

WBJEE Mathematics Sample Paper-19

Duration: 120 Minutes

Maximum Marks: 100

Instructions

- This paper contains **75** Multiple Choice Questions divided into **3 Sections**.
- **Section A (Q1–Q50):** Each correct answer carries **+1 mark**. Incorrect answer: **–0.25** marks. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries **+2 marks**. Incorrect answer: **–0.5** marks. Only **one** correct option.
- **Section C (Q66–Q75):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: –0.25) [Single Correct]

Q1. If the p th, q th and r th terms of an arithmetic progression are a , b and c respectively, then the value of $a(q - r) + b(r - p) + c(p - q)$ is:

- (A) 0
- (B) $p + q + r$
- (C) $a + b + c$
- (D) $(p - q)(q - r)(r - p)$

Q2. For positive real numbers x and y , if $\log_2 x + \log_2 y = 4$ and $\log_2(x + y) = 3$, then xy equals:

- (A) 8
- (B) 16



(C) 32

(D) 4

Q3. The number of five-digit numbers formed from the digits 0, 1, 2, 3, 4, 5 without repetition and divisible by 5 is:

(A) 180

(B) 192

(C) 216

(D) 240

Q4. The coefficient of x^8 in $(1 + x^2)^6(1 + x)^4$ is:

(A) 90

(B) 120

(C) 135

(D) 150

Q5. If S_n denotes the sum of the first n terms of an A.P. and $S_5 = 40$, $S_{10} = 130$, then the 11th term is:

(A) 20

(B) 24

(C) 26

(D) 28

Q6. If $x > 0$, $x \neq 1$, and $\log_x 16 + \log_4 x = 5$, then the sum of all possible values of $\log_4 x$ is:

(A) 2

(B) 4

(C) 5

(D) 6



Q7. The number of integral values of x satisfying $|x - 2| + |x + 1| \leq 7$ is:

- (A) 5
- (B) 6
- (C) 7
- (D) 8

Q8. If $z = x + 2i$ satisfies $|z - 1| = |z + 3|$, then x equals:

- (A) 1
- (B) -1
- (C) 2
- (D) -2

Q9. The principal argument of $\frac{(1+i)^6}{(1-i)^4}$ is:

- (A) 0
- (B) π
- (C) $\frac{\pi}{2}$
- (D) $-\frac{\pi}{2}$

Q10. The quadratic equation $x^2 - 2(k+1)x + k^2 + 3 = 0$ has two distinct real roots when:

- (A) $k > 1$
- (B) $k \geq 1$
- (C) $k < 1$
- (D) $k \leq 1$

Q11. The determinant $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ equals:

- (A) $(a-b)(b-c)(c-a)$



- (B) $(a - b)(a - c)(b - c)$
- (C) $(b - a)(c - a)(c - b)$
- (D) $(a + b + c)^2$

Q12. If a square matrix A satisfies $A^2 = 3A - 2I$ and A is invertible, then A^{-1} is:

- (A) $A - 3I$
- (B) $\frac{A - 3I}{2}$
- (C) $3I - A$
- (D) $\frac{3I - A}{2}$

Q13. If A is a 3×3 matrix with $|A| = 2$, then $|\text{adj}(2A)|$ equals:

- (A) 256
- (B) 128
- (C) 64
- (D) 32

Q14. For $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, the value of $|A^2 - 5A + I|$ is:

- (A) 3
- (B) 9
- (C) -3
- (D) -9

Q15. If $A = \{x \in \mathbb{R} : x^2 - 5x + 6 \leq 0\}$ and $B = \{x \in \mathbb{R} : |x - 2| < 2\}$, then $A \cap B$ is:

- (A) $(0, 4)$
- (B) $[2, 3]$
- (C) $(2, 3)$
- (D) $[0, 4]$



- Q16.** On the set of integers, define aRb if and only if $a - b$ is divisible by 3. Then R is:
- (A) reflexive only
 - (B) symmetric but not transitive
 - (C) an equivalence relation
 - (D) a partial order
- Q17.** If $f(x) = \frac{x-1}{x+1}$, then $f(f(x))$ equals:
- (A) x
 - (B) $\frac{1}{x}$
 - (C) $-x$
 - (D) $-\frac{1}{x}$
- Q18.** Three fair dice are thrown. The probability that the sum of the numbers obtained is 5 equals:
- (A) $\frac{1}{36}$
 - (B) $\frac{1}{18}$
 - (C) $\frac{5}{108}$
 - (D) $\frac{1}{24}$
- Q19.** A bag contains 4 red and 3 blue balls. Two balls are drawn without replacement. Given that at least one ball is red, the probability that both balls are red is:
- (A) $\frac{2}{7}$
 - (B) $\frac{1}{3}$
 - (C) $\frac{3}{7}$
 - (D) $\frac{1}{2}$



Q20. The mean of 5 observations is 12 and their variance is 8. The sum of the squares of the observations is:

- (A) 720
- (B) 740
- (C) 760
- (D) 780

Q21. For $x \in (0, \pi)$, if $\sin x + \cos x = \sqrt{2} \cos x$, then x equals:

- (A) $\frac{\pi}{12}$
- (B) $\frac{\pi}{8}$
- (C) $\frac{3\pi}{8}$
- (D) $\frac{\pi}{4}$

Q22. The value of $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{8}$ is:

- (A) $\frac{\pi}{4}$
- (B) $\frac{\pi}{3}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{\pi}{2}$

Q23. If $\tan A$ and $\tan B$ are the roots of $x^2 - 3x + 1 = 0$, where $A, B \in (0, \pi/2)$, then $A + B$ equals:

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{2}$



- Q24.** The line passing through the intersection of $2x - y + 3 = 0$ and $x + 2y - 4 = 0$ and parallel to $x - y = 0$ is:
- (A) $x - y + 3 = 0$
(B) $5x + 5y - 13 = 0$
(C) $5x - 5y + 13 = 0$
(D) $x + y - 1 = 0$
- Q25.** A circle has its centre on the x -axis, passes through $(2, 3)$ and touches the y -axis. Its radius is:
- (A) $\frac{9}{4}$
(B) 3
(C) $\frac{11}{4}$
(D) $\frac{13}{4}$
- Q26.** The tangent to the parabola $y^2 = 8x$ at the point with parameter $t = 2$ is:
- (A) $x + 2y - 8 = 0$
(B) $x - 2y + 8 = 0$
(C) $2x - y + 8 = 0$
(D) $x - 4y + 8 = 0$
- Q27.** The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is:
- (A) $\frac{4}{5}$
(B) $\frac{3}{5}$
(C) $\frac{2}{5}$
(D) $\frac{1}{5}$
- Q28.** The asymptotes of the hyperbola $4x^2 - 9y^2 = 36$ are:



- (A) $y = \pm \frac{3}{2}x$
(B) $y = \pm \frac{4}{3}x$
(C) $y = \pm \frac{2}{3}x$
(D) $y = \pm \frac{3}{4}x$

Q29. The length of the chord cut by the line $3x + 4y = 15$ from the circle $x^2 + y^2 = 25$ is:

- (A) 4
(B) 6
(C) 7
(D) 8

Q30. The focus of the parabola $y^2 - 4y - 8x + 12 = 0$ is:

- (A) (3, 2)
(B) (1, 4)
(C) (2, 3)
(D) (3, 1)

Q31. The area of the triangle formed by the coordinate axes and the line $\frac{x}{3} + \frac{y}{4} = 1$ is:

- (A) 4
(B) 6
(C) 8
(D) 12

Q32. The plane through $(1, -2, 3)$ and normal to the vector $2\hat{i} + \hat{j} - \hat{k}$ is:

- (A) $2x + y - z - 3 = 0$
(B) $x + 2y - z + 6 = 0$



(C) $2x + y - z + 3 = 0$

(D) $2x - y + z + 3 = 0$

Q33. The distance of the point $(2, -1, 3)$ from the plane $2x - y + 2z - 7 = 0$ is:

(A) $\frac{2}{3}$

(B) 1

(C) $\frac{5}{3}$

(D) $\frac{4}{3}$

Q34. The direction cosines of a line equally inclined to the positive coordinate axes are:

(A) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

(B) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(C) $(1, 1, 1)$

(D) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

Q35. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, then $|\vec{a} \times \vec{b}|^2$ equals:

(A) 63

(B) 69

(C) 75

(D) 81

Q36. The scalar projection of $\vec{a} = 3\hat{i} + 4\hat{j}$ on $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is:

(A) 3

(B) $\frac{11}{3}$

(C) $\frac{7}{3}$

(D) 11



- Q37.** The volume of the parallelepiped formed by $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ and $\vec{c} = \hat{k} + \hat{i}$ is:
- (A) 0
(B) 1
(C) $\sqrt{2}$
(D) 2
- Q38.** The value of $\lim_{x \rightarrow 0} \frac{\sin 3x - 3 \sin x}{x^3}$ is:
- (A) -4
(B) -3
(C) 0
(D) 4
- Q39.** If $f(x) = \frac{e^{kx} - 1}{x}$ for $x \neq 0$ and $f(0) = 2$, then f is continuous at $x = 0$ for:
- (A) $k = 1$
(B) $k = 2$
(C) $k = -2$
(D) $k = 0$
- Q40.** If $y = x^x$, then $\left. \frac{dy}{dx} \right|_{x=1}$ equals:
- (A) 0
(B) $\frac{1}{2}$
(C) 1
(D) 2
- Q41.** If $x = t^2 + 1$ and $y = t^3 - t$, then $\left. \frac{dy}{dx} \right|_{t=2}$ equals:
- (A) 2
(B) $\frac{5}{2}$



- (C) 3
(D) $\frac{11}{4}$

Q42. The value of $\int_0^1 \frac{2x}{1+x^2} dx$ is:

- (A) $\ln 2$
(B) $2 \ln 2$
(C) $\frac{1}{2} \ln 2$
(D) 1

Q43. The value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ is:

- (A) $\frac{\pi}{8}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$
(D) 1

Q44. The solution of $\frac{dy}{dx} + y \tan x = \sec x$, satisfying $y(0) = 1$, gives $y\left(\frac{\pi}{4}\right)$ equal to:

- (A) 1
(B) 2
(C) $\sqrt{2}$
(D) $\frac{1}{\sqrt{2}}$

Q45. The value of $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x + \frac{x^2}{2}}{x^3}$ is:

- (A) 1
(B) $\frac{1}{2}$
(C) $\frac{1}{4}$



(D) $\frac{1}{3}$

Q46. $\int \frac{dx}{x^2 + 4x + 8}$ equals:

(A) $\tan^{-1} \left(\frac{x+2}{2} \right) + C$

(B) $\frac{1}{2} \tan^{-1} \left(\frac{x+2}{2} \right) + C$

(C) $\frac{1}{4} \tan^{-1} \left(\frac{x+2}{2} \right) + C$

(D) $\ln(x^2 + 4x + 8) + C$

Q47. The function $f(x) = x^3 - 6x^2 + 9x + 1$ is strictly increasing on:

(A) $(1, 3)$

(B) $(-\infty, 3)$

(C) $(-\infty, 1) \cup (3, \infty)$

(D) $(3, \infty)$ only

Q48. For $x > 0$, the minimum value of $x + \frac{1}{x}$ is:

(A) 2

(B) 1

(C) $\frac{1}{2}$

(D) 4

Q49. The area bounded by the curves $y = x$ and $y = x^2$ is:

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) $\frac{1}{2}$

(D) 1



Q50. A particle moves along a line with displacement $s(t) = t^3 - 6t^2 + 9t$. Its acceleration at the first time $t > 0$ when velocity is zero is:

- (A) 6
- (B) 0
- (C) -3
- (D) -6

Section-B — 15 Questions × 2 Marks Each
(Negative Marking: -0.5) [Single Correct]

Q51. Positive numbers a, b, c are in G.P. If $a + b + c = 21$ and $abc = 216$, then b equals:

- (A) 6
- (B) 7
- (C) 8
- (D) 9

Q52. The number of arrangements of the letters of the word *BANANA* in which no two *N*'s are adjacent is:

- (A) 30
- (B) 36
- (C) 40
- (D) 60

Q53. If α, β are the roots of $x^2 - 2x + 5 = 0$, then $\alpha^4 + \beta^4$ equals:

- (A) -16
- (B) -14
- (C) 14



(D) 16

Q54. If A is a 2×2 matrix with trace 4 and determinant 3, then $|A^2 - I|$ equals:

(A) 8

(B) 4

(C) 1

(D) 0

Q55. A bag contains 5 white and 4 black balls. Two balls are drawn without replacement. Given that at least one is black, the probability that the other ball is white is:

(A) $\frac{10}{13}$

(B) $\frac{5}{9}$

(C) $\frac{4}{9}$

(D) $\frac{9}{13}$

Q56. If the mean of observations x_i is 5 and the mean of x_i^2 is 29, then the variance of $2x_i - 3$ is:

(A) 8

(B) 12

(C) 16

(D) 20

Q57. The point on the line $3x + 4y = 25$ nearest to the origin is:

(A) (4, 3)

(B) (3, 4)

(C) (5, 0)

(D) $(0, \frac{25}{4})$



- Q58.** A circle passes through the origin, cuts the x -axis again at a point 6 units from the origin, and has its centre on the line $y = x$. Its radius is:
- (A) 3
(B) 6
(C) $2\sqrt{3}$
(D) $3\sqrt{2}$
- Q59.** The normal to the parabola $y^2 = 4x$ at parameter $t = 1$ meets the x -axis at:
- (A) (3, 0)
(B) (2, 0)
(C) (1, 0)
(D) (-1, 0)
- Q60.** The shortest distance between the lines $\vec{r} = (1, 0, 0) + s(1, 1, 0)$ and $\vec{r} = (0, 1, 1) + t(0, 1, 1)$ is:
- (A) 1
(B) $\frac{1}{2}$
(C) $\frac{1}{\sqrt{3}}$
(D) $\sqrt{3}$
- Q61.** If $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $|\vec{a} + \vec{b}| = 5$, then $|\vec{a} - \vec{b}|$ equals:
- (A) 3
(B) 5
(C) 7
(D) 1
- Q62.** If $f(x) = x^3 + x$, then $(f^{-1})'(2)$ equals:
- (A) 4



- (B) 2
- (C) $\frac{1}{2}$
- (D) $\frac{1}{4}$

Q63. The value of $\int_0^a x(a-x) dx$ is:

- (A) $\frac{a^3}{6}$
- (B) $\frac{a^3}{3}$
- (C) $\frac{a^2}{6}$
- (D) $\frac{a^2}{3}$

Q64. The general solution of $\frac{dy}{dx} = (x+y)^2$ is:

- (A) $y = \tan x + C$
- (B) $y = \tan(x+y) + C$
- (C) $y = \tan(x+C) - x$
- (D) $y = \cot(x+C) - x$

Q65. A rectangle is inscribed under the curve $y = 12 - x^2$ in the first quadrant with two sides along the coordinate axes. Its maximum area is:

- (A) 12
- (B) 16
- (C) 18
- (D) 24

Q66. Let z be a complex number satisfying $z^2 + 2z + 2 = 0$. Which of the following statements are correct?

