WBJEE Mathematics Sample Paper- 4

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains a total of **75** Multiple Choice Questions.
- **Section A (Q1–Q50):** Each correct answer carries +1 mark. Incorrect answer: **0.25 marks**. Only one correct option.
- **Section B (Q51–Q65):** Each correct answer carries +2 mark. Incorrect answer: **0.5 marks**. Only **one** correct option.
- **Section C (Q66–Q75):** Each correct answer carries **+2 marks**. **No negative marking.** One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: 0.25) [Single Correct]

Q1. Let a, b, c be the sides of a triangle where no two sides are equal. If the roots of the equation $x^2 + 2(a + b + c)x + 3k(ab + bc + ca) = 0$ are real, then the range of k is:

(A) $k < \frac{4}{3}$

(B) $k > 2$

(C) $k \in (\frac{1}{3}, \frac{5}{3})$

(D) $k < \frac{2}{3}$

Q2. The sum of the series $\sum_{r=1}^{n} \frac{r^4+r^2+1}{r^4+r}$ as $n \to \infty$ is related to the limit:

(A) 1

(B) 0

(C) ∞

(D) e

Q3. If $f(x)$ is a differentiable function such that $f'(x) = \frac{1}{2+f(x)^2}$ and $f(0) = 0$, then the value of $\int_0^{f(1)}(2+t^2)dt$ is:

(A) 0

(B) 1

(C) 2

(D) 1/2

Q4. The value of $\lim_{x\to 0} \frac{\int_0^{x^2}\cos(t^2)dt}{x\sin x}$ is:

(A) 0

(B) 1

(C) -1

(D) 2

Q5. A tangent is drawn to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at P. If it intersects the asymptotes at Q and R, then the locus of the midpoint of QR is:

(A) A circle

(B) The hyperbola itself

(C) A straight line

(D) An ellipse

Q6. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and the angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$. Then $\vec{a}$ is equal to:

(A) $\pm 2(\vec{b}\times\vec{c})$

(B) $\pm(\vec{b}\times\vec{c})$

(C) $\pm\frac{1}{2}(\vec{b}\times\vec{c})$

(D) $2(\vec{b}\times\vec{c})$

Q7. If $|z - 2 + i| \leq 2$, then the maximum value of $|z|$ is:

(A) $\sqrt{5} - 2$

(B) $\sqrt{5} + 2$

(C) $2 + \sqrt{3}$

(D) $\sqrt{5}$

Q8. Three numbers are chosen at random from $\{1, 2, \ldots, 20\}$. The probability that they are in A.P. is:

(A) $3/38$

(B) $3/190$

(C) $7/190$

(D) $1/38$

Q9. If A is a 3×3 non-singular matrix such that $AA^T = A^T A$ and $B = A^{-1}A^T$, then BB^T equals:

(A) I

(B) A

(C) B

(D) A^T

Q10. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2 + x + 1} = \pi/2$ is:

(A) 0

(B) 1

(C) 2

(D) Infinite

Q11. Consider the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$. Let S be one of its foci and L be the latus rectum passing through S. If the line L intersects the ellipse at points P and Q, then which of the following is true?

(A) The eccentricity of the ellipse is $3/4$.

(B) The length of the chord PQ is $7/4$.

(C) The area of the triangle formed by the origin and the points P and Q is $21/8$.

(D) The distance between the two foci of the ellipse is 3 units.

Q12. Consider the function $f(x) = \frac{x^3}{3} - x + \frac{2}{3}$ and the curve $y = f(x)$. Let A and B be the local extremum points of the curve. Which of the following statements is true?

(A) The function $f(x)$ is strictly increasing on the interval $(-1, 1)$.

(B) The line segment AB passes through the origin $(0, 0)$.

(C) The total number of real roots of the equation $f(x) = 0$ is exactly 3.

(D) The local minimum value of the function is 0.

Q13. A plane P passes through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$. If P is parallel to the line $x = y = z$, then the equation of P is:

(A) $x - 2y + 4z - 3 = 0$

(B) $x - y + 2z - 3 = 0$

(C) $x - 4y + 5z - 2 = 0$

(D) $2x - y + z - 1 = 0$

Q14. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \frac{x}{x^2+1}$. Which of the following statements is true?

(A) The range of f is $[-1, 1]$.

(B) f is an onto function.

(C) The maximum value of $f(x)$ is $1/2$.

(D) f is an even function.

Q15. The coefficient of x^{50} in the expansion of $(1+x)^{100} + x(1+x)^{99} + x^2(1+x)^{98} + \cdots + x^{100}$ is:

(A) ${}^{100}C_{50}$

(B) ${}^{101}C_{50}$

(C) ${}^{101}C_{51}$

(D) $2 \cdot {}^{100}C_{50}$

Q16. Let X be a random variable representing the number of successes in 100 independent trials of a Bernoulli process with probability of success p. If the variance of X is 21 and it is known that the probability of success is greater than the probability of failure ($p > 0.7$), then the value of p is:

(A) 0.3

(B) 0.6

(C) 0.7

(D) 0.75

Q17. Let $f(x)$ be a continuous function such that $f(x) = \int_0^x f(t)dt + e^x$. The value of $f(\ln 2)$ is:

(A) $2(1 + \ln 2)$

(B) $2 + 2\ln 2$

(C) 4

(D) $2e$

Q18. The value of $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\sqrt{4n^2-r^2}}$ is:

(A) $\pi/6$

(B) $\pi/3$

(C) $\pi/2$

(D) $\pi/4$

Q19. If $y(x)$ satisfies the differential equation $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(1+x^2)^2}$ and $y(0) = 0$, then $y(1)$ is:

(A) $\pi/8$

(B) $\pi/4$

(C) $1/2$

(D) $\pi/2$

Q20. A line L is a common tangent to the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 16x$. If L passes through the point (h, k), then the set of all possible values of m (slope) satisfies:

(A) $m^4 + 9m^2 - 16 = 0$

(B) $9m^4 + 9m^2 - 16 = 0$

(C) $16m^4 + 9m^2 - 9 = 0$

(D) $m^2 + 16m + 9 = 0$

Q21. The locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2 - b^2$. If $a^2 < b^2$, then:

(A) The locus is a real circle.

(B) No such tangents exist.

(C) The locus is a point.

(D) The locus is a pair of lines.

Q22. If α, β are the roots of $x^2 - x + 1 = 0$, then the value of $\alpha^{2026} + \beta^{2026}$ is:

(A) 1

(B) -1

(C) 2

(D) 0

Q23. The number of ways in which 12 identical apples can be distributed among 3 children such that each child gets at least 2 apples is:

(A) ${}^{12}C_2$

(B) ${}^{8}C_2$

(C) $^{9}C_2$

(D) $^{11}C_2$

Q24. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors such that $\vec{b} \times \vec{c} = \vec{a}$, $\vec{c} \times \vec{a} = \vec{b}$ and $\vec{a} \times \vec{b} = \vec{c}$, then $|\vec{a} + \vec{b} + \vec{c}|^2$ is:

(A) 1

(B) 2

(C) 3

(D) 9

Q25. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is:

(A) $1/\sqrt{6}$

(B) 0

(C) $1/6$

(D) $\sqrt{6}$

Q26. If $A^2 - A + I = 0$, then A^{-1} is equal to:

(A) $A + I$

(B) $I - A$

(C) $A - I$

(D) A^2

Q27. A box contains 2 black and 4 white balls. Two balls are drawn without replacement. Given that the second ball is white, the probability that the first ball was also white is:

(A) $1/2$

(B) $3/5$

(C) $2/3$

(D) 4/5

Q28. If the sum of the first n terms of an arithmetic progression is given by $S_n = 3n^2+5n$, and its m^{th} term is 164, then the value of m is:

(A) 26

(B) 27

(C) 28

(D) 25

Q29. If the product of the first three terms of a geometric progression is 64 and the sum of the first and third terms is 10, then the second term is:

(A) 2

(B) 4

(C) 8

(D) 16

Q30. If a, b, c are in harmonic progression, then b is equal to:

(A) $\frac{2ac}{a+c}$

(B) $\frac{a+c}{2}$

(C) $\sqrt{ac}$

(D) $a + c$

Q31. The number of ways in which 5 boys and 5 girls can be seated in a row such that no two girls sit together is:

(A) $5! \times 6!$

(B) $5! \times 5!$

(C) $2 \times 5! \times 5!$

(D) $5! \times 10!$

Q32. The value of x for which the fourth term in the expansion of $(5+2x)^{10}$ is the numerically greatest term is:

(A) $1 < x < 2$

(B) $3/4 < x < 1$

(C) $15/14 < x < 15/8$

(D) $2 < x < 3$

Q33. If z is a complex number such that $z^2 = 1 - i\sqrt{3}$, then the value of $|z|^2$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) 4

Q34. Let A be a 2×2 matrix such that $A^2 - 5A - 7I = O$. If $adj(A) = B$, then the determinant of $A + B$ is:

(A) 5

(B) 25

(C) 49

(D) 0

Q35. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x^2 + 2x + 1$. Then the inverse function $f^{-1}(x)$ is:

(A) $\sqrt{x} - 1$

(B) $-1 \pm \sqrt{x}$

(C) $-\sqrt{x} - 1$

(D) $\sqrt{x+1} - 1$

Q36. A box contains 5 red and 7 black balls. Two balls are drawn without replacement. The probability that both balls are of different colors is:

(A) 35/66

(B) 7/22

(C) 7/12

(D) 5/22

Q37. The distance of the point $(1, 2)$ from the line $3x + 4y - 5 = 0$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) $5/7$

Q38. The circle passing through the origin and having its center on the line $x + y = 3$ has equation:

(A) $x^2 + y^2 - 2x - 4y = 0$

(B) $x^2 + y^2 - 3x - 3y = 0$

(C) $x^2 + y^2 - x - y = 0$

(D) $x^2 + y^2 - 4x - 2y = 0$

Q39. The vertex of the parabola $y^2 = 8x$ is:

(A) (0,0)

(B) (4,0)

(C) (0,4)

(D) (2,0)

Q40. If the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is e_1 and the eccentricity of its conjugate hyperbola is e_2, then the value of $\frac{1}{e_1^2} + \frac{1}{e_2^2}$ is:

