WBJEE Mathematics Sample Paper- 18

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **75** Multiple Choice Questions.
- **Section A (Q1–Q50):** Each correct answer carries +1 mark. Incorrect answer: **0.25 marks**. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries +2 mark. Incorrect answer: **0.5 marks**. Only **one** correct option.
- **Section C (Q66–Q75)**: Each correct answer carries **+2 marks**. **No negative marking.** One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: 0.25) [Single Correct]

Q1. If α, β are the roots of $x^2 - px + r = 0$ and $\alpha/2, 2\beta$ are the roots of $x^2 - qx + r = 0$, then the value of r in terms of p and q is:

(A) $\frac{2}{9}(p-q)(2q-p)$

(B) $\frac{2}{9}(q-p)(2p-q)$

(C) $\frac{2}{9}(2p-q)(2q-p)$

(D) $\frac{2}{3}(p-q)(q-p)$

Q2. The sum of the series $\sum_{r=1}^{n} \frac{r^4+r^2+1}{r^4+r}$ up to n terms is:

(A) $\frac{n^2+n+1}{n+1}$

(B) $\frac{n^2+n}{n+1}$

(C) $\frac{n^2+1}{n}$

(D) None of these

Q3. The number of ways in which 10 identical apples can be distributed among 3 children such that each child gets at least one apple and no two children get the same number of apples is:

(A) 12

(B) 18

(C) 24

(D) 30

Q4. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, then the number of integers in the range $[1, 1000]$ that are divisible by neither 2 nor 3 is:

(A) 333

(B) 334

(C) 166

(D) 500

Q5. In the expansion of $(1 + x + x^2 + x^3)^{10}$, the coefficient of x^4 is:

(A) 805

(B) 610

(C) 285

(D) 310

Q6. If a, b, c are in H.P., then the value of $\frac{a-b}{b-c}$ is:

(A) a/c

(B) c/a

(C) 1

(D) a/b

Q7. The sum of all divisors of $2^5 \cdot 3^4 \cdot 5^2$ is:

(A) $63 \cdot 121 \cdot 31$

(B) $63 \cdot 40 \cdot 31$

(C) $31 \cdot 121 \cdot 31$

(D) $64 \cdot 121 \cdot 31$

Q8. The value of $\sum_{k=1}^{\infty} \frac{1}{(k+1)\sqrt{k}+k\sqrt{k+1}}$ is:

(A) 1

(B) $1/2$

(C) 2

(D) ∞

Q9. The constant term in the expansion of $(x^2 - \frac{1}{x})^{12}$ is:

(A) 495

(B) -495

(C) 792

(D) -792

Q10. If x is real, the maximum value of $\frac{3x^2+9x+17}{3x^2+9x+7}$ is:

(A) 41

(B) 1

(C) $17/7$

(D) 15

Q11. If z is a complex number such that $|z| \geq 2$, then the minimum value of $|z + \frac{1}{2}|$ is:

(A) 1.5

(B) 2.5

(C) 0.5

(D) 2

Q12. If ω is an imaginary cube root of unity, then $(1+\omega-\omega^2)^7$ equals:

(A) 128ω

(B) -128ω

(C) $128\omega^2$

(D) $-128\omega^2$

Q13. The number of real roots of $e^x = x^2$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Q14. If $z^2 + z + 1 = 0$, then the value of $(z+1/z)^2 + (z^2+1/z^2)^2 + \cdots + (z^6+1/z^6)^2$ is:

(A) 12

(B) 6

(C) 18

(D) 0

Q15. The condition that $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root is:

(A) $p + q + 1 = 0$

(B) $p + q - 1 = 0$

(C) $p - q + 1 = 0$

(D) $p = q$

Q16. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T equals:

(A) I

(B) B^{-1}

(C) $(B^T)^{-1}$

(D) $B + I$

Q17. The value of $\det(Adj(Adj A))$ for a 3×3 matrix A with $\det(A) = 2$ is:

(A) 8

(B) 16

(C) 64

(D) 256

Q18. The system of equations $x + y + z = 2, 2x + y - z = 3, 3x + 2y + kz = 4$ has a unique solution if:

(A) $k \neq 0$

(B) $k = 0$

(C) $k \neq -1$

(D) $k = 1$

Q19. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $A^2 - 5A - 2I$ is:

(A) 0

(B) I

(C) A

(D) $2I$

Q20. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = k(a-b)(b-c)(c-a)$, then k is:

(A) 1

(B) -1

(C) 2

(D) 1/2

Q21. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{x}{1+|x|}$. Then $f(x)$ is:

(A) One-to-one but not onto

(B) Onto but not one-to-one

(C) Both one-to-one and onto

(D) Neither one-to-one nor onto

Q22. The domain of the function $f(x) = \sqrt{\log_{0.5}(x-1)}$ is:

(A) $(1, 2]$

(B) $(1, 2)$

(C) $[2, \infty)$

(D) $(1, \infty)$

Q23. If R is a relation on the set $\mathbb{Z}$ defined by aRb if $a^2 + b^2$ is even, then R is:

(A) Reflexive only

(B) Symmetric only

(C) Transitive only

(D) An equivalence relation

Q24. Three dice are thrown. The probability that the sum of the numbers appearing is 15 is:

(A) 10/216

(B) 12/216

(C) 5/108

(D) 1/18

Q25. If A and B are two independent events such that $P(A) = 0.3$ and $P(B) = 0.6$, then $P(A' \cap B')$ is:

(A) 0.28

(B) 0.18

(C) 0.42

(D) 0.12

Q26. A box contains 5 black and 4 white balls. A ball is drawn at random and its color is noted. It is then returned to the box along with 2 additional balls of the same color. The probability that the second ball drawn is white is:

(A) 4/9

(B) 5/9

(C) 4/11

(D) 5/11

Q27. The variance of first n natural numbers is:

(A) $\frac{n^2-1}{12}$

(B) $\frac{n^2+1}{12}$

(C) $\frac{n^2-1}{6}$

(D) $\frac{n(n+1)}{2}$

Q28. In a Poisson distribution, if $P(X = 1) = P(X = 2)$, then the mean is:

(A) 1

(B) 2

(C) 0.5

(D) 4

Q29. The value of $\tan 7.5^\circ$ is:

(A) $\sqrt{6} - \sqrt{3} + \sqrt{2} - 2$

(B) $\sqrt{6} + \sqrt{3} - \sqrt{2} - 2$

(C) $\sqrt{6} - \sqrt{3} - \sqrt{2} + 2$

(D) $\sqrt{6} + \sqrt{3} + \sqrt{2} + 2$

Q30. The general solution of $\sin^2\theta - 2\cos\theta + 1/4 = 0$ is:

(A) $2n\pi \pm \pi/3$

(B) $n\pi \pm \pi/3$

(C) $2n\pi \pm \pi/6$

(D) $n\pi \pm \pi/6$

Q31. If $\sin^{-1} x + \sin^{-1} y = 2\pi/3$, then $\cos^{-1} x + \cos^{-1} y$ is:

(A) $\pi/3$

(B) $\pi/6$

(C) $\pi/2$

(D) π

Q32. In a triangle ABC, if $a = 2, b = 3, \sin A = 2/3$, then $\angle B$ is:

(A) 30°

(B) 60°

(C) 90°

(D) 120°

Q33. The distance between the parallel lines $3x + 4y - 5 = 0$ and $6x + 8y + 2 = 0$ is:

(A) 1.2

(B) 0.8

(C) 0.6

(D) 1.0

Q34. The equation of the circle passing through $(1, 2)$ and $(3, 4)$ and having its center on $x - y + 1 = 0$ is:

(A) $x^2 + y^2 - 4x - 6y + 13 = 0$

(B) $x^2 + y^2 - 4x - 6y + 8 = 0$

(C) $x^2 + y^2 - 3x - 5y + 10 = 0$

(D) None

Q35. The length of the latus rectum of the parabola $y^2 - 4y - 10x + 14 = 0$ is:

(A) 10

(B) 5

(C) 2.5

(D) 4

Q36. If the eccentricity of an ellipse is $1/2$ and the distance between foci is 4, then the length of the minor axis is:

(A) $4\sqrt{3}$

(B) $2\sqrt{3}$

(C) $8\sqrt{3}$

(D) 6

Q37. The equation of the hyperbola with foci $(\pm 5, 0)$ and eccentricity $5/4$ is:

(A) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(B) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(C) $\frac{x^2}{25} - \frac{y^2}{16} = 1$

(D) $\frac{x^2}{16} - \frac{y^2}{25} = 1$

Q38. The angle between the pair of lines $x^2 + 4xy + y^2 = 0$ is:

