{4!}{4!0!} = 1$.

The expansion is $1\cdot x^0 + 4\cdot x^1 + 6\cdot x^2 + 4\cdot x^3 + 1\cdot x^4$.
So, $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$.

Step 4: Identify the coefficients that appear in the expansion.
The coefficients are 1, 4, 6, 4, 1.

Step 5: Check the given options.
- Option (A): 1. Appears in the expansion.
- Option (B): 4. Appears in the expansion.
- Option (C): 6. Appears in the expansion.
- Option (D): 8. Does not appear in the expansion.

Step 6: Identify the coefficients that appear.
Coefficients 1, 4, and 6 appear in the expansion.

Final Answer:

$$\boxed{1, 4, 6}$$

Answer: (A,B,C)

Go Back to Question 68

Q69.

Solution

Concept: This question asks about the fundamental properties of the imaginary unit $i = \sqrt{-1}$, specifically its powers. We need to evaluate i^2, i^3, i^4, i^5.

Solution: Step 1: Recall the definition of i.
$i = \sqrt{-1}$.

Step 2: Evaluate i^2.
$i^2 = (\sqrt{-1})^2 = -1$.
Statement (A) $i^2 = -1$ is true.

Step 3: Evaluate i^3.
$i^3 = i^2 \cdot i = (-1) \cdot i = -i$.
Statement (C) $i^3 = -i$ is true.

Step 4: Evaluate i^4.
$i^4 = i^2 \cdot i^2 = (-1) \cdot (-1) = 1$.
Statement (B) $i^4 = 1$ is true.

Step 5: Evaluate i^5.
$i^5 = i^4 \cdot i = (1) \cdot i = i$.
Statement (D) $i^5 = 1$ is false.

Step 6: Identify the true statements.
Statements (A), (B), and (C) are true.

Final Answer:

$$\boxed{i^2 = -1, \quad i^4 = 1, \quad i^3 = -i}$$

Answer: (A,B,C)

Go Back to Question 69

Q70.

Solution

Concept: This question asks to identify values equal to $\frac{\sqrt{3}}{2}$. We need to recall the values of sine, cosine, and tangent for standard angles.

Solution: Step 1: Recall standard trigonometric values.
- $\sin 60^\circ = \frac{\sqrt{3}}{2}$
- $\cos 30^\circ = \frac{\sqrt{3}}{2}$
- $\tan 60^\circ = \sqrt{3}$
- $\sin 30^\circ = \frac{1}{2}$

Step 2: Analyze option (A): $\sin 60^\circ$.
$\sin 60^\circ = \frac{\sqrt{3}}{2}$. This value is equal to $\frac{\sqrt{3}}{2}$.

Step 3: Analyze option (B): $\cos 30^\circ$.
$\cos 30^\circ = \frac{\sqrt{3}}{2}$. This value is equal to $\frac{\sqrt{3}}{2}$.

Step 4: Analyze option (C): $\tan 60^\circ$.
$\tan 60^\circ = \sqrt{3}$. This value is not equal to $\frac{\sqrt{3}}{2}$.

Step 5: Analyze option (D): $\sin 30^\circ$.
$\sin 30^\circ = \frac{1}{2}$. This value is not equal to $\frac{\sqrt{3}}{2}$.

Step 6: Identify the values equal to $\frac{\sqrt{3}}{2}$.
Values (A) and (B) are equal to $\frac{\sqrt{3}}{2}$.

Final Answer:

$$\boxed{\sin 60^\circ, \quad \cos 30^\circ}$$

Answer: (A,B)

Go Back to Question 70

Q71.

Solution

Concept: Inverse trigonometric functions have fixed principal value ranges:

$$\sin^{-1} x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos^{-1} x \in [0, \pi]$$

$$\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Solution: Step 1: Check (A):

$$\sin^{-1}(0) = 0$$

Since:

$$\sin 0 = 0$$

this statement is correct.

Step 2: Check (B):

$$\cos^{-1}(1) = 0$$

Since:

$$\cos 0 = 1$$

this statement is correct.

Step 3: Check (C):

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Since:

$$\tan \frac{\pi}{4} = 1$$

this statement is correct.

Step 4: Check (D):

$$\sin^{-1}(1) = \pi$$

The correct value is:

$$\sin^{-1}(1) = \frac{\pi}{2}$$

Hence, this statement is incorrect. Therefore, statements (A), (B), and (C) are correct.

Final Answer:

$$\boxed{\begin{gathered}\sin^{-1}(0) = 0 \\ \cos^{-1}(1) = 0 \\ \tan^{-1}(1) = \frac{\pi}{4}\end{gathered}}$$

Answer: (A,B,C)

Go Back to Question 71

Q72.

Solution

Concept: A line passes through the origin if its equation is satisfied when $x = 0$ and $y = 0$. For linear equations of the form $Ax + By + C = 0$, this means C must be zero.

Solution: Step 1: Condition for passing through the origin.
A line passes through the origin $(0, 0)$ if substituting $x = 0$ and $y = 0$ into its equation results in a true statement. For an equation $Ax + By + C = 0$, this implies $C = 0$.

Step 2: Analyze equation (A): $y = 2x$.
Substitute $x = 0, y = 0$: $0 = 2(0) \implies 0 = 0$. This is true. The line passes through the origin.

