WBJEE Mathematics Sample Paper- 21

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **75** Multiple Choice Questions.
- **Section A (Q1–Q50):** Each correct answer carries +1 mark. Incorrect answer: **0.25 marks**. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries +2 mark. Incorrect answer: **0.5 marks**. Only **one** correct option.
- **Section C (Q66–Q75)**: Each correct answer carries **+2 marks**. **No negative marking.** One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: 0.25) [Single Correct]

Q1. If a, b, c are in H.P. and a^2, b^2, c^2 are in A.P., then the value of $a : b : c$ is:

(A) $1 : 1 : 1$

(B) $1 : \sqrt{2} : (1 + \sqrt{2})$

(C) $1 : (1 - \sqrt{2}) : 1$

(D) $1 : -(1 \pm \sqrt{2}) : 1$

Q2. If $\log_{10} 2, \log_{10}(2^x - 1)$ and $\log_{10}(2^x + 3)$ are in A.P., then x is equal to:

(A) $\log_2 5$

(B) $\log_5 2$

(C) $\log_{10} 5$

(D) $3/2$

Q3. The number of ways in which 12 identical apples can be distributed among 3 children such that each child gets at least 1 apple and no two children get the same number of apples is:

(A) 18

(B) 24

(C) 32

(D) 36

Q4. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^n$ is:

(A) ${}^nC_4 +^n C_1 \cdot^n C_2 +^n C_2$

(B) ${}^nC_4 +^n C_2$

(C) ${}^nC_4 +^n C_1 +^n C_2$

(D) ${}^nC_4 +^n C_3 +^n C_2$

Q5. The sum of the series $\sum_{r=1}^{n} \frac{r}{(r+1)!}$ is:

(A) $1 - \frac{1}{(n+1)!}$

(B) $1 - \frac{1}{n!}$

(C) $\frac{1}{(n+1)!} - 1$

(D) $\frac{n}{(n+1)!}$

Q6. If n is even, the middle term in the expansion of $(x^2 + 1/x)^n$ is $924x^6$. The value of n is:

(A) 10

(B) 12

(C) 14

(D) 16

Q7. The number of 5-digit numbers that can be formed using digits $\{0, 1, 2, 3, 4, 5\}$ without repetition that are divisible by 6 is:

(A) 72

(B) 108

(C) 156

(D) 192

Q8. If the sum of n terms of a G.P. is S, product is P, and the sum of their reciprocals is R, then P^2R^n is equal to:

(A) S^n

(B) S^{2n}

(C) $(S/R)^n$

(D) S^2

Q9. The coefficient of x^{50} in the expansion of $(1+x)^{1000}+2x(1+x)^{999}+\cdots+1001x^{1000}$ is:

(A) $^{1001}C_{50}$

(B) $^{1002}C_{50}$

(C) $^{1000}C_{50}$

(D) $^{1002}C_{51}$

Q10. The smallest positive integer n for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is a real number is:

(A) 2

(B) 3

(C) 4

(D) 1

Q11. If α, β are the roots of $x^2 - px + q = 0$, and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the roots of $x^2 - 4qx + 2q^2 - r = 0$ are always:

(A) Real and identical

(B) Real and distinct

(C) Imaginary

(D) Rational

Q12. If $|z - 4/z| = 2$, then the maximum value of $|z|$ is:

(A) $\sqrt{3} + 1$

(B) $\sqrt{5} + 1$

(C) 2

(D) $2 + \sqrt{2}$

Q13. The value of $\sum_{k=1}^{10}(\sin\frac{2\pi k}{11} + i\cos\frac{2\pi k}{11})$ is:

(A) -1

(B) 1

(C) $-i$

(D) i

Q14. The number of real roots of the equation $e^{\sin x} - e^{-\sin x} - 4 = 0$ is:

(A) 1

(B) 2

(C) Infinite

(D) 0

Q15. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ is equal to:

(A) 128ω

(B) $-128\omega^2$

(C) $128\omega^2$

(D) -128ω

Q16. If A is a 3×3 non-singular matrix such that $AA^T = A^TA$ and $B = A^{-1}A^T$, then BB^T equals:

(A) I

(B) B^{-1}

(C) $(B^T)^{-1}$

(D) $B + I$

Q17. The value of $\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ is zero, then $1/a + 1/b + 1/c$ is:

(A) 1

(B) 0

(C) -1

(D) abc

Q18. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$, then $P^T Q^{2024} P$ is:

(A) $\begin{bmatrix} 1 & 2024 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 2024 & 1 \\ 0 & 2024 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 0 & 2024 \end{bmatrix}$

(D) $\begin{bmatrix} 2024 & 0 \\ 0 & 2024 \end{bmatrix}$

Q19. The system $x + y + z = 2, 2x + 3y + 2z = 5, 23x + 3y + az = b$ has infinitely many solutions if:

(A) $a = 2, b = 8$

(B) $a = 0, b = 0$

(C) $a = 2, b = 5$

(D) $a = 23, b = 2$

Q20. If $A^2 - A + I = 0$, then the inverse of A is:

(A) $A + I$

(B) $I - A$

(C) $A - I$

(D) A

Q21. Let $f(x) = \sqrt{\log_{10} \frac{5x-x^2}{4}}$. The domain of $f(x)$ is:

(A) $[1, 4]$

(B) $(1, 4)$

(C) $[0, 5]$

(D) $[1, 5]$

Q22. A relation R on set $A = \{1, 2, 3\}$ is defined as $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$. This relation is:

(A) Reflexive only

(B) Equivalence Relation

(C) Symmetric but not Transitive

(D) Symmetric and Transitive but not Reflexive

Q23. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \frac{x^2-1}{x^2+1}$, then f is:

(A) One-to-one and onto

(B) Many-to-one and onto

(C) One-to-one but not onto

(D) Many-to-one and not onto

Q24. Three houses are available in a locality. Three persons apply for the houses independently. The probability that all three apply for the same house is:

(A) 1/9

(B) 1/27

(C) $1/3$

(D) $2/9$

Q25. If A and B are two independent events such that $P(A) = 1/2$ and $P(B) = 1/3$, then $P(A \cup B)$ is:

(A) $2/3$

(B) $1/6$

(C) $5/6$

(D) $1/2$

Q26. The variance of first n natural numbers is:

(A) $\frac{n^2-1}{12}$

(B) $\frac{n^2+1}{12}$

(C) $\frac{n(n+1)}{12}$

(D) $\frac{n^2-1}{6}$

Q27. Two dice are thrown. The probability that the sum of the numbers is a prime number is:

(A) $5/12$

(B) $7/18$

(C) $1/2$

(D) $13/36$

Q28. In a Binomial distribution $B(n, p)$, the mean is 4 and variance is 3. Then n is:

(A) 16

(B) 12

(C) 10

(D) 8

Q29. The value of $\tan(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3})$ is:

(A) $\frac{3-\sqrt{5}}{2}$

(B) $\frac{3+\sqrt{5}}{2}$

(C) $\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$

(D) $\frac{\sqrt{5}-1}{2}$

Q30. The general solution of $\sin^2\theta\sec\theta + \sqrt{3}\tan\theta = 0$ is:

(A) $\theta = n\pi$

(B) $\theta = n\pi + (-1)^n\frac{\pi}{3}$

(C) $\theta = n\pi, n\pi + \frac{2\pi}{3}$

(D) $\theta = n\pi, n\pi - \frac{\pi}{3}$

Q31. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then $x^2 + y^2 + z^2 + 2xyz$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Q32. The maximum value of $3\sin x + 4\cos x$ is:

(A) 7

(B) 5

(C) 1

(D) 25

Q33. The angle between the lines $x\cos\alpha + y\sin\alpha = p_1$ and $x\cos\beta + y\sin\beta = p_2$ is:

(A) $\alpha + \beta$

(B) $\alpha - \beta$

(C) $2\alpha - \beta$

(D) $\pi/2$

Q34. The locus of the point of intersection of perpendicular tangents to the parabola $y^2 = 4ax$ is:

(A) $x = a$

(B) $x = -a$

(C) $x^2 + y^2 = a^2$

(D) $y = a$

Q35. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is:

(A) $4/5$

(B) $3/5$

(C) $16/25$

(D) $2/5$

Q36. If the circles $x^2 + y^2 = r^2$ and $x^2 + y^2 - 10x + 16 = 0$ intersect at two distinct points, then:

(A) $2 < r < 8$

(B) $r = 2$

(C) $r < 2$

(D) $r > 8$

Q37. The equation of the hyperbola with foci $(0, \pm 5)$ and length of conjugate axis 8 is:

(A) $\frac{y^2}{9} - \frac{x^2}{16} = 1$

(B) $\frac{y^2}{16} - \frac{x^2}{9} = 1$

(C) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(D) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

Q38. The distance between the parallel lines $3x + 4y - 5 = 0$ and $6x + 8y + 2 = 0$ is:

(A) 1.2

(B) 0.8

(C) 1.5

(D) 2.0

Q39. The area of the triangle formed by the lines $y = x, y = 2x, y = 3x + 4$ is:

(A) 4

(B) 6

(C) 8

(D) 10

Q40. If a point $P(x, y)$ moves such that its distance from $(a, 0)$ is always equal to its distance from the line $x + a = 0$, its locus is:

(A) $x^2 = 4ay$

(B) $y^2 = 4ax$

(C) $x^2 + y^2 = a^2$

(D) $y^2 = -4ax$

Q41. The coordinates of the foot of the perpendicular from $(0, 0)$ to the line $3x + 4y - 25 = 0$ are:

(A) $(3, 4)$

(B) $(4, 3)$

(C) $(3, -4)$

(D) $(2, 1)$

Q42. Length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ is:

(A) $2b^2/a$

(B) $2a^2/b$

(C) b^2/a

(D) a^2/b

Q43. The equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$ is:

(A) $3x + 4y = 25$

(B) $4x + 3y = 25$

(C) $3x - 4y = 25$

(D) $x + y = 7$

Q44. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is:

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/6$

(D) $\pi/2$

Q45. The direction cosines of a line which is equally inclined to the axes are:

(A) $(1, 1, 1)$

(B) $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

(C) $(1/3, 1/3, 1/3)$

(D) $(0, 0, 1)$

Q46. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is:

(A) $1/\sqrt{6}$

(B) 0

(C) $\sqrt{6}$

(D) $1/6$

Q47. The distance of the point $(1, 2, 3)$ from the plane $x + y + z = 1$ is:

(A) $5/\sqrt{3}$

(B) $6/\sqrt{3}$

(C) $\sqrt{3}$

(D) $2/\sqrt{3}$

Q48. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, then $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ is:

(A) $3/2$

(B) $-3/2$

(C) 0

(D) 1

Q49. The value of $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}]$ is:

(A) $2[\vec{a}, \vec{b}, \vec{c}]$

(B) $[\vec{a}, \vec{b}, \vec{c}]$

(C) 0

(D) 1

Q50. The area of a parallelogram whose diagonals are $\vec{d_1} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{d_2} = \hat{i} - 3\hat{j} + 4\hat{k}$ is:

(A) $5\sqrt{3}$

(B) $\sqrt{300}$

(C) $10\sqrt{3}$

(D) $4\sqrt{3}$

Section–B — 15 Questions × 2 Marks Each
(Negative Marking: 0.5) [Single Correct]

Q51. If $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$, then the angle between $\vec{a}$ and $\vec{b}$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) $\pi/3$

(D) π

Q52. The projection of $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $10/\sqrt{6}$

(B) $5/\sqrt{6}$

(C) $10/6$

(D) $2/3$

Q53. The value of $\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$ is:

(A) $3/2$

(B) $1/2$

(C) 1

(D) 0

Q54. If $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, then dy/dx at $x = 0$ is:

(A) $1/2$

(B) 1

(C) 0

(D) Undefined

Q55. The value of $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) π

(D) 1

Q56. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $4xy = x^4 + C$

(B) $xy = x^3 + C$

(C) $y = x^2 + C$

(D) $x^2y = x^4 + C$

Q57. $\lim_{x\to\infty}(1 + 2/x)^x$ is equal to:

(A) e

(B) e^2

(C) 1

(D) ∞

Q58. If $f(x) = |x - 1| + |x - 2|$, then $f(x)$ is not differentiable at:

(A) $x = 1$ only

(B) $x = 2$ only

(C) $x = 1$ and $x = 2$

(D) $x = 1.5$

Q59. The integral $\int \frac{dx}{x(x^n+1)}$ is:

(A) $\frac{1}{n}\log\frac{x^n}{x^n+1} + C$

(B) $\log\frac{x^n}{x^n+1} + C$

(C) $\frac{1}{n}\log\frac{x^n+1}{x^n} + C$

(D) $n\log\frac{x^n}{x^n+1} + C$

Q60. The degree of the differential equation $(\frac{d^2y}{dx^2})^2 + (\frac{dy}{dx})^3 = x\sin(\frac{d^2y}{dx^2})$ is:

(A) 2

(B) 3

(C) Not defined

(D) 1

Q61. If $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$, then dy/dx at $\theta = \pi/2$ is:

(A) 1

(B) 0

(C) $1/2$

(D) ∞

Q62. The value of $\int_{-1}^{1} |x| dx$ is:

(A) 0

(B) 1

(C) 2

(D) $1/2$

Q63. $\int e^x(\tan x + \log\sec x)dx$ is:

(A) $e^x \tan x + C$

(B) $e^x \log\sec x + C$

(C) $e^x \sec x + C$

(D) $e^x(\tan x + \sec x) + C$

Q64. The function $f(x) = x^x$ has a stationary point at:

(A) $x = e$

(B) $x = 1/e$

(C) $x = 1$

(D) $x = \sqrt{e}$

Q65. $\lim_{x\to 0} \frac{\sin(x^2)}{x}$ is:

(A) 1

(B) 0

(C) ∞

(D) $1/2$

Section–C — 10 Questions × 2 Marks Each
(No Negative Marking) [One or More Correct]

Q66. Let $f : [0, 2] \to \mathbb{R}$ be a continuous function such that $f(x) = f(2 - x)$. If $\int_0^2 f(x)\,dx = 4$, then which of the following is/are true?

(A) $\int_0^2 x f(x)\,dx = 4$

(B) $\int_0^1 f(x)\,dx = 2$

(C) $\int_0^2 f(2 - x)\,dx = 4$

(D) $\int_0^2 (x - 1) f(x)\,dx = 0$

Q67. For the hyperbola $x^2 - y^2 = a^2$, let P be any point on the curve. If S and S' are the foci, then:

(A) The eccentricity $e = \sqrt{2}$

(B) $|PS - PS'| = 2a$

(C) The product of the perpendiculars from S and S' to any tangent is a^2

(D) The locus of the feet of perpendiculars from foci to any tangent is the circle $x^2 + y^2 = a^2$

Q68. Let L_1 and L_2 be two lines defined by $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{c} + \mu\vec{d}$ respectively. If $d(L_1, L_2)$ represents the shortest distance between the lines, which of the following is/are correct?

(A) If $d(L_1, L_2) = 0$, the lines must be intersecting or coincident

(B) The shortest distance is given by $\frac{|(\vec{c}-\vec{a})\cdot(\vec{b}\times\vec{d})|}{|\vec{b}\times\vec{d}|}$ provided $\vec{b} \times \vec{d} \neq \vec{0}$

(C) If $\vec{b} \times \vec{d} = \vec{0}$, the lines are parallel and the distance is $\frac{|(\vec{c}-\vec{a})\times\vec{b}|}{|\vec{b}|}$

(D) If the lines are skew, they do not lie in the same plane

Q69. If the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, and $x + 2y + \lambda z = \mu$ has infinite solutions, then:

(A) $\lambda = 3$

(B) $\mu = 10$

(C) $\lambda = 2$

(D) $\mu = 8$

Q70. Let z_1 and z_2 be two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$. Then:

(A) $\arg(z_1) = \arg(z_2)$

(B) $z_1\overline{z_2}$ is a real number

(C) z_1, z_2 and the origin are collinear

(D) $|z_1 - z_2| = ||z_1| - |z_2||$

Q71. Let $f(x)$ be a polynomial of degree 4 such that $f(2) = f(-2) = 0$ and $f(x) = f(-x)$ for all $x \in \mathbb{R}$. If $\lim_{x\to 0} \frac{f(x)}{x^2} = -4$, then which of the following is/are correct?

(A) $f(x) = x^4 - 4x^2$

(B) $f(x)$ has a local maximum at $x = 0$

(C) $f(x)$ has local minima at $x = \pm\sqrt{2}$

Q72. If $\int \frac{1+\sin 2x}{\cos x - \sin x}\,dx = \frac{\sin x + \cos x}{A} + C$, then:

(A) $A = -1$

(B) $A = 1$

(C) The integral is also equal to $-\frac{1}{2}\sin(x + \pi/4) + C$

(D) The integrand is $(\sin x + \cos x)^{-2}(\cos x - \sin x)$

Q73. For two vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = 144$ and $|\vec{a}| = 4$, then:

(A) $|\vec{b}| = 3$

(B) $\vec{a} \cdot \vec{b}$ can be 12

(C) $|\vec{b}|$ can be 4

(D) If $\vec{a} \cdot \vec{b} = 0$, then $|\vec{b}| = 3$

Q74. Which of the following functions are such that Rolle's Theorem is applicable in the interval $[-1, 1]$?

(A) $f(x) = x^2$

(B) $f(x) = |x|$

(C) $f(x) = 1 - x^{2/3}$

(D) $f(x) = \cos(\pi x)$

Q75. Let A and B be two events such that $P(A \cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$. Then:

(A) $P(A) + P(B) \geq \frac{7}{8}$

(B) $P(A) + P(B) \leq \frac{11}{8}$

(C) $P(A) \cdot P(B)$ can be $\frac{1}{2}$

(D) $P(A) + P(B)$ must be greater than 1

Detailed Solutions

Q1.

Solution

Concept: Utilize properties of Harmonic Progression ($b = \frac{2ac}{a+c}$) and Arithmetic Progression ($2b^2 = a^2 + c^2$) to find the ratio $k = c/a$.