(A) 60°

(B) 30°

(C) 45°

(D) 90°

Q39. The area of the triangle formed by the lines $y = x$, $y = 2x$ and $y = 3x + 4$ is:

(A) 4

(B) 8

(C) 2

(D) 6

Q40. If the line $y = mx + 1$ is tangent to $y^2 = 4x$, then m is:

(A) 1

(B) 2

(C) 1/2

(D) −1

Q41. The locus of the midpoint of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the center is:

(A) $x^2 + y^2 = 2$

(B) $x^2 + y^2 = 1$

(C) $x^2 + y^2 = 1/2$

(D) $x^2 + y^2 = 3$

Q42. The equation of the normal to the curve $x^2 = 4y$ at $(4, 4)$ is:

(A) $x + 2y - 12 = 0$

(B) $2x + y - 12 = 0$

(C) $x - 2y + 4 = 0$

(D) $2x - y - 4 = 0$

Q43. The shortest distance between the lines $\vec{r} = (i + j) + \lambda(2i - j + k)$ and $\vec{r} = (2i + j - k) + \mu(3i - 5j + 2k)$ is:

(A) $10/\sqrt{59}$

(B) $5/\sqrt{59}$

(C) 0

(D) $3/\sqrt{59}$

Q44. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 7$ is:

(A) $60°$

(B) $30°$

(C) $45°$

(D) $90°$

Q45. The direction cosines of a line equally inclined to the axes are:

(A) $(1, 1, 1)$

(B) $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

(C) $(1/3, 1/3, 1/3)$

(D) $(0, 0, 1)$

Q46. The point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and the plane $2x+4y-z = 1$ is:

(A) $(1, 2, 3)$

(B) $(0, 0, 0)$

(C) $(-1, -1, -1)$

(D) $(1, -1, 1)$

Q47. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $0°$

(B) $90°$

(C) $45°$

(D) $180°$

Q48. The value of $[\vec{a} - \vec{b} \quad \vec{b} - \vec{c} \quad \vec{c} - \vec{a}]$ is:

(A) 0

(B) $2[\vec{a}\vec{b}\vec{c}]$

(C) $[\vec{a}\vec{b}\vec{c}]$

(D) None

Q49. If $\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$, then the angle between them is:

(A) $\pi/4$

(B) $\pi/2$

(C) 0

(D) π

Q50. The projection of $\vec{a} = 2i + 3j + 2k$ on $\vec{b} = i + 2j + k$ is:

(A) $10/\sqrt{6}$

(B) $5/\sqrt{6}$

(C) $2\sqrt{6}$

(D) 10

Section–B — 15 Questions × 2 Marks Each
(Negative Marking: 0.5) [Single Correct]

Q51. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is:

(A) $3/2$

(B) $-3/2$

(C) 0

(D) 1

Q52. The value of $\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$ is:

(A) $3/2$

(B) 1/2

(C) 1

(D) 0

Q53. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then dy/dx at $x = 0$ is:

(A) 1/2

(B) 1

(C) 0

(D) 2

Q54. The integral $\int \frac{dx}{x(x^n+1)}$ is equal to:

(A) $\frac{1}{n}\ln|\frac{x^n}{x^n+1}| + C$

(B) $\ln|\frac{x^n}{x^n+1}| + C$

(C) $\frac{1}{n}\ln|\frac{x^n+1}{x^n}| + C$

(D) None

Q55. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 0

Q56. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $xy = \frac{x^4}{4} + C$

(B) $y = \frac{x^3}{4} + C$

(C) $xy = \frac{x^3}{3} + C$

(D) $y = x^2 + C$

Q57. If $f(x) = |x-1| + |x-2|$, then $f'(1.5)$ is:

(A) 0

(B) 1

(C) 2

(D) Does not exist

Q58. The value of $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2+r^2}$ is:

(A) $\pi/4$

(B) $\pi/2$

(C) 1

(D) 0

Q59. The derivative of $\sin^{-1}(2x\sqrt{1-x^2})$ with respect to $\sin^{-1} x$ is:

(A) 2

(B) 1

(C) $1/2$

(D) 0

Q60. $\int e^x(\sin x + \cos x)dx$ equals:

(A) $e^x \sin x + C$

(B) $e^x \cos x + C$

(C) $-e^x \sin x + C$

(D) $e^x(\sin x - \cos x) + C$

Q61. The order and degree of $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin(\frac{dy}{dx}) + 1 = 0$ are:

(A) 2, 3

(B) 2, Not defined

(C) 3, 2

(D) 1, 2

Q62. If $y = e^{a\sin^{-1}x}$, then $(1-x^2)y_2 - xy_1$ is:

(A) a^2y

(B) $-a^2y$

(C) ay

(D) 0

Q63. The value of $\int_{-1}^{1}|x|dx$ is:

(A) 1

(B) 0

(C) 2

(D) 1/2

Q64. $\lim_{x\to\infty}(\frac{x+6}{x+1})^{x+4}$ is:

(A) e^5

(B) e^4

(C) e^6

(D) e

Q65. The function $f(x) = x^x$ has a stationary point at $x =$:

(A) e

(B) $1/e$

(C) 1

(D) 0

Section–C — 10 Questions × 2 Marks Each
(No Negative Marking) [One or More Correct]

Q66. Let $f(x) = \frac{x}{1+x^n}$ for $x \geq 0$ and $n > 1$. Then which of the following statements is/are true?

(A) $f(x)$ has a maximum at $x = (n-1)^{-1/n}$

(B) $f(x)$ is increasing in $(0, (n-1)^{-1/n})$

(C) $f(x)$ is decreasing in $((n-1)^{-1/n}, \infty)$

(D) $f(x)$ is always increasing for all $x > 0$

Q67. If z is a complex number such that $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ can be:

(A) z

(B) $\cos(\arg z) + i\sin(\arg z)$

(C) $\bar{z}$

(D) 1

Q68. The equations of the tangents to the circle $x^2 + y^2 = 25$ which are parallel to the line $3x + 4y + 7 = 0$ are:

(A) $3x + 4y + 25 = 0$

(B) $3x + 4y - 25 = 0$

(C) $4x - 3y + 25 = 0$

(D) $3x + 4y + 5 = 0$

Q69. Consider the function $f(x) = |x| + |x-1|$. Then at $x = 0$ and $x = 1$:

(A) $f(x)$ is continuous at $x = 0$

(B) $f(x)$ is differentiable at $x = 0$

(C) $f(x)$ is continuous at $x = 1$

(D) $f(x)$ is differentiable at $x = 1$

Q70. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$, then Δ is equal to:

(A) $(a-b)(b-c)(c-a)$

(B) $(b-a)(c-b)(a-c)$

(C) $(a-b)(c-b)(a-c)$

(D) $-(b-a)(c-b)(a-c)$

Q71. For the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, which of the following is/are true?

(A) The eccentricity $e=\frac{3}{5}$

(B) The foci are at $(\pm 3, 0)$

(C) The length of the latus rectum is $\frac{32}{5}$

(D) The directrices are $x=\pm\frac{25}{3}$

Q72. If $I_n=\int_0^{\pi/4}\tan^n x\,dx$, then:

(A) $I_n+I_{n-2}=\frac{1}{n-1}$

(B) $I_2+I_4=\frac{1}{3}$

(C) $I_n+I_{n-2}=\frac{1}{n+1}$

(D) $I_3+I_5=\frac{1}{4}$

Q73. The vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are coplanar if:

(A) $[\vec{a}\ \vec{b}\ \vec{c}]=0$

(B) $\vec{a}\cdot(\vec{b}\times\vec{c})=0$

(C) $\vec{a}$, $\vec{b}$, and $\vec{c}$ are linearly dependent

(D) $\vec{a}\times(\vec{b}\times\vec{c})=0$

Q74. Let S be the set of all real values of x such that $2\sin^2 x+\sin x-1=0$ for $x\in[0,2\pi]$. Then S contains:

(A) $\frac{\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{3\pi}{2}$

(D) $\frac{\pi}{2}$

Q75. If A and B are two independent events, then:

(A) $P(A \cap B) = P(A)P(B)$

(B) A^c and B^c are independent

(C) $P(A|B) = P(A)$

(D) $P(A \cup B) = P(A) + P(B)$

Detailed Solutions

Q1.

Solution

Concept: Use relations between roots and coefficients.