- (A) The roots are $-1 + i$ and $-1 - i$.



- (B) The modulus of each root is $\sqrt{2}$.
- (C) $z^4 + 4 = 0$ for both roots.
- (D) The principal argument of one of the roots is $\frac{\pi}{4}$.

Q67. For $\theta \in (0, \pi/2)$, if $\tan \theta = 2$, which of the following expressions are correct?

- (A) $\sin 2\theta = \frac{4}{5}$.
- (B) $\cos 2\theta = -\frac{3}{5}$.
- (C) $\tan(\frac{\pi}{4} + \theta) = -3$.
- (D) $\sin \theta + \cos \theta = \frac{3}{\sqrt{5}}$.

Q68. Let \vec{a} and \vec{b} be unit vectors such that the angle between them is 60° . Which of the following must be true?

- (A) $|\vec{a} - \vec{b}| = 1$.
- (B) $|\vec{a} + \vec{b}| = \sqrt{3}$.
- (C) $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 0$.
- (D) $|\vec{a} \times \vec{b}| = \frac{\sqrt{3}}{2}$.

Q69. For the function $f(x) = x^2 e^{-x}$ defined on $[0, \infty)$, which of the following hold true?

- (A) $f(x)$ is strictly increasing in $(0, 2)$.
- (B) $x = 2$ is a point of local maximum.
- (C) $\int_0^\infty x^2 e^{-x} dx = 2$.
- (D) $\lim_{x \rightarrow \infty} f(x) = 0$.

Q70. Let $I_n = \int_0^{\pi/2} \sin^n x dx$. Which of the following relations are correct?

- (A) $I_n = \frac{n-1}{n} I_{n-2}$ for $n > 1$.
- (B) $I_3 = \frac{2}{3}$.
- (C) $I_4 = \frac{3\pi}{16}$.
- (D) $I_2 = \frac{\pi}{4}$.



- Q71.** Consider the function $f(x) = |x - 1| + |x + 1|$. Which of the following statements are correct?
- (A) $f(x) = 2$ for all $x \in [-1, 1]$.
 - (B) The function is differentiable at $x = 1$.
 - (C) $f(x)$ is an even function.
 - (D) The derivative $f'(x) = 2$ for all $x > 0$.
- Q72.** For the function $f(x) = \ln(1 + x^2)$, which of the following properties hold true?
- (A) $f'(x)$ is an odd function.
 - (B) $f(x)$ is concave down for all $x \in \mathbb{R}$.
 - (C) $f''(0) = 2$.
 - (D) The function has a global maximum at $x = 0$.
- Q73.** Let f be a continuous function on $[0, 1]$ such that $f(x) + f(1 - x) = 1$. Which of the following must be true?
- (A) $\int_0^1 f(x) dx = \frac{1}{2}$.
 - (B) $f(1/2) = 1/2$.
 - (C) $f(x)$ must be a linear function.
 - (D) $\int_0^1 f(x) dx = 1$.
- Q74.** Let $g(x) = \frac{\sin x}{x}$ for $x \neq 0$ and $g(0) = 1$. Which of the following statements are correct?
- (A) $g(x)$ is continuous at $x = 0$.
 - (B) $g(x)$ is an odd function.
 - (C) $\lim_{x \rightarrow 0} g'(x) = 0$.
 - (D) $g''(0) = 1$.
- Q75.** For the cubic polynomial $f(x) = x^3 - 3ax$ where $a > 0$, identify the correct statements:



- (A) The slope of the tangent at $x = 0$ is $-3a$.
- (B) $x = \sqrt{a}$ is a point of local minimum.
- (C) $f(x)$ is strictly increasing on the interval $(-\sqrt{a}, \sqrt{a})$.
- (D) The local maximum value of the function is $3a$.



Detailed Solutions

Q1.

Solution

Concept:

In an Arithmetic Progression (A.P.), the n th term is given by $T_n = A + (n - 1)D$, where A is the first term and D is the common difference. For p th, q th, and r th terms, we can express a , b , and c in terms of A and D , and then substitute these into the cyclic expression $a(q - r) + b(r - p) + c(p - q)$ to evaluate it.

Solution:

Step 1: Let the first term of the A.P. be A and the common difference be D . The given terms are:

$$a = A + (p - 1)D \quad b = A + (q - 1)D \quad c = A + (r - 1)D$$

Step 2: Substitute these values into the given expression $a(q - r) + b(r - p) + c(p - q)$:

$$[A + (p - 1)D](q - r) + [A + (q - 1)D](r - p) + [A + (r - 1)D](p - q)$$

Step 3: Expand the expression by grouping terms with A and terms with D : $A[(q - r) + (r - p) + (p - q)] + D[(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)]$

Step 4: Simplify the coefficient of A : $(q - r + r - p + p - q) = 0$

Step 5: Simplify the coefficient of D by expanding the products: $[pq - pr - q + r] + [qr - qp - r + p] + [rp - rq - p + q]$ Combining like terms: $(pq - qp) + (qr - rq) + (rp - pr) + (-q + q) + (r - r) + (-p + p) = 0$

Step 6: Calculate the final value: $A(0) + D(0) = 0$

Final Answer:

Answer: (A)

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Q2.

Solution**Concept:**

This problem utilizes fundamental logarithmic properties: the product rule $\log_b m + \log_b n = \log_b(mn)$ and the definition of a logarithm $\log_b a = c \iff b^c = a$. We need to isolate the product xy using these rules.

Solution:

Step 1: Consider the first given equation: $\log_2 x + \log_2 y = 4$. Applying the product rule for logarithms, we get: $\log_2(xy) = 4$

Step 2: Convert the logarithmic form from Step 1 into exponential form using the base 2: $xy = 2^4$
 $xy = 16$

Step 3: Consider the second given equation: $\log_2(x + y) = 3$. Converting this to exponential form gives: $x + y = 2^3 = 8$

Step 4: The question specifically asks for the value of xy . From Step 2, we have already determined that $xy = 16$.

Step 5: Verify the consistency. We have $x + y = 8$ and $xy = 16$. These are roots of the equation $t^2 - 8t + 16 = 0$, which is $(t - 4)^2 = 0$. Thus, $x = 4$ and $y = 4$. Since both are positive real numbers, the condition is satisfied.

Final Answer:

Answer: (B)

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Q3.

Solution**Concept:**

A number is divisible by 5 if its last digit is either 0 or 5. Since we are forming five-digit numbers from the set $\{0, 1, 2, 3, 4, 5\}$ without repetition, we must consider two mutually exclusive cases based on whether the last digit is 0 or 5, because the presence of 0 affects the choices for the first digit (which cannot be 0).

Solution:

Step 1: Case 1 - Last digit is 0. The last position is fixed (1 way). The remaining 4 positions must be filled by 4 digits chosen from $\{1, 2, 3, 4, 5\}$. Number of ways = $P(5, 4) = 5 \times 4 \times 3 \times 2 = 120$.

Step 2: Case 2 - Last digit is 5. The last position is fixed (1 way). For the first digit, we cannot use 5 (already used) and we cannot use 0 (to maintain a 5-digit number). Available digits for the 1st position: $\{1, 2, 3, 4\}$ (4 ways). For the remaining 3 middle positions, we have 4 digits remaining (including 0). Number of ways = $4 \times P(4, 3) = 4 \times (4 \times 3 \times 2) = 4 \times 24 = 96$.

Step 3: Total number of ways. Sum the results from both cases: $120 + 96 = 216$.

Final Answer:

Answer: (C)

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Q4.

Solution

Concept:

To find the coefficient of x^8 in the product $(1 + x^2)^6(1 + x)^4$, we use the general term of a binomial expansion $T_{r+1} = \binom{n}{r}a^{n-r}b^r$. We expand both binomials and find combinations of powers that sum to 8.

Solution:

Step 1: Expand $(1 + x^2)^6$ using the general term $C(6, r)(x^2)^r = C(6, r)x^{2r}$. Possible powers of x are 0, 2, 4, 6, 8, 10, 12 with coefficients $\binom{6}{0}, \binom{6}{1}, \binom{6}{2}, \binom{6}{3}, \binom{6}{4}, \binom{6}{5}, \binom{6}{6}$.

Step 2: Expand $(1 + x)^4$ using the general term $C(4, k)x^k$. Possible powers of x are 0, 1, 2, 3, 4 with coefficients $\binom{4}{0}, \binom{4}{1}, \binom{4}{2}, \binom{4}{3}, \binom{4}{4}$.

Step 3: Identify pairs $(2r, k)$ such that $2r + k = 8$:

1. $2r = 4 \implies r = 2$ and $k = 4$. 2. $2r = 6 \implies r = 3$ and $k = 2$. 3. $2r = 8 \implies r = 4$ and $k = 0$.

Step 4: Calculate the coefficients for each pair:

1. $\binom{6}{2} \times \binom{4}{4} = 15 \times 1 = 15$ 2. $\binom{6}{3} \times \binom{4}{2} = 20 \times 6 = 120$ 3. $\binom{6}{4} \times \binom{4}{0} = 15 \times 1 = 15$

Step 5: Sum the coefficients: $15 + 120 + 15 = 150$.

Final Answer:

Answer: (D) [Go Back to Question 4](#)

Q5.

Solution

Concept:

The sum of the first n terms of an A.P. is $S_n = \frac{n}{2}[2a + (n - 1)d]$. We are given two sums (S_5 and S_{10}), which creates a system of two linear equations. Solving these gives us the first term a and the common difference d . The 11th term is then found using $T_{11} = a + 10d$.

Solution:

Step 1: Use $S_5 = 40$. $\frac{5}{2}[2a + 4d] = 40 \implies 5(a + 2d) = 40 \implies a + 2d = 8$ (Eq. 1)

Step 2: Use $S_{10} = 130$. $\frac{10}{2}[2a + 9d] = 130 \implies 5(2a + 9d) = 130 \implies 2a + 9d = 26$ (Eq. 2)

Step 3: Solve the system. Multiply Eq. 1 by 2: $2a + 4d = 16$ (Eq. 3) Subtract Eq. 3 from Eq. 2: $(2a + 9d) - (2a + 4d) = 26 - 16 \implies 5d = 10 \implies d = 2$

Step 4: Substitute $d = 2$ into Eq. 1: $a + 2(2) = 8 \implies a + 4 = 8 \implies a = 4$

Step 5: Find the 11th term: $T_{11} = a + (11 - 1)d = 4 + 10(2) = 4 + 20 = 24$

Final Answer:

Answer: (B) [Go Back to Question 5](#)



Q6.

Solution

Concept:

This problem involves the change of base property and the substitution method for logarithmic equations. Specifically, we use $\log_a b = \frac{1}{\log_b a}$ and $\log_a (b^n) = n \log_a b$. By setting $\log_4 x = y$, we can transform the logarithmic equation into a quadratic equation.

Solution:

Step 1: Express $\log_x 16$ in terms of $\log_4 x$. Using the base change rule: $\log_x 16 = \frac{\log_4 16}{\log_4 x}$

Step 2: Simplify $\log_4 16$. Since $16 = 4^2$, we have: $\log_4 16 = 2$. So, the equation becomes: $\frac{2}{\log_4 x} + \log_4 x = 5$.

Step 3: Let $y = \log_4 x$. Substitute y into the equation: $\frac{2}{y} + y = 5$

Step 4: Multiply the entire equation by y to form a quadratic equation: $2 + y^2 = 5y \implies y^2 - 5y + 2 = 0$

Step 5: The question asks for the sum of all possible values of $\log_4 x$, which are the roots y_1 and y_2 of this quadratic equation. For any quadratic $ay^2 + by + c = 0$, the sum of the roots is given by $-\frac{b}{a}$.

Step 6: Identify $a = 1$ and $b = -5$. Sum = $-\frac{-5}{1} = 5$.

Final Answer: 5

Answer: (C)

[Go Back to Question 6](#)

Q7.

Solution

Concept:

To solve an absolute value inequality involving multiple terms, we divide the number line into intervals based on the critical points where each absolute value expression equals zero. For $|x - 2| + |x + 1| \leq 7$, the critical points are $x = 2$ and $x = -1$.

Solution:

Step 1: Define the intervals: $x < -1$, $-1 \leq x \leq 2$, and $x > 2$.

Step 2: Case 1 ($x < -1$): Both terms are negative. $-(x - 2) - (x + 1) \leq 7 \implies -2x + 1 \leq 7 \implies -2x \leq 6 \implies x \geq -3$. In this interval, $x \in [-3, -1)$. Integers: $-3, -2$.

Step 3: Case 2 ($-1 \leq x \leq 2$): $(x + 1)$ is non-negative, $(x - 2)$ is non-positive. $-(x - 2) + (x + 1) \leq 7 \implies -x + 2 + x + 1 \leq 7 \implies 3 \leq 7$. This is always true. In this interval, $x \in [-1, 2]$. Integers: $-1, 0, 1, 2$.

Step 4: Case 3 ($x > 2$): Both terms are positive. $(x - 2) + (x + 1) \leq 7 \implies 2x - 1 \leq 7 \implies 2x \leq 8 \implies x \leq 4$. In this interval, $x \in (2, 4]$. Integers: $3, 4$.