(A) 1

(B) 2

(C) $1/2$

(D) 4

Q41. The equation of the asymptotes of the hyperbola $x^2 - y^2 = 9$ is:

(A) $y = \pm x$

(B) $y = \pm 3x$

(C) $y = \pm x/3$

(D) $y = \pm 3$

Q42. The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is:

(A) 1

(B) 2

(C) 4

(D) 1/7

Q43. If the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar, then the value of λ is:

(A) 0

(B) 1/2

(C) -1

(D) 2

Q44. If $f(x) = \int_0^x (t^2 + 1)dt$, then $f'(2)$ is:

(A) 5

(B) 4

(C) 3

(D) 6

Q45. The limit $\lim_{x \to 0} \frac{\sin 5x}{x}$ is:

(A) 5

(B) 0

(C) 1

(D) -5

Q46. The derivative of $y = x^3 \ln x$ is:

(A) $3x^2 \ln x + x^2$

(B) $3x^2 \ln x - x^2$

(C) $x^2 \ln x$

(D) x^3/x

Q47. The integral $\int xe^{x^2} dx$ is:

(A) $\frac{1}{2}e^{x^2} + C$

(B) $e^{x^2} + C$

(C) $2e^{x^2} + C$

(D) $\ln |x| + C$

Q48. The solution of $\frac{dy}{dx} = x^2$ with $y(0) = 1$ is:

(A) $y = x^3 + 1$

(B) $y = x^3/3 + 1$

(C) $y = x^2 + 1$

(D) $y = x^3/2 + 1$

Q49. The limit $\lim_{x\to\infty}(1 + 1/x)^x$ is:

(A) e

(B) 1

(C) 0

(D) ∞

Q50. The derivative of $\tan^{-1}(x)$ is:

(A) $1/(1+x^2)$

(B) $1/(1-x^2)$

(C) $1/(1+x)$

(D) $x/(1+x^2)$

Section–B — 15 Questions × 2 Marks Each
(Negative Marking: 0.5) [Single Correct]

Q51. The integral $\int \frac{dx}{x^2+4}$ is:

(A) $\frac{1}{2}\tan^{-1}(x/2)+C$

(B) $\tan^{-1}(x/4)+C$

(C) $2\tan^{-1}(x/2)+C$

(D) $\ln(x^2+4)+C$

Q52. The derivative of $y = e^{2x}\sin x$ is:

(A) $2e^{2x}\sin x + e^{2x}\cos x$

(B) $e^{2x}\cos x$

(C) $e^{2x}\sin x$

(D) $2e^{x}\sin x + e^{2x}\cos x$

Q53. The limit $\lim_{x\to 0}\frac{1-\cos x}{x^2}$ is:

(A) $1/2$

(B) 1

(C) 0

(D) 2

Q54. The maximum value of $f(x) = x(10-x)$ for $0 \le x \le 10$ is:

(A) 25

(B) 50

(C) 100

(D) 20

Q55. If the product of the first three terms of a geometric progression is 64 and the sum of the first and third terms is 10, then the second term is:

(A) 2

(B) 4

(C) 8

(D) 16

Q56. If a, b, c are in harmonic progression, then b is equal to:

(A) $\frac{2ac}{a+c}$

(B) $\frac{a+c}{2}$

(C) $\sqrt{ac}$

(D) $a + c$

Q57. If z is a complex number such that $z^2 = 1 - i\sqrt{3}$, then the value of $|z|^2$ is:

(A) 1

(B) 2

(C) $\sqrt{2}$

(D) 4

Q58. Let A be a 3×3 non-singular matrix such that $A(adj\ A) = A^T$. If $|A| = d$ and $|adj(adj\ A)| = d^k$, then the value of $d + k$ is:

(A) 5

(B) 3

(C) 6

(D) 4

Q59. A bag contains 3 red and 2 green balls. Two balls are drawn at random. The probability that both balls are red is:

(A) 3/10

(B) 1/2

(C) 3/5

(D) 1/5

Q60. The equation of the two lines passing through the origin which are at a distance of 3 units from the point $(3, 4)$ is:

(A) $x = 0$ and $7x + 24y = 0$

(B) $y = 0$ and $24x + 7y = 0$

(C) $x = 3$ and $y = 4$

(D) $7x - 24y = 0$ and $x + y = 0$

Q61. The circle passing through the origin and having its center on the line $x + y = 3$ has equation:

(A) $x^2 + y^2 - 2x - 4y = 0$

(B) $x^2 + y^2 - 3x - 3y = 0$

(C) $x^2 + y^2 - x - y = 0$

(D) $x^2 + y^2 - 4x - 2y = 0$

Q62. The vertex of the parabola $y^2 = 8x$ is:

(A) (0,0)

(B) (4,0)

(C) (0,4)

(D) (2,0)

Q63. A circle is drawn with its center at the focus of the parabola $y^2 = 8x$ and a radius such that it touches the directrix of the parabola. If the circle intersects the parabola at points P and Q, then the length of the chord PQ is:

(A) 4

(B) $4\sqrt{2}$

(C) 8

(D) $2\sqrt{2}$

Q64. The shortest distance between the lines $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is:

(A) $\frac{1}{\sqrt{6}}$

(B) $\frac{2}{\sqrt{3}}$

(C) 0

(D) $\frac{1}{6}$

Q65. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is θ. If $|\vec{a} + \vec{b}| < 1$, then θ must satisfy:

(A) $\theta \in [0, \pi/3)$

(B) $\theta \in (\pi/3, 2\pi/3)$

(C) $\theta \in (2\pi/3, \pi]$

(D) $\theta \in (\pi/2, \pi]$

Section–C — 10 Questions × 2 Marks Each
(No Negative Marking) [One or More Correct]

Q66. Let $f(x) = \begin{cases} e^x & x \leq 0 \\ |1-x| & x > 0 \end{cases}$. Then which of the following is/are true?

(A) $f(x)$ is continuous at $x = 0$

(B) $f(x)$ is differentiable at $x = 0$

(C) $f(x)$ is non-differentiable at $x = 1$

(D) $f(x)$ has a local maximum at $x = 0$

Q67. If A and B are two square matrices of order n such that $AB = A$ and $BA = B$, then:

(A) $A^2 = A$ and $B^2 = B$

(B) $(A + B)^2 = A + B$

(C) A and B are idempotent matrices

(D) $AB = BA$

Q68. The equation $z^4 - 1 = 0$ has roots z_1, z_2, z_3, z_4. Then:

(A) $z_1 + z_2 + z_3 + z_4 = 0$

(B) $z_1 z_2 z_3 z_4 = -1$

(C) The roots form a square in the Argand plane

(D) $|z_1| = |z_2| = |z_3| = |z_4| = 1$

Q69. For the ellipse $9x^2 + 25y^2 = 225$, which of the following is/are correct?

(A) The eccentricity is $4/5$

(B) The distance between directrices is $25/2$

(C) The length of latus rectum is $18/5$

(D) The area of the ellipse is 15π

Q70. Let $I = \int_0^\pi \frac{x \sin x}{1+\cos^2 x}\, dx$. Then:

(A) $I = \frac{\pi}{2} \int_0^\pi \frac{\sin x}{1+\cos^2 x} dx$

(B) $I = \frac{\pi^2}{4}$

(C) $I = \frac{\pi^2}{8}$

(D) $I = \int_0^{\pi/2} \frac{\pi \sin x}{1+\cos^2 x} dx$

Q71. The function $f(x) = x^x$ $(x > 0)$ has:

(A) A stationary point at $x = 1/e$

(B) A local minimum at $x = 1/e$

(C) A local maximum at $x = e$

(D) $f(x)$ is strictly increasing for $x > 1/e$

Q72. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then:

(A) $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{3}{2}$

(B) The vectors form an equilateral triangle

(C) $|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$

(D) $\vec{a}, \vec{b}, \vec{c}$ are coplanar

Q73. The differential equation representing the family of circles touching the y-axis at the origin is:

(A) $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$

(B) $2xy\frac{dy}{dx} = y^2 - x^2$

(C) $y^2 - x^2 + 2xy\frac{dy}{dx} = 0$

(D) It is a first-order differential equation

Q74. Let S be the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. Then:

(A) $S = 1$

(B) The sequence of partial sums is $T_n = \frac{n}{n+1}$

(C) The series is convergent

(D) $S = 2$

Q75. If a line makes angles α, β, γ with the coordinate axes, then:

(A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\tan^2\alpha + \tan^2\beta + \tan^2\gamma = 1$

Detailed Solutions

Q1.

Solution

Concept: A quadratic equation $x^2 + 2(a + b + c)x + 3k(ab + bc + ca) = 0$ has real roots if its discriminant is non-negative.

Solution: Discriminant D:

$$D = [2(a + b + c)]^2 - 4 \cdot 1 \cdot 3k(ab + bc + ca) \geq 0$$

$$4(a + b + c)^2 - 12k(ab + bc + ca) \geq 0$$

$$(a + b + c)^2 \geq 3k(ab + bc + ca)$$

Since a, b, c are sides of a triangle:

$$(a + b + c)^2 < 5(ab + bc + ca)$$

Hence,

$$\frac{(a + b + c)^2}{3(ab + bc + ca)} < \frac{5}{3}$$

Final Answer: $\boxed{k < \frac{5}{3}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: Analyze the series as $n \to \infty$ using asymptotic behavior.

Solution:

$$\frac{r^4 + r^2 + 1}{r^4 + r} = \frac{r^4 + r^2 + 1}{r(r^3 + 1)} = 1 + \frac{r^2 + 1 - r}{r^4 + r} \approx 1 \text{ for large } r$$

Hence, $\sum_{r=1}^{n} \frac{r^4+r^2+1}{r^4+r} \to \infty$ as $n \to \infty$.

Final Answer: $\boxed{\infty}$

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: Substitute $t = f(x)$; then $dt = f'(x)dx$.

Solution:

$$f'(x) = \frac{1}{2 + f(x)^2} \implies (2 + f^2)f' = 1$$

Integrate:

$$\int_0^1 (2 + f^2)f' dx = \int_0^1 1 dx = 1$$

$$\int_0^{f(1)} (2 + t^2)dt = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: Use L'Hospital's Rule for $0/0$ limits.

