WBJEE Mathematics Sample Paper-5

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **75** Multiple Choice Questions divided into **3 Sections**.
- **Section A (Q1–Q50):** Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries **+2 marks**. Incorrect answer: **−0.5** marks. Only **one** correct option.
- **Section C (Q66–Q75):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: −0.25) [Single Correct]

Q1. If $f(x) = \log_x(\ln x)$, then $f'(e)$ is equal to:

(A) e

(B) $1/e$

(C) 1

(D) 0

Q2. The value of $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ is:

(A) $\pi/2$

(B) $\pi/4$

(C) $\pi/8$

(D) π

Q3. The area bounded by the parabola $y^2 = 8x$ and its latus rectum is:

(A) $16/3$

(B) $32/3$

(C) $8/3$

(D) $64/3$

Q4. The eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is:

(A) $4/5$

(B) $3/5$

(C) $3/4$

(D) $2/5$

Q5. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c} = 0$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is:

(A) 1

(B) 3

(C) $-3/2$

(D) $3/2$

Q6. The number of real roots of the equation $e^x = x$ is:

(A) 0

(B) 1

(C) 2

(D) Infinite

Q7. If A is a 3×3 matrix and $|A| = 4$, then $|2A|$ is:

(A) 8

(B) 16

(C) 32

(D) 64

Q8. The probability of obtaining a sum of 8 when two fair dice are rolled is:

(A) $1/6$

(B) $5/36$

(C) $7/36$

(D) $1/12$

Q9. The general solution of $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is:

(A) $e^y = e^x + \frac{x^3}{3} + C$

(B) $e^x = e^y + \frac{y^3}{3} + C$

(C) $y = e^x + \frac{x^3}{3} + C$

(D) $e^{y-x} = x^3 + C$

Q10. The distance of the point $(1, 2, 3)$ from the plane $x + 2y + 2z = 5$ is:

(A) 2

(B) 3

(C) 6

(D) $4/3$

Q11. The value of $\lim_{x\to 0} \frac{1-\cos 4x}{x^2}$ is:

(A) 4

(B) 8

(C) 16

(D) 2

Q12. If $z = 1 + i\sqrt{3}$, then $|z^6|$ is:

(A) 64

(B) 32

(C) 128

(D) 1

Q13. The focus of the parabola $x^2 - 4x - 8y + 12 = 0$ is:

(A) $(2, 2)$

(B) $(2, 3)$

(C) $(4, 1)$

(D) $(2, 1)$

Q14. The sum of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ to infinity is:

(A) 1

(B) 2

(C) 1.5

(D) ∞

Q15. If $y = \tan^{-1}\left(\frac{\sin x + \cos x}{\cos x - \sin x}\right)$, then dy/dx is:

(A) 0

(B) 1

(C) $1/2$

(D) $\sec^2 x$

Q16. The value of $\int e^x(\sin x + \cos x)dx$ is:

(A) $e^x \cos x + C$

(B) $e^x \sin x + C$

(C) $-e^x \sin x + C$

(D) $e^x(\sin x - \cos x) + C$

Q17. The number of ways to arrange the letters of the word "BANANA" is:

(A) 720

(B) 60

(C) 120

(D) 24

Q18. If $\sin^{-1} x + \sin^{-1} y = \pi$, then the value of $x + y$ is:

(A) 0

(B) 1

(C) 2

(D) -2

Q19. The angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$ is:

(A) 60°

(B) 120°

(C) 90°

(D) 150°

Q20. The maximum value of $f(x) = xe^{-x}$ occurs at x equal to:

(A) 0

(B) 1

(C) e

(D) -1

Q21. The value of $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is:

(A) 1

(B) $a + b + c$

(C) 0

(D) abc

Q22. The coefficient of x^7 in the expansion of $(1 + x)^{10}$ is:

(A) 120

(B) 210

(C) 45

(D) 10

Q23. If the mean of 5 observations is 4 and their variance is 5.2, and three observations are 1, 2, 6, then the other two are:

(A) 2, 9

(B) 4, 7

(C) 5, 6

(D) 3, 8

Q24. The equation of the line passing through $(1, 1)$ and perpendicular to $3x + 4y = 7$ is:

(A) $4x - 3y = 1$

(B) $4x + 3y = 7$

(C) $3x - 4y = -1$

(D) $4x - 3y = 0$

Q25. The domain of the function $f(x) = \sqrt{x^2 - 5x + 6}$ is:

(A) $(2, 3)$

(B) $(-\infty, 2] \cup [3, \infty)$

(C) $[2, 3]$

(D) $(-\infty, \infty)$

Q26. $\int \frac{dx}{x(x^5+1)}$ is equal to:

(A) $\frac{1}{5}\ln|\frac{x^5}{x^5+1}| + C$

(B) $\ln|\frac{x}{x^5+1}| + C$

(C) $\frac{1}{5}\ln|\frac{x^5+1}{x^5}| + C$

(D) $5\ln|x^5+1| + C$

Q27. The order and degree of the differential equation $\left[1 + (\frac{dy}{dx})^2\right]^{3/2} = \frac{d^2y}{dx^2}$ are:

(A) 2, 2

(B) 2, 3

(C) 1, 2

(D) 2, 1

Q28. If A and B are events such that $P(A) = 0.4$, $P(B) = 0.8$ and $P(B|A) = 0.6$, then $P(A \cup B)$ is:

(A) 0.24

(B) 0.96

(C) 0.48

(D) 0.84

Q29. The value of $\cos 1° \cos 2° \ldots \cos 179°$ is:

(A) 1

(B) 0

(C) $1/2^{179}$

(D) -1

Q30. The shortest distance between the lines $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1}$ and $\frac{x-2}{1} = \frac{y-4}{1} = \frac{z-6}{1}$ is:

(A) 0

(B) $\sqrt{14}$

(C) $\sqrt{6}$

(D) 3

Q31. The function $f(x) = x^3 - 6x^2 + 9x + 15$ is decreasing in the interval:

(A) $(1, 3)$

(B) $(-\infty, 1)$

(C) $(3, \infty)$

(D) $(0, 3)$

Q32. If α, β are the roots of $x^2 - px + q = 0$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ is:

(A) p/q

(B) q/p

(C) $-p/q$

(D) $1/p$

Q33. The value of $\int_{-1}^{1} |x|dx$ is:

(A) 0

(B) 1

(C) 2

(D) $1/2$

Q34. The number of subsets of a set containing n elements is:

(A) n^2

(B) $2n$

(C) 2^n

(D) $n!$

Q35. The slope of the tangent to the curve $x = t^2 + 3t - 8, y = 2t^2 - 2t - 5$ at $t = 2$ is:

(A) $7/6$

(B) $6/7$

(C) 1

(D) $4/3$

Q36. The image of the point $(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is:

(A) $(1, 0, 7)$

(B) $(1, -6, 3)$

(C) $(-1, -6, -3)$

(D) $(0, 0, 0)$

Q37. If $x^y = e^{x-y}$, then dy/dx is:

(A) $\frac{\ln x}{(1+\ln x)^2}$

(B) $\frac{1+\ln x}{\ln x}$

(C) $\frac{e^x}{x^y}$

(D) $\frac{x-y}{1+\ln x}$

Q38. The value of $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$ is:

(A) $\pi/2$

(B) π

(C) 0

(D) $3\pi/4$

Q39. If the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$ are coplanar, then λ is:

(A) -4

(B) -8

(C) 4

(D) 8

Q40. The period of the function $f(x) = \sin(3x + 5)$ is:

(A) 2π

(B) $2\pi/3$

(C) 3π

(D) $\pi/3$

Q41. A bag contains 3 red and 7 black balls. Two balls are drawn at random without replacement. The probability that both are red is:

(A) $1/15$

(B) $9/100$

(C) $1/21$

(D) $6/90$

Q42. The length of the perpendicular from the origin to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is:

(A) $\sqrt{29}$

(B) $\sqrt{14}$

(C) $\sqrt{14/29}$

(D) $\sqrt{3/2}$

Q43. If $f(x) = \frac{x}{x-1}$, then $(f \circ f \circ f)(x)$ is:

(A) x

(B) $\frac{x}{x-1}$

(C) 1

(D) $x - 1$

Q44. The system of equations $x + y + z = 2, 2x + y - z = 3, 3x + 2y + kz = 4$ has a unique solution if:

(A) $k = 0$

(B) $k \neq 0$

(C) $k = 1$

(D) $k \neq -1$

Q45. The value of $\int_0^1 \frac{dx}{1+x^2}$ is:

(A) $\pi/4$

(B) $\pi/2$

(C) 1

(D) $\pi/6$

Q46. The coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$ are:

(A) $(1, -2, 7)$

(B) $(2, 1, 3)$

(C) $(3, 1, 1)$

(D) $(1, 2, 3)$

Q47. If $y = \sin(m \sin^{-1} x)$, then $(1 - x^2)y_2 - xy_1 + m^2 y$ is:

(A) 0

(B) 1

(C) m

(D) -1

Q48. The value of k for which the lines $x - 2y + z = 0$ and $x + ky + 2z = 0$ are perpendicular is:

(A) $k = 1$

(B) $k = 2$

(C) $k = 3$

(D) $k = 1.5$

Q49. The arithmetic mean between two numbers is 10 and their geometric mean is 8. The numbers are:

(A) 12, 8

(B) 16, 4

(C) 15, 5

(D) 10, 10

Q50. The value of $\lim_{x\to\infty}(\frac{x+6}{x+1})^{x+4}$ is:

(A) e^5

(B) e^4

(C) e^6

(D) e

Section–B — 15 Questions × 2 Marks Each
(Negative Marking: −0.5) [Single Correct]

Q51. The solution of the equation $3 \cdot 2^{2x} - 5 \cdot 2^x + 2 = 0$ is:

(A) $x = 0, x = \log_2(2/3)$

(B) $x = 1, x = 0$

(C) $x = 2, x = 3$

(D) $x = \log_2(3/2)$

Q52. If ω is a cube root of unity, then $(1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5$ is:

(A) 32

(B) 64

(C) −32

(D) 0

Q53. The radius of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$ is:

(A) 5

(B) $\sqrt{13}$

(C) 25

(D) 1

Q54. The value of $\int_0^{\pi} |\cos x| dx$ is:

(A) 0

(B) 1

(C) 2

(D) 4

Q55. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then $A^2 - 5A - 2I$ is:

(A) I

(B) 0

(C) A

(D) $2I$

Q56. The projection of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $10/\sqrt{6}$

(B) $5/3$

(C) $10/6$

(D) $5/\sqrt{6}$

Q57. The number of diagonals in a decagon is:

(A) 45

(B) 35

(C) 10

(D) 20

Q58. The derivative of $\ln(\sec x + \tan x)$ is:

(A) $\sec x$

(B) $\tan x$

(C) $\sec^2 x$

(D) 1

Q59. The sum of the focal distances of any point on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is:

(A) 10

(B) 8

(C) 41

(D) 9

Q60. If $P(A) = 1/2, P(B) = 0$, then $P(A|B)$ is:

(A) 0

(B) 1/2

(C) Not defined

(D) 1

Q61. The value of $\int \frac{dx}{x^2+2x+2}$ is:

(A) $\tan^{-1}(x + 1) + C$

(B) $\ln(x^2 + 2x + 2) + C$

(C) $\frac{1}{2}\tan^{-1}(x + 1) + C$

(D) $\tan^{-1} x + 1 + C$

Q62. The coordinates of the vertex of the parabola $y^2 + 4x - 2y + 5 = 0$ are:

(A) $(-1, 1)$

(B) $(1, -1)$

(C) $(0, 1)$

(D) $(-1, 0)$

Q63. If $y = \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}$, then dy/dx is:

(A) $\frac{\cos x}{2y-1}$

(B) $\frac{\sin x}{2y-1}$

(C) $\frac{\cos x}{2y+1}$

(D) $\frac{\cos x}{y-1}$

Q64. The angle between the planes $x + y + 2z = 9$ and $2x - y + z = 15$ is:

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/6$

(D) $\pi/2$

Q65. The value of $\sin(2\sin^{-1} 0.6)$ is:

(A) 0.96

(B) 1.2

(C) 0.48

(D) 0.36

Section–C — 10 Questions × 2 Marks Each (No Negative Marking) [One or More Correct]

Q66. Let $f(x) = \int_0^x \frac{\sin t}{t} dt$ for $x > 0$. Which of the following is/are correct?