Step 5: Combine all integers: $\{-3, -2, -1, 0, 1, 2, 3, 4\}$. Total count = 8.

Final Answer: 8

Answer: (D)

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Q8.

Solution

Concept:

The modulus of a complex number $z = a + bi$ is defined as $|z| = \sqrt{a^2 + b^2}$. The condition $|z - 1| = |z + 3|$ geometrically represents the locus of points equidistant from $(1, 0)$ and $(-3, 0)$ in the complex plane, which is the perpendicular bisector of the segment joining them.

Solution:

Step 1: Substitute $z = x + 2i$ into the given equation: $|(x + 2i) - 1| = |(x + 2i) + 3|$ $|(x - 1) + 2i| = |(x + 3) + 2i|$

Step 2: Apply the modulus formula $|a + bi| = \sqrt{a^2 + b^2}$: $\sqrt{(x - 1)^2 + 2^2} = \sqrt{(x + 3)^2 + 2^2}$

Step 3: Square both sides to eliminate the radicals: $(x - 1)^2 + 4 = (x + 3)^2 + 4$ $(x - 1)^2 = (x + 3)^2$

Step 4: Expand the squares: $x^2 - 2x + 1 = x^2 + 6x + 9$

Step 5: Cancel x^2 and solve for x : $-2x + 1 = 6x + 9 - 8 = 8x \implies x = -1$

Step 6: Geometrically, the midpoint of $(1, 0)$ and $(-3, 0)$ is $\frac{1-3}{2} = -1$, confirming that $x = -1$ is the perpendicular bisector $x = -1$.

Final Answer:

Answer: (B) [Go Back to Question 8](#)

Q9.

Solution

Concept:

The argument of a quotient of complex numbers is given by $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$. Additionally, $\arg(z^n) = n \arg(z)$. We first convert the base complex numbers $1 + i$ and $1 - i$ into their polar forms or simply find their individual arguments.

Solution:

Step 1: Find the argument of $(1 + i)$. Since it is in the first quadrant, $\arg(1 + i) = \tan^{-1}(1/1) = \frac{\pi}{4}$. Then, $\arg((1 + i)^6) = 6 \times \frac{\pi}{4} = \frac{3\pi}{2}$.

Step 2: Find the argument of $(1 - i)$. Since it is in the fourth quadrant, $\arg(1 - i) = \tan^{-1}(-1/1) = -\frac{\pi}{4}$. Then, $\arg((1 - i)^4) = 4 \times (-\frac{\pi}{4}) = -\pi$.

Step 3: Calculate the argument of the full expression: $\arg\left(\frac{(1+i)^6}{(1-i)^4}\right) = \arg((1 + i)^6) - \arg((1 - i)^4) = \frac{3\pi}{2} - (-\pi) = \frac{3\pi}{2} + \pi = \frac{5\pi}{2}$.

Step 4: Normalize to the principal argument range $(-\pi, \pi]$. $\frac{5\pi}{2}$ is coterminal with $\frac{5\pi}{2} - 2\pi = \frac{\pi}{2}$.

Step 5: Alternative check using powers: $(1 + i)^2 = 2i$ and $(1 - i)^2 = -2i$. $\frac{(2i)^3}{(-2i)^2} = \frac{8i^3}{4i^2} = \frac{-8i}{-4} = 2i$. The argument of $2i$ (a purely imaginary positive number) is $\frac{\pi}{2}$.

Final Answer:

Answer: (C) [Go Back to Question 9](#)



Q10.

Solution**Concept:**

For a quadratic equation $ax^2 + bx + c = 0$ to have two distinct real roots, its discriminant $D = b^2 - 4ac$ must be strictly greater than zero ($D > 0$). If $D = 0$, the roots are real and equal; if $D < 0$, the roots are non-real complex numbers.

Solution:

Step 1: Identify the coefficients from $x^2 - 2(k + 1)x + (k^2 + 3) = 0$: $a = 1$ $b = -2(k + 1)$
 $c = k^2 + 3$

Step 2: Set up the condition for distinct real roots: $b^2 - 4ac > 0$. $[-2(k + 1)]^2 - 4(1)(k^2 + 3) > 0$

Step 3: Expand the inequality: $4(k + 1)^2 - 4(k^2 + 3) > 0$

Step 4: Divide by 4 to simplify: $(k + 1)^2 - (k^2 + 3) > 0$ $k^2 + 2k + 1 - k^2 - 3 > 0$

Step 5: Simplify the remaining terms: $2k - 2 > 0$

Step 6: Solve for k : $2k > 2 \implies k > 1$

Step 7: Verify the logic. If $k = 1$, $D = 0$ (roots are $x = 2, 2$), which are not distinct. For any value $k \leq 1$, the condition for "distinct" real roots is not met.

Final Answer: $k > 1$

Answer: (A)

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Q11.

Solution

Concept:

This is the famous Vandermonde determinant of order 3. To solve it, we use elementary row or column operations to create zeros in a row or column, making it easier to expand. The goal is to extract common factors to reveal the cyclic product of the differences between the variables.

Solution:

Step 1: Perform column operations to create zeros in the first row. Let $C_2 \rightarrow C_2 - C_1$ and $C_3 \rightarrow C_3 - C_1$:

$$\begin{vmatrix} 1 & 0 & 0 \\ a & b - a & c - a \\ a^2 & b^2 - a^2 & c^2 - a^2 \end{vmatrix}$$

Step 2: Note that $(b^2 - a^2) = (b - a)(b + a)$ and $(c^2 - a^2) = (c - a)(c + a)$. Factor out $(b - a)$ from the second column and $(c - a)$ from the third column:

$$= (b - a)(c - a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b + a & c + a \end{vmatrix}$$

Step 3: Expand along the first row:

$$\begin{aligned} &= (b - a)(c - a)[1((c + a) - (b + a))] \\ &= (b - a)(c - a)(c - b) \end{aligned}$$

Step 4: Rearrange the terms to match the standard cyclic form. Since $(b - a) = -(a - b)$ and $(c - a) = -(a - c)$ and $(c - b) = -(b - c)$, we get:

$$= [-(a - b)][-(a - c)][-(b - c)] = (a - b)(b - c)(a - c)$$

Alternatively, rewriting $(b - a)(c - a)(c - b)$ as $-(a - b)(c - a)(c - b)$, we see Option A is $(a - b)(b - c)(c - a)$. By multiplying the signs, we find the identity:

$$(b - a)(c - a)(c - b) = (a - b)(b - c)(c - a)$$

Final Answer: $(a - b)(b - c)(c - a)$

Answer: (A)

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Q12.

Solution**Concept:**

For an invertible matrix A , the inverse A^{-1} exists such that $AA^{-1} = A^{-1}A = I$. To find an expression for A^{-1} from a matrix equation, we can manipulate the equation to isolate I on one side and then multiply the entire equation by A^{-1} .

Solution:

Step 1: Start with the given matrix equation: $A^2 = 3A - 2I$

Step 2: Rearrange the equation to place the term involving the identity matrix I on one side: $2I = 3A - A^2$

Step 3: Since A is invertible, we can multiply both sides of the equation by A^{-1} from either the left or the right: $2I(A^{-1}) = (3A - A^2)A^{-1}$

Step 4: Use the distributive property and the definition of the inverse ($AA^{-1} = I$): $2A^{-1} = 3(AA^{-1}) - A(AA^{-1})$ $2A^{-1} = 3I - AI$

Step 5: Simplify the expression on the right: $2A^{-1} = 3I - A$

Step 6: Divide by 2 to solve for A^{-1} : $A^{-1} = \frac{3I - A}{2}$

This matches Option D exactly.

Final Answer: $\frac{3I - A}{2}$

Answer: (D)

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Q13.

Solution**Concept:**

This problem requires the application of three important determinant properties:

1. $|kA| = k^n|A|$ for an $n \times n$ matrix. 2. $|\text{adj}(M)| = |M|^{n-1}$ for an $n \times n$ matrix. 3. The determinant of a scalar multiple within the adjoint.

Solution:

Step 1: Let $M = 2A$. We need to find $|\text{adj}(M)|$. Since A is a 3×3 matrix, M is also a 3×3 matrix.

Step 2: Apply the property $|\text{adj}(M)| = |M|^{3-1} = |M|^2$. So, $|\text{adj}(2A)| = |2A|^2$.

Step 3: Calculate $|2A|$ using the property $|kA| = k^n|A|$ where $n = 3$: $|2A| = 2^3|A| = 8|A|$.

Step 4: Substitute the given value $|A| = 2$ into the expression: $|2A| = 8 \times 2 = 16$.

Step 5: Now, calculate $|2A|^2$ as determined in Step 2: $|\text{adj}(2A)| = (16)^2 = 256$.

Step 6: Double-check the logic. $|2A| = 2^3 \times 2 = 16$. Adjoint of a 3×3 matrix M has determinant $|M|^2$. Thus $16^2 = 256$.

Final Answer: 256

Answer: (A)

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Q14.

Solution

Concept:

The Cayley-Hamilton Theorem states that every square matrix satisfies its own characteristic equation, defined by $|A - \lambda I| = 0$. For a 2×2 matrix, the equation is $A^2 - \text{tr}(A)A + |A|I = 0$. We can use this to simplify the given expression $A^2 - 5A + I$ before finding its determinant.

Solution:

Step 1: Find the trace and determinant of $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. $\text{tr}(A) = 1 + 4 = 5$ $|A| = (1 \times 4) - (2 \times 3) = 4 - 6 = -2$

Step 2: Write the characteristic equation using Cayley-Hamilton: $A^2 - (\text{tr}(A))A + |A|I = O$
 $A^2 - 5A - 2I = O \implies A^2 - 5A = 2I$

Step 3: Substitute $A^2 - 5A = 2I$ into the required expression: $A^2 - 5A + I = 2I + I = 3I$

Step 4: Find the determinant of the resulting matrix $3I$: $|3I| = \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} |3I| = 3^2 |I| = 9 \times 1 = 9$

Step 5: Alternatively, compute A^2 directly: $A^2 = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}$. $A^2 - 5A + I = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} - \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$. The determinant is $3 \times 3 = 9$.

Final Answer:

Answer: (B)

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Q15.

Solution**Concept:**

To find the intersection of two sets A and B , we must first determine the intervals of x that satisfy the given inequalities. Set A is defined by a quadratic inequality, while set B is defined by an absolute value inequality. The intersection $A \cap B$ consists of all real numbers that satisfy both conditions simultaneously.

Solution:

Step 1: Solve the inequality for set A . Given $x^2 - 5x + 6 \leq 0$, we factor the quadratic expression:

$$(x - 2)(x - 3) \leq 0$$

The roots are $x = 2$ and $x = 3$. Testing the intervals, the inequality holds for $x \in [2, 3]$. Thus, $A = [2, 3]$.

Step 2: Solve the inequality for set B . Given $|x - 2| < 2$, we rewrite this as:

$$-2 < x - 2 < 2$$

Adding 2 to all parts of the inequality gives:

$$0 < x < 4$$

Thus, $B = (0, 4)$.

Step 3: Find the intersection $A \cap B$. We look for the common region between the closed interval $[2, 3]$ and the open interval $(0, 4)$. Since both 2 and 3 lie within the boundaries of $(0, 4)$, the entire set A is contained within B .

$$[2, 3] \cap (0, 4) = [2, 3]$$

The resulting set is the closed interval from 2 to 3.

Final Answer: $[2, 3]$

Answer: (B)

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Q16.

Solution**Concept:**

An equivalence relation must satisfy three properties: Reflexivity (aRa), Symmetry (if aRb then bRa), and Transitivity (if aRb and bRc then aRc). Here, the relation is defined by congruence modulo 3, where aRb means $a - b = 3k$ for some integer k .

Solution:

Step 1: Check for Reflexivity. For any integer a , consider $a - a = 0$. Since 0 is divisible by 3 ($0 = 3 \times 0$), aRa holds for all $a \in \mathbb{Z}$. Thus, R is reflexive.

Step 2: Check for Symmetry. Assume aRb , which means $a - b = 3k$ for some integer k . Then, $b - a = -(a - b) = -3k = 3(-k)$. Since $-k$ is also an integer, $b - a$ is divisible by 3, so bRa holds. Thus, R is symmetric.

Step 3: Check for Transitivity. Assume aRb and bRc . Then $a - b = 3k$ and $b - c = 3m$ for integers k, m . Adding these equations:

$$(a - b) + (b - c) = 3k + 3m \implies a - c = 3(k + m)$$

Since $k + m$ is an integer, $a - c$ is divisible by 3, so aRc holds. Thus, R is transitive.

Since R is reflexive, symmetric, and transitive, it is an equivalence relation.

Final Answer:

Answer: (C)

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Q17.

Solution**Concept:**

A composite function $f(f(x))$ is obtained by substituting the expression for $f(x)$ into the variable x within the function f itself. This requires careful algebraic simplification of complex fractions to reach the final form.

Solution:

Step 1: Write down the expression for $f(f(x))$ by substituting $f(x) = \frac{x-1}{x+1}$ into the function:

$$f(f(x)) = \frac{f(x) - 1}{f(x) + 1} = \frac{\frac{x-1}{x+1} - 1}{\frac{x-1}{x+1} + 1}$$

Step 2: Simplify the numerator and the denominator by taking the common denominator $(x + 1)$:

$$\text{Numerator: } \frac{(x-1) - (x+1)}{x+1} = \frac{x-1-x-1}{x+1} = \frac{-2}{x+1}$$

$$\text{Denominator: } \frac{(x-1) + (x+1)}{x+1} = \frac{x-1+x+1}{x+1} = \frac{2x}{x+1}$$

Step 3: Combine the simplified numerator and denominator:

$$f(f(x)) = \frac{\frac{-2}{x+1}}{\frac{2x}{x+1}} = \frac{-2}{2x}$$

Step 4: Cancel the common factor of 2:

$$f(f(x)) = -\frac{1}{x}$$

This shows that the composite function results in the negative reciprocal of the input x .