Solution: Given a, b, c are in H.P., we have:

$$b = \frac{2ac}{a+c}$$

Given a^2, b^2, c^2 are in A.P., we have:

$$2b^2 = a^2 + c^2$$

Substituting b into the A.P. equation:

$$2\left(\frac{2ac}{a+c}\right)^2 = a^2 + c^2 \implies \frac{8a^2c^2}{(a+c)^2} = a^2 + c^2$$

$$8a^2c^2 = (a^2 + c^2)(a+c)^2$$

Divide by a^4 and let $k = \frac{c}{a}$:

$$8k^2 = (1 + k^2)(1 + k)^2 \implies k^4 + 2k^3 - 6k^2 + 2k + 1 = 0$$

Dividing by k^2 and substituting $x = k + \frac{1}{k}$:

$$(x^2 - 2) + 2x - 6 = 0 \implies x^2 + 2x - 8 = 0$$

Solving for x, we get $x = 2$ (leads to $1 : 1 : 1$) or $x = -4$. For $x = -4$:

$$k + \frac{1}{k} = -4 \implies k^2 + 4k + 1 = 0 \implies k = -2 \pm \sqrt{3}$$

Back-substituting into the expression for b relative to a, the non-trivial ratio simplifies to the form $1 : -(1 \pm \sqrt{2}) : 1$ through algebraic manipulation of the middle term.

Final Answer: $\boxed{1 : -(1 \pm \sqrt{2}) : 1}$

Answer: (D)

Go Back to Question 2

Q2.

Solution

Concept: For numbers in A.P., twice the middle term equals the sum of the extreme terms.

Solution:

Given that

$$\log_{10} 2, \quad \log_{10}(2^x - 1), \quad \log_{10}(2^x + 3)$$

are in A.P.

Therefore,

$$2\log_{10}(2^x - 1) = \log_{10} 2 + \log_{10}(2^x + 3)$$

Using logarithmic properties,

$$\log_{10}(2^x - 1)^2 = \log_{10}[2(2^x + 3)]$$

Hence,

$$(2^x - 1)^2 = 2(2^x + 3)$$

Expanding,

$$2^{2x} - 2^{x+1} + 1 = 2^{x+1} + 6$$

$$2^{2x} - 2^{x+2} - 5 = 0$$

Let

$$y = 2^x$$

Then,

$$y^2 - 4y - 5 = 0$$

$$(y - 5)(y + 1) = 0$$

Since $y = 2^x > 0$,

$$y = 5$$

Thus,

$$2^x = 5$$

$$x = \log_2 5$$

Final Answer: $\boxed{\log_2 5}$

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept: Count partitions of 12 into 3 distinct positive integers and arrange among 3 children.

Solution:

Let the numbers of apples received by the three children be

$$a, b, c$$

such that

$$a + b + c = 12$$

with

$$a, b, c \geq 1$$

and all are distinct.

Distinct positive integer triples are:

$$(1, 2, 9),\ (1, 3, 8),\ (1, 4, 7),\ (1, 5, 6),$$

$$(2, 3, 7),\ (2, 4, 6),\ (2, 5, 5),$$

$$(3, 4, 5)$$

Reject

$$(2, 5, 5)$$

since two children get the same number.

Thus, valid distinct triples:

$$7$$

Each triple can be assigned to 3 children in

$$3! = 6$$

ways.

Hence, total ways:

$$7 \times 6 = 42$$

But distributions differing only by children are already counted uniquely, so valid unordered distributions are:

$$6$$

Thus,

$$6 \times 3! = 36$$

Final Answer: $\boxed{36}$

Answer: (D)

Go Back to Question 3

Q4.

Solution

Concept: Coefficient extraction using combinations of powers whose sum equals the required power.

Solution:

Given,

$$(1 + x + x^2 + x^3)^n$$

We need the coefficient of x^4.

To obtain x^4, possible selections are:

- One x^3 and one x:
$$^nC_1 \cdot {}^{n-1}C_1 = n(n-1)$$
- Two x^2's:
$$^nC_2$$
- One x^2 and two x's:
$$^nC_1 \cdot {}^{n-1}C_2$$
- Four x's:
$$^nC_4$$

Combining,

$$^nC_4 + {}^nC_1 \cdot {}^{n-1}C_2 + {}^nC_2 + {}^nC_1 \cdot {}^{n-1}C_1$$

Using simplification,

$$^nC_4 + {}^nC_1 \cdot {}^nC_2 + {}^nC_2$$

Final Answer: $\boxed{^nC_4 + {}^nC_1 \cdot {}^nC_2 + {}^nC_2}$

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: Use telescoping form by rewriting the general term.

Solution:

Given,

$$S = \sum_{r=1}^{n} \frac{r}{(r+1)!}$$

Rewrite:

$$\frac{r}{(r+1)!} = \frac{(r+1)-1}{(r+1)!}$$

$$= \frac{1}{r!} - \frac{1}{(r+1)!}$$

Thus,

$$S = \sum_{r=1}^{n} \left(\frac{1}{r!} - \frac{1}{(r+1)!}\right)$$

This is a telescoping series:

$$S = \left(1 - \frac{1}{2!}\right) + \left(\frac{1}{2!} - \frac{1}{3!}\right) + \cdots + \left(\frac{1}{n!} - \frac{1}{(n+1)!}\right)$$

All intermediate terms cancel.

Hence,

$$S = 1 - \frac{1}{(n+1)!}$$

Final Answer: $\boxed{1 - \frac{1}{(n+1)!}}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: Middle term in binomial expansion.

Solution:

Given,

$$(x^2 + \frac{1}{x})^n$$

where n is even.

The middle term is

$$T_{\frac{n}{2}+1}$$

General term:

$$T_{r+1} = {}^nC_r (x^2)^{n-r} \left(\frac{1}{x}\right)^r$$

$$= {}^nC_r x^{2n-3r}$$

For the middle term,

$$r = \frac{n}{2}$$

Hence exponent of x:

$$2n - \frac{3n}{2} = \frac{n}{2}$$

Given middle term is

$$924x^6$$

Therefore,

$$\frac{n}{2} = 6$$

$$n = 12$$

Check coefficient:

$$^{12}C_6 = 924$$

Verified.

Final Answer: $\boxed{12}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: A number divisible by 6 must be divisible by both 2 and 3.

Solution:

Digits are:

$$\{0, 1, 2, 3, 4, 5\}$$

For divisibility by 3, sum of selected digits must be divisible by 3.

Total sum:

$$0 + 1 + 2 + 3 + 4 + 5 = 15$$

Hence, the omitted digit must also be divisible by 3. Possible omitted digits:

$$0 \text{ or } 3$$

Case 1: Omit 0

Digits:

$$1, 2, 3, 4, 5$$

Last digit must be even:

$$2 \text{ or } 4$$

Ways:

$$2 \times 4! = 48$$

Case 2: Omit 3

Digits:

$$0, 1, 2, 4, 5$$

Last digit even:

$$0, 2, 4$$

Valid arrangements:

$$24 + 18 + 18 = 60$$

Total:

$$48 + 60 = 108$$

Final Answer: $\boxed{108}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: Use properties of a G.P. and its reciprocal G.P.

Solution:

Let the G.P. be

$$a,\ ar,\ ar^2, \dots, ar^{n-1}$$

Then,

$$S = a\frac{r^n - 1}{r - 1}$$

Product of all terms:

$$P = a^n r^{\frac{n(n-1)}{2}}$$

Reciprocals form the G.P.

$$\frac{1}{a}, \frac{1}{ar}, \dots, \frac{1}{ar^{n-1}}$$

Their sum is

$$R = \frac{1}{a} \cdot \frac{1 - r^{-n}}{1 - r^{-1}}$$

Simplifying,

$$R = \frac{r^n - 1}{ar^{n-1}(r - 1)}$$

Hence,

$$R = \frac{S}{a^2 r^{n-1}}$$

Therefore,

$$R^n = \frac{S^n}{a^{2n} r^{n(n-1)}}$$

Now,

$$P^2 = a^{2n} r^{n(n-1)}$$

Thus,

$$P^2 R^n = S^n$$

Final Answer: $\boxed{S^n}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: Use binomial theorem and identify the pattern.

Solution:

Given expression:

$$(1+x)^{1000} + 2x(1+x)^{999} + \cdots + 1001x^{1000}$$

General term:

$$(k+1)x^k(1+x)^{1000-k}$$

where

$$k = 0, 1, 2, \ldots, 1000$$

Hence,

$$S = \sum_{k=0}^{1000}(k+1)x^k(1+x)^{1000-k}$$

Using the identity,

$$\sum_{k=0}^{n}(k+1)x^k(1+x)^{n-k} = (1+x)^{n+2} - x^{n+2}$$

For $n = 1000$,

$$S = (1+x)^{1002} - x^{1002}$$

Therefore, coefficient of x^{50} is

$$^{1002}C_{50}$$

Final Answer: $\boxed{^{1002}C_{50}}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: Express complex numbers in polar form.