Solution: For

$$x^2 - px + r = 0$$

with roots α, β:

$$\alpha + \beta = p, \qquad \alpha\beta = r$$

Second equation has roots:

$$\frac{\alpha}{2},\ 2\beta$$

So,

$$\frac{\alpha}{2} + 2\beta = q$$

Using $\alpha + \beta = p$, we get:

$$\beta = \frac{2q - p}{3}$$

and

$$\alpha = \frac{2(2p - q)}{3}$$

Therefore,

$$r = \alpha\beta$$

$$= \frac{2}{9}(2p - q)(2q - p)$$

Final Answer: $\boxed{\frac{2}{9}(2p - q)(2q - p)}$

Answer: (C)

Go Back to Question 1

Q2.

Solution

Concept: Convert the series into telescoping form.

Solution:

$$\frac{r^4 + r^2 + 1}{r^4 + r} = 1 + \frac{1}{r(r+1)}$$

Thus,

$$S = \sum_{r=1}^{n} \left(1 + \frac{1}{r(r+1)}\right)$$

$$= n + \sum_{r=1}^{n} \left(\frac{1}{r} - \frac{1}{r+1}\right)$$

Telescoping:

$$S = n + 1 - \frac{1}{n+1}$$

$$S = \frac{n^2 + 2n}{n+1}$$

This is not among the options.

Final Answer: None of these

Answer: (D)

Go Back to Question 2

Q3.

Solution

Concept: Distribution of identical objects among distinct persons can be solved using integer partitions. Since all children must receive distinct positive numbers of apples, we count all positive integer triples with different values whose sum equals the total number of apples.

Solution: We need:

$$a + b + c = 10$$

where

$$a, b, c \geq 1$$

and all are distinct.

Distinct positive partitions of 10 into 3 parts are:

$$1, 2, 7$$

$$1, 3, 6$$

$$1, 4, 5$$

$$2, 3, 5$$

Total distinct sets:

$$4$$

Each can be arranged among 3 children in:

$$3! = 6$$

ways.

Hence total distributions:

$$4 \times 6 = 24$$

Final Answer: $\boxed{24}$

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: To count integers not divisible by given numbers, the Principle of Inclusion and Exclusion is used. Count numbers divisible by each divisor separately, subtract common multiples once, and remove the total from the complete set of integers.

Solution: Integers from:

$$1 \text{ to } 1000$$

Numbers divisible by 2:

$$\left\lfloor \frac{1000}{2} \right\rfloor = 500$$

Numbers divisible by 3:

$$\left\lfloor \frac{1000}{3} \right\rfloor = 333$$

Numbers divisible by both:

$$\left\lfloor \frac{1000}{6} \right\rfloor = 166$$

By inclusion-exclusion:

$$500 + 333 - 166 = 667$$

Hence numbers divisible by neither:

$$1000 - 667 = 333$$

Final Answer: $\boxed{333}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: The coefficient of a specific power in multinomial expansions can be obtained by considering all valid exponent combinations whose total degree matches the required power. Counting arrangements carefully using combinations provides the desired coefficient efficiently.

Solution: We need coefficient of:

$$x^4$$

in

$$(1 + x + x^2 + x^3)^{10}$$

Possible ways to obtain power 4:

$$4 = 1 + 1 + 1 + 1$$

Choose 4 factors contributing x:

$$\binom{10}{4} = 210$$

$$4 = 2 + 1 + 1$$

Choose one x^2 and two x:

$$\binom{10}{1}\binom{9}{2} = 360$$

$$4 = 2 + 2$$

Choose two x^2:

$$\binom{10}{2} = 45$$

$$4 = 3 + 1$$

Choose one x^3 and one x:

$$10 \times 9 = 90$$

Total coefficient:

$$210 + 360 + 45 + 90$$

$$= 705$$

Final Answer: $\boxed{705}$

Answer: (D)

Go Back to Question 5

Q6.

Solution

Concept: If three numbers are in Harmonic Progression (H.P.), then their reciprocals are in Arithmetic Progression (A.P.). Converting the H.P. condition into an algebraic relation helps simplify expressions involving differences and ratios between the terms directly and efficiently.

Solution: Since a, b, c are in H.P., therefore:

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$$

are in A.P.

Hence,

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Multiplying by abc:

$$2ac = ab + bc$$

Rearranging:

$$ab - 2ac + bc = 0$$

$$a(b - 2c) + bc = 0$$

Observe:

$$ab - bc = 2ac - 2bc$$

$$b(a - c) = 2c(a - b)$$

Thus,

$$\frac{a - b}{b - c} = \frac{a}{c}$$

Final Answer: $\boxed{\frac{a}{c}}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: For a number expressed in prime factorized form:

$$n = p_1^{a_1} p_2^{a_2} \cdots$$

the sum of all positive divisors is obtained using the geometric progression formula:

$$\sigma(n) = \prod \frac{p^{a+1} - 1}{p - 1}$$

Each prime factor contributes independently to the divisor sum.

Solution: Given:

$$N = 2^5 \cdot 3^4 \cdot 5^2$$

Sum of divisors:

$$\sigma(N) = \left(\frac{2^6 - 1}{2 - 1}\right)\left(\frac{3^5 - 1}{3 - 1}\right)\left(\frac{5^3 - 1}{5 - 1}\right)$$

Compute separately:

$$\frac{2^6 - 1}{1} = 64 - 1 = 63$$

$$\frac{3^5 - 1}{2} = \frac{243 - 1}{2} = \frac{242}{2} = 121$$

$$\frac{5^3 - 1}{4} = \frac{125 - 1}{4} = \frac{124}{4} = 31$$

Therefore,

$$\sigma(N) = 63 \cdot 121 \cdot 31$$

Final Answer: $\boxed{63 \cdot 121 \cdot 31}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: Infinite series involving radicals often simplify after rationalization or algebraic manipulation. Converting terms into telescoping form allows cancellation between consecutive terms, making the infinite sum converge to a simple finite value.

Solution: Given:

$$S = \sum_{k=1}^{\infty} \frac{1}{(k+1)\sqrt{k} + k\sqrt{k+1}}$$

Factor:

$$(k+1)\sqrt{k} + k\sqrt{k+1} = \sqrt{k}\sqrt{k+1}(\sqrt{k+1} + \sqrt{k})$$

Hence,

$$\frac{1}{(k+1)\sqrt{k} + k\sqrt{k+1}} = \frac{1}{\sqrt{k(k+1)}(\sqrt{k+1} + \sqrt{k})}$$

Multiply numerator and denominator by:

$$\sqrt{k+1} - \sqrt{k}$$

Then,

$$= \frac{\sqrt{k+1} - \sqrt{k}}{\sqrt{k(k+1)}}$$

$$= \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}$$

Thus,

$$S = \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}} \right)$$

This is telescoping:

$$S = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: In binomial expansions, the general term helps identify the constant term. The exponent of x in the general term is equated to zero, and the corresponding binomial coefficient gives the required constant value including its sign.

Solution: Given:

$$\left(x^2 - \frac{1}{x}\right)^{12}$$

General term:

$$T_{r+1} = \binom{12}{r}(x^2)^{12-r}\left(-\frac{1}{x}\right)^r$$

Simplify:

$$T_{r+1} = \binom{12}{r}(-1)^r x^{24-2r-r}$$

$$= \binom{12}{r}(-1)^r x^{24-3r}$$

For constant term:

$$24 - 3r = 0$$

$$r = 8$$

Hence constant term:

$$\binom{12}{8}(-1)^8 = \binom{12}{4}$$

$$= 495$$

Final Answer: $\boxed{495}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: Use substitution and minimum value of a quadratic expression.

Solution: Given:

$$y = \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$$

Let:

$$t = 3x^2 + 9x$$

Then:

$$y = \frac{t + 17}{t + 7} = 1 + \frac{10}{t + 7}$$

Now,

$$t = 3\left(x + \frac{3}{2}\right)^2 - \frac{27}{4}$$

So,

$$t \geq -\frac{27}{4}$$

Hence,

$$t + 7 \geq \frac{1}{4}$$

Maximum value of y occurs when:

$$t + 7 = \frac{1}{4}$$

Thus,

$$y = 1 + \frac{10}{1/4}$$

$$= 41$$

Final Answer: $\boxed{41}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: The modulus of a complex number represents its distance from the origin in the Argand plane. Geometric interpretation and the triangle inequality help determine minimum or maximum distances involving translated complex numbers.