Final Answer: $-\frac{1}{x}$

Answer: (D)

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Q18.

Solution**Concept:**

When three fair dice are thrown, the total number of possible outcomes is $6^3 = 216$. We need to find the number of favorable outcomes where the sum of the numbers on the three faces equals 5 and then divide by the total outcomes.

Solution:

Step 1: Determine the total number of outcomes. Since each die has 6 faces, for 3 dice, total outcomes = $6 \times 6 \times 6 = 216$.

Step 2: List the combinations of three numbers (d_1, d_2, d_3) such that their sum is 5, where $1 \leq d_i \leq 6$: * $(1, 1, 3)$: This can occur in $\frac{3!}{2!} = 3$ ways: $(1, 1, 3), (1, 3, 1), (3, 1, 1)$. * $(1, 2, 2)$: This can occur in $\frac{3!}{2!} = 3$ ways: $(1, 2, 2), (2, 1, 2), (2, 2, 1)$.

Step 3: Calculate the total number of favorable outcomes. Summing the permutations from Step 2:

$$\text{Favorable outcomes} = 3 + 3 = 6$$

Step 4: Calculate the probability $P(S = 5)$:

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{6}{216}$$

Step 5: Simplify the fraction:

$$\frac{6}{216} = \frac{1}{36}$$

The probability of obtaining a sum of 5 with three dice is therefore $1/36$.

Final Answer: $\frac{1}{36}$

Answer: (A)

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Q19.

Solution**Concept:**

This is a conditional probability problem. We use the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$, where A is the event "both balls are red" and B is the event "at least one ball is red". Drawing is done without replacement from a total of 7 balls (4 Red, 3 Blue).

Solution:

Step 1: Calculate the total number of ways to draw 2 balls from 7:

$${}^7C_2 = \frac{7 \times 6}{2 \times 1} = 21$$

Step 2: Find the number of ways to draw at least one red ball (Event B). It is easier to subtract the case of drawing no red balls (all blue) from the total:

$$\text{No Red (2 Blue)} = {}^3C_2 = 3$$

$$\text{At least one Red} = 21 - 3 = 18$$

Step 3: Find the number of ways both balls are red (Event $A \cap B$). Since "both red" is a subset of "at least one red", this is just the number of ways to pick 2 red balls from 4:

$${}^4C_2 = \frac{4 \times 3}{2 \times 1} = 6$$

Step 4: Apply the conditional probability formula:

$$P(\text{Both Red} | \text{At least one Red}) = \frac{6}{18} = \frac{1}{3}$$

The probability is $1/3$ given the constraint.

Final Answer: $\frac{1}{3}$

Answer: (B)

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Q20.

Solution**Concept:**

The variance σ^2 of a set of observations x_i is related to the mean μ and the sum of the squares of the observations by the formula:

$$\sigma^2 = \frac{\sum x_i^2}{n} - \mu^2$$

where n is the number of observations. We are given $n = 5$, $\mu = 12$, and $\sigma^2 = 8$.

Solution:

Step 1: Identify the given values from the problem: $n = 5$, \bar{x} (mean) = 12, and $\text{Var}(x) = 8$.

Step 2: Rearrange the variance formula to solve for the sum of squares $\sum x_i^2$:

$$\text{Var}(x) + \bar{x}^2 = \frac{\sum x_i^2}{n}$$

$$\sum x_i^2 = n \times (\text{Var}(x) + \bar{x}^2)$$

Step 3: Substitute the known values into the equation:

$$\sum x_i^2 = 5 \times (8 + 12^2)$$

$$\sum x_i^2 = 5 \times (8 + 144)$$

Step 4: Perform the final calculation:

$$\sum x_i^2 = 5 \times 152$$

$$\sum x_i^2 = 760$$

Thus, the sum of the squares of the 5 observations is 760.

Final Answer:

Answer: (C)

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Q21.

Solution**Concept:**

To solve a trigonometric equation involving $\sin x$ and $\cos x$, we can rearrange the terms to form a single trigonometric ratio, such as $\tan x$. This simplifies the equation into a standard form $\tan x = \text{constant}$, which can then be solved within the given interval $(0, \pi)$.

Solution:

Step 1: Write down the given equation:

$$\sin x + \cos x = \sqrt{2} \cos x$$

Step 2: Isolate the $\sin x$ term by moving $\cos x$ to the right-hand side:

$$\sin x = \sqrt{2} \cos x - \cos x$$

$$\sin x = (\sqrt{2} - 1) \cos x$$

Step 3: Divide both sides by $\cos x$ (noting that $\cos x \neq 0$ for this equality to hold) to find $\tan x$:

$$\frac{\sin x}{\cos x} = \sqrt{2} - 1$$

$$\tan x = \sqrt{2} - 1$$

Step 4: Identify the value of x . We know from standard half-angle or specific trigonometric values that $\tan(\pi/8) = \sqrt{2} - 1$. Since $x \in (0, \pi)$ and the value is positive, x must be in the first quadrant.

$$x = \frac{\pi}{8}$$

Thus, the value of x satisfying the equation is $\pi/8$.

Final Answer: $\frac{\pi}{8}$

Answer: (B)

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Q22.

Solution**Concept:**

To find the sum of inverse tangent functions, we use the identity $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$, provided $xy < 1$. We can apply this formula iteratively to simplify multiple terms into a single inverse tangent expression.

Solution:

Step 1: Apply the formula to the first two terms $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{5}$:

$$\tan^{-1} \left(\frac{\frac{1}{2} + \frac{1}{5}}{1 - \frac{1}{10}} \right) = \tan^{-1} \left(\frac{\frac{7}{10}}{\frac{9}{10}} \right) = \tan^{-1} \frac{7}{9}$$

Step 2: Add the result to the third term $\tan^{-1} \frac{1}{8}$:

$$\tan^{-1} \frac{7}{9} + \tan^{-1} \frac{1}{8} = \tan^{-1} \left(\frac{\frac{7}{9} + \frac{1}{8}}{1 - \frac{7}{72}} \right)$$

Step 3: Simplify the numerator and denominator:

$$\text{Numerator: } \frac{56 + 9}{72} = \frac{65}{72}$$

$$\text{Denominator: } \frac{72 - 7}{72} = \frac{65}{72}$$

Step 4: Calculate the final ratio:

$$\tan^{-1} \left(\frac{65/72}{65/72} \right) = \tan^{-1}(1)$$

Step 5: Determine the angle:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

The simplified value of the expression is $\pi/4$.

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (A)

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Q23.

Solution**Concept:**

If x_1 and x_2 are roots of the quadratic equation $ax^2 + bx + c = 0$, then the sum of roots is $-b/a$ and the product is c/a . Here, $\tan A$ and $\tan B$ are the roots. We use the identity $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ to find $A + B$.

Solution:

Step 1: Identify the sum and product of the roots from the equation $x^2 - 3x + 1 = 0$:

$$\text{Sum: } \tan A + \tan B = -\frac{-3}{1} = 3$$

$$\text{Product: } \tan A \tan B = \frac{1}{1} = 1$$

Step 2: Substitute these values into the compound angle formula for tangent:

$$\tan(A + B) = \frac{3}{1 - 1}$$

$$\tan(A + B) = \frac{3}{0}$$

Step 3: Interpret the result. When the denominator of the tangent ratio is zero, the tangent of the angle is undefined (approaching infinity).

$$\tan(A + B) \rightarrow \infty$$

Step 4: Find the angle $A + B$. Since $A, B \in (0, \pi/2)$, their sum must be in $(0, \pi)$. The only angle in this range where the tangent is undefined is $\pi/2$.

$$A + B = \frac{\pi}{2}$$

Thus, the sum of the angles is $\pi/2$.

Final Answer: $\frac{\pi}{2}$

Answer: (D)

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Q24.

Solution

Concept:

A line passing through the intersection of two lines L_1 and L_2 can be represented as $L_1 + \lambda L_2 = 0$. Alternatively, we can find the specific point of intersection first. Since the required line is parallel to $x - y = 0$, its slope must be the same (slope $m = 1$).

Solution:

Step 1: Find the intersection point of $2x - y + 3 = 0$ and $x + 2y - 4 = 0$. Multiply the first equation by 2: $4x - 2y + 6 = 0$. Add to the second equation: $(4x - 2y + 6) + (x + 2y - 4) = 0 \implies 5x + 2 = 0 \implies x = -2/5$.

Step 2: Substitute $x = -2/5$ into the first equation to find y : $2(-2/5) - y + 3 = 0 \implies -4/5 + 3 = y \implies y = 11/5$. The point is $(-2/5, 11/5)$.

Step 3: Write the equation of a line parallel to $x - y = 0$. Such a line has the form $x - y + k = 0$.

Step 4: Substitute the intersection point into this form to find k : $-2/5 - 11/5 + k = 0 \implies -13/5 + k = 0 \implies k = 13/5$.

Step 5: Write the final equation: $x - y + 13/5 = 0 \implies 5x - 5y + 13 = 0$.

This line passes through the required intersection and maintains the necessary slope.

Final Answer: $5x - 5y + 13 = 0$

Answer: (C)

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Q25.

Solution**Concept:**

A circle with its center on the x -axis has the coordinates $(h, 0)$. If it touches the y -axis, the distance from the center to the y -axis must equal the radius. This implies the radius r is equal to $|h|$. The equation of such a circle is $(x - h)^2 + y^2 = h^2$.

Solution:

Step 1: Set up the equation of the circle. Since the center is $(h, 0)$ and it touches the y -axis, the radius is h . The equation is:

$$(x - h)^2 + y^2 = h^2$$

Expanding this: $x^2 - 2xh + h^2 + y^2 = h^2$, which simplifies to $x^2 + y^2 - 2xh = 0$.

Step 2: Substitute the given point $(2, 3)$ into the equation because the circle passes through it:

$$(2)^2 + (3)^2 - 2(2)h = 0$$

$$4 + 9 - 4h = 0$$

Step 3: Solve for h :

$$13 - 4h = 0 \implies 4h = 13 \implies h = \frac{13}{4}$$

Step 4: Since the radius $r = |h|$, we find:

$$r = \frac{13}{4}$$

The distance from the center $(13/4, 0)$ to the point $(2, 3)$ is indeed $13/4$, confirming the consistency of the result.

Final Answer: $\frac{13}{4}$

Answer: (D)

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Q26.

Solution**Concept:**

For a parabola $y^2 = 4ax$, the parametric coordinates of any point are given by $(at^2, 2at)$. The equation of the tangent at a point with parameter t is given by the formula $ty = x + at^2$. This formula allows us to find the linear equation representing the tangent line directly from the parameter.

Solution:

Step 1: Identify the value of a from the parabola $y^2 = 8x$. Comparing with $y^2 = 4ax$, we get $4a = 8$, which means $a = 2$.

Step 2: Use the given parameter $t = 2$ to find the coordinates of the point of tangency: $x = at^2 = 2(2^2) = 8$ $y = 2at = 2(2)(2) = 8$ The point is $(8, 8)$.

Step 3: Apply the tangent formula $ty = x + at^2$ with $a = 2$ and $t = 2$:

$$2y = x + 2(2^2)$$

$$2y = x + 8$$

Step 4: Rearrange the equation into the standard form $Ax + By + C = 0$:

$$x - 2y + 8 = 0$$

This linear equation represents the line that just touches the parabola at the point $(8, 8)$.

Final Answer: $x - 2y + 8 = 0$

Answer: (B) [Go Back to Question 26](#)



Q27.

Solution**Concept:**

The eccentricity e of an ellipse measures how much it deviates from being a circle. For an ellipse in standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if $a > b$, the eccentricity is calculated using the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution:

Step 1: Convert the given equation $9x^2 + 25y^2 = 225$ into the standard form by dividing both sides by 225:

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1$$

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Step 2: Identify the values of a^2 and b^2 . Here, $a^2 = 25$ and $b^2 = 9$. Since $a^2 > b^2$, the major axis is along the x -axis.

Step 3: Substitute the values into the eccentricity formula:

$$e = \sqrt{1 - \frac{9}{25}}$$

Step 4: Simplify the expression inside the square root:

$$e = \sqrt{\frac{25 - 9}{25}} = \sqrt{\frac{16}{25}}$$

Step 5: Take the square root:

$$e = \frac{4}{5}$$

The eccentricity is $4/5$, indicating a relatively elongated ellipse.

Final Answer: $\frac{4}{5}$

Answer: (A)

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Q28.

Solution**Concept:**

Asymptotes are lines that the branches of a hyperbola approach as x and y tend to infinity. For a hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the equations of the asymptotes are given by $y = \pm \frac{b}{a}x$.

Solution:

Step 1: Write the given equation $4x^2 - 9y^2 = 36$ in standard form by dividing by 36:

$$\frac{4x^2}{36} - \frac{9y^2}{36} = 1$$

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

Step 2: Identify a^2 and b^2 from the denominators. $a^2 = 9 \implies a = 3$ $b^2 = 4 \implies b = 2$

Step 3: Substitute a and b into the asymptote formula $y = \pm \frac{b}{a}x$:

$$y = \pm \frac{2}{3}x$$

Step 4: Verify the result. The asymptotes pass through the center $(0, 0)$ and their slopes are determined by the ratio of the semi-conjugate axis to the semi-transverse axis. Here, the slopes are $+2/3$ and $-2/3$.

Final Answer: $y = \pm \frac{2}{3}x$

Answer: (C)

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Q29.

Solution**Concept:**

To find the length of a chord cut from a circle by a line, we use the geometric property of a circle: the perpendicular from the center to a chord bisects the chord. If r is the radius and d is the perpendicular distance from the center to the line, the length of the chord is $2\sqrt{r^2 - d^2}$.

Solution:

Step 1: Identify the center and radius of the circle $x^2 + y^2 = 25$. The center is $C(0, 0)$ and the radius is $r = \sqrt{25} = 5$.