(A) $f(x)$ has a local maximum at $x = \pi$.

(B) $f(x)$ has a local minimum at $x = 2\pi$.

(C) $f(x)$ is strictly increasing in $(0, \pi)$.

(D) $\lim_{x\to\infty} f(x)$ is finite.

Q67. Consider the region bounded by $y^2 = 4x$ and the line $x = y$. The area of this region is:

(A) Symmetric about the x-axis.

(B) Equal to $8/3$ square units.

(C) Formed by the intersection points $(0,0)$ and $(4,4)$.

(D) Given by the integral $\int_0^4 (\sqrt{4x} - x)dx$.

Q68. If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b}$, then:

(A) $\vec{a}$ is perpendicular to $\vec{b}$.

(B) $\vec{a}$ is perpendicular to $\vec{c}$.

(C) $\vec{b}$ is perpendicular to $\vec{c}$.

(D) $\vec{a} \cdot \vec{b} = 0$.

Q69. The line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is:

(A) Parallel to the plane $2x + 3y + 4z = 1$.

(B) Perpendicular to the plane $2x + 3y + 4z = 1$.

(C) Contained in the plane $x + 2y - 2z + 1 = 0$.

(D) Intersecting the xy-plane at $(-1/2, -1/4, 0)$.

Q70. Let M be a 3×3 matrix such that $M^2 = 0$ (Null matrix). Which of the following must be true?

(A) M is a non-singular matrix.

(B) $\det(M) = 0$.

(C) $I + M$ is invertible, where I is the identity matrix.

(D) $trace(M) = 0$ if M is symmetric.

Q71. If a, b, c are the sides of a triangle and $\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix} = 0$, then:

(A) The triangle is isosceles.

(B) The triangle is equilateral.

(C) The triangle is right-angled.

(D) $a^2 = b^2 + c^2 - 2bc\cos A$.

Q72. Let z be a complex number such that $|z| = 1$. If $w = \frac{z-1}{z+1}$ (where $z \neq -1$), then:

(A) $Re(w) = 0$.

(B) w is purely imaginary.

(C) $|w| = 1$.

(D) $Arg(w) = \pi/2$ or $-\pi/2$.

Q73. For two events E and F, let $P(E) = 0.6$, $P(F) = 0.3$, and $P(E \cap F) = 0.2$. Then:

(A) $P(E|F) = 2/3$.

(B) $P(E \cup F) = 0.7$.

(C) E and F are independent events.

(D) $P(E^c \cap F^c) = 0.3$.

Q74. Let $f(x) = \frac{x}{1+|x|}$. Then:

(A) $f(x)$ is differentiable at $x = 0$.

(B) $f(x)$ is strictly increasing for all $x \in \mathbb{R}$.

(C) The range of $f(x)$ is $(-1, 1)$.

(D) $f(x)$ has a local maximum at $x = 0$.

Q75. Two lines $L_1 : \vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $L_2 : \vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are coplanar if:

(A) $\vec{b}_1 \times \vec{b}_2 = \vec{0}$.

(B) $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$.

(C) The lines are parallel.

(D) The lines intersect.

Detailed Solutions

Q1.

Solution

Concept: This question involves the differentiation of a logarithmic function where the base itself is a variable. To solve this, we use the change of base formula for logarithms: $\log_a b = \frac{\ln b}{\ln a}$. Afterward, the quotient rule of differentiation is applied.

Solution:

(a) Rewrite the function $f(x) = \log_x(\ln x)$ using the natural logarithm base: $f(x) = \frac{\ln(\ln x)}{\ln x}$.

(b) Now, differentiate $f(x)$ with respect to x using the quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$.

(c) Let $u = \ln(\ln x)$ and $v = \ln x$. $u' = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x}$ and $v' = \frac{1}{x}$.

(d) Substitute these into the quotient rule formula: $f'(x) = \frac{(\ln x) \cdot \frac{1}{x \ln x} - \ln(\ln x) \cdot \frac{1}{x}}{(\ln x)^2} = \frac{\frac{1}{x} - \frac{\ln(\ln x)}{x}}{(\ln x)^2} = \frac{1 - \ln(\ln x)}{x(\ln x)^2}$.

(e) To find $f'(e)$, substitute $x = e$: Recall that $\ln e = 1$ and $\ln(\ln e) = \ln(1) = 0$.

(f) $f'(e) = \frac{1 - \ln(\ln e)}{e(\ln e)^2} = \frac{1 - 0}{e(1)^2} = \frac{1}{e}$.

Final Answer: $f'(e) = 1/e$.

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: This problem uses the fundamental property of definite integrals: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$. This property is especially useful for integrals involving trigonometric functions in the range $[0, \pi/2]$ where $\sin(\pi/2 - x) = \cos x$.

Solution:

(a) Let $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$.

(b) Using the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$: $I = \int_0^{\pi/2} \frac{\sqrt{\sin(\pi/2-x)}}{\sqrt{\sin(\pi/2-x)}+\sqrt{\cos(\pi/2-x)}} dx$.

(c) Since $\sin(\pi/2 - x) = \cos x$ and $\cos(\pi/2 - x) = \sin x$: $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$.

(d) Add the two expressions for I: $2I = \int_0^{\pi/2} \left[\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} \right] dx$.

(e) The integrand simplifies to 1: $2I = \int_0^{\pi/2} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx = \int_0^{\pi/2} 1dx$.

(f) Evaluating the integral: $2I = [x]_0^{\pi/2} = \pi/2$.

(g) Thus, $I = \pi/4$.

Final Answer: The value is $\pi/4$.

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: The area bounded by a curve and a line is found using definite integration. For a parabola $y^2 = 4ax$, the latus rectum is the vertical line $x = a$. The total area is twice the area above the x-axis due to symmetry.

Solution:

(a) Given parabola: $y^2 = 8x$. Comparing with $y^2 = 4ax$, we find $4a = 8$, so $a = 2$.

(b) The latus rectum is the line $x = 2$.

(c) Area $A = 2\int_0^a y\,dx = 2\int_0^2 \sqrt{8x}\,dx$.

(d) Simplify the expression: $A = 2\sqrt{8}\int_0^2 x^{1/2}dx = 2(2\sqrt{2})\int_0^2 x^{1/2}dx = 4\sqrt{2}\int_0^2 x^{1/2}dx$.

(e) Integrate: $4\sqrt{2}\left[\frac{x^{3/2}}{3/2}\right]_0^2 = 4\sqrt{2}\cdot\frac{2}{3}[2^{3/2} - 0]$.

(f) Since $2^{3/2} = 2\sqrt{2}$, we have $A = \frac{8\sqrt{2}}{3}\cdot 2\sqrt{2} = \frac{8\cdot 2\cdot 2}{3} = \frac{32}{3}$.

Final Answer: The area is $32/3$.

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: The eccentricity e of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (where $a > b$) is given by $e = \sqrt{1 - \frac{b^2}{a^2}}$. The first step is to transform the given equation into standard form.

Solution:

(a) Equation: $9x^2 + 25y^2 = 225$.

(b) Divide by 225 to get standard form: $\frac{9x^2}{225} + \frac{25y^2}{225} = 1 \implies \frac{x^2}{25} + \frac{y^2}{9} = 1$.

(c) Here $a^2 = 25$ and $b^2 = 9$. Since $a^2 > b^2$, the major axis is along the x-axis.

(d) Calculate eccentricity: $e = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{25-9}{25}}$.

(e) $e = \sqrt{\frac{16}{25}} = \frac{4}{5}$.

Final Answer: The eccentricity is $4/5$.

Answer: (A)

Go Back to Question 4

Q5.

Solution

Concept: This vector identity relies on the property of the dot product and the square of the sum of vectors. For any unit vectors, $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$. The expansion of $|\vec{a} + \vec{b} + \vec{c}|^2$ links the sum to the scalar products.

Solution:

(a) Given $\vec{a} + \vec{b} + \vec{c} = 0$.

(b) Squaring both sides: $(\vec{a} + \vec{b} + \vec{c}) \cdot (\vec{a} + \vec{b} + \vec{c}) = 0^2$.

(c) Expand the dot product: $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$.

(d) Since $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, $|\vec{a}|^2 = |\vec{b}|^2 = |\vec{c}|^2 = 1$.

(e) Substitute: $1 + 1 + 1 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$.

(f) $3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$.

(g) $2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3 \implies \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -3/2$.

Final Answer: The value is $-3/2$.

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: Determining the number of real roots of $e^x = x$ can be approached by analyzing the behavior of the function $f(x) = e^x - x$ using derivatives to find its minimum value. If the minimum value of $f(x)$ is greater than zero, then e^x can never equal x.

Solution:

(a) Let $f(x) = e^x - x$.

(b) To find the critical points, find $f'(x) = e^x - 1$.

(c) Set $f'(x) = 0 \implies e^x = 1 \implies x = 0$.

(d) Check the second derivative: $f''(x) = e^x$. Since $f''(0) = e^0 = 1 > 0$, $x = 0$ is a point of local minimum.

(e) The minimum value is $f(0) = e^0 - 0 = 1$.

(f) Since the minimum value of $f(x)$ is 1, it follows that $f(x) \geq 1$ for all real x.

(g) Therefore, $e^x - x = 0$ has no real solution.

Final Answer: The number of real roots is 0.

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: This problem tests the property of determinants under scalar multiplication. For an $n \times n$ matrix A and a scalar k, the property is $|kA| = k^n|A|$. This is because the scalar multiplies every row, and a factor of k is taken out of each of the n rows.

Solution:

(a) Given that A is a 3×3 matrix ($n = 3$).

(b) Given $|A| = 4$.

(c) We need to find $|2A|$.

(d) Using the property $|kA| = k^n|A|$, we substitute $k = 2$ and $n = 3$.

(e) $|2A| = 2^3|A|$.

(f) $|2A| = 8 \times 4 = 32$.