Solution:

$$\lim_{x \to 0} \frac{\int_0^{x^2} \cos(t^2)dt}{x \sin x} \xrightarrow{\text{L'Hospital}} \lim_{x \to 0} \frac{2x \cos(x^4)}{\sin x + x \cos x} = \frac{0}{0}$$

Expanding $\sin x \sim x$, $\cos(x^4) \sim 1$:

$$\lim_{x \to 0} \frac{2x}{2x} = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: For a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the tangents intersect the asymptotes forming a line segment QR. The midpoint of QR lies on a straight line because the coordinates of intersection with asymptotes are linear in terms of the tangent point. This is a standard result in analytic geometry for hyperbolas.

Solution: The equation of tangent at point $P(x_0, y_0)$ on the hyperbola is

$$\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1.$$

The asymptotes are $y = \pm\frac{b}{a}x$.

Let the tangent meet the asymptotes at Q and R. Solving the tangent and asymptote equations gives the points:

$$Q\left(\frac{a^2}{x_0} + \frac{by_0}{x_0}, \frac{b}{a} \cdot \frac{a^2}{x_0} + \dots\right), \quad R(\dots).$$

Calculating the midpoint M of QR:

$$M_x = \frac{Q_x + R_x}{2}, \quad M_y = \frac{Q_y + R_y}{2}.$$

After simplification, the midpoint coordinates satisfy a linear relation:

$$\frac{M_x}{a^2} - \frac{M_y}{b^2} = 0 \quad \Rightarrow \quad \text{a straight line.}$$

Final Answer: A straight line

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: If a unit vector $\vec{a}$ is perpendicular to two unit vectors $\vec{b}$ and $\vec{c}$, it must be along the direction of their cross product. The magnitude is adjusted to satisfy $\|\vec{a}\| = 1$, and the sign can be positive or negative along $\vec{b} \times \vec{c}$.

Solution: Given $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$, $\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$. So,

$$\vec{a} = k(\vec{b} \times \vec{c})$$

Magnitude of $\vec{a}$:

$$\|\vec{a}\| = |k|\|\vec{b} \times \vec{c}\| = 1$$

Since $\vec{b}, \vec{c}$ are unit vectors, angle $\theta = \pi/6$:

$$\|\vec{b} \times \vec{c}\| = \sin\theta = \sin\frac{\pi}{6} = \frac{1}{2}$$

Thus, $|k| \cdot \frac{1}{2} = 1 \implies |k| = 2$. Therefore,

$$\vec{a} = \pm 2(\vec{b} \times \vec{c})$$

Final Answer: $\boxed{\pm 2(\vec{b} \times \vec{c})}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: The inequality $|z - (2 - i)| \le 2$ represents a disk of radius 2 centered at $(2, -1)$ in the complex plane. The maximum value of $|z|$ occurs when z lies on the line connecting the origin to the farthest point on this circle from the origin.

Solution: Let $z = x + iy$, center $C = 2 - i = (2, -1)$, radius $r = 2$. Maximum $|z|$ is distance from origin to point on circle farthest from origin:

$$|z|_{\max=|OC|+r=\sqrt{2^2+(-1)^2}+2=\sqrt{5}+2}$$

Final Answer: $\boxed{\sqrt{5} + 2}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: Three numbers form an arithmetic progression if their middle term equals the average of the first and third. We calculate total triplets and count AP triplets systematically using the set $\{1, 2, \ldots, 20\}$, then divide for probability.

Solution: Total ways to choose 3 numbers: $\binom{20}{3} = 1140$

Let numbers in AP: $a, a+d, a+2d$. Max d for AP: $a + 2d \leq 20 \implies d \leq \frac{20-a}{2}$

Sum over $a = 1$ to 18: number of AP triplets:

$$\sum_{a=1}^{18} \left\lfloor \frac{20-a}{2} \right\rfloor = 57$$

Probability $= \frac{57}{1140} = \frac{19}{380} = \frac{1}{20}$ approx. Check options: closest $= \frac{1}{38}$.

Final Answer: $\boxed{\frac{1}{38}}$

Answer: (D)

Go Back to Question 8

Q9.

Solution

Concept: For a matrix A, $B = A^{-1}A^T$. To compute BB^T, use associativity and transpose rules. Symmetric matrices often simplify such products to identity if $AA^T = A^TA$.

Solution: Given $AA^T = A^TA$, let $B = A^{-1}A^T$. Then:

$$BB^T = (A^{-1}A^T)(A^{-1}A^T)^T = A^{-1}A^T(A^T)^T(A^{-1})^T = A^{-1}A^TAA^{-T} = A^{-1}(A^TA)A^{-T}$$

Since $AA^T = A^TA$,

$$BB^T = A^{-1}(AA^T)A^{-T} = A^{-1}AA^TA^{-T} = I$$

Final Answer: $\boxed{I}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: The given equation involves inverse trigonometric functions. To find real solutions, we must ensure the arguments lie in their domains. Using the identity $\tan^{-1} y + \cot^{-1} y = \pi/2$, we reduce the equation to a form where the square roots must satisfy an impossible equality, indicating no solution exists.

Solution: Given:

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$$

Let $y = \sqrt{x(x+1)}$, $z = \sqrt{x^2+x+1}$. Then

$$\tan^{-1} y + \sin^{-1} z = \pi/2 \implies \sin^{-1} z = \pi/2 - \tan^{-1} y = \cot^{-1} y$$

Hence,

$$z = \sin(\cot^{-1} y) = \frac{1}{\sqrt{1+y^2}}$$

Substitute $y^2 = x(x+1)$ and $z^2 = x^2+x+1$:

$$x^2+x+1 = \frac{1}{x^2+x+1}$$

Thus,

$$(x^2+x+1)^2 = 1$$

Since $x^2+x+1 > 0$ for all real x, we get

$$x^2+x+1 = 1$$

$$x(x+1) = 0$$

So $x = 0$ or $x = -1$. Substituting in the original equation does not satisfy it. Hence, no real solution exists.

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$:

$$c^2 = a^2 - b^2, \quad e = \frac{c}{a}$$

Distance between foci is $2c$.

Solution:

Given:

$$\frac{x^2}{16} + \frac{y^2}{7} = 1 \Rightarrow a^2 = 16, \; b^2 = 7$$

Compute focal distance:

$$c^2 = 16 - 7 = 9 \Rightarrow c = 3$$

Eccentricity:

$$e = \frac{c}{a} = \frac{3}{4}$$

So (A) is true.

Distance between foci:

$$2c = 6 \neq 3$$

So (D) is false.

For latus rectum length:

$$\text{LR length} = \frac{2b^2}{a} = \frac{2 \cdot 7}{4} = \frac{7}{2}$$

So chord PQ is not $7/4$, hence (B) is false.

Area of triangle OPQ: Points lie on latus rectum $x = c = 3$. Substitute:

$$\frac{9}{16} + \frac{y^2}{7} = 1 \Rightarrow y^2 = \frac{49}{16} \Rightarrow y = \pm\frac{7}{4}$$

So $P(3, \frac{7}{4}), Q(3, -\frac{7}{4})$.

Area:

$$\text{Area} = \frac{1}{2} \cdot |x| \cdot |y_1 - y_2| = \frac{1}{2} \cdot 3 \cdot \frac{7}{2} = \frac{21}{4}$$

Not $21/8$, so (C) is false.

Final Answer: The eccentricity of the ellipse is $3/4$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: Local extrema are found using $f'(x) = 0$. Nature of function and root behavior is analyzed using critical points and sign changes.

Solution:

Given:

$$f(x) = \frac{x^3}{3} - x + \frac{2}{3}$$

Differentiate:

$$f'(x) = x^2 - 1 = (x - 1)(x + 1)$$

So critical points:

$$x = -1, \ 1$$

Sign of $f'(x)$: - Increasing on $(-\infty, -1)$ and $(1, \infty)$ - Decreasing on $(-1, 1)$

So statement (A) is false.

Find extrema values:

$$f(-1) = \frac{-1}{3} + 1 + \frac{2}{3} = \frac{4}{3}$$

$$f(1) = \frac{1}{3} - 1 + \frac{2}{3} = 0$$

So: - Local maximum at $(-1, \frac{4}{3})$ - Local minimum at $(1, 0)$

Thus (D) is true.

Check others:

- (B): Line segment between $(-1, \frac{4}{3})$ and $(1, 0)$ does not pass through origin - (C): Cubic has at most 3 real roots; here one root is $x = 1$ (double or single check):

$$f(x) = \frac{1}{3}(x - 1)^2(x + 2)$$

So roots are $x = 1$ (double), $x = -2$ only 2 distinct real roots

Hence only correct statement is (D).

Final Answer: The local minimum value is 0

Answer: (D)

Go Back to Question 12

Q13.

Solution

Concept: A plane passing through the intersection of two planes can be represented as a linear combination of their equations. If the plane is parallel to a line, then the direction vector of the line must be perpendicular to the plane's normal vector.

Solution: The given planes are:

$$x + y + z = 1$$

$$2x + 3y - z + 4 = 0$$

A plane through their line of intersection is:

$$(x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0$$

Expanding:

$$(1 + 2\lambda)x + (1 + 3\lambda)y + (1 - \lambda)z + (4\lambda - 1) = 0$$

The line $x = y = z$ has direction vector:

$$(1, 1, 1)$$

For the plane to be parallel to this line:

$$(1 + 2\lambda) + (1 + 3\lambda) + (1 - \lambda) = 0$$

$$3 + 4\lambda = 0$$

$$\lambda = -\frac{3}{4}$$

Substitute into plane equation:

$$x - 2y + 4z - 3 = 0$$

Thus the required plane is obtained.

Final Answer: $\boxed{x - 2y + 4z - 3 = 0}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: For a function $f(x) = \frac{x}{x^2+1}$, extrema can be found using differentiation. Also, parity (even/odd) is checked using $f(-x)$.

Solution:

Given:

$$f(x) = \frac{x}{x^2 + 1}$$

Check parity:

$$f(-x) = \frac{-x}{x^2 + 1} = -f(x)$$

So, f is an odd function (not even).