Step 3: Analyze equation (B): $x - y = 0$.
Substitute $x = 0, y = 0$: $0 - 0 = 0 \implies 0 = 0$. This is true. The line passes through the origin.

Step 4: Analyze equation (C): $x + y = 1$.
Substitute $x = 0, y = 0$: $0 + 0 = 1 \implies 0 = 1$. This is false. The line does not pass through the origin.

Step 5: Analyze equation (D): $3x - 2y = 0$.
Substitute $x = 0, y = 0$: $3(0) - 2(0) = 0 \implies 0 = 0$. This is true. The line passes through the origin.

Step 6: Identify the equations representing lines passing through the origin.
Equations (A), (B), and (D) represent lines passing through the origin.

Final Answer:

$$\boxed{y = 2x, \quad x - y = 0, \quad 3x - 2y = 0}$$

Answer: (A,B,D)

Go Back to Question 72

Q73.

Solution

Concept:
The standard equation of a circle centered at the origin is:

$$x^2 + y^2 = r^2$$

Solution:
Step 1: Check (A):

$$x^2 + y^2 = 16$$

This is in the form:

$$x^2 + y^2 = r^2$$

Hence, it represents a circle centered at the origin.
Step 2: Check (B):

$$x^2 + y^2 - 9 = 0$$

Rearranging:

$$x^2 + y^2 = 9$$

This is also a circle centered at the origin.
Step 3: Check (C):

$$(x - 1)^2 + y^2 = 4$$

The center is:

$$(1, 0)$$

So, it is not centered at the origin.
Step 4: Check (D):

$$x^2 + y^2 + 4x = 0$$

Completing the square:

$$(x + 2)^2 + y^2 = 4$$

The center is:

$$(-2, 0)$$

So, it is not centered at the origin.
Therefore, equations (A) and (B) represent circles centered at the origin.
Final Answer:

$$\boxed{\begin{array}{c} x^2 + y^2 = 16 \\ x^2 + y^2 - 9 = 0 \end{array}}$$

Answer: (A,B)

Go Back to Question 73

Q74.

Solution

Concept:
For a fair die:

$$P(E) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

Solution: Sample space:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Step 1: Check (A): Multiple of 3. Favorable outcomes:

$$\{3, 6\}$$

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

Hence, (A) is correct.
Step 2: Check (B): Prime number. Prime numbers:

$$\{2, 3, 5\}$$

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

Hence, (B) is incorrect.
Step 3: Check (C): Number less than 3. Favorable outcomes:

$$\{1, 2\}$$

$$P(C) = \frac{2}{6} = \frac{1}{3}$$

Hence, (C) is correct.
Step 4: Check (D): Even number greater than 4. Favorable outcome:

$$\{6\}$$

$$P(D) = \frac{1}{6}$$

Hence, (D) is incorrect. Therefore, events (A) and (C) have probability $\frac{1}{3}$.
Final Answer:

$$\boxed{\begin{array}{c}\text{Getting a multiple of 3}\\ \text{Getting a number less than 3}\end{array}}$$

Answer: (A,C)

Go Back to Question 74

Q75.

Solution

Concept: Two vectors are perpendicular if their dot product is zero.

Solution: Step 1: Let:

$$\vec{v} = \hat{i} + \hat{j}$$

Then:

$$\vec{v} = (1, 1, 0)$$

Step 2: Check (A):

$$\hat{i} - \hat{j} = (1, -1, 0)$$

Dot product:

$$(1)(1) + (1)(-1) = 0$$

Hence, this vector is perpendicular to $\hat{i} + \hat{j}$.
Step 3: Check (B):

$$2\hat{i} - 2\hat{j} = (2, -2, 0)$$

Dot product:

$$(1)(2) + (1)(-2) = 0$$

Hence, this vector is perpendicular to $\hat{i} + \hat{j}$.
Step 4: Check (C):

$$\hat{i} + \hat{j} = (1, 1, 0)$$

Dot product:

$$(1)(1) + (1)(1) = 2$$

Since the dot product is not zero, this vector is not perpendicular.
Step 5: Check (D):

$$3\hat{j} - 3\hat{i} = (-3, 3, 0)$$

Dot product:

$$(1)(-3) + (1)(3) = 0$$

Hence, this vector is perpendicular to $\hat{i} + \hat{j}$. Therefore, vectors (A), (B), and (D) are perpendicular to $\hat{i} + \hat{j}$.
Final Answer:

$$\boxed{\begin{array}{c} \hat{i} - \hat{j} \\ 2\hat{i} - 2\hat{j} \\ 3\hat{j} - 3\hat{i} \end{array}}$$

Answer: (A,B,D)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	A	4	B	5	A
6	A	7	C	8	C	9	C	10	C
11	A	12	C	13	A	14	C	15	C
16	B	17	A	18	A	19	B	20	B
21	A	22	B	23	B	24	C	25	B
26	B	27	B	28	A	29	A	30	D
31	A	32	A	33	C	34	A	35	B
36	A	37	B	38	B	39	A	40	B
41	B	42	B	43	B	44	A	45	B
46	C	47	B	48	B	49	B	50	D
51	B	52	B	53	C	54	A	55	B
56	A	57	B	58	B	59	B	60	C
61	B	62	B	63	B	64	D	65	B
66	A,C	67	A,B,D	68	A,B,C	69	A,B,C	70	A,B
71	A,B,C	72	A,B,D	73	A,B	74	A,C	75	A,B,D