WBJEE Mathematics Sample Paper-8

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **75** Multiple Choice Questions divided into **3 Categories**.
- **Section A (Q1–Q50):** Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries **+2 marks**. Incorrect answer: **−0.5** marks. Only **one** correct option.
- **Section C (Q66–Q75):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: −0.25) [Single Correct]

Q1. The value of $\lim_{x\to 0} \frac{\tan 2x - \sin 3x}{x^3}$ is:

(A) $\frac{1}{2}$

(B) $\frac{5}{2}$

(C) $\frac{7}{2}$

(D) $\frac{9}{2}$

Q2. If $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then k equals:

(A) 0

(B) 1

(C) -1

(D) 2

Q3. The maximum value of $x(6 - x)$ is:

(A) 6

(B) 8

(C) 9

(D) 12

Q4. The value of $\int_0^1 (3x^2 + 2x + 1)\, dx$ is:

(A) 2

(B) 3

(C) 4

(D) 5

Q5. The solution of $\frac{dy}{dx} = 2x$ is:

(A) $y = x^2 + C$

(B) $y = 2x + C$

(C) $y = x + C$

(D) $y = 2x^2 + C$

Q6. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, then $\det(A)$ equals:

(A) 5

(B) 6

(C) 7

(D) 8

Q7. The value of $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ is:

(A) -2

(B) 2

(C) 4

(D) -4

Q8. If $z = 3 + 4i$, then $|z|$ equals:

(A) 3

(B) 4

(C) 5

(D) 7

Q9. The roots of $x^2 - 7x + 10 = 0$ are:

(A) 2 and 5

(B) 3 and 4

(C) 1 and 10

(D) 5 and 5

Q10. The sum of first 10 natural numbers is:

(A) 45

(B) 50

(C) 55

(D) 60

Q11. A card is drawn from a pack of 52 cards. The probability of getting an ace is:

(A) $\frac{1}{13}$

(B) $\frac{1}{26}$

(C) $\frac{1}{4}$

(D) $\frac{4}{13}$

Q12. The coefficient of x^2 in $(1+x)^5$ is:

(A) 5

(B) 10

(C) 15

(D) 20

Q13. The value of $\sin^2\theta + \cos^2\theta$ is:

(A) 0

(B) 1

(C) 2

(D) -1

Q14. The principal value of $\sin^{-1}(1)$ is:

(A) 0

(B) $\frac{\pi}{2}$

(C) π

(D) 2π

Q15. The slope of the line joining $(1,2)$ and $(3,6)$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q16. The equation of the line passing through $(1, 2)$ and having slope 3 is:

(A) $y = 3x - 1$

(B) $y = 3x + 1$

(C) $y = x + 1$

(D) $y = 2x + 3$

Q17. The radius of the circle $x^2 + y^2 - 6x + 8y + 9 = 0$ is:

(A) 3

(B) 4

(C) 5

(D) 6

Q18. The vertex of the parabola $y^2 = 8x$ is:

(A) $(0, 0)$

(B) $(2, 0)$

(C) $(0, 2)$

(D) $(1, 1)$

Q19. The eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is:

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{5}{4}$

(D) $\frac{1}{2}$

Q20. The equation $x^2 - y^2 = 1$ represents a:

(A) Circle

(B) Ellipse

(C) Hyperbola

(D) Parabola

Q21. If $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = \hat{i} - \hat{j}$, then $\vec{a} + \vec{b}$ equals:

(A) $3\hat{i} + 2\hat{j}$

(B) $\hat{i} + 4\hat{j}$

(C) $2\hat{i} + 2\hat{j}$

(D) $3\hat{i} - 2\hat{j}$

Q22. The distance between the points $(1, 2, 3)$ and $(4, 6, 3)$ is:

(A) 3

(B) 4

(C) 5

(D) 6

Q23. The value of $^5P_2$ is:

(A) 10

(B) 20

(C) 25

(D) 30

Q24. The middle term in the expansion of $(1 + x)^6$ is:

(A) $15x^2$

(B) $20x^3$

(C) $15x^3$

(D) $6x^2$

Q25. The sum of the GP $1 + 2 + 4 + 8 + \cdots + 64$ is:

(A) 63

(B) 127

(C) 128

(D) 255

Q26. The value of $\lim_{x\to 0} \frac{1-\cos x}{x^2}$ is:

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

Q27. If $y = x^3$, then $\frac{dy}{dx}$ equals:

(A) x^2

(B) $2x$

(C) $3x^2$

(D) $3x$

Q28. The minimum value of $x^2 + 4x + 5$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Q29. The value of $\int x^2\,dx$ is:

(A) $\frac{x^2}{2} + C$

(B) $\frac{x^3}{3} + C$

(C) $2x + C$

(D) $x^3 + C$

Q30. The value of $\int_0^1 x\,dx$ is:

(A) $\frac{1}{2}$

(B) 1

(C) $\frac{3}{2}$

(D) 2

Q31. The order of the differential equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q32. Two dice are thrown simultaneously. The probability of getting sum equal to 7 is:

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{5}{36}$

(D) $\frac{1}{9}$

Q33. If $i = \sqrt{-1}$, then i^{20} equals:

(A) 1

(B) -1

(C) i

(D) $-i$

Q34. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A^2 equals:

(A) A

(B) 0

(C) $2A$

(D) $I + A$

Q35. If two rows of a determinant are identical, then the determinant is:

(A) 0

(B) 1

(C) -1

(D) undefined

Q36. If $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $A \cap B$ is:

(A) $\{1, 2\}$

(B) $\{3\}$

(C) $\{4, 5\}$

(D) ϕ

Q37. If $f(x) = 2x + 1$, then $f(3)$ equals:

(A) 5

(B) 6

(C) 7

(D) 8

Q38. The value of $\tan 45°$ is:

(A) 0

(B) 1

(C) $\sqrt{3}$

(D) undefined

Q39. The principal value of $\cos^{-1}(0)$ is:

(A) 0

(B) $\frac{\pi}{2}$

(C) π

(D) $\frac{3\pi}{2}$

Q40. The equation of the x-axis is:

(A) $x = 0$

(B) $y = 0$

(C) $x = y$

(D) $x + y = 0$

Q41. The centre of the circle $x^2 + y^2 - 4x + 6y + 9 = 0$ is:

(A) $(2, -3)$

(B) $(-2, 3)$

(C) $(4, -6)$

(D) $(0, 0)$

Q42. The focus of the parabola $y^2 = 4ax$ is:

(A) $(a, 0)$

(B) $(0, a)$

(C) $(-a, 0)$

(D) $(0, -a)$

Q43. The length of major axis of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is:

(A) 4

(B) 6

(C) 8

(D) 16

Q44. The transverse axis of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is along:

(A) x-axis

(B) y-axis

(C) line $y = x$

(D) line $y = -x$

Q45. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, then $|\vec{a}|$ equals:

(A) 1

(B) $\sqrt{2}$

(C) $\sqrt{3}$

(D) 3

Q46. The direction ratios of the z-axis are:

(A) $(1, 0, 0)$

(B) $(0, 1, 0)$

(C) $(0, 0, 1)$

(D) $(1, 1, 1)$

Q47. The value of ${}^6C_2$ is:

(A) 10

(B) 12

(C) 15

(D) 20

Q48. The 10th term of the AP $3, 7, 11, \ldots$ is:

(A) 35

(B) 37

(C) 39

(D) 41

Q49. The coefficient of x in $(1 + x)^4$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q50. The value of $\lim_{x \to 0} \frac{\sin x}{x}$ is:

(A) 0

(B) 1

(C) ∞

(D) undefined

Section–B — 15 Questions × 1 Mark Each
(Negative Marking: −0.5) [Single Correct]

Q51. The function $f(x) = x^2$ is continuous at:

(A) only at $x = 0$

(B) only at positive x

(C) all real x

(D) no real x

Q52. If $y = \sin x$, then $\frac{dy}{dx}$ equals:

(A) $\cos x$

(B) $-\cos x$

(C) $\sin x$

(D) $-\sin x$

Q53. The function $f(x) = x^2$ has minimum value at:

(A) $x = -1$

(B) $x = 0$

(C) $x = 1$

(D) $x = 2$

Q54. The value of $\int \cos x \, dx$ is:

(A) $\sin x + C$

(B) $-\sin x + C$

(C) $\cos x + C$

(D) $-\cos x + C$

Q55. The value of $\int_0^{\pi} \sin x \, dx$ is:

(A) 0

(B) 1

(C) 2

(D) π

Q56. The degree of the differential equation $\left(\frac{dy}{dx}\right)^2 + 1 = 0$ is:

(A) 1

(B) 2

(C) 3

(D) undefined

Q57. A coin is tossed once. The probability of getting head is:

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

Q58. The conjugate of $3 + 2i$ is:

(A) $3 - 2i$

(B) $-3 + 2i$

(C) $-3 - 2i$

(D) $2 + 3i$

Q59. The transpose of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is:

(A) $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

Q60. The determinant of $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ is:

(A) 5

(B) 6

(C) 0

(D) 1

Q61. If $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then $A \cup B$ is:

(A) $\{1, 2, 3, 4\}$

(B) $\{3, 4\}$

(C) $\{1, 2, 3, 4, 5, 6\}$

(D) $\{5, 6\}$

Q62. If $f(x) = x^2 + 1$, then $f(2)$ equals:

(A) 3

(B) 4

(C) 5

(D) 6

Q63. The value of $\cos 60°$ is:

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) 1

Q64. The principal value of $\tan^{-1}(1)$ is:

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Q65. The slope of the line $2x + 3y = 6$ is:

(A) $\frac{2}{3}$

(B) $-\frac{2}{3}$

(C) $\frac{3}{2}$

(D) $-\frac{3}{2}$

Section C — 10 Questions × 2 Marks Each (No Negative Marking) [One or More Correct]

Q66. Which of the following limits are equal to 0?

(A) $\lim_{x\to 0} x \sin x$

(B) $\lim_{x\to 0} \frac{\sin x}{x}$

(C) $\lim_{x\to 0} x^2 \cos x$

(D) $\lim_{x\to 0} (1 - \cos x)$

Q67. Which of the following functions are increasing for all real x?

(A) e^x

(B) x^3

(C) $-x$

(D) x^2

Q68. For the function $f(x) = x^2 - 4x + 3$, which of the following statements are true?

(A) The graph opens upwards

(B) Vertex lies at $(2, -1)$

(C) Minimum value is -1

(D) Axis of symmetry is $x = -2$

Q69. Which of the following integrals are equal to $\ln|x| + C$?

(A) $\int \frac{1}{x}\, dx$

(B) $\int \frac{\sec^2 x}{\tan x}\, dx$

(C) $\int \frac{2}{x}\,dx$

(D) $\int \frac{1}{|x|}\,dx$

Q70. Which of the following matrices are diagonal matrices?

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$

Q71. Which of the following statements about determinants are true?

(A) Interchanging two rows changes the sign of determinant

(B) If two rows are equal, determinant is zero

(C) Determinant of identity matrix is zero

(D) Determinant of a triangular matrix is product of diagonal elements

Q72. Which of the following complex numbers are purely imaginary?

(A) $3i$

(B) $-5i$

(C) $2+i$

(D) $-i$

Q73. A card is drawn from a standard deck of 52 cards. Which of the following events have probability $\frac{1}{13}$?

(A) Drawing a king

(B) Drawing a queen

(C) Drawing a heart

(D) Drawing an ace

Q74. Which of the following equations represent straight lines?

(A) $2x + 3y = 6$

(B) $x^2 + y^2 = 1$

(C) $y = mx + c$

(D) $x - y + 5 = 0$

Q75. Which of the following are periodic functions with period 2π?

(A) $\sin x$

(B) $\cos x$

(C) $\tan x$

(D) $\sec x$

Detailed Solutions

Q1.

Solution

Concept: Standard limits: $\lim_{\theta\to0}\dfrac{\sin\theta}{\theta}=1$ and $\lim_{\theta\to0}\dfrac{\tan\theta}{\theta}=1$. For $\lim_{x\to0}\dfrac{\tan(nx)}{x^3}$, Taylor expansion is needed: $\tan u\approx u+\frac{u^3}{3}$. For $\lim_{x\to0}\dfrac{\sin(nx)}{x^3}$, Taylor expansion is needed: $\sin u\approx u-\frac{u^3}{6}$.

Solution: The limit $\lim\limits_{x\to0}\dfrac{\tan 2x-\sin 3x}{x^3}$ is of the indeterminate form $\frac{0}{0}$.
Step 1: Use Taylor series expansions around $x=0$: $\tan(2x)=2x+\frac{(2x)^3}{3}+O(x^5)=2x+\frac{8x^3}{3}+O(x^5)$ $\sin(3x)=3x-\frac{(3x)^3}{6}+O(x^5)=3x-\frac{9x^3}{2}+O(x^5)$
Step 2: Substitute these expansions into the limit expression.

$$\lim_{x\to0}\frac{(2x+\frac{8x^3}{3})-(3x-\frac{9x^3}{2})}{x^3}$$

$$=\lim_{x\to0}\frac{-x+(\frac{8}{3}+\frac{9}{2})x^3}{x^3}$$

$$=\lim_{x\to0}\frac{-x+\frac{43}{6}x^3}{x^3}$$

$$=\lim_{x\to0}\left(-\frac{1}{x^2}+\frac{43}{6}\right)$$

Step 3: Evaluate the limit. As $x\to0$, $-\frac{1}{x^2}\to-\infty$. Thus, the limit diverges.

However, since multiple-choice options are provided and are finite, there is likely a typo in the question. A common variant leading to finite answers involves equal arguments for tan and sin and a denominator of x^3. For instance, $\lim_{x\to0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2}$. Given the options, it is probable that the question intended a form that results in one of these values. Without clarification or correction, the stated problem leads to divergence. If we assume a typo and the question was meant to be $\lim_{x\to0}\frac{\tan x-\sin x}{x^3}$, then the answer is $\frac{1}{2}$.
Final Answer: $\boxed{1/2}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: For a function $f(x)$ to be continuous at a point $x = c$, the following three conditions must be met: 1. $f(c)$ is defined. 2. $\lim_{x \to c} f(x)$ exists. 3. $\lim_{x \to c} f(x) = f(c)$.
In this case, $f(x)$ is defined piecewise. For continuity at $x = 0$, we need $\lim_{x \to 0} f(x) = f(0)$.

Solution: Step 1: Identify the value of $f(0)$. From the definition of $f(x)$, when $x = 0$, $f(x) = k$. So, $f(0) = k$.
Step 2: Calculate the limit of $f(x)$ as x approaches 0. For $x \neq 0$, $f(x) = \frac{\sin x}{x}$. We need to find $\lim_{x \to 0} \frac{\sin x}{x}$. This is a standard limit result: $\lim_{x \to 0} \frac{\sin x}{x} = 1$.
Step 3: Apply the condition for continuity. For $f(x)$ to be continuous at $x = 0$, the limit of $f(x)$ as x approaches 0 must be equal to the value of $f(x)$ at $x = 0$.

$$\lim_{x \to 0} f(x) = f(0)$$

$$1 = k$$

Step 4: State the value of k. Therefore, k must be equal to 1 for the function to be continuous at $x = 0$.
Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: To find maximum or minimum values of a function:

(a) Find $f'(x)$ and set it equal to zero.

(b) Find critical points.

(c) Use $f''(x)$:

- $f''(x) < 0 \Rightarrow$ maximum
- $f''(x) > 0 \Rightarrow$ minimum

For quadratic functions, completing the square is another quick method.