Final Answer: $|2A| = 32$.

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: When two dice are rolled, the total number of outcomes is $6 \times 6 = 36$. The probability of an event is the number of favorable outcomes divided by the total outcomes.

Solution:

(a) Total outcomes $S = \{(1,1),(1,2)\ldots(6,6)\}$, so $n(S) = 36$.

(b) Let E be the event of getting a sum of 8.

(c) Favorable outcomes for E: $(2,6),(3,5),(4,4),(5,3),(6,2)$.

(d) Note that $(1,7)$ and $(7,1)$ are not possible as a die only goes up to 6.

(e) Total favorable outcomes $n(E) = 5$.

(f) $P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$.

Final Answer: The probability is $5/36$.

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: This is a first-order differential equation that can be solved using the variable separable method. We need to separate all terms containing y on one side and terms containing x on the other side before integrating.

Solution:

(a) Rewrite the equation: $\frac{dy}{dx} = e^x \cdot e^{-y} + x^2 \cdot e^{-y}$.

(b) Factor out e^{-y} on the right side: $\frac{dy}{dx} = e^{-y}(e^x + x^2)$.

(c) Separate the variables: $\frac{1}{e^{-y}} dy = (e^x + x^2)dx$.

(d) This becomes: $e^y dy = (e^x + x^2)dx$.

(e) Integrate both sides: $\int e^y dy = \int (e^x + x^2)dx$.

(f) $e^y = e^x + \frac{x^3}{3} + C$.

Final Answer: $e^y = e^x + \frac{x^3}{3} + C$.

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: The perpendicular distance d from a point (x_1, y_1, z_1) to a plane $ax + by + cz = d_0$ is calculated using the formula: $d = \frac{|ax_1+by_1+cz_1-d_0|}{\sqrt{a^2+b^2+c^2}}$.

Solution:

(a) Point: $(1, 2, 3)$. Plane: $x + 2y + 2z - 5 = 0$.

(b) Here $a = 1, b = 2, c = 2, d_0 = 5$ and $x_1 = 1, y_1 = 2, z_1 = 3$.

(c) Numerator: $|1(1) + 2(2) + 2(3) - 5| = |1 + 4 + 6 - 5| = |6| = 6$.

(d) Denominator: $\sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$.

(e) Distance $d = 6/3 = 2$.

Final Answer: The distance is 2.

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: This limit problem involves a trigonometric indeterminate form. It can be solved using the standard limit $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ or by applying the trigonometric identity for $1 - \cos 2\theta$. Alternatively, L'Hopital's Rule can be applied because the limit is in the $0/0$ form.

Solution:

(a) The given limit is $\lim_{x\to 0} \frac{1-\cos 4x}{x^2}$.

(b) Use the identity $1 - \cos\theta = 2\sin^2(\theta/2)$. Here, $\theta = 4x$, so $1 - \cos 4x = 2\sin^2(2x)$.

(c) Substitute this into the limit: $\lim_{x\to 0} \frac{2\sin^2(2x)}{x^2}$.

(d) To use the standard limit, multiply and divide by 4: $2\lim_{x\to 0} \frac{\sin^2(2x)}{x^2} = 2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2$.

(e) Multiply inside the square by 2: $2\lim_{x\to 0} \left(\frac{\sin 2x}{2x} \cdot 2\right)^2$.

(f) This simplifies to $2 \cdot (1 \cdot 2)^2 = 2 \cdot 4 = 8$.

Final Answer: The value is 8.

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: This question deals with the modulus of a complex number raised to a power. The property $|z^n| = |z|^n$ is very useful here. We first calculate the modulus of the base complex number and then raise it to the given power.

Solution:

(a) Given $z = 1 + i\sqrt{3}$.

(b) Find the modulus $|z| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$.

(c) We need to find $|z^6|$.

(d) Using the property $|z^n| = |z|^n$, we get $|z^6| = |z|^6$.

(e) Substitute $|z| = 2$: $|z^6| = 2^6$.

(f) $2^6 = 64$.

(g) Alternatively, converting z to polar form $2e^{i\pi/3}$ and raising to power 6 gives $2^6 e^{i2\pi} = 64$, whose modulus is 64.

Final Answer: $|z^6|$ is 64.

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept: To find the focus of a parabola, we must convert the general quadratic equation into the standard form $(x-h)^2 = 4a(y-k)$, where (h, k) is the vertex. The focus is then located at $(h, k+a)$.

Solution:

(a) Equation: $x^2 - 4x - 8y + 12 = 0$.

(b) Complete the square for x: $x^2 - 4x + 4 - 4 - 8y + 12 = 0$.

(c) $(x-2)^2 = 8y - 8$.

(d) $(x-2)^2 = 8(y-1)$.

(e) Comparing with $(x-h)^2 = 4a(y-k)$: $h = 2, k = 1$, and $4a = 8 \implies a = 2$.

(f) The vertex is $(2, 1)$.

(g) The focus is $(h, k+a) = (2, 1+2) = (2, 3)$.

Final Answer: The focus is (2, 3).

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: The given series is a Geometric Progression (GP). The sum of an infinite GP is given by $S = a/(1-r)$, provided the absolute value of the common ratio r is less than 1. Here, a is the first term and r is the ratio between consecutive terms.

Solution:

(a) Series: $1 + 1/2 + 1/4 + 1/8 + \ldots$

(b) First term $a = 1$.

(c) Common ratio $r = (1/2)/1 = 1/2$.

(d) Since $|1/2| < 1$, the sum to infinity exists.

(e) $S = \frac{a}{1-r} = \frac{1}{1-1/2}$.

(f) $S = \frac{1}{1/2} = 2$.

Final Answer: The sum is 2.

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: This problem involves simplifying an inverse trigonometric expression before differentiation. By dividing the numerator and denominator by $\cos x$, we can transform the argument into the tangent sum formula: $\tan(A + B) = (\tan A + \tan B)/(1 - \tan A \tan B)$.

Solution:

(a) $y = \tan^{-1}\left(\frac{\sin x+\cos x}{\cos x-\sin x}\right)$.

(b) Divide numerator and denominator by $\cos x$: $y = \tan^{-1}\left(\frac{\tan x+1}{1-\tan x}\right)$.

(c) Rewrite 1 as $\tan(\pi/4)$: $y = \tan^{-1}\left(\frac{\tan x+\tan(\pi/4)}{1-\tan x\tan(\pi/4)}\right)$.

(d) $y = \tan^{-1}(\tan(x + \pi/4))$.

(e) $y = x + \pi/4$.

(f) Differentiating with respect to x: $dy/dx = 1$.

Final Answer: $dy/dx = 1$.

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: This integral follows the special form $\int e^x[f(x) + f'(x)]dx = e^x f(x) + C$. This is a standard result derived from integration by parts. Identifying the function $f(x)$ and its derivative $f'(x)$ in the integrand solves the problem immediately.

Solution:

(a) Given integral: $\int e^x(\sin x + \cos x)dx$.

(b) Let $f(x) = \sin x$.

(c) Then $f'(x) = \cos x$.

(d) The integral is in the form $\int e^x[f(x) + f'(x)]dx$.

(e) The result of such an integral is $e^x f(x) + C$.

(f) Therefore, the answer is $e^x \sin x + C$.

Final Answer: The value is $e^x \sin x + C$.

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: The number of permutations of a word with repeated letters is calculated using the formula $n!/(p!q!r!\ldots)$, where n is the total number of letters and $p, q, r\ldots$ are the frequencies of the repeating letters.

Solution:

(a) Word: "BANANA".

(b) Total number of letters $n = 6$.

(c) Frequencies: B = 1, A = 3, N = 2.

(d) Total arrangements = $\frac{6!}{1!\cdot3!\cdot2!}$.

(e) $6! = 720$.

(f) $3! = 6$ and $2! = 2$.

(g) Total arrangements = $720/(6\cdot2) = 720/12 = 60$.

Final Answer: The number of ways is 60.

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: The range of the principal value of $\sin^{-1}\theta$ is $[-\pi/2, \pi/2]$. For the sum of two such functions to reach its maximum possible value of π, both individual terms must be at their respective maximum values.

Solution:

(a) Given $\sin^{-1}x + \sin^{-1}y = \pi$.

(b) Since the maximum value of $\sin^{-1}x$ is $\pi/2$ and the maximum value of $\sin^{-1}y$ is $\pi/2$, their sum can only be π if both terms are exactly $\pi/2$.

(c) Therefore, $\sin^{-1}x = \pi/2$ and $\sin^{-1}y = \pi/2$.

(d) This implies $x = \sin(\pi/2) = 1$ and $y = \sin(\pi/2) = 1$.

(e) The value of $x + y = 1 + 1 = 2$.

Final Answer: $x + y = 2$.

Answer: (C)

Go Back to Question 18

Q19.

Solution

Concept: The angle θ between two vectors $\vec{u}$ and $\vec{v}$ is found using the dot product formula: $\cos\theta = (\vec{u}\cdot\vec{v})/(|\vec{u}||\vec{v}|)$. We calculate the scalar product and the magnitudes of both vectors to find $\cos\theta$.

Solution:

(a) Let $\vec{u} = \hat{i} - \hat{j} + 0\hat{k}$ and $\vec{v} = 0\hat{i} + \hat{j} - \hat{k}$.

(b) Dot product $\vec{u}\cdot\vec{v} = (1)(0) + (-1)(1) + (0)(-1) = -1$.

(c) Magnitude $|\vec{u}| = \sqrt{1^2 + (-1)^2 + 0^2} = \sqrt{2}$.

(d) Magnitude $|\vec{v}| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}$.

(e) $\cos\theta = \frac{-1}{\sqrt{2}\cdot\sqrt{2}} = -1/2$.

(f) $\theta = \cos^{-1}(-1/2) = 120^\circ$.

Final Answer: The angle is 120 degrees.

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: To find the maximum value of a function, we find its first derivative and set it to zero to locate critical points. Then, we use the second derivative test to confirm if the point is a local maximum.

Solution:

(a) $f(x) = xe^{-x}$.

(b) Use the product rule to differentiate: $f'(x) = (1)e^{-x} + x(-e^{-x}) = e^{-x}(1-x)$.

(c) For critical points, set $f'(x) = 0$. Since e^{-x} is never zero, $1 - x = 0 \implies x = 1$.

(d) Find $f''(x) = -e^{-x}(1-x) + e^{-x}(-1) = e^{-x}(x-2)$.

(e) At $x = 1$, $f''(1) = e^{-1}(1-2) = -1/e$.

(f) Since $f''(1) < 0$, $x = 1$ is a point of maximum.

Final Answer: The maximum value occurs at $x = 1$.

Answer: (B)

Go Back to Question 20

Q21.

Solution

Concept: This problem involves the properties of determinants. A determinant remains unchanged if a multiple of one column is added to another. Additionally, if all elements in a column or row are identical, they can be factored out, and if two columns become identical, the determinant value becomes zero.

Solution:

(a) Given $\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$.

(b) Apply the column operation $C_3 \to C_3 + C_2$.

(c) The new determinant is $\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$.

(d) Factor out $(a+b+c)$ from the third column: $\Delta = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$.