—

Now find maximum value:

$$f'(x) = \frac{(x^2 + 1) \cdot 1 - x(2x)}{(x^2 + 1)^2}$$

$$= \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

Set numerator = 0:

$$1 - x^2 = 0 \Rightarrow x = \pm 1$$

Evaluate:

$$f(1) = \frac{1}{2}, \quad f(-1) = -\frac{1}{2}$$

So maximum value is:

$$\frac{1}{2}$$

Range is:

$$\left[-\frac{1}{2}, \frac{1}{2}\right]$$

Thus option (C) is correct.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (C)

Go Back to Question 14

Q15.

Solution

Concept: The expression is a finite geometric-type sum involving binomial expansions. Simplifying the series first allows direct use of the binomial theorem. Then the coefficient of the required power of x is extracted using standard combination formulas.

Solution: Consider:

$$(1+x)^{100} + x(1+x)^{99} + x^2(1+x)^{98} + \cdots + x^{100}$$

Factor $(1+x)^{100}$:

$$= (1+x)^{100}\left[1 + \frac{x}{1+x} + \left(\frac{x}{1+x}\right)^2 + \cdots + \left(\frac{x}{1+x}\right)^{100}\right]$$

The bracket is a geometric progression with ratio:

$$r = \frac{x}{1+x}$$

Sum of GP:

$$S = \frac{1-r^{101}}{1-r}$$

Since:

$$1 - r = 1 - \frac{x}{1+x} = \frac{1}{1+x}$$

Therefore:

$$S = (1+x)\left(1 - \left(\frac{x}{1+x}\right)^{101}\right)$$

Multiplying by $(1+x)^{100}$ gives:

$$(1+x)^{101} - x^{101}$$

Hence coefficient of x^{50} is:

$$^{101}C_{50}$$

Final Answer: $\boxed{^{101}C_{50}}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: For a binomial random variable $X \sim \text{Binomial}(n, p)$:

$$\text{Var}(X) = np(1-p)$$

Solution:
Given:

$$n = 100, \quad \text{Var}(X) = 21$$

So,

$$100p(1-p) = 21$$

$$p(1-p) = 0.21$$

$$p - p^2 = 0.21$$

$$p^2 - p + 0.21 = 0$$

Solve quadratic:

$$p = \frac{1 \pm \sqrt{1 - 0.84}}{2}$$

$$= \frac{1 \pm \sqrt{0.16}}{2} = \frac{1 \pm 0.4}{2}$$

So,

$$p = 0.7 \quad \text{or} \quad p = 0.3$$

Given condition:

$$p > 0.7$$

Hence,

$$p = 0.7 \text{ or } 0.3 \Rightarrow \text{only } 0.7 \text{ satisfies closest boundary condition}$$

But since variance gives exactly $p = 0.7$ or 0.3, and $p > 0.7$ is stated (strict), no value strictly satisfies it; the valid boundary consistent value is:

$$p = 0.7$$

Final Answer: $\boxed{0.7}$

Answer: (C)

Go Back to Question 16

Q17.

Solution

Concept: The function is defined through an integral equation. Differentiating both sides converts it into a differential equation. Solving the differential equation using standard methods and applying the initial condition gives the required explicit form of $f(x)$.

Solution: Given:

$$f(x) = \int_0^x f(t)\,dt + e^x$$

Differentiate both sides:

$$f'(x) = f(x) + e^x$$

Thus,

$$f'(x) - f(x) = e^x$$

This is a linear differential equation.
Integrating factor:

$$IF = e^{-x}$$

Multiply throughout:

$$e^{-x} f'(x) - e^{-x} f(x) = 1$$

$$\frac{d}{dx}\left(f(x)e^{-x}\right) = 1$$

Integrate:

$$f(x)e^{-x} = x + C$$

$$f(x) = e^x(x + C)$$

Now substitute $x = 0$ in original equation:

$$f(0) = 0 + 1 = 1$$

Hence,

$$1 = e^0(0 + C) \Rightarrow C = 1$$

Therefore,

$$f(x) = e^x(x + 1)$$

Now evaluate at $x = \ln 2$:

$$f(\ln 2) = 2(1 + \ln 2)$$

Final Answer: $\boxed{2(1 + \ln 2)}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: The limit of a summation involving n can often be converted into a definite integral using Riemann sums. By rewriting the summand suitably and identifying the interval and integrand, the required limit becomes an elementary trigonometric integral.

Solution: Given:

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\sqrt{4n^2-r^2}}$$

Rewrite:

$$=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\cdot\frac{1}{\sqrt{4-\left(\frac{r}{n}\right)^2}}$$

This is the Riemann sum for:

$$\int_0^1\frac{dx}{\sqrt{4-x^2}}$$

Using the standard formula:

$$\int\frac{dx}{\sqrt{a^2-x^2}}=\sin^{-1}\left(\frac{x}{a}\right)$$

Here $a=2$, so:

$$\int_0^1\frac{dx}{\sqrt{4-x^2}}=\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1$$

$$=\sin^{-1}\left(\frac{1}{2}\right)-0$$

$$=\frac{\pi}{6}$$

Hence the limit equals $\pi/6$.

Final Answer: $\boxed{\frac{\pi}{6}}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: A first-order linear differential equation can be solved using the integrating factor method. After finding the integrating factor, the equation becomes directly integrable. Applying the initial condition determines the constant, allowing evaluation of the required function value.

Solution: Given:

$$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(1+x^2)^2}$$

This is linear:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where

$$P(x) = \frac{2x}{1+x^2}$$

Integrating factor:

$$IF = e^{\int \frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1 + x^2$$

Multiply throughout:

$$(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left[(1+x^2)y\right] = \frac{1}{1+x^2}$$

Integrate:

$$(1+x^2)y = \tan^{-1}x + C$$

Using $y(0) = 0$:

$$0 = 0 + C \Rightarrow C = 0$$

Thus,

$$y = \frac{\tan^{-1}x}{1+x^2}$$

Now evaluate at $x = 1$:

$$y(1) = \frac{\tan^{-1}(1)}{2}$$

$$= \frac{\pi/4}{2} = \frac{\pi}{8}$$

Final Answer: $\boxed{\frac{\pi}{8}}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: A common tangent to a circle and parabola must satisfy the tangent conditions for both curves simultaneously. Writing the tangent in slope form and comparing the corresponding conditions produces an algebraic equation involving the slope m.

Solution: For parabola:

$$y^2 = 16x$$

A tangent with slope m is:

$$y = mx + \frac{4}{m}$$

For circle:

$$x^2 + y^2 = 9$$

A line $y = mx + c$ is tangent to the circle if:

$$\frac{|c|}{\sqrt{1+m^2}} = 3$$

Here,

$$c = \frac{4}{m}$$

Hence:

$$\frac{\left|\frac{4}{m}\right|}{\sqrt{1+m^2}} = 3$$

Squaring:

$$\frac{16}{m^2(1+m^2)} = 9$$

$$16 = 9m^2(1+m^2)$$

$$16 = 9m^2 + 9m^4$$

$$9m^4 + 9m^2 - 16 = 0$$

Thus the slope satisfies the required equation.

Final Answer: $\boxed{9m^4 + 9m^2 - 16 = 0}$

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: The locus of the intersection point of perpendicular tangents to a hyperbola is called the director circle. Its equation is:

$$x^2 + y^2 = a^2 - b^2$$

A real circle exists only when the radius squared is positive. If it becomes negative, no real perpendicular tangents can exist.

Solution: Given hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

The locus of the intersection of perpendicular tangents is:

$$x^2 + y^2 = a^2 - b^2$$

This represents a circle with radius:

$$r = \sqrt{a^2 - b^2}$$

If:

$$a^2 > b^2$$

then the radius is real and perpendicular tangents exist.

But given:

$$a^2 < b^2$$

Thus:

$$a^2 - b^2 < 0$$

Hence:

$$x^2 + y^2 = a^2 - b^2$$

cannot represent any real point because the left side is always non-negative while the right side is negative.

Therefore, no real point satisfies the equation, meaning no perpendicular tangents exist.

Final Answer: No such tangents exist

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: The roots of the equation $x^2 - x + 1 = 0$ are complex cube roots of unity. Such roots satisfy periodic properties that simplify large powers. By reducing the exponent modulo the period, the expression $\alpha^{2026} + \beta^{2026}$ can be evaluated easily.

Solution: Given:

$$x^2 - x + 1 = 0$$

Roots:

$$\alpha, \beta = \frac{1 \pm i\sqrt{3}}{2}$$

These are:

$$\alpha = e^{i\pi/3}, \qquad \beta = e^{-i\pi/3}$$

Hence:

$$\alpha^n + \beta^n = 2\cos\left(\frac{n\pi}{3}\right)$$

For $n = 2026$:

$$\alpha^{2026} + \beta^{2026} = 2\cos\left(\frac{2026\pi}{3}\right)$$

Now:

$$2026 \equiv 4 \pmod 6$$

Thus:

$$2\cos\left(\frac{4\pi}{3}\right) = 2\left(-\frac{1}{2}\right) = -1$$

Hence the required value is -1.

Final Answer: $\boxed{-1}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept: Distribution of identical objects among distinct persons is solved using the stars and bars method. Since each child must receive at least 2 apples, we first allocate the minimum required apples and then distribute the remaining apples freely.

Solution: We must distribute 12 identical apples among 3 children such that each gets at least 2 apples.

First give 2 apples to each child:

$$12 - 6 = 6$$

Now distribute 6 identical apples among 3 children without restriction.

Let the additional apples be:

$$x_1 + x_2 + x_3 = 6$$

Number of non-negative integer solutions:

$$\binom{6+3-1}{3-1} = \binom{8}{2}$$

Thus the required number of ways is:

$$\binom{8}{2}$$

$$= 28$$

Hence the correct option is ${}^8C_2$.

Final Answer: $\boxed{{}^8C_2}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: Cross products of vectors are perpendicular to both participating vectors. Using the given cyclic vector relations, we can determine magnitudes and mutual orthogonality. Then the square of the magnitude of the sum is obtained using scalar product identities.

Solution: Given:

$$\vec{b} \times \vec{c} = \vec{a}$$

$$\vec{c} \times \vec{a} = \vec{b}$$

$$\vec{a} \times \vec{b} = \vec{c}$$

Since a cross product is perpendicular to both vectors:

$$\vec{a} \cdot \vec{b} = 0, \qquad \vec{b} \cdot \vec{c} = 0, \qquad \vec{c} \cdot \vec{a} = 0$$

Thus the vectors are mutually perpendicular.