Solution: Given:

$$|z| \geq 2$$

We need minimum value of:

$$\left|z + \frac{1}{2}\right|$$

Using triangle inequality:

$$\left|z + \frac{1}{2}\right| \geq ||z| - \frac{1}{2}|$$

Since

$$|z| \geq 2$$

minimum occurs at:

$$|z| = 2$$

Thus,

$$\left|z + \frac{1}{2}\right|_{\text{min}} = 2 - \frac{1}{2}$$

$$= \frac{3}{2}$$

Final Answer: $\boxed{1.5}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: Imaginary cube roots of unity satisfy:

$$1 + \omega + \omega^2 = 0, \qquad \omega^3 = 1$$

These identities simplify higher powers and expressions involving ω. Converting expressions into powers of ω helps evaluate complicated exponential forms quickly.

Solution: Using:

$$1 + \omega + \omega^2 = 0$$

We get:

$$1 + \omega = -\omega^2$$

Hence,

$$1 + \omega - \omega^2 = -2\omega^2$$

Therefore,

$$(1 + \omega - \omega^2)^7 = (-2\omega^2)^7$$

$$= (-2)^7(\omega^2)^7$$

$$= -128\omega^{14}$$

Since:

$$\omega^3 = 1$$

$$\omega^{14} = \omega^2$$

Thus,

$$(1 + \omega - \omega^2)^7 = -128\omega^2$$

Final Answer: $\boxed{-128\omega^2}$

Answer: (D)

Go Back to Question 12

Q13.

Solution

Concept: The number of real roots of transcendental equations can be determined graphically by comparing curves. Intersections between exponential and polynomial graphs indicate real solutions, while derivative analysis confirms the exact count.

Solution: Given:

$$e^x = x^2$$

Consider:

$$f(x) = e^x - x^2$$

We check sign changes.

At:

$$x = -1$$

$$f(-1) = e^{-1} - 1 < 0$$

At:

$$x = 0$$

$$f(0) = 1 > 0$$

So one root lies in:

$$(-1, 0)$$

Again,

$$x = 1$$

$$f(1) = e - 1 > 0$$

$$x = 2$$

$$f(2) = e^2 - 4 > 0$$

No positive sign change initially.

Now check:

$$x = -2$$

$$f(-2) = e^{-2} - 4 < 0$$

Thus only one crossing occurs.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: Complex roots of unity possess cyclic properties. Using identities satisfied by roots of quadratic equations allows repeated powers to simplify into periodic values, making long algebraic sums manageable through pattern recognition.

Solution: Given:

$$z^2 + z + 1 = 0$$

Hence,

$$z^3 = 1, \qquad z \neq 1$$

Also,

$$\frac{1}{z} = z^2$$

Therefore,

$$z + \frac{1}{z} = z + z^2 = -1$$

Thus,

$$\left(z + \frac{1}{z}\right)^2 = 1$$

Similarly,

$$z^2 + \frac{1}{z^2} = z^2 + z = -1$$

Hence every term equals:

$$1$$

There are 6 such terms:

$$1 + 1 + 1 + 1 + 1 + 1 = 6$$

Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: If two quadratic equations have a common root, subtracting the equations eliminates the quadratic term. The resulting linear relation gives the common root, which is then substituted back into either equation to obtain the required condition among coefficients.

Solution: Given:

$$x^2 + px + q = 0$$

and

$$x^2 + qx + p = 0$$

Suppose α is a common root.
Then,

$$\alpha^2 + p\alpha + q = 0$$

and

$$\alpha^2 + q\alpha + p = 0$$

Subtracting:

$$(p - q)\alpha + (q - p) = 0$$

$$(p - q)(\alpha - 1) = 0$$

If $p \neq q$, then:

$$\alpha = 1$$

Substituting into:

$$x^2 + px + q = 0$$

$$1 + p + q = 0$$

Thus,

$$p + q + 1 = 0$$

Final Answer: $\boxed{p + q + 1 = 0}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: Properties of transpose and inverse matrices simplify matrix expressions significantly. If a matrix commutes with its transpose, then products involving inverse and transpose can often be reduced using identities like $(AB)^T = B^T A^T$ and $AA^{-1} = I$.

Solution: Given:

$$B = A^{-1}A^T$$

We compute:

$$B^T = (A^{-1}A^T)^T$$

Using transpose property:

$$(AB)^T = B^T A^T$$

$$B^T = A(A^{-1})^T$$

Now,

$$BB^T = (A^{-1}A^T)\big(A(A^{-1})^T\big)$$

Since:

$$A^T A = AA^T$$

we get:

$$BB^T = A^{-1}(AA^T)(A^{-1})^T$$

$$= (A^{-1}A)A^T(A^{-1})^T$$

$$= IA^T(A^{-1})^T$$

$$= (A^{-1}A)^T$$

$$= I$$

Final Answer: $\boxed{I}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: For an $n \times n$ matrix:

$$\det(\text{Adj } A) = (\det A)^{n-1}$$

Repeated application of adjoint properties allows evaluation of determinants involving nested adjoint matrices directly without expanding determinants manually.

Solution: Given:

$$\det(A) = 2$$

and A is a 3×3 matrix.

We know:

$$\det(\text{Adj } A) = (\det A)^{3-1}$$

$$= 2^2$$

$$= 4$$

Now consider:

$$\det(\text{Adj}(\text{Adj } A))$$

Again applying formula:

$$= (\det(\text{Adj } A))^2$$

$$= 4^2$$

$$= 16$$

Final Answer: $\boxed{16}$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: A system of linear equations has a unique solution if the determinant of its coefficient matrix is non-zero. Evaluating the determinant and imposing the non-zero condition gives the required restriction on the parameter.

Solution: Coefficient matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{bmatrix}$$

For unique solution:

$$|A| \neq 0$$

Compute determinant:

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}$$

Expanding along first row:

$$= 1\begin{vmatrix} 1 & -1 \\ 2 & k \end{vmatrix} - 1\begin{vmatrix} 2 & -1 \\ 3 & k \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix}$$

$$= (k+2) - (2k+3) + (4-3)$$

$$= k + 2 - 2k - 3 + 1$$

$$= -k$$

For unique solution:

$$-k \neq 0$$

$$k \neq 0$$

Final Answer: $\boxed{k \neq 0}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: The Cayley–Hamilton theorem states that every square matrix satisfies its own characteristic equation. Computing the characteristic polynomial of the matrix allows direct simplification of higher powers of matrices into linear combinations of lower powers.

Solution: Given:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Characteristic equation:

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 \\ 3 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda) - 6 = 0$$

$$\lambda^2 - 5\lambda - 2 = 0$$

By Cayley–Hamilton theorem:

$$A^2 - 5A - 2I = 0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The determinant

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

is a Vandermonde determinant. Its value equals the product of pairwise differences of variables, with sign depending on the arrangement of factors.

Solution: Given:

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

The Vandermonde determinant formula gives:

$$\Delta = (b - a)(c - a)(c - b)$$

Now,

$$(b - a) = -(a - b)$$

and

$$(c - b) = -(b - c)$$

Thus,

$$\Delta = (a - b)(b - c)(c - a)$$

Hence,

$$k = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: To determine whether a function is one-to-one or onto, we analyze injectivity and range. Monotonic functions are one-to-one, while onto property depends on whether every real number is achieved in the codomain.

Solution: Given:

$$f(x) = \frac{x}{1 + |x|}$$

For $x \geq 0$:

$$f(x) = \frac{x}{1 + x}$$

For $x < 0$:

$$f(x) = \frac{x}{1 - x}$$

The function is strictly increasing on both intervals and overall monotonic, hence it is one-to-one.
Now determine range.
As:

$$x \to \infty$$

$$f(x) \to 1$$

As:

$$x \to -\infty$$

$$f(x) \to -1$$

Thus range is:

$$(-1, 1)$$

Since codomain is $\mathbb{R}$, not every real number is attained.
Hence function is not onto.
Final Answer: One-to-one but not onto

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: For logarithmic functions, the argument must be positive. Additionally, since the logarithm lies inside a square root, its value must be non-negative. Solving both inequalities together determines the domain of the function.

Solution: Given:

$$f(x) = \sqrt{\log_{0.5}(x-1)}$$

Conditions:

First,

$$x - 1 > 0$$

$$x > 1$$

Second,

$$\log_{0.5}(x-1) \geq 0$$

Since base:

$$0.5 < 1$$

the logarithmic function is decreasing.

Thus,

$$0 < x - 1 \leq 1$$

$$1 < x \leq 2$$

Hence domain is:

$$(1, 2]$$

Final Answer: $\boxed{(1, 2]}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: A relation is an equivalence relation if it is reflexive, symmetric, and transitive. Expressions involving parity become simpler because squares preserve evenness and oddness of integers.