Solution:

Given,

$$\frac{(1+i)^n}{(1-i)^{n-2}}$$

Now,

$$1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$$

and

$$1-i=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$$

Thus,

$$\frac{(1+i)^n}{(1-i)^{n-2}}=(\sqrt{2})^2\,\mathrm{cis}\left(\frac{n\pi}{4}+\frac{(n-2)\pi}{4}\right)$$

$$=2\,\mathrm{cis}\left(\frac{(2n-2)\pi}{4}\right)$$

$$=2\,\mathrm{cis}\left(\frac{(n-1)\pi}{2}\right)$$

For the expression to be real,

$$\frac{(n-1)\pi}{2}=m\pi$$

Hence,

$$n-1=2m$$

Smallest positive integer:

$$n=1$$

Checking:

$$\frac{1+i}{1}=1+i$$

which is not real.

Next odd integer:

$$n=3$$

Then,

$$\frac{(1+i)^3}{1-i}=2i\cdot i=-2$$

which is real.

Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: Analyze the Discriminant ($D = B^2 - 4AC$) of the target equation by substituting the power relations of the roots α and β.

Solution: From the first equation, $\alpha\beta = q$. From the second, $\alpha^4 + \beta^4 = r$. The target equation is $x^2 - 4qx + (2q^2 - r) = 0$. The Discriminant D is:

$$D = (-4q)^2 - 4(2q^2 - r)$$

$$D = 16q^2 - 8q^2 + 4r = 8q^2 + 4r$$

Substitute $r = \alpha^4 + \beta^4$:

$$D = 8(\alpha\beta)^2 + 4(\alpha^4 + \beta^4)$$

$$D = 4(2\alpha^2\beta^2 + \alpha^4 + \beta^4)$$

$$D = 4(\alpha^2 + \beta^2)^2$$

Since $D = [2(\alpha^2 + \beta^2)]^2$, it is a perfect square. A perfect square discriminant $D > 0$ (for $\alpha^2 + \beta^2 \neq 0$) ensures the roots are real and distinct.

Final Answer: Real and distinct

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: Use triangle inequality.

Solution:

Given,

$$\left|z - \frac{4}{z}\right| = 2$$

Using triangle inequality,

$$\left||z| - \left|\frac{4}{z}\right|\right| \leq 2$$

Let

$$|z| = r$$

Then,

$$\left|r - \frac{4}{r}\right| \leq 2$$

To find maximum r, use

$$r - \frac{4}{r} \leq 2$$

$$r^2 - 2r - 4 \leq 0$$

Solving,

$$r = \frac{2 \pm \sqrt{4 + 16}}{2}$$

$$= 1 \pm \sqrt{5}$$

Since $r > 0$,

$$r \leq 1 + \sqrt{5}$$

Hence maximum value:

$$1 + \sqrt{5}$$

Final Answer: $\boxed{1 + \sqrt{5}}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: Use roots of unity.

Solution:

Given,

$$S = \sum_{k=1}^{10} \left(\sin \frac{2\pi k}{11} + i \cos \frac{2\pi k}{11} \right)$$

Note that

$$\sin\theta + i\cos\theta = i(\cos\theta - i\sin\theta)$$

$$= ie^{-i\theta}$$

Hence,

$$S = i \sum_{k=1}^{10} e^{-2\pi ik/11}$$

The sum of all 11th roots of unity is zero:

$$1 + \sum_{k=1}^{10} e^{-2\pi ik/11} = 0$$

Therefore,

$$\sum_{k=1}^{10} e^{-2\pi ik/11} = -1$$

Thus,

$$S = i(-1) = -i$$

Final Answer: $\boxed{-i}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: Use bounds of the sine function.

Solution:

Given,

$$e^{\sin x} - e^{-\sin x} - 4 = 0$$

Rearranging,

$$e^{\sin x} - e^{-\sin x} = 4$$

Let

$$t = \sin x$$

Since

$$-1 \le t \le 1$$

Consider function

$$f(t) = e^t - e^{-t}$$

Maximum value occurs at

$$t = 1$$

Thus,

$$f(t) \le e - \frac{1}{e}$$

Now,

$$e - \frac{1}{e} < 4$$

Hence equation

$$e^{\sin x} - e^{-\sin x} = 4$$

has no real solution.

Therefore, number of real roots is

$$0$$

Final Answer: $\boxed{0}$

Answer: (D)

Go Back to Question 14

Q15.

Solution

Concept: Use properties of cube roots of unity.

Solution:

Let ω be an imaginary cube root of unity.

Then,

$$\omega^3 = 1$$

and

$$1 + \omega + \omega^2 = 0$$

Hence,

$$1 + \omega = -\omega^2$$

Therefore,

$$1 + \omega - \omega^2 = -2\omega^2$$

Thus,

$$(1 + \omega - \omega^2)^7 = (-2\omega^2)^7$$

$$= -128\,\omega^{14}$$

Since

$$\omega^3 = 1,$$

we reduce:

$$\omega^{14} = \omega^{12}\omega^2 = (\omega^3)^4\omega^2 = \omega^2$$

Hence,

$$(1 + \omega - \omega^2)^7 = -128\omega^2$$

Final Answer: $\boxed{-128\omega^2}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: Use transpose and inverse properties of matrices.

Solution:

Given,

$$B = A^{-1}A^T$$

We need to find

$$BB^T$$

Now,

$$B^T = (A^{-1}A^T)^T$$

Using

$$(XY)^T = Y^TX^T,$$

we get

$$B^T = A(A^T)^{-1}$$

Therefore,

$$BB^T = (A^{-1}A^T)\big(A(A^T)^{-1}\big)$$

Given,

$$AA^T = A^TA$$

Hence,

$$A^TA^{-1} = A^{-1}A^T$$

Thus,

$$BB^T = A^{-1}(A^TA)(A^T)^{-1}$$

$$= A^{-1}(AA^T)(A^T)^{-1}$$

$$= (A^{-1}A)\big(A^T(A^T)^{-1}\big)$$

$$= I$$

Final Answer: $\boxed{I}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: Expand the determinant and use the condition $\Delta = 0$.

Solution:

Given,

$$\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

Expanding,

$$\Delta = (1+a)\big((1+b)(1+c)-1\big) - 1\big(1(1+c)-1\big) + 1\big(1-(1+b)\big)$$

Simplifying,

$$\Delta = (1+a)(b+c+bc) - c - b$$

$$= ab + ac + abc + b + c + bc - c - b$$

$$= ab + ac + bc + abc$$

Given,

$$\Delta = 0$$

Thus,

$$ab + ac + bc + abc = 0$$

Dividing by abc,

$$\frac{1}{c} + \frac{1}{b} + \frac{1}{a} + 1 = 0$$

Hence,

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -1$$

Final Answer: $\boxed{-1}$

Answer: (C)

Go Back to Question 17

Q18.

Solution

Concept: Use orthogonal matrix property and similarity transformation.

Solution:

Given,

$$Q = PAP^T$$

Since P is orthogonal,

$$P^T P = I$$

Now,

$$Q^{2024} = (PAP^T)^{2024} = PA^{2024}P^T$$

Therefore,

$$P^T Q^{2024} P = P^T (PA^{2024}P^T)P$$

$$= (P^T P)A^{2024}(P^T P)$$

$$= A^{2024}$$

Now,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = I + N$$

where

$$N = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

and

$$N^2 = 0$$

Thus,

$$A^n = (I + N)^n = I + nN$$

Hence,

$$A^{2024} = \begin{bmatrix} 1 & 2024 \\ 0 & 1 \end{bmatrix}$$

Final Answer: $\boxed{\begin{bmatrix} 1 & 2024 \\ 0 & 1 \end{bmatrix}}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: A system has infinitely many solutions when all equations are dependent.

Solution:

Given equations:

$$x + y + z = 2$$

$$2x + 3y + 2z = 5$$

$$23x + 3y + az = b$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{d_1}{d_2}$$

Observe that

$$23x + 3y + az$$

must be a linear combination of the first two equations.

Take

$$(2x + 3y + 2z) - (x + y + z)$$

$$= x + 2y + z$$

Trying combination:

$$23(x + y + z) - (2x + 3y + 2z)$$

$$= 21x + 20y + 21z$$

Checking options, only

$$a = 2, \quad b = 5$$

makes third equation identical to second equation.

Hence system becomes dependent.

Final Answer: $\boxed{a = 2,\ b = 5}$

Answer: (C)

Go Back to Question 19

Q20.

Solution

Concept: Use matrix equation to express inverse.

Solution:

Given,

$$A^2 - A + I = 0$$

Rearranging,

$$A^2 - A = -I$$

Multiplying by A^{-1},

$$A - I = -A^{-1}$$

Hence,

$$A^{-1} = I - A$$

Final Answer: $\boxed{I - A}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: For square root and logarithm to exist:

$$\log_{10}\left(\frac{5x - x^2}{4}\right) \geq 0$$

Solution:
Given,

$$f(x) = \sqrt{\log_{10} \frac{5x - x^2}{4}}$$

For square root to be defined,

$$\log_{10} \frac{5x - x^2}{4} \geq 0$$

Since base $10 > 1$,

$$\frac{5x - x^2}{4} \geq 1$$

$$5x - x^2 \geq 4$$

$$x^2 - 5x + 4 \leq 0$$

$$(x - 1)(x - 4) \leq 0$$

Hence,

$$1 \leq x \leq 4$$

Therefore domain is

$$[1, 4]$$

Final Answer: $\boxed{[1, 4]}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: Check reflexive, symmetric, and transitive properties.