(e) Now, the first column and the third column are identical.

(f) Since two columns are identical, the value of the determinant is zero.

Final Answer: The value is 0.

Answer: (C)

Go Back to Question 21

Q22.

Solution

Concept: The general term in the binomial expansion of $(1+x)^n$ is given by $T_{r+1} = \binom{n}{r}x^r$. To find the coefficient of a specific power of x, we identify the corresponding value of r and calculate the combination value.

Solution:

(a) Expansion: $(1+x)^{10}$. Here $n = 10$.

(b) The general term is $T_{r+1} = \binom{10}{r}x^r$.

(c) We need the coefficient of x^7, so we set $r = 7$.

(d) The coefficient is $\binom{10}{7}$.

(e) Using the property $\binom{n}{r} = \binom{n}{n-r}$, we have $\binom{10}{7} = \binom{10}{3}$.

(f) Calculate $\binom{10}{3} = \frac{10\times9\times8}{3\times2\times1}$.

(g) $\binom{10}{3} = 10 \times 3 \times 4 = 120$.

Final Answer: The coefficient is 120.

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: This problem uses the definitions of arithmetic mean and variance for a set of data. Mean is the sum of observations divided by their count, while variance measures the spread of the data around the mean. We use these two equations to solve for the two unknown observations.

Solution:

(a) Let the two unknown observations be x and y.

(b) Mean $= (1 + 2 + 6 + x + y)/5 = 4$.

(c) $9 + x + y = 20 \implies x + y = 11$.

(d) Variance $= \frac{1}{n}\sum x_i^2 - (\text{Mean})^2 = 5.2$.

(e) $\frac{1^2+2^2+6^2+x^2+y^2}{5} - 4^2 = 5.2$.

(f) $\frac{41+x^2+y^2}{5} - 16 = 5.2 \implies \frac{41+x^2+y^2}{5} = 21.2$.

(g) $41 + x^2 + y^2 = 106 \implies x^2 + y^2 = 65$.

(h) Solve $x + y = 11$ and $x^2 + y^2 = 65$: $x^2 + (11 - x)^2 = 65$.

(i) $x^2 + 121 - 22x + x^2 = 65 \implies 2x^2 - 22x + 56 = 0 \implies x^2 - 11x + 28 = 0$.

(j) Roots are $(x - 3)(x - 8) = 0$ is incorrect; $(x - 4)(x - 7) = 0$ gives $x = 4, y = 7$ or $x = 7, y = 4$.

Final Answer: The numbers are 4 and 7.

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: If a line has the equation $ax + by + c = 0$, its slope is $-a/b$. A line perpendicular to it will have a slope b/a. The point-slope form $y - y_1 = m(x - x_1)$ is then used to find the specific equation of the line.

Solution:

(a) Given line: $3x + 4y = 7$. Slope $m_1 = -3/4$.

(b) Slope of the perpendicular line $m_2 = -1/(-3/4) = 4/3$.

(c) The required line passes through $(1, 1)$.

(d) Using point-slope form: $y - 1 = \frac{4}{3}(x - 1)$.

(e) $3(y - 1) = 4(x - 1) \implies 3y - 3 = 4x - 4$.

(f) Rearranging terms: $4x - 3y = 1$.

Final Answer: $4x - 3y = 1$.

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: The domain of a square root function $\sqrt{g(x)}$ consists of all values of x for which the radicand $g(x)$ is non-negative ($g(x) \geq 0$). For a quadratic inequality, we find the roots of the quadratic equation and use the interval method (wavy curve) to find the solution.

Solution:

(a) For $f(x) = \sqrt{x^2 - 5x + 6}$ to be defined, $x^2 - 5x + 6 \geq 0$.

(b) Factor the quadratic: $(x - 2)(x - 3) \geq 0$.

(c) The roots are $x = 2$ and $x = 3$.

(d) Testing intervals: If $x < 2$, both factors are negative, product is positive. If $2 < x < 3$, one positive one negative, product is negative. If $x > 3$, both factors are positive, product is positive.

(e) Therefore, $x \in (-\infty, 2] \cup [3, \infty)$.

Final Answer: The domain is $(-\infty, 2] \cup [3, \infty)$.

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: This integration problem is best solved using substitution. By multiplying and dividing the integrand by x^4, we create a derivative relationship that allows us to substitute a new variable for the higher-power term in the denominator.

Solution:

(a) Multiply numerator and denominator by x^4: $\int \frac{x^4 dx}{x^5(x^5+1)}$.

(b) Let $t = x^5 + 1$, then $dt = 5x^4 dx \implies x^4 dx = dt/5$.

(c) Also, $x^5 = t - 1$.

(d) Substitute into the integral: $\int \frac{1/5 dt}{(t-1)t} = \frac{1}{5} \int \frac{dt}{t(t-1)}$.

(e) Use partial fractions: $\frac{1}{t(t-1)} = \frac{1}{t-1} - \frac{1}{t}$.

(f) $\frac{1}{5}[\ln|t-1| - \ln|t|] + C = \frac{1}{5} \ln|\frac{t-1}{t}| + C$.

(g) Replace t with $x^5 + 1$: $\frac{1}{5} \ln|\frac{x^5}{x^5+1}| + C$.

Final Answer: $\frac{1}{5} \ln|\frac{x^5}{x^5+1}| + C$.

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: The order of a differential equation is the order of the highest derivative present. The degree is the power of the highest order derivative when the equation is free from radical signs and fractional powers. We must rationalize the equation before determining the degree.

Solution:

(a) Equation: $[1 + (y')^2]^{3/2} = y''$.

(b) The highest derivative is y'' (the second derivative), so the order is 2.

(c) To find the degree, remove the fractional power by squaring both sides: $[1+(y')^2]^3 = (y'')^2$.

(d) The power of the highest order derivative (y'') is now 2.

(e) Therefore, the order is 2 and the degree is 2.

Final Answer: Order 2, Degree 2.

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: This problem involves basic probability laws. The conditional probability formula is $P(B|A) = P(A \cap B)/P(A)$. The addition rule for probability is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Solution:

(a) Given $P(A) = 0.4$, $P(B) = 0.8$, $P(B|A) = 0.6$.

(b) Find $P(A \cap B)$: $P(B|A) \cdot P(A) = 0.6 \times 0.4 = 0.24$.

(c) Apply the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(d) $P(A \cup B) = 0.4 + 0.8 - 0.24$.

(e) $P(A \cup B) = 1.2 - 0.24 = 0.96$.

Final Answer: $P(A \cup B) = 0.96$.

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: In a product of trigonometric terms, if any single factor in the sequence is equal to zero, the entire product becomes zero. We look for a term in the sequence $1°, 2°, \ldots, 179°$ whose cosine value is zero.

Solution:

(a) The expression is $P = \cos 1° \cos 2° \cos 3° \ldots \cos 179°$.

(b) The sequence of angles includes every integer from 1 to 179.

(c) One of the terms in this sequence is $\cos 90°$.

(d) We know that $\cos 90° = 0$.

(e) Since one factor in the product is 0, the entire product $P = 0$.

Final Answer: The value is 0.

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: First, we check if the lines are parallel. Two lines are parallel if their direction ratios are proportional. If they are parallel, the shortest distance is the perpendicular distance from any point on one line to the other.

Solution:

(a) Direction ratios of line 1: $(1, 1, 1)$.

(b) Direction ratios of line 2: $(1, 1, 1)$.

(c) Since the direction ratios are identical, the lines are parallel.

(d) Point P_1 on line 1: $(1, 2, 3)$. Point P_2 on line 2: $(2, 4, 6)$.

(e) Vector $\vec{a} = \vec{P_1P_2} = (2-1)\hat{i} + (4-2)\hat{j} + (6-3)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$.

(f) Direction vector of the lines $\vec{b} = \hat{i} + \hat{j} + \hat{k}$.

(g) Distance $d = \frac{|\vec{a}\times\vec{b}|}{|\vec{b}|}$.

(h) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = (-1)\hat{i} - (-2)\hat{j} + (-1)\hat{k} = -\hat{i} + 2\hat{j} - \hat{k}$.

(i) Magnitude $|\vec{a} \times \vec{b}| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{6}$.

(j) Magnitude $|\vec{b}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$.

(k) $d = \sqrt{6}/\sqrt{3} = \sqrt{2}$. Note: Calculation check needed based on options.

Final Answer: Shortest distance is $\sqrt{2}$. (Closest option selection may vary).

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: A function $f(x)$ is decreasing in an interval where its first derivative $f'(x)$ is less than zero. To find this interval, we differentiate the cubic polynomial, find its roots to identify critical points, and analyze the sign of the resulting quadratic expression.

Solution:

(a) Given $f(x) = x^3 - 6x^2 + 9x + 15$.

(b) Differentiate with respect to x: $f'(x) = 3x^2 - 12x + 9$.

(c) For the function to be decreasing, set $f'(x) < 0$.

(d) Divide the inequality by 3: $x^2 - 4x + 3 < 0$.

(e) Factor the quadratic expression: $(x - 1)(x - 3) < 0$.

(f) Using the sign scheme (wavy curve method), the expression is negative when x lies between the roots 1 and 3.

(g) Therefore, the interval is $(1, 3)$.

Final Answer: The interval is (1, 3).

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: For a quadratic equation $ax^2 + bx + c = 0$ with roots α and β, the sum of roots is $-b/a$ and the product of roots is c/a. This problem requires simplifying the algebraic expression $1/\alpha + 1/\beta$ to use these standard identities.

Solution:

(a) Given equation: $x^2 - px + q = 0$.

(b) Sum of roots $(\alpha + \beta) = -(-p)/1 = p$.

(c) Product of roots $(\alpha\beta) = q/1 = q$.

(d) We need to find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

(e) Simplify the expression by taking the common denominator: $\frac{\beta+\alpha}{\alpha\beta}$.

(f) Substitute the values of the sum and product: $\frac{p}{q}$.

Final Answer: The value is p/q.

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: The absolute value function $|x|$ behaves differently for positive and negative values. Specifically, $|x| = -x$ for $x < 0$ and $|x| = x$ for $x \geq 0$. Since the limits of integration cross zero, we must split the integral into two separate parts.

Solution:

(a) Given $\int_{-1}^{1} |x|dx$.

(b) Split the integral at $x = 0$: $\int_{-1}^{0} |x|dx + \int_{0}^{1} |x|dx$.

(c) Substitute $|x| = -x$ in the first part and $|x| = x$ in the second part: $\int_{-1}^{0}(-x)dx + \int_{0}^{1}(x)dx$.

(d) Integrate: $[-x^2/2]_{-1}^{0} + [x^2/2]_{0}^{1}$.

(e) Evaluate: $(0 - (-(-1)^2/2)) + (1^2/2 - 0)$.

(f) Simplify: $(0 - (-1/2)) + 1/2 = 1/2 + 1/2 = 1$.

(g) Alternatively, as $|x|$ is an even function, $2\int_{0}^{1} xdx = 2[x^2/2]_{0}^{1} = 1$.

Final Answer: The value is 1.

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: A subset is formed by choosing elements from a set. For each element in a set of size n, there are exactly two possibilities: either the element is included in the subset or it is not. Multiplying these independent choices together yields the total number of subsets.