Solution: Given,

$$f(x) = x(6 - x) = 6x - x^2$$

Method 1: Using Derivatives Step 1: Find the first derivative.

$$f'(x) = 6 - 2x$$

Step 2: Set $f'(x) = 0$.

$$6 - 2x = 0 \Rightarrow x = 3$$

Step 3: Find the second derivative.

$$f''(x) = -2$$

Since $f''(3) < 0$, the function has a maximum at $x = 3$. Step 4: Find the maximum value.

$$f(3) = 3(6 - 3) = 9$$

Method 2: Completing the Square

$$f(x) = 6x - x^2 = -(x^2 - 6x)$$

$$f(x) = -(x^2 - 6x + 9 - 9)$$

$$f(x) = -(x - 3)^2 + 9$$

Since $(x - 3)^2 \geq 0$,

$$-(x - 3)^2 \leq 0$$

Thus, the maximum value of $f(x)$ is

$$9$$

Final Answer: $\boxed{9}$

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: To evaluate a definite integral $\int_a^b f(x)dx$, we first find the antiderivative (or indefinite integral) of $f(x)$, let's call it $F(x)$. Then, we apply the Fundamental Theorem of Calculus, which states that $\int_a^b f(x)dx = F(b) - F(a)$.

The power rule for integration states that $\int x^n dx = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$. The integral of a sum is the sum of the integrals: $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$.

Solution: We need to evaluate the definite integral $\int_0^1 (3x^2 + 2x + 1)\,dx$.

Step 1: Find the indefinite integral of the function $f(x) = 3x^2 + 2x + 1$. We integrate each term separately using the power rule:

$$\int (3x^2 + 2x + 1)\,dx = \int 3x^2\,dx + \int 2x\,dx + \int 1\,dx$$

$$= 3\int x^2\,dx + 2\int x^1\,dx + \int x^0\,dx$$

Applying the power rule $\int x^n dx = \frac{x^{n+1}}{n+1}$:

$$= 3\left(\frac{x^{2+1}}{2+1}\right) + 2\left(\frac{x^{1+1}}{1+1}\right) + \left(\frac{x^{0+1}}{0+1}\right) + C$$

$$= 3\left(\frac{x^3}{3}\right) + 2\left(\frac{x^2}{2}\right) + \left(\frac{x^1}{1}\right) + C$$

$$= x^3 + x^2 + x + C$$

So, the antiderivative is $F(x) = x^3 + x^2 + x$.

Step 2: Apply the Fundamental Theorem of Calculus. The definite integral is given by $F(1) - F(0)$.

First, evaluate $F(x)$ at the upper limit $x = 1$:

$$F(1) = (1)^3 + (1)^2 + (1) = 1 + 1 + 1 = 3$$

Next, evaluate $F(x)$ at the lower limit $x = 0$:

$$F(0) = (0)^3 + (0)^2 + (0) = 0 + 0 + 0 = 0$$

Step 3: Subtract the value at the lower limit from the value at the upper limit.

$$\int_0^1 (3x^2 + 2x + 1)\,dx = F(1) - F(0) = 3 - 0 = 3$$

Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The problem asks for the solution to a first-order ordinary differential equation. The equation is given by $\frac{dy}{dx} = 2x$. This is a separable differential equation. To solve it, we integrate both sides with respect to their respective variables.

Solution: The given differential equation is:

$$\frac{dy}{dx} = 2x$$

Step 1: Separate the variables. Multiply both sides by dx to separate y and x:

$$dy = 2x\,dx$$

Step 2: Integrate both sides of the equation. Integrate the left side with respect to y and the right side with respect to x:

$$\int dy = \int 2x\,dx$$

Step 3: Perform the integration. The integral of dy with respect to y is y. The integral of $2x$ with respect to x is found using the power rule for integration ($\int x^n dx = \frac{x^{n+1}}{n+1} + C$):

$$\int 2x\,dx = 2\int x^1\,dx = 2\left(\frac{x^{1+1}}{1+1}\right) + C$$

$$= 2\left(\frac{x^2}{2}\right) + C$$

$$= x^2 + C$$

Here, C is the constant of integration.
Step 4: Combine the results. Equating the results from both sides:

$$y = x^2 + C$$

This is the general solution to the differential equation.
Comparing this with the given options: (A) $y = x^2 + C$ (B) $y = 2x + C$ (C) $y = x + C$ (D) $y = 2x^2 + C$
The obtained solution matches option (A).
Final Answer: $\boxed{y = x^2 + C}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: The determinant of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by the formula $\det(A) = ad - bc$.

Solution: We are given the matrix $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$. Here, $a = 2$, $b = 1$, $c = 3$, and $d = 4$.

Step 1: Apply the formula for the determinant of a 2×2 matrix.

$$\det(A) = ad - bc$$

Step 2: Substitute the values from the matrix into the formula.

$$\det(A) = (2)(4) - (1)(3)$$

Step 3: Perform the multiplication and subtraction.

$$\det(A) = 8 - 3$$

$$\det(A) = 5$$

The determinant of the matrix A is 5.

Final Answer: $\boxed{5}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: The determinant of a 2×2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is calculated as $ad - bc$.

Solution: We need to calculate the value of the determinant $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$. Here, $a = 1$, $b = 2$, $c = 3$, and $d = 4$.

Step 1: Apply the formula for the determinant of a 2×2 matrix.

$$\det = ad - bc$$

Step 2: Substitute the given values into the formula.

$$\det = (1)(4) - (2)(3)$$

Step 3: Perform the calculations.

$$\det = 4 - 6$$

$$\det = -2$$

The value of the determinant is -2.

Final Answer: $\boxed{-2}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: For a complex number $z = a + bi$, where a is the real part and b is the imaginary part, the modulus (or magnitude) of z, denoted as $|z|$, is calculated using the formula:

$$|z| = \sqrt{a^2 + b^2}$$

This formula represents the distance of the complex number from the origin in the complex plane.

Solution: We are given the complex number $z = 3 + 4i$. Here, the real part is $a = 3$ and the imaginary part is $b = 4$.
Step 1: Identify the real and imaginary parts of the complex number. Real part, $a = 3$. Imaginary part, $b = 4$.
Step 2: Apply the formula for the modulus of a complex number.

$$|z| = \sqrt{a^2 + b^2}$$

Step 3: Substitute the values of a and b into the formula.

$$|z| = \sqrt{(3)^2 + (4)^2}$$

Step 4: Perform the squaring and addition.

$$|z| = \sqrt{9 + 16}$$

$$|z| = \sqrt{25}$$

Step 5: Calculate the square root.

$$|z| = 5$$

The modulus of the complex number $z = 3 + 4i$ is 5.
Final Answer: $\boxed{5}$

Answer: (C)
Go Back to Question 8

Q9.

Solution

Concept: To find the roots of a quadratic equation of the form $ax^2 + bx + c = 0$, we can use factorization, completing the square, or the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$. For simple equations, factorization is often the quickest method.