Step 2: Calculate the perpendicular distance d from the center $(0, 0)$ to the line $3x + 4y - 15 = 0$:

$$d = \frac{|3(0) + 4(0) - 15|}{\sqrt{3^2 + 4^2}} = \frac{|-15|}{\sqrt{9 + 16}} = \frac{15}{5} = 3$$

Step 3: Use the chord length formula $L = 2\sqrt{r^2 - d^2}$:

$$L = 2\sqrt{5^2 - 3^2}$$

$$L = 2\sqrt{25 - 9}$$

Step 4: Calculate the final value:

$$L = 2\sqrt{16} = 2 \times 4 = 8$$

The line $3x + 4y = 15$ cuts a chord of length 8 units from the circle.

Final Answer:

Answer: (D)

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Q30.

Solution**Concept:**

To find the focus of a parabola given in general form, we must first convert it into the standard form $(y - k)^2 = 4a(x - h)$ by completing the square for the y -terms. Once in this form, the vertex is (h, k) and the focus is $(h + a, k)$.

Solution:

Step 1: Start with $y^2 - 4y - 8x + 12 = 0$ and complete the square for y :

$$y^2 - 4y = 8x - 12$$

Add $(4/2)^2 = 4$ to both sides:

$$y^2 - 4y + 4 = 8x - 12 + 4$$

$$(y - 2)^2 = 8x - 8$$

Step 2: Factor the right-hand side to get the standard form:

$$(y - 2)^2 = 8(x - 1)$$

Step 3: Identify h, k , and a . By comparison with $(y - k)^2 = 4a(x - h)$: $k = 2, h = 1$, and $4a = 8 \implies a = 2$.

Step 4: Determine the coordinates of the focus. For a parabola opening to the right, the focus is $(h + a, k)$:

$$\text{Focus} = (1 + 2, 2) = (3, 2)$$

The vertex is at $(1, 2)$ and shifting a units to the right gives the focus at $(3, 2)$.

Final Answer: $(3, 2)$

Answer: (A)

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Q31.

Solution**Concept:**

A line given in the intercept form $\frac{x}{a} + \frac{y}{b} = 1$ intersects the x -axis at $(a, 0)$ and the y -axis at $(0, b)$. These points, along with the origin $(0, 0)$, form a right-angled triangle. The area of such a triangle is given by the formula $\text{Area} = \frac{1}{2}|a \cdot b|$.

Solution:

Step 1: Identify the x -intercept and y -intercept from the given equation:

$$\frac{x}{3} + \frac{y}{4} = 1$$

Here, the x -intercept $a = 3$ and the y -intercept $b = 4$.

Step 2: Determine the vertices of the triangle. The triangle is formed by the points $(0, 0)$, $(3, 0)$, and $(0, 4)$.

Step 3: Calculate the area using the area formula for a right-angled triangle:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$\text{Area} = \frac{1}{2} \times 3 \times 4$$

Step 4: Perform the multiplication:

$$\text{Area} = \frac{12}{2} = 6$$

The area of the triangle formed by the line and the coordinate axes is 6 square units.

Final Answer:

Answer: (B)

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Q32.

Solution**Concept:**

The equation of a plane passing through a point (x_1, y_1, z_1) with a normal vector $\vec{n} = A\hat{i} + B\hat{j} + C\hat{k}$ is given by the formula $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$. This represents all points (x, y, z) such that the vector from the fixed point to (x, y, z) is perpendicular to the normal.

Solution:

Step 1: Identify the given point and normal vector: Point $(x_1, y_1, z_1) = (1, -2, 3)$ Normal vector coefficients: $A = 2, B = 1, C = -1$

Step 2: Substitute these values into the point-normal form of the plane equation:

$$2(x - 1) + 1(y - (-2)) + (-1)(z - 3) = 0$$

Step 3: Expand the parentheses:

$$2x - 2 + y + 2 - z + 3 = 0$$

Step 4: Simplify and group the terms:

$$2x + y - z + 3 = 0$$

This linear equation in three variables represents the unique plane perpendicular to the given vector that passes through the specified point.

Final Answer: $2x + y - z + 3 = 0$

Answer: (C)

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Q33.

Solution**Concept:**

The perpendicular distance d from a point (x_1, y_1, z_1) to a plane $Ax + By + Cz + D = 0$ is calculated using the formula:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

This formula computes the shortest distance from the point to the surface of the plane.

Solution:

Step 1: Identify the coordinates of the point and the coefficients of the plane: Point: $(2, -1, 3)$

Plane: $2x - y + 2z - 7 = 0$ Here, $A = 2, B = -1, C = 2, D = -7$.

Step 2: Substitute the point into the numerator of the distance formula:

$$\text{Numerator} = |2(2) - 1(-1) + 2(3) - 7|$$

$$\text{Numerator} = |4 + 1 + 6 - 7| = |4| = 4$$

Step 3: Calculate the denominator (the magnitude of the normal vector):

$$\text{Denominator} = \sqrt{2^2 + (-1)^2 + 2^2}$$

$$\text{Denominator} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

Step 4: Divide the numerator by the denominator to find the distance:

$$d = \frac{4}{3}$$

The distance of the point $(2, -1, 3)$ from the given plane is $4/3$ units.

Final Answer: $\frac{4}{3}$

Answer: (D)

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Q34.

Solution

Concept:

Direction cosines (l, m, n) of a line are the cosines of the angles α, β, γ that the line makes with the positive $x, y,$ and z axes respectively. They satisfy the fundamental identity $l^2 + m^2 + n^2 = 1$.

If a line is equally inclined to the axes, then $\alpha = \beta = \gamma$.

Solution:

Step 1: Let the angles be $\alpha = \beta = \gamma$. Consequently, the direction cosines are equal:

$$l = m = n = \cos \alpha$$

Step 2: Substitute these into the identity $l^2 + m^2 + n^2 = 1$:

$$\cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$3 \cos^2 \alpha = 1$$

Step 3: Solve for $\cos \alpha$:

$$\cos^2 \alpha = \frac{1}{3}$$

$$\cos \alpha = \pm \frac{1}{\sqrt{3}}$$

Step 4: Since the question specifies inclination to the "positive" coordinate axes, we take the positive square root:

$$l = \frac{1}{\sqrt{3}}, m = \frac{1}{\sqrt{3}}, n = \frac{1}{\sqrt{3}}$$

The direction cosines are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

Final Answer: $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

Answer: (A)

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Q35.

Solution**Concept:**

The cross product $\vec{a} \times \vec{b}$ of two vectors results in a third vector perpendicular to both. The square of the magnitude of the cross product, $|\vec{a} \times \vec{b}|^2$, can be calculated either by finding the determinant for the cross product first or by using Lagrange's Identity: $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2$.

Solution:

Step 1: Find the cross product $\vec{a} \times \vec{b}$ using the determinant method:

$$\begin{aligned}\vec{a} \times \vec{b} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{vmatrix} \\ &= \hat{i}(6 - 1) - \hat{j}(3 - (-2)) + \hat{k}(-1 - 4) \\ &= 5\hat{i} - 5\hat{j} - 5\hat{k}\end{aligned}$$

Step 2: Calculate the magnitude squared of the resulting vector:

$$\begin{aligned}|\vec{a} \times \vec{b}|^2 &= (5)^2 + (-5)^2 + (-5)^2 \\ |\vec{a} \times \vec{b}|^2 &= 25 + 25 + 25\end{aligned}$$

Step 3: Sum the squares:

$$|\vec{a} \times \vec{b}|^2 = 75$$

Alternatively, using $|\vec{a}|^2 = 6$, $|\vec{b}|^2 = 14$, $\vec{a} \cdot \vec{b} = 2 - 2 - 3 = -3$, then $6(14) - (-3)^2 = 84 - 9 = 75$.

Final Answer: 75

Answer: (C)

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Q36.

Solution**Concept:**

The scalar projection of a vector \vec{a} onto a vector \vec{b} is the magnitude of the orthogonal projection of \vec{a} in the direction of \vec{b} . It is given by the formula $\text{Proj}_{\vec{b}}\vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$. This represents how much of vector \vec{a} lies along the line defined by vector \vec{b} .

Solution:

Step 1: Calculate the dot product of \vec{a} and \vec{b} . Given $\vec{a} = 3\hat{i} + 4\hat{j} + 0\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$:

$$\vec{a} \cdot \vec{b} = (3)(1) + (4)(2) + (0)(2)$$

$$\vec{a} \cdot \vec{b} = 3 + 8 + 0 = 11$$

Step 2: Calculate the magnitude of vector \vec{b} :

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2}$$

$$|\vec{b}| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

Step 3: Substitute the values into the scalar projection formula:

$$\text{Scalar Projection} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{11}{3}$$

The result indicates that the length of the projection of \vec{a} onto the direction of \vec{b} is $11/3$ units.

Final Answer: $\frac{11}{3}$

Answer: (B)

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Q37.

Solution**Concept:**

The volume of a parallelepiped formed by three vectors \vec{a} , \vec{b} , and \vec{c} as coterminous edges is equal to the magnitude of their scalar triple product, denoted by $[\vec{a}\vec{b}\vec{c}]$. This can be calculated using the determinant of a matrix whose rows are the components of the three vectors.

Solution:

Step 1: Write the components of the given vectors: $\vec{a} = 1\hat{i} + 1\hat{j} + 0\hat{k}$ $\vec{b} = 0\hat{i} + 1\hat{j} + 1\hat{k}$ $\vec{c} = 1\hat{i} + 0\hat{j} + 1\hat{k}$

Step 2: Set up the determinant for the scalar triple product:

$$\text{Volume} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

Step 3: Expand the determinant along the first row:

$$\text{Volume} = 1(1(1) - 0(1)) - 1(0(1) - 1(1)) + 0(\dots)$$

$$\text{Volume} = 1(1) - 1(-1)$$

Step 4: Calculate the final value:

$$\text{Volume} = 1 + 1 = 2$$

Since the determinant is 2, the volume of the parallelepiped is 2 cubic units.

Final Answer:

Answer: (D)

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Q38.

Solution**Concept:**

To solve this limit, we can use the triple-angle identity $\sin 3x = 3 \sin x - 4 \sin^3 x$. Alternatively, L'Hôpital's Rule can be applied iteratively since the limit is in the indeterminate form $0/0$. Using the trigonometric identity is usually more direct for powers of x .

Solution:

Step 1: Substitute the identity $\sin 3x = 3 \sin x - 4 \sin^3 x$ into the limit expression:

$$\lim_{x \rightarrow 0} \frac{(3 \sin x - 4 \sin^3 x) - 3 \sin x}{x^3}$$

Step 2: Simplify the numerator by canceling the $3 \sin x$ terms:

$$\lim_{x \rightarrow 0} \frac{-4 \sin^3 x}{x^3}$$

Step 3: Rewrite the expression to isolate the standard limit:

$$-4 \cdot \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^3$$

Step 4: Apply the standard result $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$:

$$-4 \cdot (1)^3 = -4$$

The limit evaluates to -4 . If L'Hôpital's Rule were used, differentiating three times would yield the same result.

Final Answer:

Answer: (A)

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Q39.

Solution**Concept:**

A function $f(x)$ is continuous at a point $x = c$ if the limit of $f(x)$ as x approaches c is equal to the functional value $f(c)$. For this problem, we must find k such that $\lim_{x \rightarrow 0} \frac{e^{kx} - 1}{x} = f(0)$.

Solution:

Step 1: Identify the given functional value at the point of continuity: $f(0) = 2$

Step 2: Evaluate the limit of the function as $x \rightarrow 0$:

$$L = \lim_{x \rightarrow 0} \frac{e^{kx} - 1}{x}$$

This is a $0/0$ form. We use the standard limit $\lim_{\theta \rightarrow 0} \frac{e^\theta - 1}{\theta} = 1$.

Step 3: Adjust the limit to match the standard form by multiplying and dividing by k :

$$L = \lim_{x \rightarrow 0} \frac{e^{kx} - 1}{kx} \cdot k = 1 \cdot k = k$$

Step 4: Set the limit equal to the functional value for continuity:

$$L = f(0) \implies k = 2$$

Therefore, the function is continuous at $x = 0$ when $k = 2$.

Final Answer: $k = 2$

Answer: (B)

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Q40.

Solution

Concept:

To differentiate a function where the variable is in both the base and the exponent, such as $y = x^x$, we use logarithmic differentiation. By taking the natural logarithm of both sides, we can use the properties of logs to move the exponent and then differentiate implicitly.

Solution:

Step 1: Take the natural log of both sides:

$$\ln y = \ln(x^x) \implies \ln y = x \ln x$$

Step 2: Differentiate both sides with respect to x using the Product Rule on the right:

$$\frac{1}{y} \frac{dy}{dx} = \frac{d}{dx}(x) \cdot \ln x + x \cdot \frac{d}{dx}(\ln x)$$

$$\frac{1}{y} \frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

Step 3: Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1)$$

Step 4: Substitute $x = 1$ into the derivative:

$$\left. \frac{dy}{dx} \right|_{x=1} = 1^1(\ln 1 + 1)$$

Since $\ln 1 = 0$:

$$\left. \frac{dy}{dx} \right|_{x=1} = 1(0 + 1) = 1$$

The slope of the curve $y = x^x$ at $x = 1$ is 1.

Final Answer:

Answer: (C)

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Q41.

Solution**Concept:**

When variables x and y are expressed in terms of a third variable t (the parameter), the derivative of y with respect to x is found using the formula $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. This allows us to find the slope of the curve at any value of t without necessarily eliminating the parameter.

Solution:

Step 1: Differentiate $x = t^2 + 1$ with respect to t :

$$\frac{dx}{dt} = 2t$$

Step 2: Differentiate $y = t^3 - t$ with respect to t :

$$\frac{dy}{dt} = 3t^2 - 1$$

Step 3: Combine these using the chain rule for parametric functions:

$$\frac{dy}{dx} = \frac{3t^2 - 1}{2t}$$

Step 4: Substitute the given value $t = 2$ into the derivative expression:

$$\left. \frac{dy}{dx} \right|_{t=2} = \frac{3(2)^2 - 1}{2(2)}$$

$$\left. \frac{dy}{dx} \right|_{t=2} = \frac{3(4) - 1}{4} = \frac{11}{4}$$

The rate of change of y with respect to x when $t = 2$ is $11/4$.