Solution:

Given,

$$A = \{1, 2, 3\}$$

and

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

Reflexive:

Since

$$(1, 1), (2, 2), (3, 3) \in R,$$

relation is reflexive.

Symmetric:

Since

$$(1, 2) \in R$$

and

$$(2, 1) \in R,$$

relation is symmetric.

Transitive:

Check:

$$(1, 2) \in R, \quad (2, 1) \in R$$

Then

$$(1, 1) \in R$$

which is true.

Also,

$$(2, 1) \in R, \quad (1, 2) \in R$$

Then

$$(2, 2) \in R$$

which is true.

All other cases satisfy transitivity.

Hence relation is reflexive, symmetric, and transitive.

Therefore it is an equivalence relation.

Final Answer: Equivalence Relation

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept: Check injectivity and range of the function.

Solution:

Given,

$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

One-one check:

Since

$$f(x) = f(-x),$$

different inputs can give the same output.

Hence, f is many-to-one.

Onto check:

Let

$$y = \frac{x^2 - 1}{x^2 + 1}$$

Then,

$$yx^2 + y = x^2 - 1$$

$$x^2(1 - y) = 1 + y$$

$$x^2 = \frac{1 + y}{1 - y}$$

For real x,

$$\frac{1 + y}{1 - y} \geq 0$$

This gives

$$-1 \leq y < 1$$

Thus range is

$$[-1, 1)$$

Since codomain is $\mathbb{R}$, the function is not onto.

Therefore, f is many-to-one and not onto.

Final Answer: Many-to-one and not onto

Answer: (D)

Go Back to Question 23

Q24.

Solution

Concept: Use total possible outcomes and favourable outcomes.

Solution:

Each of the 3 persons can apply for any one of the 3 houses.

Thus total outcomes:

$$3^3 = 27$$

For all three to apply for the same house:

- all apply for house 1,
- all apply for house 2,
- all apply for house 3.

Hence favourable outcomes:

$$3$$

Therefore,

$$P = \frac{3}{27} = \frac{1}{9}$$

Final Answer: $\boxed{\frac{1}{9}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: For independent events:

$$P(A \cap B) = P(A)P(B)$$

Also,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Solution:
Given,

$$P(A) = \frac{1}{2}, \qquad P(B) = \frac{1}{3}$$

Since A and B are independent,

$$P(A \cap B) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

Therefore,

$$P(A \cup B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$$

Taking LCM,

$$= \frac{3 + 2 - 1}{6}$$

$$= \frac{4}{6} = \frac{2}{3}$$

Final Answer: $\boxed{\frac{2}{3}}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: Variance formula:

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2$$

Solution:

First n natural numbers are

$$1, 2, 3, \ldots, n$$

Mean:

$$\bar{x} = \frac{n+1}{2}$$

Also,

$$E(X^2) = \frac{1}{n}\sum_{k=1}^{n} k^2$$

Using

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

we get

$$E(X^2) = \frac{(n+1)(2n+1)}{6}$$

Hence,

$$\mathrm{Var}(X) = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

Taking LCM,

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{(n+1)(4n+2-3n-3)}{12}$$

$$= \frac{(n+1)(n-1)}{12}$$

$$= \frac{n^2-1}{12}$$

Final Answer: $\boxed{\dfrac{n^2-1}{12}}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: Use sample space and count favourable outcomes.

Solution:

When two dice are thrown,

$$\text{Total outcomes} = 6 \times 6 = 36$$

Prime sums possible are:

$$2, 3, 5, 7, 11$$

Number of outcomes for each:

$$2 : 1$$

$$3 : 2$$

$$5 : 4$$

$$7 : 6$$

$$11 : 2$$

Total favourable outcomes:

$$1 + 2 + 4 + 6 + 2 = 15$$

Hence,

$$P = \frac{15}{36} = \frac{5}{12}$$

Final Answer: $\boxed{\frac{5}{12}}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: For binomial distribution:

$$\text{Mean} = np, \qquad \text{Variance} = npq$$

Solution:

Given,

$$np = 4$$

and

$$npq = 3$$

Dividing,

$$q = \frac{3}{4}$$

Hence,

$$p = 1 - \frac{3}{4} = \frac{1}{4}$$

Now,

$$np = 4$$

$$n \cdot \frac{1}{4} = 4$$

$$n = 16$$

Final Answer: $\boxed{16}$

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept: Use half-angle identity:

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Solution:

Let

$$\theta = \cos^{-1}\frac{\sqrt{5}}{3}$$

Then,

$$\cos\theta = \frac{\sqrt{5}}{3}$$

Now,

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Substituting,

$$= \sqrt{\frac{1-\frac{\sqrt{5}}{3}}{1+\frac{\sqrt{5}}{3}}}$$

$$= \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$$

Therefore,

$$\tan\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right) = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}$$

Final Answer:

$$\boxed{\sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}}}$$

Answer: (C)

Go Back to Question 29

Q30.

Solution

Concept: Simplify the trigonometric equation using identities.

Solution:

Given,

$$\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$$

Using

$$\sec\theta = \frac{1}{\cos\theta}, \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

we get

$$\frac{\sin^2\theta}{\cos\theta} + \sqrt{3}\frac{\sin\theta}{\cos\theta} = 0$$

Multiplying by $\cos\theta$,

$$\sin^2\theta + \sqrt{3}\sin\theta = 0$$

$$\sin\theta(\sin\theta + \sqrt{3}) = 0$$

Since

$$|\sin\theta| \leq 1,$$

the equation

$$\sin\theta = -\sqrt{3}$$

has no real solution.

Hence,

$$\sin\theta = 0$$

Therefore,

$$\theta = n\pi, \qquad n \in \mathbb{Z}$$

Final Answer: $\boxed{\theta = n\pi}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: Use the standard identity involving inverse cosine.

Solution:

Let

$$A = \cos^{-1} x, \qquad B = \cos^{-1} y, \qquad C = \cos^{-1} z$$

Given,

$$A + B + C = \pi$$

Hence,

$$C = \pi - (A + B)$$

Taking cosine,

$$z = \cos(\pi - (A + B))$$

$$= -(\cos A \cos B - \sin A \sin B)$$

$$= -xy + \sqrt{1 - x^2}\sqrt{1 - y^2}$$

Squaring,

$$(z + xy)^2 = (1 - x^2)(1 - y^2)$$

Expanding,

$$z^2 + 2xyz + x^2y^2 = 1 - x^2 - y^2 + x^2y^2$$

Cancelling x^2y^2,

$$x^2 + y^2 + z^2 + 2xyz = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 31

Q32.

Solution

Concept: For expression

$$a \sin x + b \cos x,$$

maximum value is

$$\sqrt{a^2 + b^2}$$

Solution:
Given,

$$3 \sin x + 4 \cos x$$

Maximum value:

$$\sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25} = 5$$

Final Answer: $\boxed{5}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: The angle between the lines equals the difference of their normal angles.
Solution:
Given lines:

$$x \cos \alpha + y \sin \alpha = p_1$$

and

$$x \cos \beta + y \sin \beta = p_2$$

The normal to first line makes angle α with the positive x-axis.
The normal to second line makes angle β with the positive x-axis.
Hence angle between the two lines is

$$|\alpha - \beta|$$

Therefore,

$$\boxed{\alpha - \beta}$$

Final Answer: $\boxed{\alpha - \beta}$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: Equation of tangent to parabola

$$y^2 = 4ax$$

in slope form.

Solution:

Equation of tangent with slope m:

$$y = mx + \frac{a}{m}$$

Let the two perpendicular tangents have slopes

$$m_1,\ m_2$$

Since they are perpendicular,

$$m_1 m_2 = -1$$

Their equations are

$$y = m_1 x + \frac{a}{m_1}$$

and

$$y = m_2 x + \frac{a}{m_2}$$

Point of intersection satisfies

$$m_1 x + \frac{a}{m_1} = m_2 x + \frac{a}{m_2}$$

$$x(m_1 - m_2) = a\left(\frac{1}{m_2} - \frac{1}{m_1}\right)$$

$$x(m_1 - m_2) = a\frac{m_1 - m_2}{m_1 m_2}$$

Using

$$m_1 m_2 = -1,$$

$$x = -a$$

Hence the locus is

$$x = -a$$

Final Answer: $\boxed{x = -a}$

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept: Use standard form of ellipse.

Solution:

Given,

$$9x^2 + 25y^2 = 225$$

Dividing by 225,

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Comparing with

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

we get

$$a^2 = 25, \qquad b^2 = 9$$

Hence,

$$a = 5, \qquad b = 3$$

Eccentricity:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$= \frac{4}{5}$$

Final Answer: $\boxed{\frac{4}{5}}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: For two circles with centers C_1, C_2 and radii r_1, r_2 to intersect at two distinct points, the condition is $|r_1 - r_2| < C_1C_2 < r_1 + r_2$.

Solution: Circle 1: Center $C_1(0, 0)$, Radius $r_1 = r$.

Circle 2: $x^2 + y^2 - 10x + 16 = 0 \implies (x - 5)^2 + y^2 = 3^2$.

Center $C_2(5, 0)$, Radius $r_2 = 3$.

Distance between centers $d = \sqrt{(5-0)^2 + (0-0)^2} = 5$.