Solution: We need to find the roots of the quadratic equation $x^2 - 7x + 10 = 0$.
Method 1: Factorization We look for two numbers that multiply to give the constant term ($c = 10$) and add up to the coefficient of the x term ($b = -7$). Let these two numbers be p and q. We need: $p \times q = 10$ $p + q = -7$
We can list the pairs of factors of 10: (1, 10), (-1, -10), (2, 5), (-2, -5).
Now, let's check the sum for each pair: $1 + 10 = 11$ $-1 + (-10) = -11$ $2 + 5 = 7$ $-2 + (-5) = -7$
The pair $(-2, -5)$ satisfies both conditions. So, we can factor the quadratic equation as:

$$(x-2)(x-5) = 0$$

To find the roots, we set each factor equal to zero: $x - 2 = 0 \implies x = 2$ $x - 5 = 0 \implies x = 5$
The roots are 2 and 5.
Method 2: Quadratic Formula For the equation $x^2 - 7x + 10 = 0$, we have $a = 1$, $b = -7$, and $c = 10$. Using the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 - 40}}{2}$$

$$x = \frac{7 \pm \sqrt{9}}{2}$$

$$x = \frac{7 \pm 3}{2}$$

We get two roots: $x_1 = \frac{7+3}{2} = \frac{10}{2} = 5$ $x_2 = \frac{7-3}{2} = \frac{4}{2} = 2$
The roots are 2 and 5.
Comparing with the options: (A) 2 and 5 (B) 3 and 4 (C) 1 and 10 (D) 5 and 5
The roots are 2 and 5, which matches option (A).
Final Answer: $\boxed{\text{2 and 5}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: The sum of the first n natural numbers (1, 2, 3, ..., n) is given by the formula for the sum of an arithmetic series:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where n is the number of terms, a_1 is the first term, and a_n is the last term.
For the first n natural numbers, $a_1 = 1$ and $a_n = n$.
So, the formula simplifies to:

$$S_n = \frac{n}{2}(1 + n) = \frac{n(n + 1)}{2}$$

Solution: We need to find the sum of the first 10 natural numbers. Here, $n = 10$.
Step 1: Use the formula for the sum of the first n natural numbers.

$$S_{10} = \frac{10(10 + 1)}{2}$$

Step 2: Perform the calculations.

$$S_{10} = \frac{10(11)}{2}$$

$$S_{10} = \frac{110}{2}$$

$$S_{10} = 55$$

The sum of the first 10 natural numbers is 55.
Let's verify by listing the numbers and summing them: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 3 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 6 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 10 + 5 + 6 + 7 + 8 + 9 + 10 = 15 + 6 + 7 + 8 + 9 + 10 = 21 + 7 + 8 + 9 + 10 = 28 + 8 + 9 + 10 = 36 + 9 + 10 = 45 + 10 = 55
The result is confirmed.
Comparing with the options: (A) 45 (B) 50 (C) 55 (D) 60
The correct option is 55.
Final Answer: $\boxed{55}$

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept: Probability is defined as the ratio of the number of favorable outcomes to the total number of possible outcomes.

$$P(\text{Event}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

A standard pack of 52 cards contains 4 suits (hearts, diamonds, clubs, spades), and each suit has 13 cards (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King). There are 4 Aces in a standard deck of 52 cards (one Ace for each suit).

Solution: We want to find the probability of drawing an ace from a standard pack of 52 cards.
Step 1: Determine the total number of possible outcomes. The total number of cards in a standard deck is 52. So, the total number of possible outcomes when drawing one card is 52.
Step 2: Determine the number of favorable outcomes. The favorable outcome is drawing an ace. In a standard deck of 52 cards, there are 4 Aces (Ace of Hearts, Ace of Diamonds, Ace of Clubs, Ace of Spades). So, the number of favorable outcomes is 4.
Step 3: Calculate the probability.

$$P(\text{getting an Ace}) = \frac{\text{Number of Aces}}{\text{Total number of cards}}$$

$$P(\text{getting an Ace}) = \frac{4}{52}$$

Step 4: Simplify the fraction. Divide both the numerator and the denominator by their greatest common divisor, which is 4.

$$\frac{4}{52} = \frac{4 \div 4}{52 \div 4} = \frac{1}{13}$$

The probability of getting an ace is $\frac{1}{13}$.
Comparing with the options: (A) $\frac{1}{13}$ (B) $\frac{1}{26}$ (C) $\frac{1}{4}$ (D) $\frac{4}{13}$
The correct option is $\frac{1}{13}$.
Final Answer: $\boxed{\frac{1}{13}}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: The Binomial Theorem states that for any non-negative integer n, the expansion of $(x+y)^n$ is given by:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

where $\binom{n}{k}$ is the binomial coefficient, calculated as $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.
In this problem, we are expanding $(1+x)^5$. This means x in the formula is 1, and y in the formula is x. The power is $n=5$. The general term in the expansion of $(1+x)^5$ is:

$$T_{k+1} = \binom{5}{k}(1)^{5-k}(x)^k = \binom{5}{k}x^k$$

We are looking for the coefficient of x^2. This corresponds to the term where $k=2$.

Solution: We need to find the coefficient of x^2 in the expansion of $(1+x)^5$.
Step 1: Use the Binomial Theorem for the expansion of $(1+x)^5$. The general term in the expansion is given by $\binom{n}{k}y^k$, where $n=5$ and $y=x$. So, the general term is $\binom{5}{k}x^k$.
Step 2: Identify the term that contains x^2. We need to find the value of k such that $x^k = x^2$. This means $k=2$.
Step 3: Calculate the binomial coefficient for $k=2$ and $n=5$. The binomial coefficient is $\binom{5}{2}$.

$$\binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{5!}{2!3!}$$

$$\binom{5}{2} = \frac{5\times4\times3\times2\times1}{(2\times1)(3\times2\times1)}$$

$$\binom{5}{2} = \frac{5\times4}{2\times1}$$

$$\binom{5}{2} = \frac{20}{2}$$

$$\binom{5}{2} = 10$$

Step 4: The coefficient of x^2 is the calculated binomial coefficient. The coefficient of x^2 in the expansion of $(1+x)^5$ is 10.
Let's write out the first few terms of the expansion to verify: $(1+x)^5 = \binom{5}{0}1^5x^0 + \binom{5}{1}1^4x^1 + \binom{5}{2}1^3x^2 + \ldots = 1\times1\times1 + 5\times1\times x + 10\times1\times x^2 + \ldots = 1 + 5x + 10x^2 + \ldots$ The coefficient of x^2 is indeed 10.
Comparing with the options: (A) 5 (B) 10 (C) 15 (D) 20
The correct option is 10.
Final Answer: $\boxed{10}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: The Pythagorean identity states:

$$\sin^2\theta + \cos^2\theta = 1$$

This follows directly from the equation of the unit circle.

Solution: On the unit circle, a point corresponding to angle θ has coordinates:

$$(\cos\theta, \sin\theta)$$

Since every point on the unit circle satisfies:

$$x^2 + y^2 = 1$$

Substituting $x = \cos\theta$ and $y = \sin\theta$:

$$\cos^2\theta + \sin^2\theta = 1$$

Rewriting:

$$\sin^2\theta + \cos^2\theta = 1$$

This holds for all values of θ.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: The inverse sine function $\sin^{-1}(x)$ gives an angle in the interval:

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Solution: We need:

$$\sin^{-1}(1)$$

Let $\theta = \sin^{-1}(1)$, so:

$$\sin\theta = 1$$

In the principal range, this occurs at:

$$\theta = \frac{\pi}{2}$$

Since $\frac{\pi}{2}$ lies within the required interval, it is valid.

Final Answer: $\boxed{\frac{\pi}{2}}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: The slope of a line through two points (x_1, y_1) and (x_2, y_2) is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Solution: Given points:

$$(1, 2), \ (3, 6)$$

Step 1: Apply the formula.

$$m = \frac{6 - 2}{3 - 1}$$

Step 2: Simplify.

$$m = \frac{4}{2}$$

$$m = 2$$

Thus, the slope of the line is:

$$2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: The point-slope form of the equation of a straight line is given by $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is a point on the line. We can rearrange this equation into the slope-intercept form, $y = mx + c$.

Solution: We are given a point $(x_1, y_1) = (1, 2)$ and the slope $m = 3$.
Step 1: Use the point-slope form of the equation of a line.

$$y - y_1 = m(x - x_1)$$

Substitute the given values:

$$y - 2 = 3(x - 1)$$

Step 2: Simplify and rearrange the equation into slope-intercept form ($y = mx + c$).

$$y - 2 = 3x - 3$$

Add 2 to both sides:

$$y = 3x - 3 + 2$$

$$y = 3x - 1$$

The equation of the line is $y = 3x - 1$.
Final Answer: $\boxed{y = 3x - 1}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: The standard equation of a circle with center (h, k) and radius r is $(x-h)^2+(y-k)^2 = r^2$. To find the radius of a circle given in the general form $x^2 + y^2 + 2gx + 2fy + c = 0$, we first convert it to the standard form by completing the square. The center of the circle is $(-g, -f)$ and the radius is $r = \sqrt{g^2 + f^2 - c}$.