Solution: Relation:

$$aRb \iff a^2 + b^2 \text{ is even}$$

Since parity of a^2 is same as parity of a:

$$a^2 + b^2$$

is even when both are even or both are odd.

Thus:

$$aRb$$

means a and b have same parity.

Reflexive:

$$a^2 + a^2 = 2a^2$$

is even.

Hence reflexive.

Symmetric: If

$$a^2 + b^2$$

is even, then

$$b^2 + a^2$$

is also even.

Hence symmetric.

Transitive: If a, b have same parity and b, c have same parity, then a, c have same parity.

Hence transitive.

Therefore R is an equivalence relation.

Final Answer: An equivalence relation

Answer: (D)

Go Back to Question 23

Q24.

Solution

Concept: Probability problems involving dice are solved by counting favorable outcomes over total outcomes. Integer solution methods help determine the number of ordered triples whose sum satisfies the required condition.

Solution: Total outcomes when 3 dice are thrown:

$$6^3 = 216$$

We need:

$$x + y + z = 15$$

where

$$1 \leq x, y, z \leq 6$$

Possible ordered triples:

$$(6, 6, 3)$$

Number of arrangements:

$$\frac{3!}{2!} = 3$$

$$(6, 5, 4)$$

Arrangements:

$$3! = 6$$

$$(5, 5, 5)$$

Arrangements:

$$1$$

Total favorable outcomes:

$$3 + 6 + 1 = 10$$

Thus probability:

$$\frac{10}{216}$$

Final Answer: $\boxed{\frac{10}{216}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: For independent events:

$$P(A \cap B) = P(A)P(B)$$

The probability of complements occurring together can be found using:

$$P(A' \cap B') = (1 - P(A))(1 - P(B))$$

when events are independent.

Solution: Given:

$$P(A) = 0.3, \qquad P(B) = 0.6$$

Thus,

$$P(A') = 1 - 0.3 = 0.7$$

and

$$P(B') = 1 - 0.6 = 0.4$$

Since A and B are independent, their complements are also independent.
Hence,

$$P(A' \cap B') = P(A')P(B')$$

$$= 0.7 \times 0.4$$

$$= 0.28$$

Final Answer: $\boxed{0.28}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: Use total probability for sequential events.

Solution:

Initially:

$$5B,\ 4W$$

If first ball is white:

$$\frac{4}{9} \times \frac{6}{11} = \frac{24}{99}$$

If first ball is black:

$$\frac{5}{9} \times \frac{4}{11} = \frac{20}{99}$$

Therefore,

$$P(\text{second white}) = \frac{24}{99} + \frac{20}{99}$$

$$= \frac{44}{99} = \frac{4}{9}$$

Final Answer: $\boxed{\frac{4}{9}}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: Variance measures the spread of observations around the mean. For the first n natural numbers, standard summation formulas for squares and arithmetic means are substituted into:

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

Solution: Numbers are:

$$1, 2, 3, \ldots, n$$

Mean:

$$\bar{x} = \frac{n+1}{2}$$

Now,

$$E(X^2) = \frac{1}{n}\sum_{k=1}^{n} k^2$$

Using:

$$\sum k^2 = \frac{n(n+1)(2n+1)}{6}$$

Thus,

$$E(X^2) = \frac{(n+1)(2n+1)}{6}$$

Variance:

$$\sigma^2 = E(X^2) - \bar{x}^2$$

$$= \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

Taking LCM:

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{(n+1)(4n+2-3n-3)}{12}$$

$$= \frac{(n+1)(n-1)}{12}$$

$$= \frac{n^2-1}{12}$$

Final Answer: $\boxed{\dfrac{n^2-1}{12}}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: For a Poisson distribution with mean λ:

$$P(X = r) = \frac{e^{-\lambda}\lambda^r}{r!}$$

Equating probabilities for different values of r allows direct determination of the parameter λ.

Solution: Given:

$$P(X = 1) = P(X = 2)$$

Using Poisson formula:

$$\frac{e^{-\lambda}\lambda}{1!} = \frac{e^{-\lambda}\lambda^2}{2!}$$

Cancel:

$$e^{-\lambda}\lambda$$

We get:

$$1 = \frac{\lambda}{2}$$

Thus,

$$\lambda = 2$$

Mean of Poisson distribution: $\boxed{2}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: Half-angle identities simplify trigonometric values of special angles. Using:

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

with known values of 15° helps evaluate $\tan 7.5^\circ$ exactly.

Solution: We use:

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

Take:

$$\theta = 15^\circ$$

Known values:

$$\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$$

$$\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Thus,

$$\tan 7.5^\circ = \frac{1-\frac{\sqrt{6}+\sqrt{2}}{4}}{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

Simplifying:

$$\tan 7.5^\circ = \sqrt{6}-\sqrt{3}+\sqrt{2}-2$$

Final Answer: $\boxed{\sqrt{6}-\sqrt{3}+\sqrt{2}-2}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: Trigonometric equations involving both sine and cosine can often be reduced to quadratic equations using:

$$\sin^2\theta = 1 - \cos^2\theta$$

Solving the resulting quadratic gives standard trigonometric solutions.

Solution: Given:

$$\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$$

Using:

$$\sin^2\theta = 1 - \cos^2\theta$$

$$1 - \cos^2\theta - 2\cos\theta + \frac{1}{4} = 0$$

$$-\cos^2\theta - 2\cos\theta + \frac{5}{4} = 0$$

Multiply by -4:

$$4\cos^2\theta + 8\cos\theta - 5 = 0$$

Let:

$$x = \cos\theta$$

Then:

$$4x^2 + 8x - 5 = 0$$

$$(2x-1)(2x+5) = 0$$

$$x = \frac{1}{2}$$

Hence,

$$\cos\theta = \frac{1}{2}$$

General solution:

$$\theta = 2n\pi \pm \frac{\pi}{3}$$

Final Answer: $\boxed{2n\pi \pm \frac{\pi}{3}}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: Inverse sine and inverse cosine functions satisfy:

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

This identity converts sums of inverse trigonometric functions into simpler expressions immediately.

Solution: Given:

$$\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$$

Using identity:

$$\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$$

Similarly,

$$\cos^{-1} y = \frac{\pi}{2} - \sin^{-1} y$$

Adding:

$$\cos^{-1} x + \cos^{-1} y$$

$$= \pi - (\sin^{-1} x + \sin^{-1} y)$$

$$= \pi - \frac{2\pi}{3}$$

$$= \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: The Law of Sines relates sides and opposite angles in a triangle:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Substituting known values allows direct determination of unknown angles.

Solution: Using sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Given:

$$a = 2, \qquad b = 3, \qquad \sin A = \frac{2}{3}$$

Thus,

$$\frac{2}{2/3} = \frac{3}{\sin B}$$

$$3 = \frac{3}{\sin B}$$

$$\sin B = 1$$

Hence,

$$B = 90^\circ$$

Final Answer: $\boxed{90^\circ}$

Answer: (C)

Go Back to Question 32

Q33.

Solution

Concept: Distance between two parallel lines:

$$ax + by + c_1 = 0$$

and

$$ax + by + c_2 = 0$$

is given by:

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

after reducing both equations to the same coefficients.

Solution: Given lines:

$$3x + 4y - 5 = 0$$

and

$$6x + 8y + 2 = 0$$

Divide second equation by 2:

$$3x + 4y + 1 = 0$$

Distance:

$$d = \frac{|1 - (-5)|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{6}{\sqrt{25}}$$

$$= \frac{6}{5}$$

$$= 1.2$$

Final Answer: $\boxed{1.2}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: The center lies on the perpendicular bisector of the chord and also on the given line.

Solution: Points:

$$A(1,2), \quad B(3,4)$$

Midpoint:

$$(2,3)$$

Slope of $AB = 1$, so perpendicular slope $= -1$.

Perpendicular bisector:

$$x + y - 5 = 0$$

Given:

$$x - y + 1 = 0$$

Solving:

$$x = 2, \quad y = 3$$

Center:

$$(2,3)$$

Radius:

$$r^2 = (2-1)^2 + (3-2)^2 = 2$$

Equation:

$$(x-2)^2 + (y-3)^2 = 2$$

$$x^2 + y^2 - 4x - 6y + 11 = 0$$

Final Answer: None

Answer: (D)

Go Back to Question 34

Q35.

Solution

Concept: The standard form of a parabola is:

$$(y-k)^2 = 4a(x-h)$$

where the length of the latus rectum equals:

$$4a$$

Completing the square converts the given equation into standard form.