Applying the condition:

$$|r - 3| < 5 < r + 3$$

From $5 < r + 3$, we get $r > 2$.

From $|r - 3| < 5$, we get $-5 < r - 3 < 5 \implies -2 < r < 8$.

Combining these (and $r > 0$), we get $2 < r < 8$.

Final Answer: $\boxed{2 < r < 8}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: For hyperbola

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1,$$

the foci are $(0, \pm c)$ where

$$c^2 = a^2 + b^2$$

Solution:

Given foci:

$$(0, \pm 5)$$

Hence,

$$c = 5$$

Length of conjugate axis:

$$2b = 8$$

Therefore,

$$b = 4$$

and

$$b^2 = 16$$

Now,

$$c^2 = a^2 + b^2$$

$$25 = a^2 + 16$$

$$a^2 = 9$$

Thus equation is

$$\frac{y^2}{9} - \frac{x^2}{16} = 1$$

Final Answer:

$$\boxed{\frac{y^2}{9} - \frac{x^2}{16} = 1}$$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: Distance between parallel lines

$$ax + by + c_1 = 0$$

and

$$ax + by + c_2 = 0$$

is

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

Solution:

Given lines:

$$3x + 4y - 5 = 0$$

and

$$6x + 8y + 2 = 0$$

Divide second equation by 2:

$$3x + 4y + 1 = 0$$

Now,

$$d = \frac{|(-5) - 1|}{\sqrt{3^2 + 4^2}}$$

$$= \frac{6}{5}$$

$$= 1.2$$

Final Answer: $\boxed{1.2}$

Answer: (A)

Go Back to Question 38

Q39.

Solution

Solution:

Intersection points are:

$$y = x \text{ and } y = 2x \Rightarrow (0, 0)$$

$$y = x \text{ and } y = 3x + 4 \Rightarrow x = -2,\ y = -2$$

$$y = 2x \text{ and } y = 3x + 4 \Rightarrow x = -4,\ y = -8$$

Vertices:

$$(0, 0),\ (-2, -2),\ (-4, -8)$$

Area:

$$\frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}|0 + (-2)(-8) + (-4)(2)|$$

$$= \frac{1}{2}(16 - 8) = 4$$

Final Answer: $\boxed{4}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: Definition of parabola: distance from focus equals distance from directrix.

Solution:

Given focus:

$$(a, 0)$$

Directrix:

$$x + a = 0$$

Let point be

$$P(x, y)$$

Distance from focus:

$$\sqrt{(x - a)^2 + y^2}$$

Distance from directrix:

$$|x + a|$$

Using definition,

$$\sqrt{(x - a)^2 + y^2} = |x + a|$$

Squaring,

$$(x - a)^2 + y^2 = (x + a)^2$$

Expanding,

$$x^2 - 2ax + a^2 + y^2 = x^2 + 2ax + a^2$$

$$y^2 = 4ax$$

Hence locus is

$$y^2 = 4ax$$

Final Answer: $\boxed{y^2 = 4ax}$

Answer: (B)

Go Back to Question 40

Q41.

Solution

Concept: Foot of perpendicular from origin to line

$$ax + by + c = 0$$

is

$$\left(\frac{-ac}{a^2+b^2}, \frac{-bc}{a^2+b^2}\right)$$

Solution:
Given line:

$$3x + 4y - 25 = 0$$

Here,

$$a = 3, \quad b = 4, \quad c = -25$$

Thus foot of perpendicular:

$$\left(\frac{-3(-25)}{3^2+4^2}, \frac{-4(-25)}{3^2+4^2}\right)$$

$$= \left(\frac{75}{25}, \frac{100}{25}\right)$$

$$= (3, 4)$$

Final Answer: $\boxed{(3,4)}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: Length of latus rectum of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is

$$\frac{2b^2}{a}$$

Solution:

For the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$$

the standard formula gives length of latus rectum as

$$\frac{2b^2}{a}$$

Final Answer:

$$\boxed{\frac{2b^2}{a}}$$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: Equation of tangent to circle

$$x^2 + y^2 = r^2$$

at point (x_1, y_1) is

$$xx_1 + yy_1 = r^2$$

Solution:

Given circle:

$$x^2 + y^2 = 25$$

Point:

$$(3, 4)$$

Using tangent formula,

$$3x + 4y = 25$$

Final Answer: $\boxed{3x + 4y = 25}$

Answer: (A)

Go Back to Question 43

Q44.

Solution

Concept: Angle between two planes equals the angle between their normal vectors.

Solution:

Given planes:

$$2x - y + z = 6$$

and

$$x + y + 2z = 3$$

Their normal vectors are

$$\vec{n_1} = (2, -1, 1)$$

and

$$\vec{n_2} = (1, 1, 2)$$

Using formula,

$$\cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}||\vec{n_2}|}$$

Now,

$$\vec{n_1} \cdot \vec{n_2} = 2(1) + (-1)(1) + 1(2) = 3$$

Also,

$$|\vec{n_1}| = \sqrt{4 + 1 + 1} = \sqrt{6}$$

$$|\vec{n_2}| = \sqrt{1 + 1 + 4} = \sqrt{6}$$

Hence,

$$\cos\theta = \frac{3}{6} = \frac{1}{2}$$

Therefore,

$$\theta = \frac{\pi}{3}$$

Final Answer: $\boxed{\frac{\pi}{3}}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: Direction cosines satisfy

$$l^2 + m^2 + n^2 = 1$$

Solution:
If a line is equally inclined to all axes, then

$$l = m = n$$

Let

$$l = m = n = k$$

Using

$$l^2 + m^2 + n^2 = 1$$

$$3k^2 = 1$$

$$k = \frac{1}{\sqrt{3}}$$

Hence direction cosines are

$$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

Final Answer:

$$\boxed{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)}$$

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: The shortest distance d between lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ is $d = \frac{|(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$.

Solution: From the lines, we have $\vec{a}_1 = (1, 2, 3)$, $\vec{a}_2 = (2, 4, 5)$, $\vec{b}_1 = (2, 3, 4)$, and $\vec{b}_2 = (3, 4, 5)$.

1. $\vec{a}_2 - \vec{a}_1 = (1, 2, 2)$.
2. $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$.
3. Magnitude $|\vec{b}_1 \times \vec{b}_2| = \sqrt{1+4+1} = \sqrt{6}$.
4. Dot product $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (1)(-1) + (2)(2) + (2)(-1) = 1$. Thus, $d = \frac{1}{\sqrt{6}}$.

Final Answer: $\boxed{\frac{1}{\sqrt{6}}}$

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: Distance from point (x_1, y_1, z_1) to plane

$$ax + by + cz + d = 0$$

is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

Solution:

Given plane:

$$x + y + z = 1$$

$$x + y + z - 1 = 0$$

Point:

$$(1, 2, 3)$$

Thus,

$$D = \frac{|1 + 2 + 3 - 1|}{\sqrt{1 + 1 + 1}}$$

$$= \frac{5}{\sqrt{3}}$$

Final Answer: $\boxed{\frac{5}{\sqrt{3}}}$

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept: Use dot product after squaring the vector equation.

Solution:

Given,

$$\vec{a} + \vec{b} + \vec{c} = 0$$

Squaring both sides,

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0$$

Expanding,

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

Since $\vec{a}, \vec{b}, \vec{c}$ are unit vectors,

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Thus,

$$1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$3 + 2S = 0$$

where

$$S = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

Hence,

$$S = -\frac{3}{2}$$

Final Answer: $\boxed{-\frac{3}{2}}$

Answer: (B)

Go Back to Question 48

Q49.

Solution

Concept: Use properties of scalar triple product.

Solution:

Given,

$$[\vec{a}-\vec{b},\ \vec{b}-\vec{c},\ \vec{c}-\vec{a}]$$

Using linearity of scalar triple product,

$$= [\vec{a},\vec{b},\vec{c}] - [\vec{a},\vec{b},\vec{a}] - [\vec{a},\vec{c},\vec{c}] + [\vec{a},\vec{c},\vec{a}]$$

$$-[\vec{b},\vec{b},\vec{c}] + [\vec{b},\vec{c},\vec{c}] + [\vec{b},\vec{b},\vec{a}] - [\vec{b},\vec{c},\vec{a}]$$

All terms having two identical vectors are zero.

Hence,

$$= [\vec{a},\vec{b},\vec{c}] - [\vec{b},\vec{c},\vec{a}]$$

Since scalar triple product is cyclic,

$$[\vec{b},\vec{c},\vec{a}] = [\vec{a},\vec{b},\vec{c}]$$

Thus,

$$= 2[\vec{a},\vec{b},\vec{c}]$$

Final Answer:

$$\boxed{2[\vec{a},\vec{b},\vec{c}]}$$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: Area of parallelogram in terms of diagonals:

$$\text{Area} = \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$$

Solution:

Given,

$$\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$$

Now,

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= (-2)\hat{i} - 14\hat{j} - 10\hat{k}$$

Magnitude:

$$\sqrt{(-2)^2 + (-14)^2 + (-10)^2}$$

$$= \sqrt{4 + 196 + 100}$$

$$= \sqrt{300} = 10\sqrt{3}$$

Hence area:

$$\frac{1}{2}(10\sqrt{3}) = 5\sqrt{3}$$

Final Answer: $\boxed{5\sqrt{3}}$

Answer: (A)

Go Back to Question 50

Q51.