Final Answer:

$$\frac{11}{4}$$

Answer: (D)

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Q42.

Solution**Concept:**

To evaluate the integral of a fraction where the numerator is the exact derivative of the denominator, we use the substitution method or the logarithmic integration rule. If $f(x) = 1 + x^2$, then $f'(x) = 2x$. The integral follows the form $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$.

Solution:

Step 1: Recognize the substitution. Let $u = 1 + x^2$. Then $du = 2x dx$.

Step 2: Change the limits of integration. When $x = 0$, $u = 1 + 0^2 = 1$. When $x = 1$, $u = 1 + 1^2 = 2$.

Step 3: Substitute and integrate with respect to u :

$$\begin{aligned}\int_0^1 \frac{2x}{1+x^2} dx &= \int_1^2 \frac{1}{u} du \\ &= [\ln |u|]_1^2\end{aligned}$$

Step 4: Apply the fundamental theorem of calculus:

$$= \ln 2 - \ln 1$$

Since $\ln 1 = 0$, the result is $\ln 2$.

Final Answer: $\ln 2$

Answer: (A)

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Q43.

Solution

Concept:

This definite integral can be solved using the property $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$. For limits 0 to $\pi/2$, $\sin x$ changes to $\cos x$ and vice versa. Adding the original integral to this transformed version often simplifies the denominator.

Solution:

Step 1: Let the given integral be I :

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \quad \text{--- (1)}$$

Step 2: Apply the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$:

$$I = \int_0^{\pi/2} \frac{\sin(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)} dx = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \quad \text{--- (2)}$$

Step 3: Add equations (1) and (2):

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\pi/2} 1 dx$$

Step 4: Integrate and solve for I :

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: $\frac{\pi}{4}$

Answer: (B)

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Q44.

Solution**Concept:**

This is a first-order linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$. We solve it by finding an integrating factor $IF = e^{\int P(x) dx}$. The general solution is $y(IF) = \int Q(x)(IF) dx$.

Solution:

Step 1: Identify $P(x) = \tan x$ and $Q(x) = \sec x$. Find the Integrating Factor:

$$IF = e^{\int \tan x dx} = e^{\ln |\sec x|} = \sec x$$

Step 2: Write the general solution:

$$y \cdot \sec x = \int \sec x \cdot \sec x dx$$

$$y \sec x = \tan x + C$$

Step 3: Apply the initial condition $y(0) = 1$:

$$1 \cdot \sec 0 = \tan 0 + C \implies 1(1) = 0 + C \implies C = 1$$

So, $y \sec x = \tan x + 1$, which means $y = \frac{\tan x + 1}{\sec x} = \sin x + \cos x$.

Step 4: Find y at $x = \pi/4$:

$$y(\pi/4) = \sin(\pi/4) + \cos(\pi/4) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Final Answer: $\sqrt{2}$

Answer: (C)

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Q45.

Solution**Concept:**

To find the limit of an expression involving $\ln(1+x)$ where $x \rightarrow 0$, we can use the Maclaurin series expansion: $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$. This allows us to cancel the polynomial terms in the numerator and isolate the leading power of x .

Solution:

Step 1: Substitute the expansion of $\ln(1+x)$ into the numerator:

$$\ln(1+x) - x + \frac{x^2}{2} = \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \right) - x + \frac{x^2}{2}$$

Step 2: Simplify the numerator: The terms x and $-x$ cancel out, as do $-x^2/2$ and $x^2/2$. We are left with:

$$\frac{x^3}{3} - \frac{x^4}{4} + \dots$$

Step 3: Divide the simplified numerator by x^3 :

$$\frac{\frac{x^3}{3} - \frac{x^4}{4} + \dots}{x^3} = \frac{1}{3} - \frac{x}{4} + \dots$$

Step 4: Take the limit as $x \rightarrow 0$:

$$\lim_{x \rightarrow 0} \left(\frac{1}{3} - \frac{x}{4} + \dots \right) = \frac{1}{3}$$

Alternatively, applying L'Hôpital's Rule three times would yield the same constant result of $1/3$.

Final Answer: $\frac{1}{3}$

Answer: (D)

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Q46.

Solution**Concept:**

To integrate a rational function with a quadratic denominator that does not have real roots, we use the method of completing the square. This transforms the integral into the standard form

$$\int \frac{dx}{(x+h)^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x+h}{a} \right) + C.$$

Solution:

Step 1: Complete the square for the quadratic expression in the denominator $x^2 + 4x + 8$:

$$x^2 + 4x + 8 = (x^2 + 4x + 4) + 4 = (x + 2)^2 + 2^2$$

Step 2: Rewrite the integral using this transformed denominator:

$$\int \frac{dx}{(x+2)^2 + 2^2}$$

Step 3: Use the standard integration formula $\int \frac{du}{u^2+a^2} = \frac{1}{a} \tan^{-1} \left(\frac{u}{a} \right) + C$. Here, $u = x + 2$ and $a = 2$.

Step 4: Substitute the values into the formula:

$$\frac{1}{2} \tan^{-1} \left(\frac{x+2}{2} \right) + C$$

This gives the antiderivative of the function, which involves the inverse tangent function due to the irreducible quadratic denominator.

Final Answer: $\frac{1}{2} \tan^{-1} \left(\frac{x+2}{2} \right) + C$

Answer: (B)

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Q47.

Solution**Concept:**

A function $f(x)$ is strictly increasing on an interval where its first derivative $f'(x)$ is strictly greater than zero. To find these intervals, we differentiate the function, find the critical points by setting $f'(x) = 0$, and test the signs of the derivative in the resulting intervals.

Solution:

Step 1: Find the first derivative of $f(x) = x^3 - 6x^2 + 9x + 1$:

$$f'(x) = 3x^2 - 12x + 9$$

Step 2: Factor the derivative and find the critical points where $f'(x) = 0$:

$$3(x^2 - 4x + 3) = 0 \implies 3(x - 1)(x - 3) = 0$$

The critical points are $x = 1$ and $x = 3$.

Step 3: Analyze the sign of $f'(x)$ in the intervals $(-\infty, 1)$, $(1, 3)$, and $(3, \infty)$. * For $x \in (-\infty, 1)$, $f'(x) > 0$ (e.g., test $x = 0$, $f'(0) = 9$). * For $x \in (1, 3)$, $f'(x) < 0$ (e.g., test $x = 2$, $f'(2) = -3$). * For $x \in (3, \infty)$, $f'(x) > 0$ (e.g., test $x = 4$, $f'(4) = 9$).

Step 4: Conclude where the function is strictly increasing ($f'(x) > 0$): The intervals are $(-\infty, 1) \cup (3, \infty)$.

Final Answer: $(-\infty, 1) \cup (3, \infty)$

Answer: (C)

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Q48.

Solution**Concept:**

The minimum value of a function can be found using calculus (derivatives) or the Arithmetic Mean-Geometric Mean (AM-GM) inequality. The AM-GM inequality states that for positive real numbers a and b , $\frac{a+b}{2} \geq \sqrt{ab}$, with equality holding if and only if $a = b$.

Solution:

Step 1: Apply the AM-GM inequality to the positive terms x and $1/x$:

$$\frac{x + \frac{1}{x}}{2} \geq \sqrt{x \cdot \frac{1}{x}}$$

Step 2: Simplify the right-hand side:

$$\sqrt{x \cdot \frac{1}{x}} = \sqrt{1} = 1$$

Step 3: Solve for the expression $x + 1/x$:

$$x + \frac{1}{x} \geq 2 \times 1 \implies x + \frac{1}{x} \geq 2$$

Step 4: Determine when the equality holds. Equality occurs when the two terms are equal:

$$x = \frac{1}{x} \implies x^2 = 1 \implies x = 1 \text{ (since } x > 0\text{)}$$

At $x = 1$, the value is $1 + 1/1 = 2$. Thus, the minimum value is 2.

Final Answer: 2

Answer: (A)

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Q49.

Solution**Concept:**

The area between two curves $f(x)$ and $g(x)$ from $x = a$ to $x = b$ is calculated using the definite integral $\int_a^b |f(x) - g(x)| dx$. We first find the points of intersection to determine the limits and identify which curve lies above the other.

Solution:

Step 1: Find the points of intersection by setting $y = x$ equal to $y = x^2$:

$$x = x^2 \implies x^2 - x = 0 \implies x(x - 1) = 0$$

The curves intersect at $x = 0$ and $x = 1$.

Step 2: Determine which curve is upper in the interval $(0, 1)$. For $x = 0.5$, $y = x$ gives 0.5 and $y = x^2$ gives 0.25. Since $x > x^2$, the line $y = x$ is the upper curve.

Step 3: Set up the integral for the area:

$$\text{Area} = \int_0^1 (x - x^2) dx$$

Step 4: Evaluate the integral:

$$\text{Area} = \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \left(\frac{1}{2} - \frac{1}{3} \right) - (0 - 0)$$

$$\text{Area} = \frac{3 - 2}{6} = \frac{1}{6}$$

The bounded area is $1/6$ square units.

Final Answer: $\frac{1}{6}$

Answer: (B)

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Q50.

Solution**Concept:**

Velocity $v(t)$ is the first derivative of displacement $s(t)$, and acceleration $a(t)$ is the first derivative of velocity (the second derivative of displacement). To solve this, we find when $v(t) = 0$, pick the first positive time t , and evaluate $a(t)$ at that moment.

Solution:

Step 1: Find the velocity function $v(t)$ by differentiating $s(t) = t^3 - 6t^2 + 9t$:

$$v(t) = s'(t) = 3t^2 - 12t + 9$$

Step 2: Set $v(t) = 0$ to find the times when the particle is at rest:

$$3(t^2 - 4t + 3) = 0 \implies 3(t - 1)(t - 3) = 0$$

The times are $t = 1$ and $t = 3$. The first time $t > 0$ is $t = 1$.

Step 3: Find the acceleration function $a(t)$ by differentiating $v(t)$:

$$a(t) = v'(t) = 6t - 12$$

Step 4: Evaluate the acceleration at $t = 1$:

$$a(1) = 6(1) - 12 = 6 - 12 = -6$$

The negative sign indicates that at $t = 1$, the particle is decelerating or experiencing a force in the opposite direction of the positive displacement.

Final Answer:

Answer: (D)

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Q51.

Solution**Concept:**

For three numbers a, b, c in Geometric Progression (G.P.), they can be represented as $\frac{b}{r}, b, br$, where r is the common ratio. This specific substitution is highly effective when the product of the terms is provided, as the ratio r cancels out, allowing for the direct calculation of the middle term.

Solution:

Step 1: Use the product information $abc = 216$. Substituting the G.P. terms $a = b/r$ and $c = br$:

$$(b/r) \cdot b \cdot (br) = 216 \implies b^3 = 216$$

Step 2: Solve for the middle term b by taking the cube root of both sides:

$$b = \sqrt[3]{216} = 6$$

Step 3: (Verification) Use the sum $a + b + c = 21$. Substituting $b = 6$:

$$\frac{6}{r} + 6 + 6r = 21 \implies \frac{6}{r} + 6r = 15$$

Dividing by 3 gives $\frac{2}{r} + 2r = 5$, which leads to the quadratic $2r^2 - 5r + 2 = 0$. Solving this gives $r = 2$ or $r = 1/2$. If $r = 2$, the numbers are 3, 6, 12, which satisfy all conditions.

The value of b is unequivocally 6.

Final Answer:

Answer: (A)

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Q52.

Solution**Concept:**

To solve problems where specific items must not be adjacent, we employ the "Gap Method." First, arrange the remaining items, then place the restricted items into the resulting spaces (gaps) between and around the already placed items. This ensures no two restricted items can touch.

Solution:

Step 1: Identify the frequency of letters in the word "BANANA": B: 1, A: 3, N: 2. Total letters = 6.

Step 2: Arrange the letters other than N (B, A, A, A). Since 'A' is repeated 3 times, the number of ways is:

$$\frac{4!}{3!} = 4 \text{ ways}$$

Step 3: Identify the gaps created by these 4 letters (e.g., _ B _ A _ A _ A _). There are $4 + 1 = 5$ possible positions for the N 's.

Step 4: Select 2 gaps out of the 5 available to place the two identical N 's:

$$\text{Ways to place } N\text{'s} = {}^5C_2 = \frac{5 \times 4}{2 \times 1} = 10 \text{ ways}$$

Step 5: Multiply the results to find the total arrangements:

$$\text{Total} = 4 \times 10 = 40$$

Thus, there are 40 unique arrangements of BANANA where the N 's are not adjacent.

Final Answer:

Answer: (C)

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Q53.

Solution**Concept:**

For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is $S = \alpha + \beta = -b/a$ and the product of the roots is $P = \alpha\beta = c/a$. We can find the value of $\alpha^4 + \beta^4$ by using algebraic identities to express it in terms of S and P .

Solution:

Step 1: Identify the sum and product from $x^2 - 2x + 5 = 0$:

$$S = \alpha + \beta = 2, \quad P = \alpha\beta = 5$$

Step 2: Find the value of $\alpha^2 + \beta^2$:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (2)^2 - 2(5) = 4 - 10 = -6$$

Step 3: Express $\alpha^4 + \beta^4$ using the value of $(\alpha^2 + \beta^2)$:

$$\alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2$$

Step 4: Substitute the known values into the equation:

$$\alpha^4 + \beta^4 = (-6)^2 - 2(5)^2$$

$$\alpha^4 + \beta^4 = 36 - 2(25) = 36 - 50 = -14$$

The calculation shows that the sum of the fourth powers of the roots is -14 .

Final Answer:

Answer: (B)

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Q54.

Solution**Concept:**

For any 2×2 matrix A , the characteristic equation is given by $\lambda^2 - \text{tr}(A)\lambda + \det(A) = 0$. The eigenvalues λ_1, λ_2 are the roots of this equation. The determinant of any matrix polynomial $f(A)$ is the product of $f(\lambda_i)$ for all eigenvalues.