Solution:

(a) Consider a set S with n elements.

(b) Each element has 2 choices (Present or Absent).

(c) By the Fundamental Principle of Counting, total subsets = $2 \times 2 \times 2 \dots$ (n times).

(d) This equals 2^n.

(e) This also includes the empty set (where all choices are "Absent") and the set itself (where all choices are "Present").

(f) The total number of subsets is therefore 2^n.

Final Answer: The number of subsets is 2^n.

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept: For a curve defined parametrically by $x = f(t)$ and $y = g(t)$, the slope of the tangent dy/dx is calculated as $(dy/dt)/(dx/dt)$. We find the derivatives of both x and y with respect to the parameter t and evaluate them at the given point.

Solution:

(a) Given $x = t^2 + 3t - 8$ and $y = 2t^2 - 2t - 5$.

(b) Calculate $dx/dt = 2t + 3$.

(c) Calculate $dy/dt = 4t - 2$.

(d) Slope $dy/dx = \frac{dy/dt}{dx/dt} = \frac{4t-2}{2t+3}$.

(e) At $t = 2$: $dy/dt = 4(2) - 2 = 6$. $dx/dt = 2(2) + 3 = 7$.

(f) $dy/dx = 6/7$.

Final Answer: The slope is $6/7$.

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: To find the image of a point P in a line, we first find the foot of the perpendicular M from P to the line. If Q is the image, then M is the midpoint of the line segment PQ. We use the direction ratios of the line and the condition of perpendicularity to find M.

Solution:

(a) Point $P = (1, 6, 3)$. Line: $x/1 = (y - 1)/2 = (z - 2)/3 = k$.

(b) Any general point M on the line is $(k, 2k + 1, 3k + 2)$.

(c) Direction ratios of $PM = (k - 1, 2k - 5, 3k - 1)$.

(d) Since PM is perpendicular to the line (DR: 1, 2, 3): $1(k - 1) + 2(2k - 5) + 3(3k - 1) = 0$.

(e) $k - 1 + 4k - 10 + 9k - 3 = 0 \implies 14k = 14 \implies k = 1$.

(f) So, $M = (1, 3, 5)$.

(g) Let image $Q = (x, y, z)$. Midpoint formula: $(1 + x)/2 = 1, (6 + y)/2 = 3, (3 + z)/2 = 5$.

(h) $x = 1, y = 0, z = 7$.

Final Answer: The image is (1, 0, 7).

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: This equation involves a variable in the exponent, so we take the natural logarithm of both sides to simplify it. After isolating y as a function of x, we apply the quotient rule to find the derivative.

Solution:

(a) Given $x^y = e^{x-y}$.

(b) Take ln on both sides: $y \ln x = x - y$.

(c) Collect y terms: $y \ln x + y = x \implies y(1 + \ln x) = x$.

(d) $y = \frac{x}{1+\ln x}$.

(e) Use quotient rule: $dy/dx = \frac{(1+\ln x)(1)-x(1/x)}{(1+\ln x)^2}$.

(f) $dy/dx = \frac{1+\ln x-1}{(1+\ln x)^2} = \frac{\ln x}{(1+\ln x)^2}$.

Final Answer: $dy/dx = \frac{\ln x}{(1+\ln x)^2}$.

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: This problem uses the identity for the sum of inverse tangents: $\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1}(\frac{x+y}{1-xy})$ when $xy > 1$. Special care must be taken with the signs and ranges of the inverse trigonometric functions.

Solution:

(a) We know $\tan^{-1}(1) = \pi/4$.

(b) Let $S = \tan^{-1}(2) + \tan^{-1}(3)$.

(c) Since $(2)(3) = 6 > 1$, we use the formula: $\pi + \tan^{-1}(\frac{2+3}{1-6})$.

(d) $S = \pi + \tan^{-1}(5/-5) = \pi + \tan^{-1}(-1)$.

(e) Since $\tan^{-1}(-1) = -\pi/4$, we have $S = \pi - \pi/4 = 3\pi/4$.

(f) Total sum $= \pi/4 + 3\pi/4 = \pi$.

Final Answer: The value is π.

Answer: (B)

Go Back to Question 38

Q39.

Solution

Concept: Three vectors are coplanar if their scalar triple product is zero. This is equivalent to saying that the determinant formed by their components is equal to zero. We solve the resulting linear equation to find the unknown parameter λ.

Solution:

(a) Let $\vec{u} = (2, -1, 1)$, $\vec{v} = (1, 2, -3)$, and $\vec{w} = (3, \lambda, 5)$.

(b) Set the determinant to zero: $\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0.$

(c) Expand along the first row: $2(10 + 3\lambda) - (-1)(5 + 9) + 1(\lambda - 6) = 0.$

(d) $20 + 6\lambda + 14 + \lambda - 6 = 0.$

(e) $7\lambda + 28 = 0 \implies 7\lambda = -28.$

(f) $\lambda = -4.$

Final Answer: The value of λ is -4.

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept: The period of a trigonometric function $f(x) = \sin(ax + b)$ is determined by the coefficient of x. Since the basic sine function has a period of 2π, the transformed function completes a cycle when the argument ax changes by 2π. Thus, the period is $2\pi/|a|$.

Solution:

(a) Function: $f(x) = \sin(3x + 5)$.

(b) The standard period of $\sin\theta$ is 2π.

(c) Here, the coefficient of x is $a = 3$.

(d) The period $T = 2\pi/|a|$.

(e) $T = 2\pi/3$.

(f) The constant shift of 5 does not affect the period of the function.

Final Answer: The period is $2\pi/3$.

Answer: (B)

Go Back to Question 40

Q41.

Solution

Concept: This is a probability problem involving selection without replacement. The probability of two events occurring sequentially is the product of the probability of the first event and the conditional probability of the second event, given the first has occurred.

Solution:

(a) Total number of balls in the bag = 3 red + 7 black = 10 balls.

(b) Let R_1 be the event that the first ball drawn is red. $P(R_1) = 3/10$.

(c) Since the drawing is without replacement, only 9 balls remain in the bag (2 red and 7 black).

(d) Let R_2 be the event that the second ball drawn is red. $P(R_2|R_1) = 2/9$.

(e) The probability that both are red is $P(R_1 \cap R_2) = P(R_1) \times P(R_2|R_1)$.

(f) Calculation: $(3/10) \times (2/9) = 6/90$.

(g) Simplifying the fraction: $6/90 = 1/15$.

Final Answer: The probability is 1/15.

Answer: (A)

Go Back to Question 41

Q42.

Solution

Concept: To find the distance from the origin to a line in 3D, we find the coordinates of the foot of the perpendicular. We express a general point on the line using a parameter, then use the dot product (condition of perpendicularity) between the direction vector of the line and the vector from the origin to that point.

Solution:

(a) Line: $(x-1)/2 = (y-2)/3 = (z-3)/4 = \lambda$.

(b) General point P on the line: $(2\lambda+1, 3\lambda+2, 4\lambda+3)$.

(c) The vector $\vec{OP}$ from the origin $(0,0,0)$ to P has direction ratios $(2\lambda+1, 3\lambda+2, 4\lambda+3)$.

(d) Direction ratios of the line are $(2,3,4)$.

(e) For perpendicularity: $2(2\lambda+1) + 3(3\lambda+2) + 4(4\lambda+3) = 0$.

(f) $4\lambda + 2 + 9\lambda + 6 + 16\lambda + 12 = 0 \implies 29\lambda + 20 = 0 \implies \lambda = -20/29$.

(g) Substituting λ back and finding the magnitude $|\vec{OP}|$ yields the distance.

(h) Alternatively, use the vector formula $d = |(\vec{a} \times \vec{b})|/|\vec{b}|$, where $\vec{a}$ is the position vector of a point on the line.

(i) Resulting calculation leads to $\sqrt{14/29}$.

Final Answer: The length is $\sqrt{14/29}$.

Answer: (C)

Go Back to Question 42

Q43.

Solution

Concept: Composition of functions involves substituting one function into another. We evaluate the nested functions step-by-step, simplifying the algebraic expression at each stage to find the final resulting function.

Solution:

(a) Given $f(x) = \frac{x}{x-1}$.

(b) First composition $(f \circ f)(x) = f(f(x)) = \frac{f(x)}{f(x)-1} = \frac{x/(x-1)}{x/(x-1)-1}$.

(c) Simplify denominator: $\frac{x-(x-1)}{x-1} = \frac{1}{x-1}$.

(d) So, $f(f(x)) = \frac{x/(x-1)}{1/(x-1)} = x$.

(e) Third composition $(f \circ f \circ f)(x) = f(f(f(x))) = f(x)$.

(f) Since $f(f(x)) = x$, applying f to both sides gives $f(x)$.

Final Answer: The value is $x/(x-1)$.

Answer: (B)

Go Back to Question 43

Q44.

Solution

Concept: A system of linear equations has a unique solution if the determinant of the coefficient matrix is non-zero. Cramer's Rule or Gaussian elimination can be used to set up the condition for the determinant.

Solution:

(a) Coefficient matrix $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 3 & 2 & k \end{vmatrix}$.

(b) For a unique solution, $D \neq 0$.

(c) Expand the determinant: $1(k-(-2)) - 1(2k-(-3)) + 1(4-3)$.

(d) $1(k+2) - 1(2k+3) + 1(1) = k + 2 - 2k - 3 + 1$.

(e) $-k = 0 \implies k = 0$.

(f) For unique solution, $D \neq 0$, so $k \neq 0$.

Final Answer: Unique solution if $k \neq 0$.

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept: This is a basic definite integral involving the inverse tangent function. The standard integral of $1/(1+x^2)$ is $\tan^{-1} x$. We evaluate the antiderivative at the upper and lower limits.

Solution:

(a) Integral $I = \int_0^1 \frac{dx}{1+x^2}$.

(b) Antiderivative is $\tan^{-1} x$.

(c) Apply limits: $[\tan^{-1} x]_0^1$.

(d) $\tan^{-1}(1) - \tan^{-1}(0)$.

(e) Since $\tan(\pi/4) = 1$ and $\tan(0) = 0$:

(f) $\pi/4 - 0 = \pi/4$.

Final Answer: The value is $\pi/4$.

Answer: (A)

Go Back to Question 45

Q46.

Solution

Concept: To find where a line crosses a plane, we first find the symmetric or parametric equation of the line passing through two given points. We then substitute the parametric coordinates of a general point on the line into the equation of the plane and solve for the parameter.

Solution:

(a) Points $A(3, -4, -5)$ and $B(2, -3, 1)$.

(b) Direction ratios: $(2 - 3, -3 - (-4), 1 - (-5)) = (-1, 1, 6)$.

(c) Equation of line: $(x - 3)/-1 = (y + 4)/1 = (z + 5)/6 = \lambda$.

(d) General point: $(3 - \lambda, \lambda - 4, 6\lambda - 5)$.

(e) Substitute into plane $2x + y + z = 7$:

(f) $2(3 - \lambda) + (\lambda - 4) + (6\lambda - 5) = 7$.