Solution

Concept: Use formulas:

$$|\vec{a} \times \vec{b}| = ab \sin\theta$$

and

$$\vec{a} \cdot \vec{b} = ab \cos\theta$$

Solution:

Given,

$$|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$$

Thus,

$$ab \sin\theta = ab \cos\theta$$

$$\sin\theta = \cos\theta$$

Hence,

$$\tan\theta = 1$$

Therefore,

$$\theta = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 51

Q52.

Solution

Concept: Projection of $\vec{a}$ on $\vec{b}$ is

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$$

Solution:

Given,

$$\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

and

$$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

First compute dot product:

$$\vec{a}\cdot\vec{b} = 2(1) + 3(2) + 2(1)$$

$$= 2 + 6 + 2 = 10$$

Magnitude of $\vec{b}$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2}$$

$$= \sqrt{6}$$

Hence projection:

$$\frac{10}{\sqrt{6}}$$

Final Answer:

$$\boxed{\frac{10}{\sqrt{6}}}$$

Answer: (A)

Go Back to Question 52

Q53.

Solution

Concept: Use standard expansions near $x = 0$.

Solution:

Using expansions,

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2} + \cdots$$

and

$$\cos x = 1 - \frac{x^2}{2} + \cdots$$

Therefore,

$$e^{x^2} - \cos x = x^2 + \frac{x^2}{2} + \cdots$$

$$= \frac{3}{2}x^2 + \cdots$$

Hence,

$$\lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = \frac{3}{2}$$

Final Answer: $\boxed{\frac{3}{2}}$

Answer: (A)

Go Back to Question 53

Q54.

Solution

Concept: Simplify the inverse trigonometric expression.

Solution:

Given,

$$y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

Rationalising,

$$\frac{\sqrt{1+x^2}-1}{x} = \frac{x}{\sqrt{1+x^2}+1}$$

Thus,

$$y = \tan^{-1}\left(\frac{x}{\sqrt{1+x^2}+1}\right)$$

Using identity,

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$

we get

$$y = \frac{1}{2}\tan^{-1}x$$

Differentiating,

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{1+x^2}$$

At $x = 0$,

$$\frac{dy}{dx} = \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept: Use the property

$$I = \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

Solution:

Let

$$I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x}\,dx$$

Replacing x by

$$\frac{\pi}{2} - x,$$

$$I = \int_0^{\pi/2} \frac{\cos^{100} x}{\sin^{100} x + \cos^{100} x}\,dx$$

Adding both,

$$2I = \int_0^{\pi/2} \frac{\sin^{100} x + \cos^{100} x}{\sin^{100} x + \cos^{100} x}\,dx$$

$$= \int_0^{\pi/2} 1\,dx$$

$$= \frac{\pi}{2}$$

Hence,

$$I = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 55

Q56.

Solution

Concept: Linear differential equation:

$$\frac{dy}{dx} + Py = Q$$

Solution:

Given,

$$\frac{dy}{dx} + \frac{y}{x} = x^2$$

Here,

$$P = \frac{1}{x}$$

Integrating factor:

$$IF = e^{\int \frac{1}{x}\,dx} = e^{\log x} = x$$

Multiplying throughout by x,

$$x\frac{dy}{dx} + y = x^3$$

$$\frac{d}{dx}(xy) = x^3$$

Integrating,

$$xy = \frac{x^4}{4} + C$$

Therefore,

$$4xy = x^4 + C$$

Final Answer: $\boxed{4xy = x^4 + C}$

Answer: (A)

Go Back to Question 56

Q57.

Solution

Concept: Use standard limit:

$$\lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x = e^a$$

Solution:

Given,

$$\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x$$

Comparing with standard form,

$$a = 2$$

Hence,

$$\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^x = e^2$$

Final Answer: $\boxed{e^2}$

Answer: (B)

Go Back to Question 57

Q58.

Solution

Concept: A modulus function is not differentiable where its argument becomes zero.

Solution:

Given,

$$f(x) = |x - 1| + |x - 2|$$

The function can fail to be differentiable at points where

$$x - 1 = 0$$

or

$$x - 2 = 0$$

Thus,

$$x = 1, \quad x = 2$$

Hence $f(x)$ is not differentiable at both points.

Final Answer:

$$\boxed{x = 1 \text{ and } x = 2}$$

Answer: (C)

Go Back to Question 58

Q59.

Solution

Concept: Use partial fraction decomposition.

Solution:

We write

$$\frac{1}{x(x^n+1)} = \frac{1}{x} - \frac{x^{n-1}}{x^n+1}$$

Thus,

$$\int \frac{dx}{x(x^n+1)} = \int \frac{dx}{x} - \int \frac{x^{n-1}}{x^n+1}\,dx$$

Now let

$$u = x^n + 1$$

Then,

$$du = nx^{n-1}dx$$

$$x^{n-1}dx = \frac{du}{n}$$

Hence,

$$\int \frac{x^{n-1}}{x^n+1}\,dx = \frac{1}{n}\int \frac{du}{u}$$

$$= \frac{1}{n}\log(x^n+1)$$

Therefore,

$$\int \frac{dx}{x(x^n+1)} = \log x - \frac{1}{n}\log(x^n+1) + C$$

$$= \frac{1}{n}\log\frac{x^n}{x^n+1} + C$$

Final Answer:

$$\boxed{\frac{1}{n}\log\frac{x^n}{x^n+1} + C}$$

Answer: (A)

Go Back to Question 59

Q60.

Solution

Concept: Degree of a differential equation is defined only when the equation is polynomial in derivatives.

Solution:

Given differential equation:

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x\sin\left(\frac{d^2y}{dx^2}\right)$$

Since

$$\sin\left(\frac{d^2y}{dx^2}\right)$$

contains the derivative inside a transcendental function, the equation is not polynomial in derivatives.

Hence degree is not defined.

Final Answer: Not defined

Answer: (C)

Go Back to Question 60

Q61.

Solution

Concept: For parametric equations,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

Solution:

Given,

$$x = a(\theta + \sin\theta)$$

and

$$y = a(1 - \cos\theta)$$

Differentiate with respect to θ:

$$\frac{dx}{d\theta} = a(1 + \cos\theta)$$

$$\frac{dy}{d\theta} = a\sin\theta$$

Hence,

$$\frac{dy}{dx} = \frac{a\sin\theta}{a(1 + \cos\theta)}$$

$$= \frac{\sin\theta}{1 + \cos\theta}$$

At

$$\theta = \frac{\pi}{2},$$

$$\sin\frac{\pi}{2} = 1, \qquad \cos\frac{\pi}{2} = 0$$

Thus,

$$\frac{dy}{dx} = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 61

Q62.

Solution

Concept: Use definition of modulus function.

Solution:

Given,

$$\int_{-1}^{1} |x| \, dx$$

Since

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Therefore,

$$\int_{-1}^{1} |x| \, dx = \int_{-1}^{0} (-x) \, dx + \int_{0}^{1} x \, dx$$

$$= \left[-\frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 62

Q63.

Solution

Concept: Recognise the integrand as derivative of a product.

Solution:

Consider

$$\frac{d}{dx}\left(e^x \log \sec x\right)$$

Using product rule,

$$= e^x \log \sec x + e^x \frac{d}{dx}(\log \sec x)$$

Now,

$$\frac{d}{dx}(\log \sec x) = \tan x$$

Hence,

$$\frac{d}{dx}\left(e^x \log \sec x\right) = e^x(\tan x + \log \sec x)$$

Therefore,

$$\int e^x(\tan x + \log \sec x)\, dx = e^x \log \sec x + C$$

Final Answer:

$$\boxed{e^x \log \sec x + C}$$

Answer: (B)

Go Back to Question 63

Q64.

Solution

Concept: Stationary points occur where

$$\frac{dy}{dx} = 0$$

Solution:

Given,

$$y = x^x$$

Taking logarithm,

$$\log y = x \log x$$

Differentiate:

$$\frac{1}{y}\frac{dy}{dx} = \log x + 1$$

Thus,

$$\frac{dy}{dx} = x^x(\log x + 1)$$

For stationary point,

$$\frac{dy}{dx} = 0$$

Since

$$x^x \neq 0,$$

$$\log x + 1 = 0$$

$$\log x = -1$$

$$x = e^{-1} = \frac{1}{e}$$

Final Answer:

$$\boxed{\frac{1}{e}}$$

Answer: (B)

Go Back to Question 64

Q65.

Solution

Concept: Use standard limit $\sin u \sim u$ as $u \to 0$.

Solution: Let $u = x^2$. Then as $x \to 0$, $u \to 0$.

$$\frac{\sin(x^2)}{x} = \frac{\sin(x^2)}{x^2} \cdot x$$

Using $\lim_{u \to 0} \frac{\sin u}{u} = 1$, we get:

$$\lim_{x \to 0} \frac{\sin(x^2)}{x^2} = 1$$

So the expression becomes:

$$\lim_{x \to 0} x \cdot 1 = 0$$

Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 65

Q66.

Solution

Concept: Use symmetry property $f(x) = f(2-x)$ and substitution in integrals to exploit even symmetry about $x = 1$.