Solution: Given:

$$y^2 - 4y - 10x + 14 = 0$$

Complete square in y:

$$y^2 - 4y = 10x - 14$$

$$y^2 - 4y + 4 = 10x - 10$$

$$(y-2)^2 = 10(x-1)$$

Comparing with:

$$(y-k)^2 = 4a(x-h)$$

we get:

$$4a = 10$$

Hence length of latus rectum: $\boxed{10}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: For an ellipse:

$$e = \frac{c}{a}$$

where c is focal distance and a is semi-major axis. Also:

$$b^2 = a^2 - c^2$$

The minor axis length equals:

$$2b$$

Solution: Given:

$$e = \frac{1}{2}$$

Distance between foci:

$$2c = 4$$

Thus,

$$c = 2$$

Using:

$$e = \frac{c}{a}$$

$$\frac{1}{2} = \frac{2}{a}$$

$$a = 4$$

Now,

$$b^2 = a^2 - c^2$$

$$= 16 - 4$$

$$= 12$$

$$b = 2\sqrt{3}$$

Length of minor axis:

$$2b = 4\sqrt{3}$$

Final Answer: $\boxed{4\sqrt{3}}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: For a hyperbola:

$$e = \frac{c}{a}$$

and

$$c^2 = a^2 + b^2$$

Using the given eccentricity and focus coordinates determines the standard equation directly.

Solution: Foci:

$$(\pm 5, 0)$$

Hence,

$$c = 5$$

Given eccentricity:

$$e = \frac{5}{4}$$

Using:

$$e = \frac{c}{a}$$

$$\frac{5}{4} = \frac{5}{a}$$

$$a = 4$$

Thus,

$$a^2 = 16$$

Now:

$$c^2 = a^2 + b^2$$

$$25 = 16 + b^2$$

$$b^2 = 9$$

Equation:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Final Answer: $\boxed{\frac{x^2}{16} - \frac{y^2}{9} = 1}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: For a homogeneous second-degree equation:

$$ax^2 + 2hxy + by^2 = 0$$

the angle between the pair of lines is given by:

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

Solution: Given:

$$x^2 + 4xy + y^2 = 0$$

Comparing with:

$$ax^2 + 2hxy + by^2 = 0$$

we get:

$$a = 1, \quad 2h = 4, \quad b = 1$$

Thus,

$$h = 2$$

Now,

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$= \frac{2\sqrt{4 - 1}}{2}$$

$$= \sqrt{3}$$

Hence,

$$\theta = 60^\circ$$

Final Answer: $\boxed{60^\circ}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept: Find intersection points of the lines and use the triangle area formula.

Solution:

Intersections:

$$y = x, \; y = 2x \Rightarrow (0, 0)$$

$$y = x, \; y = 3x + 4 \Rightarrow (-2, -2)$$

$$y = 2x, \; y = 3x + 4 \Rightarrow (-4, -8)$$

Area:

$$\frac{1}{2}\left|0 + (-2)(-8) + (-4)(2)\right|$$

$$= \frac{1}{2}(16 - 8)$$

$$= 4$$

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: A tangent to the parabola:

$$y^2 = 4ax$$

in slope form is:

$$y = mx + \frac{a}{m}$$

Comparing with the given tangent equation determines the slope parameter directly.

Solution: Given parabola:

$$y^2 = 4x$$

Thus,

$$a = 1$$

Tangent form:

$$y = mx + \frac{1}{m}$$

Given tangent:

$$y = mx + 1$$

Comparing:

$$\frac{1}{m} = 1$$

$$m = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: If a chord subtends an angle θ at the center of a circle of radius r, then the perpendicular distance from the center to the chord is:

$$r \cos \frac{\theta}{2}$$

The midpoint of the chord lies at this fixed distance from the center.

Solution: Circle:

$$x^2 + y^2 = 4$$

Radius:

$$r = 2$$

Chord subtends:

$$90^\circ$$

at center.

Distance of midpoint from center:

$$d = r \cos 45^\circ$$

$$= 2 \cdot \frac{1}{\sqrt{2}}$$

$$= \sqrt{2}$$

Hence locus is circle centered at origin with radius:

$$\sqrt{2}$$

Thus,

$$x^2 + y^2 = 2$$

Final Answer: $\boxed{x^2 + y^2 = 2}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: The normal to a curve is perpendicular to the tangent at the given point. First differentiate the curve to obtain tangent slope, then use the negative reciprocal to form the normal equation.

Solution: Given:

$$x^2 = 4y$$

Differentiate:

$$2x = 4\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{x}{2}$$

At point:

$$(4, 4)$$

Slope of tangent:

$$m_t = \frac{4}{2} = 2$$

Slope of normal:

$$m_n = -\frac{1}{2}$$

Equation of normal:

$$y - 4 = -\frac{1}{2}(x - 4)$$

$$2y - 8 = -x + 4$$

$$x + 2y - 12 = 0$$

Final Answer: $\boxed{x + 2y - 12 = 0}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: The shortest distance between two skew lines is:

$$\frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

where $\vec{b}_1, \vec{b}_2$ are direction vectors.

Solution: Lines:

$$\vec{r} = (1, 1, 0) + \lambda(2, -1, 1)$$

$$\vec{r} = (2, 1, -1) + \mu(3, -5, 2)$$

Direction vectors:

$$\vec{b}_1 = (2, -1, 1)$$

$$\vec{b}_2 = (3, -5, 2)$$

Compute cross product:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= (3 - (-5))i - (4 - 3)j + (-10 + 3)k$$

$$= (3, -1, -7)$$

Magnitude:

$$\sqrt{9 + 1 + 49} = \sqrt{59}$$

Vector joining points:

$$(2, 1, -1) - (1, 1, 0) = (1, 0, -1)$$

Dot product:

$$(1, 0, -1) \cdot (3, -1, -7) = 3 + 7 = 10$$

Shortest distance:

$$\frac{10}{\sqrt{59}}$$

Final Answer: $\boxed{\frac{10}{\sqrt{59}}}$

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept: The angle between two planes equals the angle between their normal vectors. Using the dot product formula for vectors gives:

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$$

Solution: Normals to planes:
For:

$$2x - y + z = 6$$

normal vector:

$$(2, -1, 1)$$

For:

$$x + y + 2z = 7$$

normal vector:

$$(1, 1, 2)$$

Dot product:

$$2(1) + (-1)(1) + 1(2)$$

$$= 2 - 1 + 2$$

$$= 3$$

Magnitudes:

$$\sqrt{4 + 1 + 1} = \sqrt{6}$$

$$\sqrt{1 + 1 + 4} = \sqrt{6}$$

Thus,

$$\cos\theta = \frac{3}{6} = \frac{1}{2}$$

Hence,

$$\theta = 60^\circ$$

Final Answer: $\boxed{60^\circ}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: Direction cosines satisfy:

$$l^2 + m^2 + n^2 = 1$$

If a line is equally inclined to all coordinate axes, then all direction cosines are equal.

Solution: Let:

$$l = m = n = k$$

Using:

$$l^2 + m^2 + n^2 = 1$$

$$3k^2 = 1$$

$$k = \frac{1}{\sqrt{3}}$$

Hence direction cosines:

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Final Answer: $\boxed{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: Convert symmetric form into parametric form and substitute into the plane equation.

Solution: Let

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = t$$

Then:

$$x = 1 + 2t, \quad y = 2 + 3t, \quad z = 3 + 4t$$

Substitute into:

$$2x + 4y - z = 1$$

$$2(1 + 2t) + 4(2 + 3t) - (3 + 4t) = 1$$

$$7 + 12t = 1$$

$$t = -\frac{1}{2}$$

Hence,

$$x = 0, \quad y = \frac{1}{2}, \quad z = 1$$

Final Answer: $\boxed{\left(0, \frac{1}{2}, 1\right)}$

Answer: (D)

Go Back to Question 46

Q47.

Solution

Concept: Vector magnitudes satisfy:

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$$

and

$$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$$

Comparing them determines orthogonality.

Solution: Given:

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$$

Squaring:

$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

Thus,

$$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$$

$$4\vec{a}\cdot\vec{b} = 0$$

$$\vec{a}\cdot\vec{b} = 0$$

Hence vectors are perpendicular.

Therefore angle between them:

$$90^\circ$$

Final Answer: $\boxed{90^\circ}$

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept: The scalar triple product:

$$[\vec{a}\,\vec{b}\,\vec{c}]$$

is zero if the vectors are linearly dependent. Expanding determinant properties and using column operations simplify expressions involving differences of vectors.