Solution: The equation of the circle is given as $x^2 + y^2 - 6x + 8y + 9 = 0$.
Step 1: Group the x terms and y terms, and move the constant to the right side.

$$(x^2 - 6x) + (y^2 + 8y) = -9$$

Step 2: Complete the square for the x terms. To complete the square for $x^2 - 6x$, take half of the coefficient of x (which is $-6/2 = -3$) and square it ($(-3)^2 = 9$). Add this value to both sides of the equation.

$$(x^2 - 6x + 9) + (y^2 + 8y) = -9 + 9$$

Step 3: Complete the square for the y terms. To complete the square for $y^2 + 8y$, take half of the coefficient of y (which is $8/2 = 4$) and square it ($4^2 = 16$). Add this value to both sides of the equation.

$$(x^2 - 6x + 9) + (y^2 + 8y + 16) = -9 + 9 + 16$$

Step 4: Rewrite the expressions in parentheses as squared terms.

$$(x - 3)^2 + (y + 4)^2 = 16$$

Step 5: Identify the radius from the standard equation. Comparing this to the standard form $(x - h)^2 + (y - k)^2 = r^2$, we have: $h = 3$ $k = -4$ $r^2 = 16$
Step 6: Calculate the radius. Taking the square root of r^2:

$$r = \sqrt{16}$$

$$r = 4$$

The radius of the circle is 4.
Final Answer: $\boxed{4}$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: The standard equation of a parabola with vertex at the origin $(0,0)$ and opening to the right is $y^2 = 4ax$. If it opens to the left, it's $y^2 = -4ax$. If it opens upwards, it's $x^2 = 4ay$. If it opens downwards, it's $x^2 = -4ay$.

Solution: The given equation of the parabola is $y^2 = 8x$.
Step 1: Compare the given equation with the standard forms of parabolas. The equation $y^2 = 8x$ matches the standard form $y^2 = 4ax$, which represents a parabola with its vertex at the origin $(0,0)$ and opening to the right.
Step 2: Identify the vertex. From the standard form $y^2 = 4ax$, the vertex is at $(0,0)$. In our case, $y^2 = 8x$, the vertex is at $(0,0)$.
Step 3: Determine the direction of opening (optional, but good for understanding). Comparing $y^2 = 8x$ with $y^2 = 4ax$, we have $4a = 8$, which means $a = 2$. Since $a > 0$, the parabola opens to the right.
The vertex of the parabola $y^2 = 8x$ is $(0,0)$.
Final Answer: $\boxed{(0,0)}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: An ellipse centered at the origin with its major axis along the x-axis has the standard equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > b$. The eccentricity e of such an ellipse is given by the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Solution: The equation of the ellipse is given as $\frac{x^2}{25} + \frac{y^2}{16} = 1$.

Step 1: Identify a^2 and b^2 from the equation. Comparing with the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have: $a^2 = 25 \implies a = \sqrt{25} = 5$ $b^2 = 16 \implies b = \sqrt{16} = 4$

Since $a > b$ $(5 > 4)$, the major axis is along the x-axis.

Step 2: Use the formula for eccentricity. The eccentricity e is given by:

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Step 3: Substitute the values of a^2 and b^2 into the formula.

$$e = \sqrt{1 - \frac{16}{25}}$$

Step 4: Perform the subtraction inside the square root.

$$e = \sqrt{\frac{25}{25} - \frac{16}{25}}$$

$$e = \sqrt{\frac{25 - 16}{25}}$$

$$e = \sqrt{\frac{9}{25}}$$

Step 5: Calculate the square root.

$$e = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$$

The eccentricity of the ellipse is $\frac{3}{5}$.

Final Answer: $\boxed{\frac{3}{5}}$

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept: The standard forms of second-degree conic sections are:

1. Circle: $(x-h)^2+(y-k)^2=r^2$ or $x^2+y^2+2gx+2fy+c=0$.
2. Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (centered at origin).
3. Hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ or $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ (centered at origin).
4. Parabola: $y^2=4ax$ or $x^2=4ay$ (vertex at origin).

Solution: The given equation is $x^2-y^2=1$.

Step 1: Compare the equation with the standard forms of conic sections. The equation has terms with x^2 and y^2, both with coefficients. The general form of a second-degree equation is $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$. In our case, $A=1$, $B=0$, $C=-1$, $D=0$, $E=0$, $F=-1$. The characteristic equation to identify the conic section is based on the discriminant B^2-4AC. $B^2-4AC=(0)^2-4(1)(-1)=0-(-4)=4$. Since $B^2-4AC>0$ and A and C have opposite signs, the conic section is a hyperbola.

Step 2: Rewrite the equation in a standard form. The equation $x^2-y^2=1$ can be written as:

$$\frac{x^2}{1^2}-\frac{y^2}{1^2}=1$$

This is the standard form of a hyperbola centered at the origin, with its transverse axis along the x-axis.

Therefore, the equation $x^2-y^2=1$ represents a hyperbola.

Final Answer: $\boxed{Hyperbola}$

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept: Vector addition is done by adding corresponding components. If

$$\vec{a} = a_1\hat{i} + a_2\hat{j}, \quad \vec{b} = b_1\hat{i} + b_2\hat{j}$$

then,

$$\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j}$$

Solution: We are given:

$$\vec{a} = 2\hat{i} + 3\hat{j}, \quad \vec{b} = \hat{i} - \hat{j}$$

Step 1: Add $\hat{i}$ components.

$$2 + 1 = 3$$

Step 2: Add $\hat{j}$ components.

$$3 + (-1) = 2$$

Step 3: Form the resultant vector.

$$\vec{a} + \vec{b} = (2 + 1)\hat{i} + (3 - 1)\hat{j}$$

$$\vec{a} + \vec{b} = 3\hat{i} + 2\hat{j}$$

Step 4: Interpretation. The resultant vector represents the combined displacement obtained by moving along $\vec{a}$ and then $\vec{b}$.

Final Answer: $\boxed{3\hat{i} + 2\hat{j}}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in three-dimensional space is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Solution: We need to find the distance between the points $P_1 = (1, 2, 3)$ and $P_2 = (4, 6, 3)$.
Step 1: Identify the coordinates of the two points. $(x_1, y_1, z_1) = (1, 2, 3)$ $(x_2, y_2, z_2) = (4, 6, 3)$
Step 2: Apply the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Substitute the coordinates:

$$d = \sqrt{(4 - 1)^2 + (6 - 2)^2 + (3 - 3)^2}$$

Step 3: Calculate the differences.

$$d = \sqrt{(3)^2 + (4)^2 + (0)^2}$$

Step 4: Square the differences.

$$d = \sqrt{9 + 16 + 0}$$

Step 5: Add the squared differences.

$$d = \sqrt{25}$$

Step 6: Calculate the square root.

$$d = 5$$

The distance between the points $(1, 2, 3)$ and $(4, 6, 3)$ is 5.
Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 22

Q23.

Solution

Concept: The number of permutations of n distinct objects taken r at a time, denoted by ${}^nP_r$ or $P(n, r)$, is given by the formula:

$$ {}^nP_r = \frac{n!}{(n-r)!} $$

This formula is used when the order of selection matters.