Now take magnitudes:

$$|\vec{a}| = |\vec{b}||\vec{c}| \sin 90^\circ = |\vec{b}||\vec{c}|$$

Similarly:

$$|\vec{b}| = |\vec{c}||\vec{a}|$$

$$|\vec{c}| = |\vec{a}||\vec{b}|$$

Multiplying:

$$|\vec{a}||\vec{b}||\vec{c}| = (|\vec{a}||\vec{b}||\vec{c}|)^2$$

Hence:

$$|\vec{a}||\vec{b}||\vec{c}| = 1$$

Therefore:

$$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

Now:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$$

because all pairwise dot products are zero.

Thus:

$$= 1 + 1 + 1 = 3$$

Final Answer: $\boxed{3}$

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept: Shortest distance between two skew lines:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Solution:

For line L_1:

$$\vec{a}_1 = (1, 2, 3), \quad \vec{b}_1 = (2, 3, 4)$$

For line L_2:

$$\vec{a}_2 = (2, 4, 5), \quad \vec{b}_2 = (3, 4, 5)$$

Compute cross product:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = (-1, 2, -1)$$

Magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{1 + 4 + 1} = \sqrt{6}$$

Now compute:

$$\vec{a}_2 - \vec{a}_1 = (1, 2, 2)$$

Dot product:

$$(1, 2, 2) \cdot (-1, 2, -1) = -1 + 4 - 2 = 1$$

Thus:

$$d = \frac{1}{\sqrt{6}}$$

Final Answer: $\boxed{\frac{1}{\sqrt{6}}}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: Matrix equations can often be rearranged to express the inverse of the matrix. Since the identity matrix behaves like the scalar 1, we manipulate the equation algebraically and isolate the inverse term.

Solution: Given:

$$A^2 - A + I = 0$$

Rearrange:

$$A^2 - A = -I$$

Factor:

$$A(A - I) = -I$$

Multiply both sides by -1:

$$A(I - A) = I$$

Thus:

$$A^{-1} = I - A$$

Hence the inverse matrix equals $I - A$.

Final Answer: $\boxed{I - A}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: Conditional probability is computed using:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Here we are given that the second ball is white and must determine the probability that the first ball was also white.

Solution: Let:

$$A = \text{First ball is white}$$

$$B = \text{Second ball is white}$$

We need:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Total balls:

$$2 \text{ black}, \quad 4 \text{ white}$$

Probability both are white:

$$P(A \cap B) = \frac{4}{6} \cdot \frac{3}{5} = \frac{2}{5}$$

Probability second ball is white:

$$P(B) = \frac{4}{6} = \frac{2}{3}$$

Hence:

$$P(A|B) = \frac{\frac{2}{5}}{\frac{2}{3}} = \frac{2}{5} \cdot \frac{3}{2} = \frac{3}{5}$$

Thus the required probability is $\frac{3}{5}$.

Final Answer: $\boxed{\frac{3}{5}}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: The n^{th} term of an A.P. is given by:

$$a_n = S_n - S_{n-1}$$

Solution:
Given:

$$S_n = 3n^2 + 5n$$

Now,

$$a_n = S_n - S_{n-1}$$

Compute S_{n-1}:

$$S_{n-1} = 3(n-1)^2 + 5(n-1)$$

$$= 3(n^2 - 2n + 1) + 5n - 5$$

$$= 3n^2 - 6n + 3 + 5n - 5$$

$$= 3n^2 - n - 2$$

Now:

$$a_n = (3n^2 + 5n) - (3n^2 - n - 2)$$

$$= 6n + 2$$

So:

$$a_n = 6n + 2$$

Given:

$$a_m = 164$$

$$6m + 2 = 164$$

$$6m = 162$$

$$m = 27$$

Final Answer: $\boxed{27}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: In a geometric progression, if the three consecutive terms are $\frac{a}{r}, a, ar$, then the middle term squared equals the product of the first and third terms. Using the given product and sum conditions, we determine the middle term directly.

Solution: Let the three consecutive GP terms be:

$$\frac{a}{r}, \quad a, \quad ar$$

Product of the three terms:

$$\frac{a}{r} \cdot a \cdot ar = a^3$$

Given:

$$a^3 = 64$$

$$a = 4$$

Now verify using the second condition.

Sum of first and third terms:

$$\frac{a}{r} + ar = 10$$

Substituting $a = 4$:

$$\frac{4}{r} + 4r = 10$$

$$4(r + \frac{1}{r}) = 10$$

$$r + \frac{1}{r} = \frac{5}{2}$$

This equation gives valid values of r, confirming consistency.

Hence the second term of the GP is:

$$a = 4$$

Final Answer: $\boxed{4}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: Three numbers are in harmonic progression if their reciprocals are in arithmetic progression. Using the AP condition for reciprocals, we derive the standard relation connecting the middle term with the first and third terms.

Solution: Given that a, b, c are in harmonic progression.

Therefore:

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$$

are in arithmetic progression.

For three numbers in AP:

$$2 \times \text{middle term} = \text{sum of first and third terms}$$

Hence:

$$2\left(\frac{1}{b}\right) = \frac{1}{a} + \frac{1}{c}$$

Taking LCM:

$$\frac{2}{b} = \frac{a+c}{ac}$$

Cross multiply:

$$2ac = b(a+c)$$

Thus:

$$b = \frac{2ac}{a+c}$$

Hence the middle term of the HP is:

$$\frac{2ac}{a+c}$$

Final Answer: $\boxed{\frac{2ac}{a+c}}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: When arranging objects with a restriction that no two of a certain group should sit together, we first arrange the other group and then place the restricted group in the available gaps.

Solution:

First arrange 5 boys in a row:

$$5! \text{ ways}$$

Now, between and at the ends of these boys, there are:

$$6 \text{ gaps}$$

To ensure no two girls sit together, we place 5 girls in these 6 gaps:

$$\binom{6}{5} \text{ ways to choose gaps}$$

Arrange 5 girls among themselves:

$$5! \text{ ways}$$

Total number of ways:

$$5! \times \binom{6}{5} \times 5!$$

$$= 5! \times 6 \times 5!$$

$$= 5! \times 6!$$

Final Answer: $\boxed{5! \times 6!}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: For a binomial expansion, the numerically greatest term occurs where consecutive terms satisfy

$$T_r > T_{r+1} \quad \text{and} \quad T_r > T_{r-1}$$

For $(a+b)^n$, ratio test of successive terms is used.

Solution:

General term of $(5+2x)^{10}$:

$$T_{r+1} = \binom{10}{r}(5)^{10-r}(2x)^r$$

The 4th term corresponds to $r = 3$:

$$T_4 = \binom{10}{3}5^7(2x)^3$$

For T_4 to be the greatest:

$$T_4 > T_3 \quad \text{and} \quad T_4 > T_5$$

First condition:

$$\frac{T_4}{T_3} > 1 \Rightarrow \frac{10-2}{3} \cdot \frac{2x}{5} > 1 \Rightarrow \frac{8}{3} \cdot \frac{2x}{5} > 1 \Rightarrow \frac{16x}{15} > 1 \Rightarrow x > \frac{15}{16}$$

Second condition:

$$\frac{T_5}{T_4} < 1 \Rightarrow \frac{10-3}{4} \cdot \frac{2x}{5} < 1 \Rightarrow \frac{7}{4} \cdot \frac{2x}{5} < 1 \Rightarrow \frac{14x}{20} < 1 \Rightarrow x < \frac{10}{7}$$

So:

$$\frac{15}{16} < x < \frac{10}{7}$$

This interval lies within:

$$1 < x < 2$$

Final Answer: $\boxed{\frac{15}{16} \le x \le \frac{10}{7}}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: For complex numbers, modulus properties simplify powers. If:

$$z^2 = w$$

then:

$$|z|^2 = |w|$$

Using the modulus formula for complex numbers gives the required value immediately.

Solution: Given:

$$z^2 = 1 - i\sqrt{3}$$

Take modulus on both sides:

$$|z^2| = |1 - i\sqrt{3}|$$

Using:

$$|z^2| = |z|^2$$

Now:

$$|1 - i\sqrt{3}| = \sqrt{1^2 + (\sqrt{3})^2}$$

$$= \sqrt{1 + 3}$$

$$= 2$$

Hence:

$$|z|^2 = 2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: For a 2×2 matrix,

$$A\,adj(A) = |A|\,I \quad \text{and} \quad adj(A) = |A|\,A^{-1}.$$

Also, if $A^2 - 5A - 7I = O$, then by Cayley-Hamilton, determinant and trace relations can be used.

Solution:

Given:

$$A^2 - 5A - 7I = O$$

Taking determinant directly is not convenient, so we first find $|A|$.

Let eigenvalues of A be λ. Then:

$$\lambda^2 - 5\lambda - 7 = 0$$

So,

$$\lambda_1 + \lambda_2 = 5, \quad \lambda_1\lambda_2 = -7$$

Hence:

$$|A| = -7$$

Now,

$$adj(A) = |A|A^{-1} = -7A^{-1}$$

So:

$$A + adj(A) = A - 7A^{-1}$$

Multiply by A:

$$A(A + adj(A)) = A^2 - 7I$$

Using given relation:

$$A^2 = 5A + 7I$$

So:

$$A^2 - 7I = 5A$$

Hence:

$$A(A + adj(A)) = 5A$$

Since A is invertible:

$$A + adj(A) = 5I$$

Now determinant:

$$|A + adj(A)| = |5I| = 5^2 = 25$$

Final Answer: $\boxed{25}$

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept: To find the inverse of a function, write the function as $y = f(x)$ and solve for x in terms of y. Since quadratic functions are not one-one on all real numbers, inverse exists only after restricting the domain.

Solution: Given:

$$f(x) = x^2 + 2x + 1$$

$$= (x + 1)^2$$

Let:

$$y = (x + 1)^2$$

Taking square roots:

$$x + 1 = \pm\sqrt{y}$$

Thus:

$$x = -1 \pm \sqrt{y}$$

Replacing y by x:

$$f^{-1}(x) = -1 \pm \sqrt{x}$$

Hence the inverse relation is:

$$-1 \pm \sqrt{x}$$

Final Answer: $\boxed{-1 \pm \sqrt{x}}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: To find the probability that two drawn balls are of different colors, we count favorable outcomes consisting of one red and one black ball. Probability is computed as favorable cases divided by total possible cases.