Solution: Given:

$$[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}]$$

Using linearity:

$$(\vec{a}-\vec{b})+(\vec{b}-\vec{c})+(\vec{c}-\vec{a})=0$$

Thus the three vectors are linearly dependent.
Hence scalar triple product:

$$[\vec{a}-\vec{b} \quad \vec{b}-\vec{c} \quad \vec{c}-\vec{a}]=0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 48

Q49.

Solution

Concept: For vectors:

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

and

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

Equating them gives a trigonometric relation for the angle between vectors.

Solution: Given:

$$\vec{a} \cdot \vec{b} = |\vec{a} \times \vec{b}|$$

Thus,

$$|\vec{a}||\vec{b}| \cos\theta = |\vec{a}||\vec{b}| \sin\theta$$

Assuming vectors are non-zero:

$$\cos\theta = \sin\theta$$

$$\tan\theta = 1$$

Hence,

$$\theta = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: The scalar projection of vector $\vec{a}$ on vector $\vec{b}$ is:

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

It represents the magnitude of the component of one vector along the direction of another vector.

Solution: Given:

$$\vec{a} = 2i + 3j + 2k$$

$$\vec{b} = i + 2j + k$$

Dot product:

$$\vec{a} \cdot \vec{b} = 2(1) + 3(2) + 2(1)$$

$$= 2 + 6 + 2$$

$$= 10$$

Magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2}$$

$$= \sqrt{6}$$

Projection of $\vec{a}$ on $\vec{b}$:

$$\frac{10}{\sqrt{6}}$$

Final Answer: $\boxed{\frac{10}{\sqrt{6}}}$

Answer: (A)

Go Back to Question 50

Q51.

Solution

Concept: For vectors:

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a})$$

Using unit vector properties simplifies the relation immediately.

Solution: Given:

$$\vec{a}+\vec{b}+\vec{c} = 0$$

Taking modulus squared:

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 0$$

Thus,

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$$

Since $\vec{a}, \vec{b}, \vec{c}$ are unit vectors:

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Hence,

$$1 + 1 + 1 + 2S = 0$$

where

$$S = \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$$

$$3 + 2S = 0$$

$$2S = -3$$

$$S = -\frac{3}{2}$$

Final Answer: $\boxed{-\frac{3}{2}}$

Answer: (B)

Go Back to Question 51

Q52.

Solution

Concept: Standard series expansions near zero are:

$$e^{x^2} = 1 + x^2 + \cdots$$

and

$$\cos x = 1 - \frac{x^2}{2} + \cdots$$

Substituting these expansions simplifies the limit directly.

Solution: Using expansions:

$$e^{x^2} = 1 + x^2 + \cdots$$

$$\cos x = 1 - \frac{x^2}{2} + \cdots$$

Therefore,

$$e^{x^2} - \cos x = \left(1 + x^2\right) - \left(1 - \frac{x^2}{2}\right)$$

$$= x^2 + \frac{x^2}{2}$$

$$= \frac{3x^2}{2}$$

Hence,

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = \frac{3}{2}$$

Final Answer: $\boxed{\frac{3}{2}}$

Answer: (A)

Go Back to Question 52

Q53.

Solution

Concept: Algebraic simplification often converts complicated inverse trigonometric expressions into standard forms. Rationalizing expressions near $x = 0$ helps determine derivatives conveniently.

Solution: Given:

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

Rationalize:

$$\frac{\sqrt{1+x^2}-1}{x} \cdot \frac{\sqrt{1+x^2}+1}{\sqrt{1+x^2}+1}$$

$$= \frac{x}{\sqrt{1+x^2}+1}$$

Thus,

$$y = \tan^{-1}\left(\frac{x}{\sqrt{1+x^2}+1}\right)$$

Near $x = 0$:

$$\sqrt{1+x^2} \approx 1$$

Hence,

$$y \approx \tan^{-1}\left(\frac{x}{2}\right)$$

Therefore,

$$\left.\frac{dy}{dx}\right|_{x=0} = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 53

Q54.

Solution

Concept: Partial fraction decomposition simplifies rational integrals. Expressing the integrand as simpler logarithmic derivatives allows direct integration using standard logarithm formulas.

Solution: Consider:

$$\int \frac{dx}{x(x^n + 1)}$$

Write:

$$\frac{1}{x(x^n + 1)} = \frac{1}{x} - \frac{x^{n-1}}{x^n + 1}$$

Thus,

$$I = \int \frac{dx}{x} - \int \frac{x^{n-1}}{x^n + 1} dx$$

For second integral, let:

$$u = x^n + 1$$

Then:

$$du = nx^{n-1} dx$$

Hence,

$$\int \frac{x^{n-1}}{x^n + 1} dx = \frac{1}{n} \ln |x^n + 1|$$

Therefore,

$$I = \ln |x| - \frac{1}{n} \ln |x^n + 1| + C$$

$$= \frac{1}{n} \ln \left| \frac{x^n}{x^n + 1} \right| + C$$

Final Answer: $\boxed{\frac{1}{n} \ln \left| \frac{x^n}{x^n + 1} \right| + C}$

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept: Definite integrals over complementary intervals often use symmetry:

$$I = \int_0^{\pi/2} f(x)\,dx$$

and substitution

$$x \to \frac{\pi}{2} - x$$

to combine integrals into a simpler constant value.

Solution: Let

$$I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Substitute:

$$x \to \frac{\pi}{2} - x$$

Then,

$$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

Add the two:

$$2I = \int_0^{\pi/2} \frac{\sqrt{\sin x} + \sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$$

$$= \int_0^{\pi/2} 1\,dx$$

$$= \frac{\pi}{2}$$

Thus,

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 55

Q56.

Solution

Concept: A first-order linear differential equation:

$$\frac{dy}{dx} + Py = Q$$

is solved using the integrating factor:

$$IF = e^{\int P\,dx}$$

which converts the equation into an exact derivative.

Solution: Given:

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Here,

$$P = \frac{1}{x}$$

Integrating factor:

$$IF = e^{\int \frac{1}{x}dx}$$

$$= e^{\ln x}$$

$$= x$$

Multiply equation by x:

$$x\frac{dy}{dx} + y = x^3$$

Left side becomes:

$$\frac{d}{dx}(xy) = x^3$$

Integrate:

$$xy = \frac{x^4}{4} + C$$

Final Answer: $\boxed{xy = \frac{x^4}{4} + C}$

Answer: (A)

Go Back to Question 56

Q57.

Solution

Concept: Absolute value functions are differentiable away from corner points. The derivative changes according to the sign of the quantity inside the modulus function.

Solution: Given:

$$f(x) = |x - 1| + |x - 2|$$

At:

$$x = 1.5$$

We have:

$$x - 1 > 0$$

and

$$x - 2 < 0$$

Thus,

$$f(x) = (x - 1) - (x - 2)$$

$$= x - 1 - x + 2$$

$$= 1$$

Hence,

$$f'(x) = 0$$

Therefore,

$$f'(1.5) = 0$$

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 57

Q58.

Solution

Concept: Limits of sums can often be interpreted as Riemann integrals:

$$\lim_{n\to\infty} \sum f\left(\frac{r}{n}\right)\frac{1}{n} = \int_0^1 f(x)\,dx$$

Solution: Given:

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2 + r^2}$$

Rewrite:

$$= \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n} \cdot \frac{1}{1 + \left(\frac{r}{n}\right)^2}$$

Thus,

$$= \int_0^1 \frac{dx}{1 + x^2}$$

$$= \left[\tan^{-1} x\right]_0^1$$

$$= \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (A)

Go Back to Question 58

Q59.

Solution

Concept: Using trigonometric identities:

$$2\sin\theta\cos\theta = \sin 2\theta$$

helps simplify inverse trigonometric expressions before differentiation with respect to another inverse function.

Solution: Let:

$$x = \sin\theta$$

Then:

$$\sqrt{1-x^2} = \cos\theta$$

Thus,

$$2x\sqrt{1-x^2} = 2\sin\theta\cos\theta$$

$$= \sin 2\theta$$

Hence,

$$\sin^{-1}(2x\sqrt{1-x^2}) = 2\theta$$

But:

$$\theta = \sin^{-1} x$$

Therefore,

$$y = 2\sin^{-1} x$$

Hence derivative with respect to $\sin^{-1} x$:

$$\frac{dy}{d(\sin^{-1} x)} = 2$$

Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 59

Q60.

Solution

Concept: Recognition of derivatives simplifies integration. The derivative of:

$$e^x \sin x$$

contains both $\sin x$ and $\cos x$, matching the integrand exactly.
Solution: Differentiate:

$$e^x \sin x$$

Using product rule:

$$\frac{d}{dx}(e^x \sin x) = e^x \sin x + e^x \cos x$$

$$= e^x (\sin x + \cos x)$$

Thus,

$$\int e^x (\sin x + \cos x)\, dx = e^x \sin x + C$$

Final Answer: $\boxed{e^x \sin x + C}$

Answer: (A)

Go Back to Question 60

Q61.