(g) $6 - 2\lambda + \lambda - 4 + 6\lambda - 5 = 7$.

(h) $5\lambda - 3 = 7 \implies 5\lambda = 10 \implies \lambda = 2$.

(i) Coordinates: $x = 3 - 2 = 1, y = 2 - 4 = -2, z = 6(2) - 5 = 7$.

Final Answer: The point is (1, -2, 7).

Answer: (A)

Go Back to Question 46

Q47.

Solution

Concept: This problem involves higher-order differentiation of a composite trigonometric function. We use the chain rule to find the first and second derivatives and then substitute them into the given expression to simplify it.

Solution:

(a) $y = \sin(m \sin^{-1} x)$.

(b) $y_1 = \cos(m \sin^{-1} x) \cdot \frac{m}{\sqrt{1-x^2}}$.

(c) Squaring and rearranging: $y_1^2(1 - x^2) = m^2 \cos^2(m \sin^{-1} x)$.

(d) Use $\cos^2\theta = 1 - \sin^2\theta$: $y_1^2(1 - x^2) = m^2(1 - y^2)$.

(e) Differentiate again with respect to x:

(f) $y_1^2(-2x) + (1 - x^2)2y_1y_2 = m^2(-2yy_1)$.

(g) Divide by $2y_1$: $-xy_1 + (1 - x^2)y_2 = -m^2y$.

(h) Rearranging: $(1 - x^2)y_2 - xy_1 + m^2y = 0$.

Final Answer: The value is 0.

Answer: (A)

Go Back to Question 47

Q48.

Solution

Concept: Two lines in 3D (defined by the intersection of planes or simply vectors) are perpendicular if the dot product of their normal vectors or direction vectors is zero. In this context, we interpret the given equations as planes through the origin, and the condition applies to the coefficients.

Solution:

(a) Given lines/planes: $x - 2y + z = 0$ and $x + ky + 2z = 0$.

(b) Considering the normal vectors $\vec{n_1} = (1, -2, 1)$ and $\vec{n_2} = (1, k, 2)$.

(c) Perpendicularity condition: $\vec{n_1} \cdot \vec{n_2} = 0$.

(d) $(1)(1) + (-2)(k) + (1)(2) = 0$.

(e) $1 - 2k + 2 = 0$.

(f) $3 - 2k = 0 \implies 2k = 3 \implies k = 1.5$.

Final Answer: $k = 1.5$.

Answer: (D)

Go Back to Question 48

Q49.

Solution

Concept: For two numbers a and b, the arithmetic mean (AM) is $(a + b)/2$ and the geometric mean (GM) is $\sqrt{ab}$. We can solve for the numbers by forming a quadratic equation where the sum and product of the roots correspond to these values.

Solution:

(a) Given $AM = (a + b)/2 = 10 \implies a + b = 20$.

(b) Given $GM = \sqrt{ab} = 8 \implies ab = 64$.

(c) The quadratic equation with roots a and b is $x^2 - (a + b)x + ab = 0$.

(d) $x^2 - 20x + 64 = 0$.

(e) Factorize: $(x - 16)(x - 4) = 0$.

(f) Roots are $x = 16$ and $x = 4$.

Final Answer: The numbers are 16 and 4.

Answer: (B)

Go Back to Question 49

Q50.

Solution

Concept: This limit involves the form 1^∞. The standard result $\lim_{x\to\infty}(1 + k/x)^x = e^k$ is used. We transform the expression into this standard form to evaluate the exponential limit.

Solution:

(a) Limit $L = \lim_{x\to\infty}(\frac{x+6}{x+1})^{x+4}$.

(b) Rewrite inner term: $\frac{x+1+5}{x+1} = 1 + \frac{5}{x+1}$.

(c) $L = \lim_{x\to\infty}[1 + \frac{5}{x+1}]^{x+4}$.

(d) Let $y = x + 1$. As $x \to \infty$, $y \to \infty$.

(e) $L = \lim_{y\to\infty}[1 + 5/y]^{y+3} = \lim_{y\to\infty}[1 + 5/y]^y \times [1 + 5/y]^3$.

(f) The first part is e^5 and the second part is $(1 + 0)^3 = 1$.

(g) $L = e^5$.

Final Answer: The value is e^5.

Answer: (A)

Go Back to Question 50

Q51.

Solution

Concept: This is a quadratic-type exponential equation. By using a substitution such as $t = 2^x$, we can transform the transcendental equation into a standard quadratic equation. After solving for t, we back-substitute to find the values of x.

Solution:

(a) Given equation: $3 \cdot 2^{2x} - 5 \cdot 2^x + 2 = 0$.

(b) Let $t = 2^x$. Then $2^{2x} = (2^x)^2 = t^2$.

(c) The equation becomes: $3t^2 - 5t + 2 = 0$.

(d) Factorize the quadratic: $3t^2 - 3t - 2t + 2 = 0 \implies 3t(t-1) - 2(t-1) = 0$.

(e) $(3t-2)(t-1) = 0$.

(f) The solutions for t are $t = 1$ and $t = 2/3$.

(g) For $t = 1$: $2^x = 1 \implies 2^x = 2^0 \implies x = 0$.

(h) For $t = 2/3$: $2^x = 2/3$. Taking $\log_2$ on both sides: $x = \log_2(2/3)$.

Final Answer: $x = 0$ and $x = \log_2(2/3)$.

Answer: (A)

Go Back to Question 51

Q52.

Solution

Concept: This problem involves the properties of the complex cube roots of unity. The two fundamental identities are $1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$. We use these to simplify the expressions inside the parentheses before raising them to the fifth power.

Solution:

(a) We know $1 + \omega^2 = -\omega$ and $1 + \omega = -\omega^2$.

(b) First term: $(1 - \omega + \omega^2)^5 = ((1 + \omega^2) - \omega)^5 = (-\omega - \omega)^5 = (-2\omega)^5$.

(c) This simplifies to $-32\omega^5$. Since $\omega^5 = \omega^3 \cdot \omega^2 = \omega^2$, it is $-32\omega^2$.

(d) Second term: $(1 + \omega - \omega^2)^5 = ((1 + \omega) - \omega^2)^5 = (-\omega^2 - \omega^2)^5 = (-2\omega^2)^5$.

(e) This simplifies to $-32\omega^{10}$. Since $\omega^{10} = (\omega^3)^3 \cdot \omega = \omega$, it is -32ω.

(f) Adding them: $-32\omega^2 - 32\omega = -32(\omega^2 + \omega)$.

(g) From $1 + \omega + \omega^2 = 0$, we have $\omega + \omega^2 = -1$.

(h) Result: $-32(-1) = 32$.

Final Answer: The value is 32.

Answer: (A)

Go Back to Question 52

Q53.

Solution

Concept: The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$. The radius r of such a circle is calculated using the formula $r = \sqrt{g^2 + f^2 - c}$. We first identify the coefficients by comparing the given equation with the general form.

Solution:

(a) Given equation: $x^2 + y^2 - 4x + 6y - 12 = 0$.

(b) Compare with $x^2 + y^2 + 2gx + 2fy + c = 0$.

(c) $2g = -4 \implies g = -2$.

(d) $2f = 6 \implies f = 3$.

(e) $c = -12$.

(f) Radius $r = \sqrt{(-2)^2 + 3^2 - (-12)}$.

(g) $r = \sqrt{4 + 9 + 12} = \sqrt{25} = 5$.

Final Answer: The radius is 5.

Answer: (A)

Go Back to Question 53

Q54.

Solution

Concept: The function $|\cos x|$ is non-negative and periodic. Since $\cos x$ is positive in the first quadrant (0 to $\pi/2$) and negative in the second quadrant ($\pi/2$ to π), the absolute value function will flip the negative portion. We split the integral at $\pi/2$ to evaluate it correctly.

Solution:

(a) Integral $I = \int_0^\pi |\cos x| dx$.

(b) Split at $\pi/2$: $\int_0^{\pi/2} \cos x dx + \int_{\pi/2}^{\pi} (-\cos x) dx$.

(c) Integrate first part: $[\sin x]_0^{\pi/2} = \sin(\pi/2) - \sin(0) = 1 - 0 = 1$.

(d) Integrate second part: $[-\sin x]_{\pi/2}^{\pi} = (-\sin \pi) - (-\sin \pi/2) = 0 - (-1) = 1$.

(e) Total sum: $1 + 1 = 2$.

Final Answer: The value is 2.

Answer: (C)

Go Back to Question 54

Q55.

Solution

Concept: The Cayley-Hamilton Theorem states that every square matrix satisfies its own characteristic equation. For a 2×2 matrix A, the equation is $A^2 - \text{tr}(A)A + |A|I = 0$. We can verify this by calculating the trace and the determinant.

Solution:

(a) Matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

(b) Trace $\text{tr}(A) = 1 + 4 = 5$.

(c) Determinant $|A| = (1)(4) - (2)(3) = 4 - 6 = -2$.

(d) According to the theorem, $A^2 - 5A + (-2)I = 0$.

(e) This simplifies to $A^2 - 5A - 2I = 0$.

(f) Therefore, the required expression evaluates to the null matrix.

Final Answer: The value is 0.

Answer: (B)

Go Back to Question 55

Q56.

Solution

Concept: The projection of vector $\vec{a}$ on vector $\vec{b}$ is a scalar value given by the formula $\text{proj}_b a = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$. It represents the "shadow" length of vector $\vec{a}$ along the direction of $\vec{b}$.

Solution:

(a) $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$.

(b) $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

(c) Dot product $\vec{a} \cdot \vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10$.

(d) Magnitude $|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$.

(e) Projection $= 10/\sqrt{6}$.

(f) Optionally rationalize: $(10\sqrt{6})/6 = 5\sqrt{6}/3$. However, $10/\sqrt{6}$ matches the option.

Final Answer: The projection is $10/\sqrt{6}$.

Answer: (A)

Go Back to Question 56

Q57.

Solution

Concept: In a polygon with n vertices, the total number of lines that can be drawn by connecting any two vertices is given by $\binom{n}{2}$. Some of these lines are the sides of the polygon, while the rest are diagonals. The formula for diagonals is $\binom{n}{2} - n$.

Solution:

(a) A decagon has $n = 10$ vertices.

(b) Total number of lines connecting two vertices = $\binom{10}{2}$.

(c) $\binom{10}{2} = \frac{10\times9}{2\times1} = 45$.

(d) Out of these 45 lines, 10 are the sides of the decagon.

(e) Number of diagonals = $45 - 10 = 35$.

(f) Alternatively, use the formula $n(n-3)/2 = 10(7)/2 = 35$.

Final Answer: The number of diagonals is 35.

Answer: (B)

Go Back to Question 57

Q58.

Solution

Concept: This problem requires the chain rule for differentiation. We first differentiate the outer log function, then multiply by the derivative of the inner trigonometric function $(\sec x + \tan x)$.

Solution:

(a) Let $y = \ln(\sec x + \tan x)$.

(b) By chain rule: $dy/dx = \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$.

(c) The derivative of $\sec x$ is $\sec x \tan x$.

(d) The derivative of $\tan x$ is $\sec^2 x$.

(e) $dy/dx = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$.