Solution: We need to calculate the value of ${}^5P_2$. Here, $n = 5$ (the total number of objects) and $r = 2$ (the number of objects taken at a time).
Step 1: Write down the formula for permutations.

$$ {}^nP_r = \frac{n!}{(n-r)!} $$

Step 2: Substitute the given values of n and r.

$$ {}^5P_2 = \frac{5!}{(5-2)!} $$

Step 3: Simplify the denominator.

$$ {}^5P_2 = \frac{5!}{3!} $$

Step 4: Expand the factorials. $5! = 5 \times 4 \times 3 \times 2 \times 1$ $3! = 3 \times 2 \times 1$
Step 5: Calculate the value.

$$ {}^5P_2 = \frac{5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1} $$

We can cancel out the 3! term from the numerator and the denominator:

$$ {}^5P_2 = 5 \times 4 $$

$$ {}^5P_2 = 20 $$

The value of ${}^5P_2$ is 20.
Final Answer: $\boxed{20}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: In the binomial expansion of $(x+y)^n$, if n is an even integer, there is a single middle term. The position of the middle term is given by $\frac{n}{2}+1$. The general term in the expansion of $(x+y)^n$ is $T_{k+1} = \binom{n}{k} x^{n-k} y^k$.

Solution: We need to find the middle term in the expansion of $(1+x)^6$. Here, $n = 6$, which is an even number.

Step 1: Determine the position of the middle term. The position of the middle term is $\frac{n}{2}+1 = \frac{6}{2}+1 = 3+1 = 4$. So, we are looking for the 4th term in the expansion.

Step 2: Use the general term formula $T_{k+1} = \binom{n}{k} x^{n-k} y^k$. In this case, the expansion is $(1+x)^6$, so x in the formula is 1, and y in the formula is x. The power $n = 6$. The general term is $T_{k+1} = \binom{6}{k}(1)^{6-k}(x)^k = \binom{6}{k} x^k$.

Step 3: Find the term when $k+1 = 4$. If $k+1 = 4$, then $k = 3$. Substitute $k = 3$ into the general term formula:

$$T_4 = \binom{6}{3} x^3$$

Step 4: Calculate the binomial coefficient $\binom{6}{3}$.

$$\binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6!}{3!3!}$$

$$\binom{6}{3} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(3 \times 2 \times 1)}$$

$$\binom{6}{3} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = \frac{120}{6} = 20$$

Step 5: Write the middle term. The middle term is $T_4 = 20x^3$.

Final Answer: $\boxed{20x^3}$

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept: This is a problem involving a Geometric Progression (GP). The sum of the first n terms of a GP with first term a and common ratio r is given by the formula:

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{if } r \neq 1$$

First, we need to identify the first term (a), the common ratio (r), and the number of terms (n).

Solution: The given series is $1 + 2 + 4 + 8 + \cdots + 64$.
Step 1: Identify the first term (a) and the common ratio (r). The first term is $a = 1$. To find the common ratio, divide any term by its preceding term: $r = \frac{2}{1} = 2$ $r = \frac{4}{2} = 2$ $r = \frac{8}{4} = 2$ The common ratio is $r = 2$.
Step 2: Find the number of terms (n). Let the last term be $a_n = 64$. The formula for the n-th term of a GP is $a_n = a \cdot r^{n-1}$. Substitute the values: $64 = 1 \cdot (2)^{n-1}$ $64 = 2^{n-1}$ Since $64 = 2^6$, we have: $2^6 = 2^{n-1}$ Equating the exponents: $6 = n - 1$ $n = 6 + 1 = 7$. So, there are 7 terms in the series.
Step 3: Use the formula for the sum of the first n terms of a GP. We have $a = 1$, $r = 2$, and $n = 7$. Since $r > 1$, we use the formula $S_n = \frac{a(r^n-1)}{r-1}$.

$$S_7 = \frac{1(2^7 - 1)}{2 - 1}$$

Step 4: Calculate the sum.

$$S_7 = \frac{2^7 - 1}{1}$$

$2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$.

$$S_7 = 128 - 1$$

$$S_7 = 127$$

The sum of the GP is 127.
Final Answer: $\boxed{127}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: Use standard trigonometric identities and limits:

$$1 - \cos x = 2\sin^2\left(\frac{x}{2}\right), \quad \lim_{x\to 0}\frac{\sin x}{x} = 1$$

Solution: We evaluate:

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}$$

Method 1: Identity

Using $1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$:

$$\lim_{x\to 0}\frac{2\sin^2(\frac{x}{2})}{x^2}$$

Rewrite:

$$= 2\left(\frac{\sin(\frac{x}{2})}{x}\right)^2$$

Adjust to standard form:

$$= \frac{1}{2}\left(\frac{\sin(\frac{x}{2})}{\frac{x}{2}}\right)^2$$

Let $\theta = \frac{x}{2}$, then as $x \to 0$, $\theta \to 0$:

$$= \frac{1}{2}\left(\lim_{\theta\to 0}\frac{\sin\theta}{\theta}\right)^2$$

$$= \frac{1}{2}(1)^2 = \frac{1}{2}$$

Method 2: Series

Using $\cos x = 1 - \frac{x^2}{2!} + \cdots$:

$$\frac{1-\cos x}{x^2} = \frac{\frac{x^2}{2!} - \frac{x^4}{4!} + \cdots}{x^2}$$

$$= \frac{1}{2!} - \frac{x^2}{4!} + \cdots$$

Taking limit:

$$= \frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept: The derivative measures the rate of change of a function. For a power function, the power rule is used:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

This rule is applicable for any real exponent n.

Solution: We need to differentiate:

$$y = x^3$$

Step 1: Identify the form of the function. This is a power function of the form x^n where $n = 3$.
Step 2: Apply the power rule.

$$\frac{dy}{dx} = \frac{d}{dx}(x^3)$$

Using $\frac{d}{dx}(x^n) = nx^{n-1}$:

$$\frac{dy}{dx} = 3x^{3-1}$$

Step 3: Simplify the expression.

$$\frac{dy}{dx} = 3x^2$$

Step 4: Interpretation. The result $3x^2$ represents the slope of the tangent to the curve $y = x^3$ at any point x.

Final Answer: $\boxed{3x^2}$

Answer: (C)

Go Back to Question 27

Q28.

Solution

Concept: For a quadratic function $f(x) = ax^2 + bx + c$, if $a > 0$, the parabola opens upward and has a minimum value at its vertex. The minimum value can be found using derivatives or by completing the square.

Solution: We need to find the minimum value of:

$$f(x) = x^2 + 4x + 5$$

Since the coefficient of x^2 is positive, the function has a minimum value.
Method 1: Using Calculus Step 1: Find the first derivative.

$$f'(x) = \frac{d}{dx}(x^2 + 4x + 5) = 2x + 4$$

Step 2: Set the derivative equal to zero.

$$2x + 4 = 0$$

$$x = -2$$

Step 3: Find the second derivative.

$$f''(x) = 2$$

Since $f''(x) > 0$, the function has a minimum at $x = -2$.
Step 4: Find the minimum value.

$$f(-2) = (-2)^2 + 4(-2) + 5$$

$$f(-2) = 4 - 8 + 5 = 1$$

Method 2: Completing the Square

$$f(x) = x^2 + 4x + 5$$

$$f(x) = (x^2 + 4x + 4) + 1$$

$$f(x) = (x + 2)^2 + 1$$

Since $(x + 2)^2 \geq 0$, its minimum value is 0. Hence, the minimum value of the function is:

$$1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: The indefinite integral of a power function x^n is given by the power rule for integration:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

where $n \neq -1$ and C is the constant of integration.

Solution: We need to find the indefinite integral of x^2. Here, $n = 2$.

Step 1: Apply the power rule for integration.

$$\int x^2 \, dx = \frac{x^{2+1}}{2+1} + C$$

Step 2: Simplify the expression.

$$\int x^2 \, dx = \frac{x^3}{3} + C$$

The indefinite integral of x^2 is $\frac{x^3}{3} + C$.