Solution

Concept: Order is the highest derivative present in a differential equation. Degree is defined only when the equation is polynomial in derivatives after removing radicals and transcendental functions involving derivatives.
Solution: Given:

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$

Highest derivative:

$$\frac{d^2y}{dx^2}$$

Hence order:

$$2$$

Since:

$$\sin\left(\frac{dy}{dx}\right)$$

is a transcendental function of derivative, the equation is not polynomial in derivatives.
Therefore degree is not defined.
Final Answer: $\boxed{2,\ \text{Not defined}}$

Answer: (B)

Go Back to Question 61

Q62.

Solution

Concept: Implicit differentiation together with chain rule helps derive differential equations satisfied by functions involving inverse trigonometric expressions and exponentials.

Solution: Given:

$$y = e^{a\sin^{-1}x}$$

Take logarithm:

$$\ln y = a\sin^{-1}x$$

Differentiate:

$$\frac{y_1}{y} = \frac{a}{\sqrt{1-x^2}}$$

Thus,

$$y_1 = \frac{ay}{\sqrt{1-x^2}}$$

Differentiate again:

$$y_2 = \frac{a^2y}{1-x^2} + \frac{axy}{(1-x^2)^{3/2}}$$

Now compute:

$$(1-x^2)y_2 - xy_1$$

Substituting:

$$= a^2y$$

Final Answer: $\boxed{a^2y}$

Answer: (A)

Go Back to Question 62

Q63.

Solution

Concept: For modulus functions:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Solution:

$$\int_{-1}^{1} |x| dx = 2\int_{0}^{1} x\, dx$$

$$= 2\left[\frac{x^2}{2}\right]_0^1$$

$$= 2\left(\frac{1}{2}\right)$$

$$= 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 63

Q64.

Solution

Concept: Use:

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = e^k$$

Solution:

$$\left(\frac{x+6}{x+1}\right)^{x+4} = \left(1+\frac{5}{x+1}\right)^{x+4}$$

As $x \to \infty$,

$$\left(1+\frac{5}{x+1}\right)^{x+1} \to e^5$$

Hence,

$$\left(1+\frac{5}{x+1}\right)^{x+4} \to e^5$$

Final Answer: $\boxed{e^5}$

Answer: (A)

Go Back to Question 64

Q65.

Solution

Concept: For $f(x) = x^x$,

$$\frac{d}{dx}(x^x) = x^x(1 + \ln x)$$

Stationary point occurs when:

$$f'(x) = 0$$

Solution:

$$x^x(1 + \ln x) = 0$$

Since $x^x \neq 0$,

$$1 + \ln x = 0$$

$$\ln x = -1$$

$$x = e^{-1}$$

$$x = \frac{1}{e}$$

Final Answer: $\boxed{\frac{1}{e}}$

Answer: (B)

Go Back to Question 65

Q66.

Solution

Concept: Use differentiation to analyze maxima and monotonicity.

Solution:

$$f(x) = \frac{x}{1 + x^n}$$

$$f'(x) = \frac{1 + x^n - nx^n}{(1 + x^n)^2} = \frac{1 - (n-1)x^n}{(1 + x^n)^2}$$

So,

$$f'(x) > 0 \Rightarrow x < \left(\frac{1}{n-1}\right)^{1/n}$$

$$f'(x) = 0 \Rightarrow x = \left(\frac{1}{n-1}\right)^{1/n}$$

$$f'(x) < 0 \Rightarrow x > \left(\frac{1}{n-1}\right)^{1/n}$$

Hence A, B, C are correct.

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 67

Q67.

Solution

Concept: Use properties of complex numbers with $|z| = 1 \Rightarrow \bar{z} = 1/z$.

Solution: Let $z = e^{i\theta}$, so $\bar{z} = e^{-i\theta}$.

$$\frac{1 + z}{1 + \bar{z}} = \frac{1 + e^{i\theta}}{1 + e^{-i\theta}}$$

Multiply numerator and denominator by $e^{i\theta}$:

$$= \frac{e^{i\theta} + e^{2i\theta}}{e^{i\theta} + 1} = e^{i\theta} = z$$

Thus,

$$\frac{1 + z}{1 + \bar{z}} = z = \cos\theta + i\sin\theta$$

So A and B are correct.

Final Answer: A, B

Answer: (A,B)

Go Back to Question 67

Q68.

Solution

Concept: Tangents to a circle are at distance equal to radius from center.

Solution: Given circle:

$$x^2 + y^2 = 25 \Rightarrow r = 5$$

Line parallel form:

$$3x + 4y + c = 0$$

Distance from origin:

$$\frac{|c|}{\sqrt{3^2 + 4^2}} = \frac{|c|}{5} = 5 \Rightarrow |c| = 25$$

So tangents are:

$$3x + 4y \pm 25 = 0$$

Thus A and B are correct.

Final Answer: A, B

Answer: (A,B)

Go Back to Question 68

Q69.

Solution

Concept: Check continuity and differentiability of absolute value functions.

Solution: $f(x) = |x| + |x - 1|$

At $x = 0$: Left derivative = −2, right derivative = 0 So not differentiable but continuous.

At $x = 1$: Left derivative = 0, right derivative = 2 So not differentiable but continuous.

Thus A and C are correct.

Final Answer: A, C

Answer: (A,C)

Go Back to Question 69

Q70.

Solution

Concept: Use determinant properties (Vandermonde determinant).

Solution:

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a - b)(b - c)(c - a)$$

Thus only option A is correct.

Final Answer: A

Answer: (A)

Go Back to Question 70

Q71.

Solution

Concept: Use standard formulas for ellipse properties.

Solution:

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \Rightarrow a = 5,\ b = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$$

Foci:

$$(\pm ae, 0) = (\pm 3, 0)$$

Latus rectum:

$$\frac{2b^2}{a} = \frac{32}{5}$$

Directrices:

$$x = \pm\frac{a}{e} = \pm\frac{25}{3}$$

Thus A, B, C, D are correct.

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 71

Q72.

Solution

Concept: Use standard reduction formula for $\int \tan^n x\, dx$.

Solution: For $I_n = \int_0^{\pi/4} \tan^n x\, dx$, the identity is:

$$I_n + I_{n-2} = \frac{1}{n-1}$$

Hence: - $I_2 + I_4 = \frac{1}{3}$ - $I_3 + I_5 = \frac{1}{4}$

Thus A, B, D are correct.

Final Answer: A, B, D

Answer: (A,B,D)

Go Back to Question 72

Q73.

Solution

Concept: Coplanarity condition using scalar triple product.

Solution: Vectors are coplanar if:

$$[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = 0$$

Also equivalent to linear dependence.

Thus A, B, C are correct.

But $\vec{a} \times (\vec{b} \times \vec{c}) = 0$ is not general condition for coplanarity.

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 73

Q74.

Solution

Concept: Solve trigonometric equation using quadratic form.

Solution:

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

So,

$$\sin x = \frac{1}{2}, -1$$

In $[0, 2\pi]$:

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

Thus A, B, C are correct.

Final Answer: A, B, C

Answer: (A,B,C)

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Q75.

Solution

Concept: Properties of independent events.

Solution: If A and B are independent:

$$P(A \cap B) = P(A)P(B)$$

$$P(A|B) = P(A)$$

Also complements are independent:

$$A^c \text{ and } B^c \text{ are independent}$$

But $P(A \cup B) = P(A) + P(B)$ only if disjoint, not general.

Thus A, B, C are correct.

Final Answer: A, B, C

Answer: (A,B,C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	C	4	A	5	D
6	A	7	A	8	A	9	A	10	A
11	A	12	D	13	B	14	B	15	A
16	A	17	B	18	A	19	A	20	A
21	A	22	A	23	D	24	A	25	A
26	A	27	A	28	B	29	A	30	A
31	A	32	C	33	A	34	D	35	A
36	A	37	A	38	A	39	A	40	A
41	A	42	A	43	A	44	A	45	B
46	D	47	B	48	A	49	A	50	A
51	B	52	A	53	A	54	A	55	B
56	A	57	A	58	A	59	A	60	A
61	B	62	A	63	A	64	A	65	B
66	A,B,C	67	A,B	68	A,B	69	A,C	70	A
71	A,B,C,D	72	A,B,D	73	A,B,C	74	A,B,C	75	A,B,C