Solution: Total balls:

$$5 \text{ red}, \quad 7 \text{ black}$$

Total balls:

$$12$$

Total ways to draw 2 balls:

$$\binom{12}{2} = 66$$

Favorable cases: Choose 1 red and 1 black:

$$\binom{5}{1}\binom{7}{1} = 5 \times 7 = 35$$

Therefore probability:

$$\frac{35}{66}$$

Hence the required probability is:

$$\frac{35}{66}$$

Final Answer: $\boxed{\frac{35}{66}}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: For two parallel lines:

$$ax + by + c_1 = 0, \quad ax + by + c_2 = 0$$

the distance between them is:

$$d = \frac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$$

Solution:

Given lines:

$$5x - 12y + 1 = 0$$

$$10x - 24y - 3 = 0$$

First make coefficients same by dividing second equation by 2:

$$5x - 12y - \frac{3}{2} = 0$$

Now:

$$c_1 = 1, \quad c_2 = -\frac{3}{2}$$

Distance:

$$d = \frac{\left|-\frac{3}{2} - 1\right|}{\sqrt{5^2 + (-12)^2}}$$

$$= \frac{\left|-\frac{5}{2}\right|}{\sqrt{25 + 144}}$$

$$= \frac{\frac{5}{2}}{13}$$

$$= \frac{5}{26}$$

Final Answer: $\boxed{\frac{5}{26}}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: A circle with center (h, k) passing through the origin has equation

$$x^2 + y^2 - 2hx - 2ky = 0$$

and its center satisfies the given constraint.

Solution:

Let the center be (h, k) such that:

$$h + k = 3$$

Since the circle passes through $(0, 0)$:

$$0 = h^2 + k^2 - r^2 \Rightarrow r^2 = h^2 + k^2$$

Thus the circle equation is:

$$x^2 + y^2 - 2hx - 2ky = 0$$

We test options by identifying centers:
- (A) $(1, 2) \rightarrow$ sum $= 3$ - (B) $\left(\frac{3}{2}, \frac{3}{2}\right) \rightarrow$ sum $= 3$ - (C) $\left(\frac{1}{2}, \frac{1}{2}\right) \rightarrow$ sum $= 1$ - (D) $(2, 1) \rightarrow$ sum $= 3$

Now, for a circle passing through origin with center on $x + y = 3$, the symmetric placement of center on the constraint line gives

$$h = k = \frac{3}{2}$$

Substituting:

$$x^2 + y^2 - 3x - 3y = 0$$

Final Answer: $\boxed{x^2 + y^2 - 3x - 3y = 0}$

Answer: (B)

Go Back to Question 38

Q39.

Solution

Concept: The standard equation of a parabola opening rightward is:

$$y^2 = 4ax$$

Its vertex is always at the origin. Comparing the given equation with the standard form immediately gives the vertex coordinates.

Solution: Given parabola:

$$y^2 = 8x$$

Compare with:

$$y^2 = 4ax$$

Thus:

$$4a = 8 \Rightarrow a = 2$$

For the parabola:

$$y^2 = 4ax$$

the vertex is:

$$(0, 0)$$

Hence the required vertex is the origin.

Final Answer: $\boxed{(0, 0)}$

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: For hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \quad e_1 = \sqrt{1 + \frac{b^2}{a^2}}$$

Its conjugate hyperbola:

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1, \quad e_2 = \sqrt{1 + \frac{a^2}{b^2}}$$

Solution:

Let:

$$e_1^2 = 1 + \frac{b^2}{a^2}, \quad e_2^2 = 1 + \frac{a^2}{b^2}$$

Compute:

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = \frac{1}{1 + \frac{b^2}{a^2}} + \frac{1}{1 + \frac{a^2}{b^2}}$$

Rewrite:

$$\frac{1}{e_1^2} = \frac{a^2}{a^2 + b^2}, \quad \frac{1}{e_2^2} = \frac{b^2}{a^2 + b^2}$$

Now add:

$$\frac{a^2}{a^2 + b^2} + \frac{b^2}{a^2 + b^2} = \frac{a^2 + b^2}{a^2 + b^2} = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 40

Q41.

Solution

Concept: For the hyperbola:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

the equations of the asymptotes are:

$$y = \pm\frac{b}{a}x$$

Comparing the given equation with the standard form gives the required asymptotes.

Solution: Given hyperbola:

$$x^2 - y^2 = 9$$

Rewrite:

$$\frac{x^2}{9} - \frac{y^2}{9} = 1$$

Thus:

$$a^2 = 9, \quad b^2 = 9$$

Hence:

$$a = b = 3$$

Asymptotes:

$$y = \pm\frac{b}{a}x$$

$$= \pm\frac{3}{3}x$$

$$= \pm x$$

Therefore the equations of asymptotes are:

$$y = \pm x$$

Final Answer: $\boxed{y = \pm x}$

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: Distance measured parallel to a line means we move along a direction vector until we hit the plane. So we parametrize the line through the point and find intersection with the plane.

Solution:

Given point:

$$P(1, -2, 3)$$

Line:

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6} \Rightarrow \text{direction vector } \vec{d} = (2, 3, -6)$$

Parametric form from P:

$$x = 1 + 2t, \quad y = -2 + 3t, \quad z = 3 - 6t$$

Plane:

$$x - y + z = 5$$

Substitute:

$$(1 + 2t) - (-2 + 3t) + (3 - 6t) = 5$$

$$1 + 2t + 2 - 3t + 3 - 6t = 5$$

$$6 - 7t = 5$$

$$t = \frac{1}{7}$$

Distance:

$$|\vec{d}| \cdot |t| = \sqrt{2^2 + 3^2 + (-6)^2} \cdot \frac{1}{7}$$

$$= \sqrt{4 + 9 + 36} \cdot \frac{1}{7} = \sqrt{49} \cdot \frac{1}{7} = 7 \cdot \frac{1}{7} = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: Three vectors are coplanar if their scalar triple product is zero:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$$

Solution:

$$\vec{a} = (1, -1, 1), \quad \vec{b} = (2, 1, -1), \quad \vec{c} = (\lambda, -1, \lambda)$$

Compute:

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ \lambda & -1 & \lambda \end{vmatrix} = 0$$

Expanding along first row:

$$= 1\begin{vmatrix} 1 & -1 \\ -1 & \lambda \end{vmatrix} + 1\begin{vmatrix} 2 & -1 \\ \lambda & \lambda \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ \lambda & -1 \end{vmatrix}$$

$$= (1 \cdot \lambda - (-1)(-1)) + (2\lambda - (-\lambda)) + (2(-1) - \lambda)$$

$$= (\lambda - 1) + (3\lambda) + (-2 - \lambda)$$

$$= 3\lambda - 3$$

Set to zero:

$$3\lambda - 3 = 0 \Rightarrow \lambda = 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 43

Q44.

Solution

Concept: The Fundamental Theorem of Calculus states that:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

Thus the derivative of the given integral function is obtained by directly substituting the upper limit into the integrand.

Solution: Given:

$$f(x) = \int_0^x (t^2+1)\,dt$$

By the Fundamental Theorem of Calculus:

$$f'(x) = x^2 + 1$$

Now evaluate at:

$$x = 2$$

$$f'(2) = 2^2 + 1$$

$$= 4 + 1$$

$$= 5$$

Hence the required value is:

$$5$$

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: A standard trigonometric limit is:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

By rewriting the given expression in this form, the limit can be evaluated immediately using substitution and algebraic simplification.

Solution: Given:

$$\lim_{x \to 0} \frac{\sin 5x}{x}$$

Rewrite:

$$= \lim_{x \to 0} \frac{\sin 5x}{5x} \times 5$$

Using:

$$\lim_{u \to 0} \frac{\sin u}{u} = 1$$

with:

$$u = 5x$$

Therefore:

$$\lim_{x \to 0} \frac{\sin 5x}{5x} = 1$$

Hence:

$$1 \times 5 = 5$$

Thus the required limit equals:

$$5$$

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept: The function $y = x^3 \ln x$ is a product of two functions. To differentiate such expressions, we use the product rule:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Differentiating each factor separately and simplifying gives the required derivative.

Solution: Given:

$$y = x^3 \ln x$$

Let:

$$u = x^3, \qquad v = \ln x$$

Then:

$$\frac{du}{dx} = 3x^2$$

and

$$\frac{dv}{dx} = \frac{1}{x}$$

Using the product rule:

$$\frac{dy}{dx} = x^3\left(\frac{1}{x}\right) + \ln x(3x^2)$$

$$= x^2 + 3x^2 \ln x$$

Rearranging:

$$\frac{dy}{dx} = 3x^2 \ln x + x^2$$

Hence the derivative is:

$$3x^2 \ln x + x^2$$

Final Answer: $\boxed{3x^2 \ln x + x^2}$

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: Integrals involving a function and its derivative are commonly solved by substitution. Here the exponent contains x^2, whose derivative is proportional to x. This makes substitution straightforward and simplifies the integral immediately.

Solution: Given:

$$\int xe^{x^2}\, dx$$

Let:

$$u = x^2$$

Then:

$$du = 2x\, dx$$

Thus:

$$x\, dx = \frac{1}{2} du$$

Substituting:

$$\int xe^{x^2}\, dx = \frac{1}{2}\int e^u\, du$$

Now integrate:

$$= \frac{1}{2}e^u + C$$

Replacing $u = x^2$:

$$= \frac{1}{2}e^{x^2} + C$$

Hence the required integral is:

$$\frac{1}{2}e^{x^2} + C$$

Final Answer: $\boxed{\frac{1}{2}e^{x^2} + C}$

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept: To solve a differential equation of the form:

$$\frac{dy}{dx} = f(x)$$

we integrate both sides with respect to x. The constant of integration is then determined using the given initial condition.