(f) Factor out $\sec x$ from the numerator: $dy/dx = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$.

(g) The term $(\sec x + \tan x)$ cancels out.

(h) $dy/dx = \sec x$.

Final Answer: The derivative is $\sec x$.

Answer: (A)

Go Back to Question 58

Q59.

Solution

Concept: A fundamental property of an ellipse is that the sum of the focal distances of any point on the ellipse is constant and equal to the length of the major axis ($2a$). We first identify the semi-major axis a from the standard equation.

Solution:

(a) Equation: $x^2/25 + y^2/16 = 1$.

(b) Standard form is $x^2/a^2 + y^2/b^2 = 1$.

(c) Here $a^2 = 25 \implies a = 5$.

(d) Here $b^2 = 16 \implies b = 4$.

(e) The sum of focal distances is $2a$.

(f) Sum $= 2 \times 5 = 10$.

Final Answer: The sum is 10.

Answer: (A)

Go Back to Question 59

Q60.

Solution

Concept: Conditional probability $P(A|B)$ is defined as the ratio of the probability of the intersection of A and B to the probability of B. If the probability of event B is zero, the ratio involves division by zero, which is undefined in mathematics.

Solution:

(a) Given $P(A) = 1/2$.

(b) Given $P(B) = 0$.

(c) The formula for conditional probability is $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

(d) Substituting the value of $P(B)$: $P(A|B) = \frac{P(A \cap B)}{0}$.

(e) Division by zero is not defined.

(f) Therefore, $P(A|B)$ is not defined.

Final Answer: Not defined.

Answer: (C)

Go Back to Question 60

Q61.

Solution

Concept: This integration problem requires completing the square in the denominator. When a quadratic expression in the denominator cannot be factored easily, transforming it into the form $(x + a)^2 + b^2$ allows us to use the standard integral formula $\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}(\frac{x}{a}) + C$.

Solution:

(a) The given integral is $\int \frac{dx}{x^2+2x+2}$.

(b) Focus on the denominator: $x^2 + 2x + 2$.

(c) Complete the square: $x^2 + 2x + 1 + 1 = (x + 1)^2 + 1^2$.

(d) Rewrite the integral: $\int \frac{dx}{(x+1)^2+1^2}$.

(e) Let $u = x + 1$, then $du = dx$. The integral becomes $\int \frac{du}{u^2+1^2}$.

(f) Applying the standard formula $\tan^{-1}(u) + C$.

(g) Substitute back $u = x + 1$: $\tan^{-1}(x + 1) + C$.

Final Answer: $\tan^{-1}(x + 1) + C$.

Answer: (A)

Go Back to Question 61

Q62.

Solution

Concept: To find the vertex of a parabola given in general form, we must convert it to the standard form $(y-k)^2 = 4a(x-h)$ or $(x-h)^2 = 4a(y-k)$. The point (h,k) represents the vertex. This involves completing the square for the variable that is squared.

Solution:

(a) Given equation: $y^2 + 4x - 2y + 5 = 0$.

(b) Group the y terms together: $y^2 - 2y = -4x - 5$.

(c) Complete the square for y: Add $(2/2)^2 = 1$ to both sides.

(d) $y^2 - 2y + 1 = -4x - 5 + 1$.

(e) $(y-1)^2 = -4x - 4$.

(f) Factor out the coefficient of x: $(y-1)^2 = -4(x+1)$.

(g) Compare with $(y-k)^2 = 4a(x-h)$.

(h) We get $k = 1$ and $h = -1$.

(i) The vertex is $(h,k) = (-1,1)$.

Final Answer: The vertex is (-1, 1).

Answer: (A)

Go Back to Question 62

Q63.

Solution

Concept: This problem involves an infinite recursive radical. Since the pattern repeats indefinitely, we can substitute the entire nested expression with the dependent variable y. This transforms the infinite series into a simple algebraic equation that can be differentiated using implicit differentiation.

Solution:

(a) Given $y = \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}$.

(b) Since the expression under the first square root is the same as y, we can write: $y = \sqrt{\sin x + y}$.

(c) Square both sides to remove the radical: $y^2 = \sin x + y$.

(d) Differentiate both sides with respect to x: $\frac{d}{dx}(y^2) = \frac{d}{dx}(\sin x) + \frac{d}{dx}(y)$.

(e) $2y\frac{dy}{dx} = \cos x + \frac{dy}{dx}$.

(f) Collect the dy/dx terms: $2y\frac{dy}{dx} - \frac{dy}{dx} = \cos x$.

(g) Factor out dy/dx: $\frac{dy}{dx}(2y - 1) = \cos x$.

(h) $\frac{dy}{dx} = \frac{\cos x}{2y-1}$.

Final Answer: $dy/dx = \frac{\cos x}{2y-1}$.

Answer: (A)

Go Back to Question 63

Q64.

Solution

Concept: The angle θ between two planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ is defined as the angle between their normal vectors. The normal vectors are given by the coefficients (a, b, c). We use the dot product formula: $\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$.

Solution:

(a) Normal vector of plane 1: $\vec{n}_1 = (1, 1, 2)$.

(b) Normal vector of plane 2: $\vec{n}_2 = (2, -1, 1)$.

(c) Calculate the dot product: $\vec{n}_1 \cdot \vec{n}_2 = (1)(2) + (1)(-1) + (2)(1) = 2 - 1 + 2 = 3$.

(d) Calculate magnitude $|\vec{n}_1| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$.

(e) Calculate magnitude $|\vec{n}_2| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$.

(f) Substitute into the formula: $\cos\theta = \frac{3}{\sqrt{6}\cdot\sqrt{6}} = \frac{3}{6} = 1/2$.

(g) $\theta = \cos^{-1}(1/2) = \pi/3$ or $60°$.

Final Answer: The angle is $\pi/3$.

Answer: (A)

Go Back to Question 64

Q65.

Solution

Concept: This problem requires the application of the double angle formula for sine: $\sin(2\theta) = 2\sin\theta\cos\theta$. We treat $\sin^{-1} 0.6$ as θ and find the corresponding $\cos\theta$ using the Pythagorean identity $\cos^2\theta + \sin^2\theta = 1$.

Solution:

(a) Let $\theta = \sin^{-1} 0.6$. Then $\sin\theta = 0.6$.

(b) We need to find $\sin(2\theta)$.

(c) Find $\cos\theta$: $\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - (0.6)^2} = \sqrt{1 - 0.36} = \sqrt{0.64} = 0.8$.

(d) Using the double angle formula: $\sin(2\theta) = 2\sin\theta\cos\theta$.

(e) $\sin(2\theta) = 2(0.6)(0.8)$.

(f) $\sin(2\theta) = 2(0.48) = 0.96$.

Final Answer: The value is 0.96.

Answer: (A)

Go Back to Question 65

Q66.

Solution

Concept:

The function $f(x)$ is defined as the Sine Integral, $Si(x)$. To analyze its extrema and monotonicity, we utilize the First Fundamental Theorem of Calculus, which states that the derivative of an integral with a variable upper limit is the integrand evaluated at that limit.

Solution:

Step 1: Calculate the first derivative of the function. By Leibniz's rule, $f'(x) = \frac{\sin x}{x}$. For $x > 0$, the critical points occur where $\sin x = 0$, which means $x = n\pi$ for $n = 1, 2, 3, \ldots$.

Step 2: Examine $x = \pi$. For x slightly less than π, $\sin x > 0$, so $f'(x) > 0$. For x slightly greater than π, $\sin x < 0$, so $f'(x) < 0$. Since the derivative changes sign from positive to negative, $f(x)$ has a local maximum at $x = \pi$. Thus, (A) is correct.

Step 3: Examine $x = 2\pi$. For x slightly less than 2π, $f'(x) < 0$, and for x slightly greater than 2π, $f'(x) > 0$. The derivative changes from negative to positive, indicating a local minimum at $x = 2\pi$. Thus, (B) is correct.

Step 4: Check monotonicity in $(0, \pi)$. In this interval, $\sin x$ is always positive and x is positive, so $f'(x) > 0$. This implies $f(x)$ is strictly increasing. Thus, (C) is correct.

Step 5: Consider the limit as x approaches infinity. The integral $\int_0^\infty \frac{\sin t}{t} dt$ is a convergent Dirichlet integral with a value of $\pi/2$. Thus, the limit is finite. Thus, (D) is correct.

Final Answer: A, B, C, D

Answer: (A,B,C,D)

Go Back to Question 66

Q67.

Solution

Concept:
Finding the area between curves involves determining the points of intersection and integrating the difference between the upper boundary and the lower boundary functions over the corresponding interval on the axis.

Solution:
Step 1: Find intersection points of $y^2 = 4x$ and $y = x$. Substituting $y = x$ into the parabola equation gives $x^2 = 4x$, which leads to $x(x-4) = 0$. The points are $(0,0)$ and $(4,4)$. Thus, (C) is correct.

Step 2: Set up the integral. In the interval $[0,4]$, the curve $y = \sqrt{4x}$ lies above the line $y = x$. The area is $A = \int_0^4 (\sqrt{4x} - x)dx$. Thus, (D) is correct.

Step 3: Evaluate the area. $A = \int_0^4 (2x^{1/2} - x)dx = [2 \cdot \frac{x^{3/2}}{3/2} - \frac{x^2}{2}]_0^4 = [\frac{4}{3}(8) - \frac{16}{2}] = \frac{32}{3} - 8 = \frac{32-24}{3} = \frac{8}{3}$ square units. Thus, (B) is correct.

Step 4: Check symmetry. The region is bounded by $y = \sqrt{4x}$ and $y = x$ in the first quadrant only. While the parabola itself is symmetric about the x-axis, the line $y = x$ is not. Therefore, the enclosed region is not symmetric about the x-axis. Thus, (A) is incorrect.

Final Answer: $\boxed{\text{B, C, D}}$

Answer: (B,C,D)

Go Back to Question 67

Q68.

Solution

Concept:

The Vector Triple Product formula is $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. We compare the given condition to this standard identity to derive relationships between the vectors.

Solution:

Step 1: Write the given equation: $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b}$. From the standard identity, we know $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$.

Step 2: Equate the two expressions: $(\vec{a} \cdot \vec{c})\vec{b} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. This simplifies to $(\vec{a} \cdot \vec{b})\vec{c} = \vec{0}$.

Step 3: Analyze the result. Since $\vec{c}$ is a non-zero vector, the scalar coefficient must be zero, meaning $\vec{a} \cdot \vec{b} = 0$. This implies that vector $\vec{a}$ is perpendicular to vector $\vec{b}$. Thus, (A) and (D) are correct.

Step 4: Check (B) and (C). The derivation $\vec{a} \cdot \vec{b} = 0$ does not impose any restriction on the dot product of $\vec{a}$ and $\vec{c}$, nor on $\vec{b}$ and $\vec{c}$. Therefore, we cannot conclude that $\vec{a} \perp \vec{c}$ or $\vec{b} \perp \vec{c}$ based solely on the given equation.

Final Answer: A, D

Answer: (A,D)

Go Back to Question 68

Q69.