Solution: Given $f(x) = f(2-x)$ on $[0,2]$.

$$\int_0^2 x f(x)\,dx$$

Put $x \to 2-x$:

$$= \int_0^2 (2-x) f(x)\,dx$$

Adding:

$$2\int_0^2 x f(x)\,dx = 2\int_0^2 f(x)\,dx = 8 \Rightarrow \int_0^2 x f(x)\,dx = 4$$

So (A) true.

Also symmetry about $x = 1$:

$$\int_0^1 f(x)\,dx = \int_1^2 f(x)\,dx = 2$$

So (B) true.

(C):

$$\int_0^2 f(2-x)\,dx = \int_0^2 f(x)\,dx = 4$$

So (C) true.

(D): Let $x \to 2-x$:

$$\int_0^2 (x-1) f(x)\,dx = -\int_0^2 (x-1) f(x)\,dx = 0$$

So (D) true.

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 66

Q67.

Solution

Concept: Use standard hyperbola properties: eccentricity, focal distances, and tangent geometry.
Solution: For $x^2 - y^2 = a^2$, we have $b^2 = a^2$, so:

$$c = \sqrt{a^2 + b^2} = \sqrt{2a^2} = a\sqrt{2} \Rightarrow e = \sqrt{2}$$

So (A) true.
Difference of focal distances:

$$|PS - PS'| = 2a \Rightarrow (B)$$

Product of perpendiculars from foci to tangent equals $b^2 = a^2$, not a^2 statement matches (C) true.
Locus of feet of perpendiculars from foci is circle:

$$x^2 + y^2 = a^2 \Rightarrow (D)$$

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 67

Q68.

Solution

Concept: Shortest distance between two lines in vector form depends on whether they are parallel, intersecting, or skew. Use cross product and scalar triple product for perpendicular distance.
Solution:
For two lines

$$L_1 : \vec{r} = \vec{a} + \lambda\vec{b}, \quad L_2 : \vec{r} = \vec{c} + \mu\vec{d}$$

(A) If $d(L_1, L_2) = 0$, then the lines must either intersect or be coincident. Hence (A) is true.
(B) If $\vec{b} \times \vec{d} \neq \vec{0}$, then lines are not parallel. The shortest distance is given by:

$$d = \frac{|(\vec{c} - \vec{a}) \cdot (\vec{b} \times \vec{d})|}{|\vec{b} \times \vec{d}|}$$

So (B) is true.
(C) If $\vec{b} \times \vec{d} = \vec{0}$, then directions are parallel. The distance between parallel lines is:

$$d = \frac{|(\vec{c} - \vec{a}) \times \vec{b}|}{|\vec{b}|}$$

So (C) is true.
(D) If lines are skew, they are non-parallel and non-intersecting, hence not coplanar. So (D) is true.
Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 68

Q69.

Solution

Concept: Use condition for infinite solutions: proportional coefficients.

Solution: Given system:

$$x + y + z = 6, \quad x + 2y + 3z = 10, \quad x + 2y + \lambda z = \mu$$

For infinite solutions:

$$\frac{1}{1} = \frac{2}{2} = \frac{\lambda}{3} = \frac{\mu}{10}$$

So:

$$\lambda = 3, \quad \mu = 10$$

Thus (A) and (B) true.

(C), (D) false.

Final Answer: A, B

Answer: (A,B)

Go Back to Question 69

Q70.

Solution

Concept: Use equality condition for complex modulus and triangle equality condition.

Solution: Given:

$$|z_1 + z_2| = |z_1| + |z_2|$$

Equality holds when vectors are in same direction:

$$\arg(z_1) = \arg(z_2)$$

So (A) true.

Thus $z_1\overline{z_2}$ is real (B)

They are collinear with origin (C)

Also:

$$|z_1 - z_2| = ||z_1| - |z_2||$$

So (D) true.

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 70

Q71.

Solution

Concept: An even polynomial of degree 4 depends only on even powers of x. Use factor conditions, limit condition to determine coefficients, then analyze extrema using derivatives.

Solution: Since $f(x)$ is even and degree 4, write:

$$f(x) = ax^4 + bx^2 + c$$

Given $f(2) = 0$:

$$16a + 4b + c = 0 \quad (1)$$

and $f(-2) = 0$ gives the same equation.

Also,

$$\lim_{x\to 0} \frac{f(x)}{x^2} = \lim_{x\to 0} \frac{ax^4 + bx^2 + c}{x^2} = \lim_{x\to 0}(ax^2 + b + \frac{c}{x^2})$$

For limit to exist, $c = 0$. Hence:

$$\lim_{x\to 0} \frac{f(x)}{x^2} = b = -4$$

So $b = -4$, $c = 0$. Now from (1):

$$16a + 4(-4) = 0 \Rightarrow 16a - 16 = 0 \Rightarrow a = 1$$

Thus:

$$f(x) = x^4 - 4x^2$$

So (A) is true.

Now find extrema:

$$f'(x) = 4x^3 - 8x = 4x(x^2 - 2)$$

Critical points:

$$x = 0, \ \pm\sqrt{2}$$

Second derivative:

$$f''(x) = 12x^2 - 8$$

At $x = 0$:

$$f''(0) = -8 < 0 \Rightarrow \text{local maximum}$$

So (B) true.

At $x = \pm\sqrt{2}$:

$$f''(\pm\sqrt{2}) = 12(2) - 8 = 16 > 0 \Rightarrow \text{local minima}$$

So (C) true.

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 71

Q72.

Solution

Concept: Use substitution and standard trigonometric identities for simplification.

Solution:

$$\frac{1+\sin 2x}{\cos x - \sin x}$$

Using identities leads to:

$$= \frac{(\sin x + \cos x)^2}{(\cos x - \sin x)}$$

Hence: (A) false (B) false (C) true (D) false

Final Answer: C

Answer: (C)

Go Back to Question 72

Q73.

Solution

Concept: Use identity $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$.

Solution:

$$16|\vec{b}|^2 = 144 \Rightarrow |\vec{b}| = 3$$

So (A) true.

Dot product:

$$\vec{a}\cdot\vec{b} = \pm 12$$

So (B) true.

(C) false since magnitude fixed.

If perpendicular:

$$|\vec{b}| = 3$$

So (D) true.

Final Answer: A, B, D

Answer: (A,B,D)

Go Back to Question 73

Q74.

Solution

Concept: Rolle's Theorem applies when a function is continuous on a closed interval, differentiable on the open interval, and has equal values at the endpoints. We check these conditions for each function on $[-1, 1]$. Continuity and endpoint equality are necessary, but differentiability failure at any interior point disqualifies the function from satisfying Rolle's theorem.

Solution: Rolle's Theorem requires: (i) continuity on $[-1, 1]$, (ii) differentiability on $(-1, 1)$, and (iii) $f(-1) = f(1)$.

(A) $f(x) = x^2$ This function is continuous and differentiable everywhere. Also:

$$f(-1) = 1 = f(1)$$

Hence Rolle's Theorem applies (A) correct.

(B) $f(x) = |x|$ Although continuous on $[-1, 1]$ and $f(-1) = f(1) = 1$, it is not differentiable at $x = 0$. Hence Rolle's theorem does not apply (B) incorrect.

(C) $f(x) = 1 - x^{2/3}$ The function is continuous on $[-1, 1]$ and:

$$f(-1) = f(1) = 0$$

However, derivative:

$$f'(x) = -\frac{2}{3}x^{-1/3}$$

is undefined at $x = 0$. Hence not differentiable on $(-1, 1)$ Rolle's theorem does not apply (C) incorrect.

(D) $f(x) = \cos(\pi x)$ This function is continuous and differentiable everywhere. Also:

$$f(-1) = \cos(-\pi) = -1, \quad f(1) = \cos(\pi) = -1$$

Thus endpoint values are equal and all conditions hold Rolle's theorem applies (D) correct.

Final Answer: A, D

Answer: (A,D)

Go Back to Question 74

Q75.

Solution

Concept: Use probability identities and bounds for union and intersection.

Solution:

$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

So:

$$\geq \frac{3}{4} + \frac{1}{8} = \frac{7}{8} \Rightarrow (A)$$

Upper bound:

$$\leq 1 + \frac{3}{8} = \frac{11}{8} \Rightarrow (B)$$

Product can be $\frac{1}{2}$ for suitable values (C)

Not necessarily > 1 (D) false

Final Answer: A, B, C

Answer: (A,B,C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	A	3	D	4	A	5	A
6	B	7	B	8	A	9	B	10	B
11	B	12	B	13	C	14	D	15	B
16	A	17	C	18	A	19	C	20	B
21	A	22	B	23	D	24	A	25	A
26	A	27	A	28	A	29	C	30	A
31	B	32	B	33	B	34	B	35	A
36	A	37	A	38	A	39	A	40	B
41	A	42	A	43	A	44	A	45	B
46	A	47	A	48	B	49	A	50	A
51	B	52	A	53	A	54	A	55	B
56	A	57	B	58	C	59	A	60	C
61	A	62	B	63	B	64	B	65	B
66	A,B,C,D	67	A,B,C,D	68	A,B,C,D	69	A,B	70	A,B,C,D
71	A,B,C	72	C	73	A,B,D	74	A,D	75	A,B,C