Final Answer: $\boxed{\frac{x^3}{3} + C}$

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept: To evaluate a definite integral $\int_a^b f(x)dx$, we first find the antiderivative of $f(x)$, say $F(x)$, and then apply the Fundamental Theorem of Calculus: $\int_a^b f(x)dx = F(b) - F(a)$. The power rule for integration is $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.

Solution: We need to evaluate the definite integral $\displaystyle\int_0^1 x\,dx$.
Step 1: Find the antiderivative of $f(x) = x$. Using the power rule with $n = 1$:

$$\int x\,dx = \frac{x^{1+1}}{1+1} + C = \frac{x^2}{2} + C$$

Let $F(x) = \dfrac{x^2}{2}$.
Step 2: Apply the Fundamental Theorem of Calculus.

$$\int_0^1 x\,dx = F(1) - F(0)$$

Step 3: Evaluate $F(x)$ at the upper and lower limits. At the upper limit $x = 1$:

$$F(1) = \frac{(1)^2}{2} = \frac{1}{2}$$

At the lower limit $x = 0$:

$$F(0) = \frac{(0)^2}{2} = 0$$

Step 4: Subtract the value at the lower limit from the value at the upper limit.

$$\int_0^1 x\,dx = \frac{1}{2} - 0 = \frac{1}{2}$$

The value of the definite integral is $\dfrac{1}{2}$.

Final Answer: $\boxed{\dfrac{1}{2}}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: The order of a differential equation is the order of the highest derivative present in the equation.

Solution: The given differential equation is:

$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$$

Step 1: Identify the derivatives present in the equation.

- $\frac{d^2y}{dx^2}$ is the second derivative.
- $\frac{dy}{dx}$ is the first derivative.

Step 2: Determine the highest order derivative.
The highest derivative present is:

$$\frac{d^2y}{dx^2}$$

Its order is 2.
Therefore, the order of the differential equation is:

$$2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 31

Q32.

Solution

Concept: Probability is calculated as the ratio of favorable outcomes to the total possible outcomes. When two dice are thrown, each die has 6 possible outcomes (1, 2, 3, 4, 5, 6). The total number of possible outcomes when throwing two dice is $6 \times 6 = 36$.

Solution: We need to find the probability of getting a sum of 7 when two dice are thrown simultaneously.
Step 1: Determine the total number of possible outcomes. When rolling two dice, there are 6 outcomes for the first die and 6 outcomes for the second die. The total number of possible outcomes is $6 \times 6 = 36$. These outcomes can be represented as pairs (d_1, d_2), where d_1 is the result of the first die and d_2 is the result of the second die.
Step 2: Determine the number of favorable outcomes (sum equals 7). We need to find the pairs (d_1, d_2) such that $d_1 + d_2 = 7$. These pairs are: - (1, 6) - (2, 5) - (3, 4) - (4, 3) - (5, 2) - (6, 1) There are 6 favorable outcomes.
Step 3: Calculate the probability.

$$P(\text{sum is 7}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(\text{sum is 7}) = \frac{6}{36}$$

Step 4: Simplify the fraction.

$$\frac{6}{36} = \frac{1}{6}$$

The probability of getting a sum of 7 when two dice are thrown is $\frac{1}{6}$.

Final Answer: $\boxed{\frac{1}{6}}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: The powers of the imaginary unit i repeat in a cycle of four:

$$i = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

To find higher powers of i, divide the exponent by 4 and check the remainder:

- Remainder $0 \Rightarrow 1$
- Remainder $1 \Rightarrow i$
- Remainder $2 \Rightarrow -1$
- Remainder $3 \Rightarrow -i$

Solution: We need to find:

$$i^{20}$$

Step 1: Divide the exponent by 4.

$$20 \div 4 = 5$$

The remainder is 0.
Step 2: Use the cyclic property. Since the remainder is 0,

$$i^{20} = 1$$

Alternatively,

$$i^{20} = (i^4)^5 = 1^5 = 1$$

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: The matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix of order 2, denoted by I. The identity matrix has the property that when it is multiplied by any matrix M of compatible dimensions, the result is M. That is, $MI = IM = M$. Therefore, $I^2 = I \times I = I$.

Solution: We are given the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. This is the 2×2 identity matrix, usually denoted by I.

We need to find A^2.

$$A^2 = A \times A$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Step 1: Perform matrix multiplication. To find the element in the first row and first column of A^2, we multiply the first row of the first matrix by the first column of the second matrix: $(1 \times 1) + (0 \times 0) = 1 + 0 = 1$.

To find the element in the first row and second column: $(1 \times 0) + (0 \times 1) = 0 + 0 = 0$.

To find the element in the second row and first column: $(0 \times 1) + (1 \times 0) = 0 + 0 = 0$.

To find the element in the second row and second column: $(0 \times 0) + (1 \times 1) = 0 + 1 = 1$.

So, $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Step 2: Compare the result with the given options. The resulting matrix is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which is the original matrix A. Since A is the identity matrix I, $A^2 = I \times I = I = A$.

Final Answer: $\boxed{A}$

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept: A determinant has several important properties related to its rows and columns. One such property states that if any two rows (or any two columns) of a determinant are identical, then the value of the determinant is zero.
This follows from the property that interchanging two rows changes the sign of the determinant. If the two rows are already identical, swapping them does not change the determinant, leading to the conclusion that the determinant must be zero.

Solution: Consider a determinant:

$$D = \begin{vmatrix} a & b \\ a & b \end{vmatrix}$$

Here, the two rows are identical.
Step 1: Recall the property of determinants.
If two rows of a determinant are interchanged, the determinant changes sign. Thus, after swapping the two rows:

$$D = -D$$

Step 2: Use the fact that the rows are identical.
Since the rows are exactly the same, interchanging them does not actually change the determinant. Therefore, the determinant still remains:

$$D$$

Hence,

$$D = -D$$

Step 3: Solve for D.
Adding D to both sides:

$$D + D = 0$$

$$2D = 0$$

$$D = 0$$

Therefore, a determinant with two identical rows has value zero.
Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: The intersection of two sets, denoted by $A \cap B$, is the set of all elements that are common to both sets.
If no common element exists, the intersection is the empty set ϕ.

Solution: We are given:

$$A = \{1, 2, 3\}$$

and

$$B = \{3, 4, 5\}$$

We need to find:

$$A \cap B$$

Step 1: Write the elements of set A.

$$1,\ 2,\ 3$$

Step 2: Write the elements of set B.

$$3,\ 4,\ 5$$

Step 3: Identify the common elements.
Comparing both sets, we see that only the element 3 appears in both sets.
Step 4: Form the intersection set.

$$A \cap B = \{3\}$$

Thus, the intersection contains only one common element.
Final Answer: $\boxed{\{3\}}$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: A function assigns a unique output to every input value. To evaluate a function at a particular value, substitute the given value into the expression of the function and simplify.

Solution: We are given the function:

$$f(x) = 2x + 1$$

We need to find:

$$f(3)$$

Step 1: Substitute $x = 3$ into the function.

$$f(3) = 2(3) + 1$$

Step 2: Multiply.

$$f(3) = 6 + 1$$

Step 3: Simplify.

$$f(3) = 7$$

Therefore, the value of the function at $x = 3$ is 7.

Final Answer: $\boxed{7}$

Answer: (C)

Go Back to Question 37

Q38.

Solution

Concept: The tangent of an angle in a right-angled triangle is defined as the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle. For a 45° angle, we can consider an isosceles right-angled triangle.

Solution: We need to find the value of $\tan 45^\circ$.
Consider an isosceles right-angled triangle. Let the two equal sides (opposite and adjacent to the 45° angles) have a length of 1 unit. By the definition of tangent:

$$\tan\theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

For $\theta = 45^\circ$:

$$\tan 45^\circ = \frac{\text{Length of opposite side}}{\text{Length of adjacent side}}$$

$$\tan 45^\circ = \frac{1}{1}$$

$$\tan 45^\circ = 1$$

Alternatively, using the unit circle, consider the point on the unit circle corresponding to an angle of 45°. The coordinates of this point are $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$. The tangent of an angle is the ratio of the y-coordinate to the x-coordinate:

$$\tan 45^\circ = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

The value of $\tan 45^\circ$ is 1.
Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 38

Q39.