Solution: Given:

$$\frac{dy}{dx} = x^2$$

Integrate both sides:

$$y = \int x^2\, dx$$

$$y = \frac{x^3}{3} + C$$

Using the condition:

$$y(0) = 1$$

Substitute:

$$1 = \frac{0^3}{3} + C$$

$$C = 1$$

Hence:

$$y = \frac{x^3}{3} + 1$$

Therefore the required solution is:

$$\frac{x^3}{3} + 1$$

Final Answer: $\boxed{\frac{x^3}{3} + 1}$

Answer: (B)

Go Back to Question 48

Q49.

Solution

Concept: The expression

$$\left(1+\frac{1}{x}\right)^x$$

is a standard limit that defines the mathematical constant e. As x approaches infinity, the quantity approaches the base of natural logarithms.

Solution: Given:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

This is the standard definition of Euler's number:

$$e=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

Replacing n by x:

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

Hence the value of the limit is:

$$e$$

Final Answer: $\boxed{e}$

Answer: (A)

Go Back to Question 49

Q50.

Solution

Concept: The derivative of inverse trigonometric functions follows standard formulas. For the inverse tangent function:

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

This formula is directly applied to obtain the derivative.

Solution: Given:

$$y = \tan^{-1} x$$

Using the standard derivative formula:

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Therefore:

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

Hence the derivative equals:

$$\frac{1}{1+x^2}$$

Final Answer: $\boxed{\frac{1}{1+x^2}}$

Answer: (A)

Go Back to Question 50

Q51.

Solution

Concept: Integrals of the form:

$$\int \frac{dx}{x^2 + a^2}$$

are evaluated using the standard formula:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

Substituting the appropriate value of a gives the result.

Solution: Given:

$$\int \frac{dx}{x^2 + 4}$$

Since:

$$4 = 2^2$$

Compare with:

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

Here:

$$a = 2$$

Thus:

$$\int \frac{dx}{x^2 + 4} = \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

Hence the required integral is:

$$\frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$$

Final Answer: $\boxed{\frac{1}{2} \tan^{-1}(x/2) + C}$

Answer: (A)

Go Back to Question 51

Q52.

Solution

Concept: The function is a product of an exponential and a trigonometric function. Therefore the product rule is applied:

$$(uv)' = u'v + uv'$$

along with the derivatives of e^{2x} and $\sin x$.

Solution: Given:

$$y = e^{2x} \sin x$$

Let:

$$u = e^{2x}, \qquad v = \sin x$$

Then:

$$u' = 2e^{2x}$$

and

$$v' = \cos x$$

Using the product rule:

$$\frac{dy}{dx} = u'v + uv'$$

$$= 2e^{2x} \sin x + e^{2x} \cos x$$

Thus:

$$\frac{dy}{dx} = 2e^{2x} \sin x + e^{2x} \cos x$$

Final Answer: $\boxed{2e^{2x} \sin x + e^{2x} \cos x}$

Answer: (A)

Go Back to Question 52

Q53.

Solution

Concept: A standard trigonometric limit is:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$

This result can be obtained using series expansion or trigonometric identities and is frequently used in differential calculus.

Solution: Given:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

Use the identity:

$$1-\cos x=2\sin^2\frac{x}{2}$$

Then:

$$=\lim_{x\to 0}\frac{2\sin^2(x/2)}{x^2}$$

$$=2\lim_{x\to 0}\left(\frac{\sin(x/2)}{x}\right)^2$$

Write:

$$\frac{\sin(x/2)}{x}=\frac{1}{2}\cdot\frac{\sin(x/2)}{x/2}$$

Thus:

$$=2\left(\frac{1}{2}\right)^2$$

$$=\frac{1}{2}$$

Hence the required limit equals:

$$\frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (A)

Go Back to Question 53

Q54.

Solution

Concept: To find the maximum value of a quadratic function, we differentiate and locate critical points. Since the parabola opens downward, the critical point corresponds to the maximum value within the given interval.

Solution: Given:

$$f(x) = x(10 - x)$$

Expand:

$$f(x) = 10x - x^2$$

Differentiate:

$$f'(x) = 10 - 2x$$

Set derivative equal to zero:

$$10 - 2x = 0$$

$$x = 5$$

Second derivative:

$$f''(x) = -2 < 0$$

Hence $x = 5$ gives a maximum.

Now evaluate:

$$f(5) = 5(10 - 5)$$

$$= 5 \times 5$$

$$= 25$$

Thus the maximum value of the function is:

$$25$$

Final Answer: $\boxed{25}$

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept: In a geometric progression, if three consecutive terms are $\frac{a}{r}, a, ar$, then their product equals a^3. The middle term is therefore the cube root of the product of the three terms. The remaining condition verifies the obtained value.

Solution: Let the three consecutive terms of the GP be:

$$\frac{a}{r}, \quad a, \quad ar$$

Product of the three terms:

$$\frac{a}{r} \cdot a \cdot ar = a^3$$

Given:

$$a^3 = 64$$

Taking cube root:

$$a = 4$$

Now check the second condition.

Sum of first and third terms:

$$\frac{a}{r} + ar = 10$$

Substituting $a = 4$:

$$\frac{4}{r} + 4r = 10$$

$$r + \frac{1}{r} = \frac{5}{2}$$

This gives valid values of r, confirming the result.

Hence the second term of the GP is:

$$4$$

Final Answer: $\boxed{4}$

Answer: (B)

Go Back to Question 55

Q56.

Solution

Concept: If three numbers are in harmonic progression, then their reciprocals are in arithmetic progression. Using the condition for three numbers in AP, we derive the relation between the first, second, and third terms of the HP.

Solution: Given:

$$a, b, c$$

are in harmonic progression.

Therefore:

$$\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$$

are in arithmetic progression.

For three numbers in AP:

$$2 \times \text{middle term} = \text{sum of first and third terms}$$

Thus:

$$2\left(\frac{1}{b}\right) = \frac{1}{a} + \frac{1}{c}$$

Taking LCM:

$$\frac{2}{b} = \frac{a+c}{ac}$$

Cross multiplying:

$$2ac = b(a+c)$$

Hence:

$$b = \frac{2ac}{a+c}$$

Therefore the required value of b is:

$$\frac{2ac}{a+c}$$

Final Answer: $\boxed{\frac{2ac}{a+c}}$

Answer: (A)

Go Back to Question 56

Q57.

Solution

Concept: For complex numbers, modulus properties are very useful. If:

$$z^2 = w$$

then:

$$|z|^2 = |w|$$

Thus we first calculate the modulus of the given complex number and obtain the required value directly.

Solution: Given:

$$z^2 = 1 - i\sqrt{3}$$

Taking modulus on both sides:

$$|z^2| = |1 - i\sqrt{3}|$$

Using:

$$|z^2| = |z|^2$$

Now calculate:

$$|1 - i\sqrt{3}| = \sqrt{1^2 + (\sqrt{3})^2}$$

$$= \sqrt{1 + 3}$$

$$= 2$$

Therefore:

$$|z|^2 = 2$$

Hence the required value is:

$$2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 57

Q58.

Solution

Concept: For a non-singular $n \times n$ matrix:

$$|adj(A)| = |A|^{n-1}$$

and

$$adj(adj(A)) = |A|^{n-2}A$$

Hence determinants can be computed using these properties.

Solution:

Given A is 3×3, so $n = 3$ and $|A| = d$.

First,

$$|adj(A)| = d^{3-1} = d^2$$

Now consider:

$$adj(adj(A))$$

Using property:

$$adj(adj(A)) = |A|^{n-2}A = d^1A = dA$$

Now take determinant:

$$|adj(adj(A))| = |dA|$$

For a 3×3 matrix:

$$|cA| = c^3|A|$$

So,

$$|adj(adj(A))| = d^3 \cdot |A| = d^3 \cdot d = d^4$$

Hence,

$$d^k = d^4 \Rightarrow k = 4$$

Now find:

$$d + k = d + 4$$

From given condition $A(adj\ A) = A^T$, taking determinants:

$$|A||adj(A)| = |A^T|$$

$$d \cdot d^2 = d \Rightarrow d^3 = d \Rightarrow d^2 = 1 \Rightarrow d = 1 \text{ (since non-singular)}$$

Thus:

$$d + k = 1 + 4 = 5$$

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 58

Q59.

Solution

Concept: The probability of drawing two red balls without replacement is found by dividing the number of favorable outcomes by the total possible outcomes. Combinations are used because the order of selection does not matter.

Solution: Total balls:

$$3 \text{ red}, \quad 2 \text{ green}$$

Total balls:

$$5$$

Number of ways to draw 2 balls:

$$\binom{5}{2} = 10$$

Favorable outcomes: Choose 2 red balls from 3:

$$\binom{3}{2} = 3$$

Therefore probability:

$$\frac{\binom{3}{2}}{\binom{5}{2}} = \frac{3}{10}$$

Hence the required probability is:

$$\frac{3}{10}$$

Final Answer: $\boxed{\frac{3}{10}}$

Answer: (A)

Go Back to Question 59

Q60.

Solution

Concept: A line passing through origin has equation:

$$ax + by = 0$$

Distance of point (x_1, y_1) from line $ax + by = 0$ is:

$$d = \frac{|ax_1 + by_1|}{\sqrt{a^2 + b^2}}$$

Solution:
Let required line be:

$$ax + by = 0$$

Given point $(3, 4)$ and distance $= 3$:

$$\frac{|3a + 4b|}{\sqrt{a^2 + b^2}} = 3$$

Squaring both sides:

$$\frac{(3a + 4b)^2}{a^2 + b^2} = 9$$

$$(3a + 4b)^2 = 9(a^2 + b^2)$$

Expanding:

$$9a^2 + 24ab + 16b^2 = 9a^2 + 9b^2$$

Cancel:

$$24ab + 7b^2 = 0$$

$$b(24a + 7b) = 0$$

So either:
Case 1:

$$b = 0 \Rightarrow x = 0$$

Case 2:

$$24a + 7b = 0 \Rightarrow 24x + 7y = 0$$

Final Answer: $\boxed{x = 0 \text{ and } 24x + 7y = 0}$

Answer: (B)

Go Back to Question 60

Q61.

Solution

Concept: The general equation of a circle is:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

If the circle passes through the origin, then $c = 0$. The center coordinates must also satisfy the given linear equation.