Solution:

Step 1: Write the characteristic equation using the given trace (4) and determinant (3):

$$\lambda^2 - 4\lambda + 3 = 0$$

Step 2: Solve for the eigenvalues by factoring the quadratic:

$$(\lambda - 1)(\lambda - 3) = 0 \implies \lambda_1 = 1, \lambda_2 = 3$$

Step 3: Use the property of determinants for matrix functions. The determinant $|A^2 - I|$ is:

$$|A^2 - I| = (\lambda_1^2 - 1)(\lambda_2^2 - 1)$$

Step 4: Substitute the eigenvalues into the expression:

$$|A^2 - I| = (1^2 - 1)(3^2 - 1) = (0)(8) = 0$$

Because 1 is an eigenvalue of A , the matrix $(A - I)$ is singular, which in turn makes $(A^2 - I) = (A - I)(A + I)$ singular, resulting in a determinant of zero.

Final Answer:

Answer: (D)

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Q55.

Solution**Concept:**

This is a problem of conditional probability, defined as $P(A|B) = \frac{P(A \cap B)}{P(B)}$. Let event B be "at least one ball is black" and event A be "one ball is white and the other is black". $P(A|B)$ calculates the probability of A given that we already know B has occurred.

Solution:

Step 1: Calculate the total number of ways to draw 2 balls from 9 (5W + 4B):

$$n(S) = {}^9C_2 = \frac{9 \times 8}{2} = 36$$

Step 2: Find the number of ways for event B (at least one black). This is easier via the complement (no black balls):

$$\text{No Black (2 White)} = {}^5C_2 = 10$$

$$n(B) = 36 - 10 = 26$$

Step 3: Find the number of ways for event $A \cap B$ (exactly one white and one black):

$$n(A \cap B) = {}^5C_1 \times {}^4C_1 = 5 \times 4 = 20$$

Step 4: Apply the conditional probability formula:

$$P(A|B) = \frac{n(A \cap B)}{n(B)} = \frac{20}{26} = \frac{10}{13}$$

The probability that the other ball is white, given at least one is black, is 10/13.

Final Answer:

$$\frac{10}{13}$$

Answer: (A)

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Q56.

Solution**Concept:**

The variance of a set of observations x_i is given by $\sigma_x^2 = E[x^2] - (E[x])^2$. A key property of variance is that for any linear transformation $y = ax + b$, the variance of the new set is $\text{Var}(ax + b) = a^2\text{Var}(x)$. This occurs because variance is independent of shifts in the origin (b) but is affected by changes in scale (a).

Solution:

Step 1: Calculate the variance of the original observations x_i using the given means:

$$\text{Var}(x) = E[x^2] - (E[x])^2$$

$$\text{Var}(x) = 29 - (5)^2 = 29 - 25 = 4$$

Step 2: Apply the property of variance for a linear transformation. We need to find the variance of $2x_i - 3$. Here, $a = 2$ and $b = -3$.

$$\text{Var}(2x - 3) = 2^2 \cdot \text{Var}(x)$$

Step 3: Substitute the value of $\text{Var}(x)$ calculated in Step 1:

$$\text{Var}(2x - 3) = 4 \cdot 4 = 16$$

The additive constant -3 does not affect the variance, while the multiplier 2 scales the variance by its square.

Final Answer:

Answer: (C)

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Q57.

Solution**Concept:**

The point on a line $Ax + By = C$ nearest to the origin $(0, 0)$ is the foot of the perpendicular dropped from the origin to the line. This point lies on a line passing through the origin with a slope perpendicular to the given line. Alternatively, it can be found using the formula for the foot of the perpendicular $(x, y) = \left(\frac{AC}{A^2+B^2}, \frac{BC}{A^2+B^2} \right)$.

Solution:

Step 1: Identify the coefficients from the line equation $3x + 4y = 25$: $A = 3, B = 4, C = 25$.

Step 2: Calculate the denominator $A^2 + B^2$:

$$3^2 + 4^2 = 9 + 16 = 25$$

Step 3: Apply the coordinates formula for the foot of the perpendicular:

$$x = \frac{3(25)}{25} = 3$$

$$y = \frac{4(25)}{25} = 4$$

Step 4: Verify the result. The point $(3, 4)$ satisfies the equation: $3(3) + 4(4) = 9 + 16 = 25$. Furthermore, the slope of the line joining $(0, 0)$ to $(3, 4)$ is $4/3$, which is the negative reciprocal of the given line's slope $(-3/4)$.

The nearest point to the origin is therefore $(3, 4)$.

Final Answer: $(3, 4)$

Answer: (B)

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Q58.

Solution**Concept:**

A circle passing through the origin $(0, 0)$ and another point on the x -axis $(6, 0)$ must have its center's x -coordinate exactly halfway between them due to symmetry. Once the center is found using the given line equation, the radius is the distance from the center to any point on the circle (like the origin).

Solution:

Step 1: Determine the x -coordinate of the center. Since the circle passes through $(0, 0)$ and $(6, 0)$, the perpendicular bisector of this chord is the line $x = 3$. Thus, the center C must have $x = 3$.

Step 2: Use the condition that the center lies on the line $y = x$. Since $x = 3$, it follows that $y = 3$. The center is $C(3, 3)$.

Step 3: Calculate the radius r . The radius is the distance from $C(3, 3)$ to the origin $(0, 0)$:

$$r = \sqrt{(3 - 0)^2 + (3 - 0)^2}$$

$$r = \sqrt{3^2 + 3^2} = \sqrt{18}$$

Step 4: Simplify the radical expression:

$$r = \sqrt{9 \cdot 2} = 3\sqrt{2}$$

The center $(3, 3)$ and radius $3\sqrt{2}$ satisfy the condition of passing through both $(0, 0)$ and $(6, 0)$.

Final Answer: $3\sqrt{2}$

Answer: (D)

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Q59.

Solution**Concept:**

For a parabola $y^2 = 4ax$, the equation of the normal at a point with parameter t is given by $y = -tx + 2at + at^3$. To find where this normal intersects the x -axis, we set $y = 0$ and solve for x .

Solution:

Step 1: Identify the constant a from the parabola equation $y^2 = 4x$. Comparing with $y^2 = 4ax$, we find $4a = 4 \implies a = 1$.

Step 2: Write the equation of the normal at parameter $t = 1$:

$$y = -(1)x + 2(1)(1) + (1)(1)^3$$

$$y = -x + 2 + 1 \implies y = -x + 3$$

Step 3: Find the x -intercept by setting $y = 0$:

$$0 = -x + 3$$

$$x = 3$$

Step 4: The intersection point on the x -axis is $(3, 0)$. The point on the parabola at $t = 1$ is $(at^2, 2at) = (1, 2)$. The line $y = -x + 3$ passes through $(1, 2)$ and is perpendicular to the tangent at that point.

Final Answer: $(3, 0)$

Answer: (A)

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Q60.

Solution

Concept:

The shortest distance d between two skew lines $\vec{r} = \vec{a}_1 + s\vec{b}_1$ and $\vec{r} = \vec{a}_2 + t\vec{b}_2$ is the projection of the vector joining the points $\vec{a}_2 - \vec{a}_1$ onto the common perpendicular $\vec{b}_1 \times \vec{b}_2$.

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Solution:

Step 1: Identify the vectors from the line equations: $\vec{a}_1 = (1, 0, 0)$, $\vec{b}_1 = (1, 1, 0)$ $\vec{a}_2 = (0, 1, 1)$, $\vec{b}_2 = (0, 1, 1)$ Vector $\vec{a}_2 - \vec{a}_1 = (-1, 1, 1)$.

Step 2: Calculate the cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{i}(1 - 0) - \hat{j}(1 - 0) + \hat{k}(1 - 0) = (1, -1, 1)$$

Step 3: Calculate the magnitude $|\vec{b}_1 \times \vec{b}_2|$:

$$\sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

Step 4: Calculate the dot product $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$:

$$(-1)(1) + (1)(-1) + (1)(1) = -1 - 1 + 1 = -1$$

Step 5: Find the distance:

$$d = \frac{|-1|}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

The shortest distance between the given skew lines is $1/\sqrt{3}$.

Final Answer: $\frac{1}{\sqrt{3}}$

Answer: (C)

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Q61.

Solution**Concept:**

For any two vectors \vec{a} and \vec{b} , the magnitudes of their sum and difference are related by the Parallelogram Law: $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$. This identity is derived by expanding the dot products $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$.

Solution:

Step 1: Identify the given values: $|\vec{a}| = 3$, $|\vec{b}| = 4$, and $|\vec{a} + \vec{b}| = 5$.

Step 2: Use the Parallelogram Law identity:

$$|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$$

Step 3: Substitute the known magnitudes into the equation:

$$5^2 + |\vec{a} - \vec{b}|^2 = 2(3^2 + 4^2)$$

$$25 + |\vec{a} - \vec{b}|^2 = 2(9 + 16)$$

Step 4: Solve for $|\vec{a} - \vec{b}|^2$:

$$25 + |\vec{a} - \vec{b}|^2 = 2(25)$$

$$25 + |\vec{a} - \vec{b}|^2 = 50$$

$$|\vec{a} - \vec{b}|^2 = 25$$

Step 5: Take the square root:

$$|\vec{a} - \vec{b}| = 5$$

Alternatively, since $3^2 + 4^2 = 5^2$, the vectors \vec{a} and \vec{b} are perpendicular. For perpendicular vectors, the magnitudes of the sum and difference are always equal.

Final Answer: 5

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Q62.

Solution**Concept:**

The derivative of the inverse function is given by the formula $(f^{-1})'(y) = \frac{1}{f'(x)}$, where $y = f(x)$. To evaluate this at a specific point $y = b$, we first find the value of a such that $f(a) = b$, and then calculate $1/f'(a)$.

Solution:

Step 1: Given $f(x) = x^3 + x$ and we need $(f^{-1})'(2)$. Here, $y = 2$. Set $f(x) = 2$ to find the corresponding value of x :

$$x^3 + x = 2$$

By inspection, $x = 1$ is a solution since $1^3 + 1 = 2$. Since $f(x)$ is strictly increasing ($f'(x) > 0$), this solution is unique.

Step 2: Find the derivative $f'(x)$:

$$f'(x) = 3x^2 + 1$$

Step 3: Evaluate $f'(x)$ at $x = 1$:

$$f'(1) = 3(1)^2 + 1 = 4$$

Step 4: Apply the inverse derivative formula:

$$(f^{-1})'(2) = \frac{1}{f'(1)} = \frac{1}{4}$$

The slope of the inverse function at $y = 2$ is the reciprocal of the slope of the original function at $x = 1$.

Final Answer:

$$\frac{1}{4}$$

Answer: (D)

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Q63.

Solution**Concept:**

To evaluate the definite integral $\int_0^a x(a-x) dx$, we can expand the integrand to form a simple polynomial. Alternatively, one could use the property $\int_0^a f(x) dx = \int_0^a f(a-x) dx$, though for this specific algebraic form, direct expansion is more efficient.

Solution:

Step 1: Expand the integrand $x(a-x)$:

$$\int_0^a x(a-x) dx = \int_0^a (ax - x^2) dx$$

Step 2: Apply the Power Rule for integration term by term:

$$\int (ax - x^2) dx = \frac{ax^2}{2} - \frac{x^3}{3}$$

Step 3: Evaluate the antiderivative at the upper limit $x = a$ and the lower limit $x = 0$:

$$\left[\frac{ax^2}{2} - \frac{x^3}{3} \right]_0^a = \left(\frac{a(a^2)}{2} - \frac{a^3}{3} \right) - (0 - 0)$$

Step 4: Simplify the resulting expression:

$$\frac{a^3}{2} - \frac{a^3}{3} = \frac{3a^3 - 2a^3}{6} = \frac{a^3}{6}$$

The area represented by the integral is proportional to the cube of the parameter a .

Final Answer: $\frac{a^3}{6}$

Answer: (A)

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Q64.

Solution**Concept:**

This is a differential equation where the variables are not immediately separable. However, when the derivative is a function of a linear combination of x and y (i.e., $y' = f(ax + by + c)$), we use the substitution $v = ax + by + c$ to transform it into a separable form.

Solution:

Step 1: Let $v = x + y$. Differentiate both sides with respect to x :

$$\frac{dv}{dx} = 1 + \frac{dy}{dx} \implies \frac{dy}{dx} = \frac{dv}{dx} - 1$$

Step 2: Substitute these into the original equation $\frac{dy}{dx} = (x + y)^2$:

$$\frac{dv}{dx} - 1 = v^2 \implies \frac{dv}{dx} = v^2 + 1$$

Step 3: Separate the variables v and x and integrate:

$$\int \frac{dv}{v^2 + 1} = \int dx$$

$$\tan^{-1} v = x + C$$

Step 4: Replace v with $x + y$:

$$\tan^{-1}(x + y) = x + C$$

Step 5: Solve for y :

$$x + y = \tan(x + C)$$

$$y = \tan(x + C) - x$$

This expression provides the general solution where C is the constant of integration.

Final Answer: $y = \tan(x + C) - x$

Answer: (C)

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Q65.

Solution**Concept:**

For a rectangle in the first quadrant with two sides on the coordinate axes and one vertex (x, y) on the curve $y = 12 - x^2$, the area is given by $A = x \cdot y$. To find the maximum area, we express A in terms of x , find the derivative $A'(x)$, and solve for $A'(x) = 0$.

Solution:

Step 1: Set up the area function A using the constraint $y = 12 - x^2$:

$$A(x) = x(12 - x^2) = 12x - x^3$$

Step 2: Differentiate the area function with respect to x to find the critical points:

$$A'(x) = 12 - 3x^2$$

Step 3: Set $A'(x) = 0$ to find the value of x that maximizes the area:

$$12 - 3x^2 = 0 \implies x^2 = 4 \implies x = 2 \text{ (since } x > 0\text{)}$$

Step 4: Verify that it is a maximum. $A''(x) = -6x$, which is negative for $x = 2$, confirming a maximum.

Step 5: Calculate the maximum area by substituting $x = 2$ back into $A(x)$:

$$A(2) = 12(2) - (2)^3 = 24 - 8 = 16$$

The dimensions of the rectangle with maximum area are 2 by 8, resulting in an area of 16.