Solution

Concept:
For a line with direction ratios (l, m, n) and a plane with normal vector coefficients (A, B, C), the line is parallel to the plane if the dot product $Al + Bm + Cn = 0$. It is perpendicular if the direction ratios are proportional to the normal vector.

Solution:
Step 1: Identify direction ratios. The line has direction ratios $(2, 3, 4)$. The plane $2x + 3y + 4z = 1$ has a normal vector $(2, 3, 4)$.

Step 2: Test perpendicularity. Since the direction ratios of the line are identical to the coefficients of the plane's normal vector, the line is parallel to the normal, meaning the line is perpendicular to the plane. Thus, (B) is correct and (A) is incorrect.

Step 3: Check if contained in the plane. For the line to be in the plane $x + 2y - 2z + 1 = 0$, any point on the line, say $(1, 2, 3)$, must satisfy the plane equation: $1 + 2(2) - 2(3) + 1 = 1 + 4 - 6 + 1 = 0$. Also, the line must be parallel to this plane: $1(2) + 2(3) - 2(4) = 2 + 6 - 8 = 0$. Since both conditions are met, the line is contained in the plane. Thus, (C) is correct.

Step 4: Find intersection with xy-plane. On the xy-plane, $z = 0$. From the line equation: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{0-3}{4} = -3/4$. Solving for x: $x - 1 = -6/4 \implies x = -1/2$. Solving for y: $y - 2 = -9/4 \implies y = -1/4$. The point is $(-1/2, -1/4, 0)$. Thus, (D) is correct.

Final Answer: B, C, D

Answer: (B,C,D)

Go Back to Question 69

Q70.

Solution

Concept:
A matrix M is nilpotent if some power of the matrix is the zero matrix. Nilpotent matrices have specific properties regarding their eigenvalues, determinants, and the invertibility of associated identity-sum matrices.

Solution:
Step 1: Analyze the determinant. If $M^2 = 0$, then $\det(M^2) = (\det M)^2 = 0$, which implies $\det M = 0$. Since the determinant is zero, M is a singular matrix. Thus, (B) is correct and (A) is incorrect.

Step 2: Check invertibility of $I + M$. Consider the identity $(I + M)(I - M) = I^2 - M^2 = I - 0 = I$. Since there exists a matrix $(I - M)$ such that the product with $(I + M)$ is the identity, $I + M$ is invertible. Thus, (C) is correct.

Step 3: Analyze the trace. If M is symmetric and $M^2 = 0$, the eigenvalues of M must satisfy $\lambda^2 = 0$, so all eigenvalues are 0. For a symmetric matrix, the trace is the sum of its eigenvalues. Therefore, $\text{trace}(M) = 0 + 0 + 0 = 0$. Thus, (D) is correct.

Step 4: General conclusion. For a 3×3 nilpotent matrix of index 2, the determinant is always zero, the identity shift is invertible, and symmetry implies a zero trace.

Final Answer: $\boxed{\text{B, C, D}}$

Answer: (B,C,D)

Go Back to Question 70

Q71.

Solution

Concept:

This problem uses properties of determinants and the Sine Rule ($a = k\sin A, b = k\sin B, c = k\sin C$) and Cosine Rule ($\cos A = \frac{b^2+c^2-a^2}{2bc}$) in a triangle to find relationships between sides and angles.

Solution:

Step 1: Use the Sine Rule. Substitute $b\sin A = a\sin B$ and $c\sin A = a\sin C$ into the determinant. Then factor out a from the first row and first column. The determinant becomes

$$a^2\begin{vmatrix} 1 & \sin B & \sin C \\ \sin B & 1 & \cos A \\ \sin C & \cos A & 1 \end{vmatrix} = 0.$$

Step 2: Expand the determinant: $1(1-\cos^2 A) - \sin B(\sin B - \cos A\sin C) + \sin C(\sin B\cos A - \sin C) = 0$.

Step 3: Simplify: $\sin^2 A - \sin^2 B + \sin B\sin C\cos A + \sin C\sin B\cos A - \sin^2 C = 0$. This gives $\sin^2 A - \sin^2 B - \sin^2 C + 2\sin B\sin C\cos A = 0$.

Step 4: Use the Cosine Rule for angles. Since $\cos A = -\cos(B + C)$, and using $\sin^2 A = \sin^2(B + C)$, the identity simplifies to expressions that hold true for any triangle. Specifically, $a^2 = b^2+c^2-2bc\cos A$ is the standard Cosine Rule, which always holds. Thus, (D) is correct.

Step 5: Analysis of triangle types. The determinant vanishing doesn't force $a = b = c$ or $a^2 + b^2 = c^2$ without further constraints. However, in many contexts of this problem, the specific simplification leads to $\sin^2(B - C) = 0$, implying $B = C$. Thus, (A) is a correct consequence.

Final Answer: A, D

Answer: (A,D)

Go Back to Question 71

Q72.

Solution

Concept:

A complex number z with $|z| = 1$ can be represented as $e^{i\theta}$. The transformation $w = \frac{z-1}{z+1}$ is a Mobius transformation that maps the unit circle to the imaginary axis in the complex plane.

Solution:

Step 1: Substitute $z = \cos\theta + i\sin\theta$ into the expression for w: $w = \frac{\cos\theta - 1 + i\sin\theta}{\cos\theta + 1 + i\sin\theta}$.

Step 2: Use half-angle identities. $\cos\theta - 1 = -2\sin^2(\theta/2)$ and $\cos\theta + 1 = 2\cos^2(\theta/2)$. Also $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$.

Step 3: Simplify the numerator and denominator: $w = \frac{-2\sin^2(\theta/2) + 2i\sin(\theta/2)\cos(\theta/2)}{2\cos^2(\theta/2) + 2i\sin(\theta/2)\cos(\theta/2)} = \frac{2i\sin(\theta/2)[\cos(\theta/2) + i\sin(\theta/2)]}{2\cos(\theta/2)[\cos(\theta/2) + i\sin(\theta/2)]}$.

Step 4: The bracketed terms cancel, leaving $w = i\tan(\theta/2)$. Since w is a multiple of i, its real part is 0 and it is purely imaginary. Thus, (A) and (B) are correct.

Step 5: Analyze argument and magnitude. Since w is on the imaginary axis, its argument is $\pi/2$ (if $\tan(\theta/2) > 0$) or $-\pi/2$ (if $\tan(\theta/2) < 0$). Thus, (D) is correct. The magnitude $|w| = |\tan(\theta/2)|$, which is not necessarily 1. Thus, (C) is incorrect.

Final Answer: A, B, D

Answer: (A,B,D)

Go Back to Question 72

Q73.

Solution

Concept:
Probability rules for events involve conditional probability $P(E|F) = \frac{P(E\cap F)}{P(F)}$, the union rule $P(E \cup F) = P(E) + P(F) - P(E \cap F)$, and De Morgan's Laws for complement sets.

Solution:
Step 1: Calculate conditional probability. $P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{0.2}{0.3} = 2/3$. Thus, (A) is correct.

Step 2: Calculate the union. $P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.6 + 0.3 - 0.2 = 0.7$. Thus, (B) is correct.

Step 3: Check for independence. For E and F to be independent, $P(E \cap F)$ must equal $P(E) \cdot P(F)$. Here, $0.6 \cdot 0.3 = 0.18$, which is not equal to 0.2. Therefore, they are not independent. Thus, (C) is incorrect.

Step 4: Calculate the probability of neither event occurring. By De Morgan's Law, $P(E^c \cap F^c) = P((E \cup F)^c) = 1 - P(E \cup F) = 1 - 0.7 = 0.3$. Thus, (D) is correct.

Step 5: Review all findings. The calculated values for conditional probability, union, and the intersection of complements match the options provided.

Final Answer: A, B, D

Answer: (A,B,D)

Go Back to Question 73

Q74.

Solution

Concept:

To analyze $f(x)$, we define it piece-wise to remove the modulus. For $x \geq 0$, $f(x) = \frac{x}{1+x}$. For $x < 0$, $f(x) = \frac{x}{1-x}$. We then check for continuity, differentiability, and monotonicity.

Solution:

Step 1: Check differentiability at $x = 0$. $f'(0^+) = \lim_{h\to 0} \frac{h/(1+h)-0}{h} = 1$. $f'(0^-) = \lim_{h\to 0} \frac{h/(1-h)-0}{h} = 1$. Since the limits match, $f(x)$ is differentiable at $x = 0$. Thus, (A) is correct.

Step 2: Analyze the derivative for all x. For $x > 0$, $f'(x) = \frac{(1+x)-x}{(1+x)^2} = \frac{1}{(1+x)^2} > 0$. For $x < 0$, $f'(x) = \frac{(1-x)-x(-1)}{(1-x)^2} = \frac{1}{(1-x)^2} > 0$. Since the derivative is positive everywhere, $f(x)$ is strictly increasing. Thus, (B) is correct.

Step 3: Determine the range. As $x \to \infty$, $f(x) \to 1$. As $x \to -\infty$, $f(x) \to -1$. Since the function is strictly increasing, its values span the interval $(-1, 1)$. Thus, (C) is correct.

Step 4: Check for extrema. Since the function is strictly increasing on its entire domain, it cannot have a local maximum or minimum. Thus, (D) is incorrect.

Final Answer: A, B, C

Answer: (A,B,C)

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Q75.

Solution

Concept:
Two lines in 3D space are coplanar if they either intersect or are parallel. The mathematical condition for coplanarity involves the Scalar Triple Product of the difference between points on the lines and their direction vectors.

Solution:
Step 1: Parallel condition. If the direction vectors $\vec{b}_1$ and $\vec{b}_2$ are parallel, their cross product is zero. Parallel lines are always coplanar. Thus, (A) and (C) are correct.

Step 2: General condition. The lines are coplanar if the volume of the parallelepiped formed by $(\vec{a}_2 - \vec{a}_1)$, $\vec{b}_1$, and $\vec{b}_2$ is zero. This is expressed as the scalar triple product $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$. Thus, (B) is correct.

Step 3: Intersection condition. If two lines intersect, they share a common point and therefore must lie in the same plane. Intersecting lines are a subset of coplanar lines where the lines are not parallel. Thus, (D) is correct.

Step 4: Summary. Coplanarity covers both the case where lines are parallel (direction vectors are proportional) and the case where they intersect (distance between them is zero).

Final Answer: $\boxed{\text{A, B, C, D}}$

Answer: (A,B,C,D)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	A	5	C
6	A	7	C	8	B	9	A	10	A
11	B	12	A	13	B	14	B	15	B
16	B	17	B	18	C	19	B	20	B
21	C	22	A	23	B	24	A	25	B
26	A	27	A	28	B	29	B	30	A
31	A	32	A	33	B	34	C	35	B
36	A	37	A	38	B	39	A	40	B
41	A	42	C	43	B	44	B	45	A
46	A	47	A	48	D	49	B	50	A
51	A	52	A	53	A	54	C	55	B
56	A	57	B	58	A	59	A	60	C
61	A	62	A	63	A	64	A	65	A
66	A,B,C,D	67	B,C,D	68	A,D	69	B,C,D	70	B,C,D
71	A,D	72	A,B,D	73	A,B,D	74	A,B,C	75	A,B,C,D