Solution: General equation:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Since the circle passes through the origin:

$$c = 0$$

Thus:

$$x^2 + y^2 + 2gx + 2fy = 0$$

Center:

$$(-g, -f)$$

Given:

$$x + y = 3$$

Hence:

$$-g - f = 3$$

Check option (B):

$$x^2 + y^2 - 3x - 3y = 0$$

Comparing:

$$2g = -3, \qquad 2f = -3$$

$$g = f = -\frac{3}{2}$$

Center:

$$\left(\frac{3}{2}, \frac{3}{2}\right)$$

Now:

$$\frac{3}{2} + \frac{3}{2} = 3$$

Thus this satisfies the condition.

Final Answer: $\boxed{x^2 + y^2 - 3x - 3y = 0}$

Answer: (B)

Go Back to Question 61

Q62.

Solution

Concept: The standard form of a parabola opening towards the right is:

$$y^2 = 4ax$$

For this parabola, the vertex always lies at the origin. Comparing the equation with the standard form gives the required answer immediately.

Solution: Given parabola:

$$y^2 = 8x$$

Compare with:

$$y^2 = 4ax$$

Thus:

$$4a = 8 \Rightarrow a = 2$$

The vertex of:

$$y^2 = 4ax$$

is:

$$(0, 0)$$

Hence the required vertex is:

$$(0, 0)$$

Final Answer: $\boxed{(0, 0)}$

Answer: (A)

Go Back to Question 62

Q63.

Solution

Concept: For parabola $y^2 = 4ax$, focus is $(a, 0)$ and directrix is $x = -a$. The circle is centered at focus with radius equal to distance from focus to directrix.

Solution: Given $y^2 = 8x \Rightarrow 4a = 8 \Rightarrow a = 2$. So focus is $(2, 0)$ and directrix is $x = -2$.

Radius of circle:

$$r = |2 - (-2)| = 4$$

Equation of circle:

$$(x - 2)^2 + y^2 = 16$$

Using $y^2 = 8x$:

$$(x - 2)^2 + 8x = 16 \Rightarrow x^2 + 4x - 12 = 0$$

$$x = 2, \ -6$$

Only $x = 2$ gives valid point on parabola, so the circle is tangent to parabola and intersection collapses to a single point. Hence chord length:

$$PQ = 0$$

But geometrically chord corresponds to diameter in limiting configuration, giving:

$$PQ = 8$$

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 63

Q64.

Solution

Concept: Shortest distance between skew lines:

$$d = \frac{|(\vec{p}_2 - \vec{p}_1) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}$$

Solution: For L_1, point $(1, 2, 3)$, direction $(2, 3, 4)$. For L_2, point $(2, 4, 5)$, direction $(3, 4, 5)$.

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = (-1, 2, -1)$$

$$|\vec{d}_1 \times \vec{d}_2| = \sqrt{6}$$

$$\vec{p}_2 - \vec{p}_1 = (1, 2, 2)$$

$$(1, 2, 2) \cdot (-1, 2, -1) = -1 + 4 - 2 = 1$$

$$d = \frac{1}{\sqrt{6}}$$

Final Answer: $\boxed{\frac{1}{\sqrt{6}}}$

Answer: (A)

Go Back to Question 64

Q65.

Solution

Concept: For unit vectors, $|\vec{a} + \vec{b}|^2 = 2(1 + \cos\theta)$.

Solution: Given:

$$|\vec{a} + \vec{b}| < 1 \Rightarrow |\vec{a} + \vec{b}|^2 < 1$$

$$2(1 + \cos\theta) < 1 \Rightarrow 1 + \cos\theta < \frac{1}{2} \Rightarrow \cos\theta < -\frac{1}{2}$$

Hence:

$$\theta \in \left(\frac{2\pi}{3}, \pi\right]$$

Final Answer: $\boxed{\theta \in (2\pi/3, \pi]}$

Answer: (C)

Go Back to Question 65

Q66.

Solution

Concept: Check continuity and differentiability at junction points of a piecewise function and analyze extrema using one-sided derivatives.

Solution: For $x \leq 0$, $f(x) = e^x$; for $x > 0$, $f(x) = |1 - x|$.

At $x = 0$:

$$\lim_{x \to 0^-} f(x) = 1, \quad \lim_{x \to 0^+} f(x) = 1 \Rightarrow \text{continuous} \Rightarrow (A)$$

Left derivative:

$$f'_-(0) = 1$$

Right derivative:

$$f'_+(0) = -1 \Rightarrow \text{not differentiable} \Rightarrow (B \text{ false})$$

At $x = 1$, $|1 - x|$ has a cusp non-differentiable (C)

At $x = 0$, left side increases and right side decreases local maximum (D)

Final Answer: $\boxed{\text{A, C, D}}$

Answer: (A,C,D)

Go Back to Question 66

Q67.

Solution

Concept: Use matrix algebra identities from given relations $AB = A$ and $BA = B$ to derive idempotency and commutativity properties.

Solution: Given:

$$AB = A, \quad BA = B$$

Multiply $AB = A$ by A:

$$A^2B = A^2 \Rightarrow A^2 = A$$

Similarly:

$$B^2 = B$$

So (A) true and (C) true.

Now:

$$(A + B)^2 = A^2 + B^2 + AB + BA$$

$$= A + B + A + B = 2A + 2B \neq A + B$$

So (B) false.

From given:

$$AB = A, \; BA = B \Rightarrow AB \neq BA \text{ in general}$$

So (D) false.

Final Answer: A, C

Answer: (A,C)

Go Back to Question 67

Q68.

Solution

Concept: Use roots of unity properties and Vieta's formulas for polynomial $z^4 - 1 = 0$.

Solution: Roots:

$$1, \; -1, \; i, \; -i$$

Sum:

$$1 - 1 + i - i = 0 \Rightarrow (A)$$

Product:

$$1 \cdot (-1) \cdot i \cdot (-i) = 1 \Rightarrow (B \text{ false})$$

They form a square in Argand plane (C)

All roots lie on unit circle (D)

Final Answer: A, C, D

Answer: (A,C,D)

Go Back to Question 68

Q69.

Solution

Concept: Convert ellipse to standard form and apply formulas for eccentricity, latus rectum, directrices, and area.

Solution:

$$9x^2 + 25y^2 = 225 \Rightarrow \frac{x^2}{25} + \frac{y^2}{9} = 1$$

So $a = 5, \ b = 3$

Eccentricity:

$$e = \sqrt{1 - \frac{9}{25}} = \frac{4}{5} \Rightarrow (A)$$

Directrices distance:

$$\frac{2a}{e} = \frac{10}{4/5} = \frac{25}{2} \Rightarrow (B)$$

Latus rectum:

$$\frac{2b^2}{a} = \frac{18}{5} \Rightarrow (C)$$

Area:

$$\pi ab = 15\pi \Rightarrow (D)$$

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 69

Q70.

Solution

Concept: Use symmetry property of integrals and substitution for trigonometric rational functions.

Solution:

$$I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

Using symmetry $x \to \pi - x$:

$$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx \Rightarrow (A)$$

Evaluate:

$$\int_0^{\pi} \frac{\sin x}{1 + \cos^2 x} dx = 1$$

So:

$$I = \frac{\pi}{2}$$

Thus (B) and (C) false.

Also:

$$I = \int_0^{\pi/2} \frac{\pi \sin x}{1 + \cos^2 x} dx \Rightarrow (D)$$

Final Answer: A, D

Answer: (A,D)

Go Back to Question 70

Q71.

Solution

Concept: Differentiate x^x using logarithmic differentiation.

Solution:

$$f(x) = x^x = e^{x \ln x}$$

$$f'(x) = x^x (1 + \ln x)$$

Stationary point:

$$1 + \ln x = 0 \Rightarrow x = \frac{1}{e} \Rightarrow (A)$$

Second derivative shows minimum at $x = 1/e$ (B)

No maximum at $x = e$ (C false)

For $x > 1/e$, $f'(x) > 0$ increasing (D)

Final Answer: A, B, D

Answer: (A,B,D)

Go Back to Question 71

Q72.

Solution

Concept: Use vector identity from condition $\vec{a} + \vec{b} + \vec{c} = 0$ and properties of unit vectors.

Solution:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = 0 \Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$$

$$\Rightarrow \text{sum} = -\frac{3}{2} \Rightarrow (A)$$

Vectors form equilateral triangle (B)

Equal pairwise angles cross products equal (C)

They lie in a plane (D)

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 72

Q73.

Solution

Concept: Derive differential equation by eliminating parameters for family of circles.

Solution: General form leads to:

$$x^2 + y^2 - 2xy\frac{dy}{dx} = 0 \Rightarrow (A)$$

Rearranging gives equivalent form (B false, C false)

It is first-order DE (D)

Final Answer: A, D

Answer: (A,D)

Go Back to Question 73

Q74.

Solution

Concept: Telescoping series using partial fractions.

Solution:

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$S = 1 \Rightarrow (A)$$

Partial sum:

$$T_n = \frac{n}{n+1} \Rightarrow (B)$$

Series convergent (C)

(D false)

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 74

Q75.

Solution

Concept: Direction cosines satisfy fundamental identity.

Solution:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \Rightarrow (A)$$

Using $\sin^2 = 1 - \cos^2$:

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2 \Rightarrow (B)$$

Using identities:

$$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1 \Rightarrow (C)$$

(D false)

Final Answer: A, B, C

Answer: (A,B,C)

Go Back to Question 75

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	C	3	B	4	B	5	C
6	A	7	B	8	D	9	A	10	A
11	A	12	D	13	A	14	C	15	B
16	C	17	A	18	A	19	A	20	B
21	B	22	B	23	B	24	C	25	A
26	B	27	B	28	B	29	B	30	A
31	A	32	A	33	B	34	B	35	B
36	A	37	A	38	B	39	A	40	A
41	A	42	A	43	B	44	A	45	A
46	A	47	A	48	B	49	A	50	A
51	A	52	A	53	A	54	A	55	B
56	A	57	B	58	A	59	A	60	B
61	B	62	A	63	C	64	A	65	C
66	A,C,D	67	A,C	68	A,C,D	69	A,B,C,D	70	A,D
71	A,B,D	72	A,B,C,D	73	A,D	74	A,B,C	75	A,B,C