Final Answer:

Answer: (B)

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Q66.

Solution

Concept:

For a quadratic $az^2 + bz + c = 0$, roots are $z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. For $z = x + iy$, modulus $|z| = \sqrt{x^2 + y^2}$ and $\arg(z)$ is the angle with the positive real axis. Powers of complex numbers are best handled via De Moivre's Theorem.

Solution:

Step 1: Solve $z^2 + 2z + 2 = 0$. $D = 2^2 - 4(1)(2) = -4$. $z = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i$. Option A is correct.

Step 2: Check Modulus. $|z| = \sqrt{(-1)^2 + (\pm 1)^2} = \sqrt{2}$. Option B is correct.

Step 3: Check $z^4 + 4 = 0$. In polar form, $z = \sqrt{2}e^{i(\pm 3\pi/4)}$. $z^4 = (\sqrt{2})^4 e^{i(\pm 3\pi)} = 4(\cos 3\pi \pm i \sin 3\pi) = 4(-1) = -4$. Thus $z^4 + 4 = 0$. Option C is correct.

Step 4: Check Argument. Roots are in the 2nd and 3rd quadrants. The arguments are $3\pi/4$ and $-3\pi/4$. Option D is incorrect as it states $\pi/4$.

Final Answer: A, B, C

Answer: (A, B, C) [Go Back to Question 66](#)

Q67.

Solution

Concept:

Using $\tan \theta$, we find double angles: $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$ and $\cos 2\theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$. Compound angle $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$. Individual values $\sin \theta, \cos \theta$ are found via a right triangle.

Solution:

Step 1: $\sin 2\theta = \frac{2(2)}{1+4} = \frac{4}{5}$. Option A is correct.

Step 2: $\cos 2\theta = \frac{1-4}{1+4} = -\frac{3}{5}$. Option B is correct.

Step 3: $\tan(\frac{\pi}{4} + \theta) = \frac{1+2}{1-2} = -3$. Option C is correct.

Step 4: If $\tan \theta = 2$, then $\sin \theta = \frac{2}{\sqrt{5}}$ and $\cos \theta = \frac{1}{\sqrt{5}}$. $\sin \theta + \cos \theta = \frac{2+1}{\sqrt{5}} = \frac{3}{\sqrt{5}}$. Option D is correct.

All options represent correct mathematical identities for the given condition.

Final Answer: A, B, C, D

Answer: (A, B, C, D) [Go Back to Question 67](#)



Q68.

Solution

Concept:

Unit vectors satisfy $|\vec{a}| = |\vec{b}| = 1$. The dot product $\vec{a} \cdot \vec{b} = \cos 60^\circ = 1/2$. Vector magnitudes use $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. The magnitude of the cross product represents the area of the parallelogram.

Solution:

Step 1: $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 1 + 1 - 2(1/2) = 1$. Thus $|\vec{a} - \vec{b}| = 1$. Option A is correct.

Step 2: $|\vec{a} + \vec{b}|^2 = 1 + 1 + 2(1/2) = 3$. Thus $|\vec{a} + \vec{b}| = \sqrt{3}$. Option B is correct.

Step 3: $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2 = 1 - 1 = 0$. Option C is correct.

Step 4: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin 60^\circ = 1 \cdot 1 \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$. Option D is correct.

Final Answer: A, B, C, D

Answer: (A, B, C, D) [Go Back to Question 68](#)

Q69.

Solution

Concept:

Monotonicity depends on the sign of $f'(x)$. Extrema occur where $f'(x) = 0$. The improper integral $\int_0^\infty x^n e^{-x} dx$ is the Gamma function $\Gamma(n + 1) = n!$. Limits at infinity for $x^n e^{-x}$ always approach 0 due to exponential dominance.

Solution:

Step 1: $f'(x) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2 - x)$. In $(0, 2)$, $x > 0$ and $(2 - x) > 0$, so $f'(x) > 0$. Option A is correct.

Step 2: $f'(x)$ changes sign from positive to negative at $x = 2$. So $x = 2$ is a local maximum. Option B is correct.

Step 3: $\int_0^\infty x^2 e^{-x} dx = \Gamma(3) = 2! = 2$. Option C is correct.

Step 4: $\lim_{x \rightarrow \infty} \frac{x^2}{e^x} = 0$ (applying L'Hopital's twice). Option D is correct.

Final Answer: A, B, C, D

Answer: (A, B, C, D) [Go Back to Question 69](#)



Q70.

Solution**Concept:**

Wallis' Reduction Formula for $I_n = \int_0^{\pi/2} \sin^n x \, dx$ states $I_n = \frac{n-1}{n} I_{n-2}$. Base cases: $I_0 = \pi/2$, $I_1 = 1$.

Solution:

Step 1: The reduction formula is $I_n = \frac{n-1}{n} I_{n-2}$. Option A is correct.

Step 2: $I_3 = \frac{3-1}{3} I_1 = \frac{2}{3}(1) = \frac{2}{3}$. Option B is correct.

Step 3: $I_2 = \frac{2-1}{2} I_0 = \frac{1}{2}(\frac{\pi}{2}) = \frac{\pi}{4}$. Option D is correct.

Step 4: $I_4 = \frac{4-1}{4} I_2 = \frac{3}{4}(\frac{\pi}{4}) = \frac{3\pi}{16}$. Option C is correct.

Final Answer: A, B, C, D

Answer: (A, B, C, D)

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Q71.

Solution**Concept:**

The function $f(x) = |x - 1| + |x + 1|$ is a sum of absolute value functions. To analyze it, we must identify critical points where the expressions inside the modulus become zero, namely $x = 1$ and $x = -1$. These points divide the real line into three intervals: $(-\infty, -1)$, $[-1, 1]$, and $(1, \infty)$. Differentiability is checked by comparing left-hand and right-hand derivatives at these "corners."

Solution:

Step 1: Define the function piecewise based on the critical points: If $x < -1$, $f(x) = -(x-1) - (x+1) = -2x$. If $-1 \leq x \leq 1$, $f(x) = -(x-1) + (x+1) = 2$. If $x > 1$, $f(x) = (x-1) + (x+1) = 2x$.

Step 2: Evaluate Statement I. In the interval $[-1, 1]$, the function simplifies to $f(x) = 2$. Thus, Statement I is correct.

Step 3: Evaluate Statement II. At $x = 1$, the left-hand derivative is the derivative of the constant 2, which is 0. The right-hand derivative is the derivative of $2x$, which is 2. Since $0 \neq 2$, the function is not differentiable at $x = 1$. Statement II is incorrect.

Step 4: Evaluate Statement III. $f(-x) = |-x - 1| + |-x + 1| = |-(x+1)| + |-(x-1)| = |x+1| + |x-1| = f(x)$. This confirms the function is even. Statement III is correct.

Step 5: Evaluate Statement IV. For $x \in (0, 1)$, $f'(x) = 0$. For $x > 1$, $f'(x) = 2$. Thus, $f'(x)$ is not 2 for all $x > 0$. Statement IV is incorrect.

Final Answer: A, C

Answer: (A, C)

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Q72.

Solution

Concept:

The function $f(x) = \ln(1 + x^2)$ involves a natural logarithm of a quadratic expression. Since $1 + x^2 \geq 1$ for all real x , the function is defined everywhere. Parity is determined by replacing x with $-x$, and concavity is dictated by the sign of the second derivative $f''(x)$.

Solution:

Step 1: Analyze parity of $f'(x)$. First, $f'(x) = \frac{1}{1+x^2} \cdot (2x) = \frac{2x}{1+x^2}$. Then, $f'(-x) = \frac{2(-x)}{1+(-x)^2} = -\frac{2x}{1+x^2} = -f'(x)$. Since $f'(-x) = -f'(x)$, the derivative is an odd function. Statement I is correct.

Step 2: Determine concavity. Apply the quotient rule to $f'(x)$: $f''(x) = \frac{(1+x^2)(2) - (2x)(2x)}{(1+x^2)^2} = \frac{2+2x^2-4x^2}{(1+x^2)^2} = \frac{2(1-x^2)}{(1+x^2)^2}$. Concavity is down only when $f''(x) < 0 \implies 1 - x^2 < 0 \implies |x| > 1$. It is not concave down for all x . Statement II is incorrect.

Step 3: Evaluate $f''(0)$. $f''(0) = \frac{2(1-0)}{(1+0)^2} = 2$. Statement III is correct.

Step 4: Check for global maximum. At $x = 0$, $f'(0) = 0$ and $f''(0) = 2$. Since the second derivative is positive, $x = 0$ is a local minimum, not a maximum. Statement IV is incorrect.

Final Answer: A, C

Answer: (A, C) [Go Back to Question 72](#)

Q73.

Solution

Concept:

This problem utilizes the symmetry of definite integrals on the interval $[0, 1]$. Specifically, the property $\int_0^1 f(x) dx = \int_0^1 f(1-x) dx$ is combined with the functional equation $f(x) + f(1-x) = 1$ to find the area under the curve. Individual point values are derived by substituting the midpoint.

Solution:

Step 1: Use the integral symmetry property: Let $I = \int_0^1 f(x) dx$. By substituting $u = 1 - x$, we also have $I = \int_0^1 f(1-x) dx$. Adding these two expressions: $2I = \int_0^1 [f(x) + f(1-x)] dx$.

Step 2: Apply the given condition $f(x) + f(1-x) = 1$: $2I = \int_0^1 1 dx = [x]_0^1 = 1$. Therefore, $I = 1/2$. Statement I is correct and Statement IV is incorrect.

Step 3: Evaluate the midpoint $x = 1/2$. Substitute $x = 1/2$ into the functional equation: $f(1/2) + f(1 - 1/2) = 1 \implies 2f(1/2) = 1 \implies f(1/2) = 1/2$. Statement II is correct.

Step 4: Assess linearity. The condition $f(x) + f(1-x) = 1$ implies point symmetry about $(1/2, 1/2)$. While a linear function $f(x) = x$ works, many non-linear functions (like $f(x) = \sin^2(\pi x/2)$ or cubic functions) satisfy this. Statement III is not necessarily true.

Final Answer: A, B

Answer: (A, B) [Go Back to Question 73](#)



Q74.

Solution

Concept:

The function $g(x) = \frac{\sin x}{x}$ is the unnormalized sinc function. At $x = 0$, it is defined via its limit to ensure continuity. Its properties are best examined using the Taylor series expansion: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$. Dividing by x gives $1 - \frac{x^2}{6} + \frac{x^4}{120} - \dots$.

Solution:

Step 1: Verify continuity at $x = 0$. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$. Since $g(0) = 1$ was defined, the limit equals the function value. Statement I is correct.

Step 2: Check parity. $g(-x) = \frac{\sin(-x)}{-x} = \frac{-\sin x}{-x} = \frac{\sin x}{x} = g(x)$. Because $g(-x) = g(x)$, the function is even. Statement II is incorrect as it claims the function is odd.

Step 3: Analyze the limit of the derivative. $g'(x) = \frac{x \cos x - \sin x}{x^2}$. Using the Taylor series: $g(x) = 1 - \frac{x^2}{6} + \dots \implies g'(x) = -\frac{2x}{6} + \dots = -\frac{x}{3} + \dots$. As $x \rightarrow 0$, $g'(x) \rightarrow 0$. Statement III is correct.

Step 4: Determine the second derivative at zero. Differentiating $g'(x) \approx -\frac{x}{3}$ gives $g''(0) = -1/3$. Alternatively, the coefficient of the x^2 term in the Taylor series is $\frac{g''(0)}{2!}$. Since $-1/6 = \frac{g''(0)}{2}$, we find $g''(0) = -1/3$. Statement IV is incorrect.

Final Answer: A, C

Answer: (A, C)

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Q75.

Solution

Concept:

For a cubic polynomial $f(x) = x^3 - 3ax$, the derivatives provide information about slopes and extrema. The first derivative $f'(x)$ gives the slope and critical points, while the second derivative $f''(x)$ determines the nature of those critical points (minimum or maximum).

Solution:

Step 1: Calculate the slope at the origin. $f'(x) = 3x^2 - 3a$. Substitute $x = 0$ to find the slope of the tangent: $f'(0) = 3(0)^2 - 3a = -3a$. Statement I is correct.

Step 2: Find and classify critical points. Set $f'(x) = 0 \implies 3x^2 - 3a = 0 \implies x^2 = a$. The critical points are $x = \sqrt{a}$ and $x = -\sqrt{a}$. Find $f''(x) = 6x$. For $x = \sqrt{a}$, $f''(\sqrt{a}) = 6\sqrt{a} > 0$ (since $a > 0$). Thus, $x = \sqrt{a}$ is a local minimum. Statement II is correct.

Step 3: Check monotonicity on $(-\sqrt{a}, \sqrt{a})$. In this interval, $x^2 < a$, so $3x^2 < 3a$, which means $f'(x) = 3x^2 - 3a < 0$. The function is strictly decreasing in this interval. Statement III is incorrect.

Step 4: Find the local maximum value. The local maximum occurs at $x = -\sqrt{a}$. $f(-\sqrt{a}) = (-\sqrt{a})^3 - 3a(-\sqrt{a}) = -a\sqrt{a} + 3a\sqrt{a} = 2a\sqrt{a}$. Statement IV says the value is $3a$, which is incorrect.

Final Answer: A, B

Answer: (A, B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	B
6	C	7	D	8	B	9	C	10	A
11	A	12	D	13	A	14	B	15	B
16	C	17	D	18	A	19	B	20	C
21	B	22	A	23	D	24	C	25	D
26	B	27	A	28	C	29	D	30	A
31	B	32	C	33	D	34	A	35	C
36	B	37	D	38	A	39	B	40	C
41	D	42	A	43	B	44	C	45	D
46	B	47	C	48	A	49	B	50	D
51	A	52	C	53	B	54	D	55	A
56	C	57	B	58	D	59	A	60	C
61	B	62	D	63	A	64	C	65	B
66	A, B, C	67	A, B, C, D	68	A, B, C, D	69	A, B, C, D	70	A, B, C, D
71	A, C	72	A, C	73	A, B	74	A, C	75	A, B