Solution

Concept: The inverse cosine function $\cos^{-1}(x)$, also called $\arccos(x)$, gives the angle whose cosine value is x.
The principal value range of the inverse cosine function is:

$$[0, \pi]$$

This means the answer obtained from $\cos^{-1}(x)$ must always lie between 0 and π radians, inclusive.

Solution: We need to evaluate:

$$\cos^{-1}(0)$$

This means we must find an angle θ such that:

$$\cos\theta = 0$$

From standard trigonometric values:

$$\cos\left(\frac{\pi}{2}\right) = 0$$

Now check whether the angle lies in the principal value interval:

$$[0, \pi]$$

Since

$$\frac{\pi}{2}$$

lies within this interval, it is the required principal value.
Therefore,

$$\cos^{-1}(0) = \frac{\pi}{2}$$

Final Answer: $\boxed{\frac{\pi}{2}}$

Answer: (B)

Go Back to Question 39

Q40.

Solution

Concept: In the Cartesian coordinate plane:

- The x-axis is the horizontal axis.
- The y-axis is the vertical axis.

Every point on the x-axis has y-coordinate equal to zero, while every point on the y-axis has x-coordinate equal to zero.

Solution: We need to find the equation of the x-axis.
Any point lying on the x-axis has coordinates of the form:

$$(x, 0)$$

This shows that the y-coordinate of every point on the x-axis is zero.
Therefore, the equation representing the x-axis is:

$$y = 0$$

Similarly, the equation of the y-axis is:

$$x = 0$$

Hence, the required equation is:

$$y = 0$$

Final Answer: $\boxed{y = 0}$

Answer: (B)

Go Back to Question 40

Q41.

Solution

Concept: The general equation of a circle is:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

For this equation:

- Center = $(-g, -f)$
- Radius = $\sqrt{g^2 + f^2 - c}$

The coefficients of x and y help determine the center of the circle.

Solution: The given equation of the circle is:

$$x^2 + y^2 - 4x + 6y + 9 = 0$$

Step 1: Compare the equation with the general form.

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Comparing coefficients:

$$2g = -4$$

$$g = -2$$

Similarly,

$$2f = 6$$

$$f = 3$$

Also,

$$c = 9$$

Step 2: Find the center of the circle.
The center is given by:

$$(-g, -f)$$

Substituting the values:

$$(-(-2), -(3))$$

$$(2, -3)$$

Step 3: (Optional) Verify the radius.

$$r = \sqrt{g^2 + f^2 - c}$$

$$r = \sqrt{(-2)^2 + (3)^2 - 9}$$

$$r = \sqrt{4 + 9 - 9}$$

$$r = \sqrt{4} = 2$$

Q42.

Solution

Concept: The standard equation of a parabola opening towards the positive x-axis is:

$$y^2 = 4ax$$

For this parabola:

- Vertex = $(0, 0)$
- Focus = $(a, 0)$
- Directrix = $x = -a$

The focus always lies on the axis of symmetry of the parabola.

Solution: The given equation is:

$$y^2 = 4ax$$

Step 1: Identify the standard form.
The equation already matches the standard form of a parabola with vertex at the origin and axis along the x-axis.
Step 2: Recall the coordinates of the focus.
For the parabola:

$$y^2 = 4ax$$

the focus is:

$$(a, 0)$$

Hence, the focus lies on the x-axis at a distance a units from the origin.
Final Answer: $\boxed{(a, 0)}$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: The standard form of an ellipse centered at the origin is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where $a > b$.
For an ellipse:

- Major axis length = $2a$
- Minor axis length = $2b$

The larger denominator determines the direction of the major axis.

Solution: The given ellipse is:

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Step 1: Compare with the standard form.
Here,

$$16 > 9$$

So,

$$a^2 = 16, \quad b^2 = 9$$

Since the larger denominator is under x^2, the major axis is along the x-axis.
Step 2: Find the value of a.

$$a = \sqrt{16} = 4$$

Step 3: Find the length of the major axis.
The formula is:

$$2a$$

Substituting $a = 4$:

$$2a = 2 \times 4 = 8$$

Therefore, the length of the major axis is:

$$8$$

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 43

Q44.

Solution

Concept: The standard equation of a hyperbola with transverse axis along the x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

For this hyperbola:

- Transverse axis is along the x-axis
- Vertices are $(\pm a, 0)$

The term with positive sign determines the direction of the transverse axis.

Solution: The given equation is:

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Step 1: Compare with the standard form.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

By comparison:

$$a^2 = 9, \quad b^2 = 16$$

Step 2: Determine the direction of the transverse axis.
The x^2 term has a positive sign, so the transverse axis lies along the x-axis.
Step 3: Find the vertices.
Since

$$a = \sqrt{9} = 3$$

the vertices are:

$$(\pm 3, 0)$$

Hence, the transverse axis of the hyperbola is along the x-axis.
Final Answer: x-axis

Answer: (A)

Go Back to Question 44

Q45.

Solution

Concept: The magnitude or modulus of a vector represents its length. For a vector

$$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

the magnitude is:

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

This formula is obtained using the three-dimensional distance formula.

Solution: We are given the vector:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$

Step 1: Write the vector in component form.

$$\vec{a} = 1\hat{i} + 1\hat{j} + 1\hat{k}$$

So, the components are:

$$a_1 = 1, \quad a_2 = 1, \quad a_3 = 1$$

Step 2: Apply the magnitude formula.

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Substituting the values:

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2}$$

$$|\vec{a}| = \sqrt{1 + 1 + 1}$$

$$|\vec{a}| = \sqrt{3}$$

Thus, the length or magnitude of the vector is:

$$\sqrt{3}$$

Final Answer: $\boxed{\sqrt{3}}$

Answer: (C)

Go Back to Question 45

Q46.

Solution

Concept: Direction ratios (d.r.'s) of a line are any three numbers proportional to the direction cosines of the line. They indicate the direction of the line in three-dimensional space.
For the coordinate axes:

- x-axis $\longrightarrow (1, 0, 0)$
- y-axis $\longrightarrow (0, 1, 0)$
- z-axis $\longrightarrow (0, 0, 1)$

These represent unit vectors along the respective axes.

Solution: The z-axis is the line passing through the origin and extending in the positive and negative z-directions.
A unit vector along the positive z-axis is:

$$\hat{k} = 0\hat{i} + 0\hat{j} + 1\hat{k}$$

The components of this vector are:

$$(0, 0, 1)$$

These components give the direction ratios of the z-axis.
Since direction ratios are proportional to the direction cosines, the direction ratios of the z-axis are:

$$(0, 0, 1)$$

Final Answer: $\boxed{(0, 0, 1)}$

Answer: (C)

Go Back to Question 46

Q47.

Solution

Concept: The number of combinations of n distinct objects taken r at a time, denoted by ${}^nC_r$ or $C(n,r)$ or $\binom{n}{r}$, is given by the formula:

$$^nC_r = \frac{n!}{r!(n-r)!}$$

This formula is used when the order of selection does not matter.

Solution: We need to calculate the value of ${}^6C_2$. Here, $n = 6$ (the total number of objects) and $r = 2$ (the number of objects to choose).
Step 1: Write down the formula for combinations.

$$^nC_r = \frac{n!}{r!(n-r)!}$$

Step 2: Substitute the given values of n and r.

$$^6C_2 = \frac{6!}{2!(6-2)!}$$

Step 3: Simplify the expression.

$$^6C_2 = \frac{6!}{2!4!}$$

Step 4: Expand the factorials. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$ $2! = 2 \times 1$ $4! = 4 \times 3 \times 2 \times 1$
Step 5: Calculate the value.

$$^6C_2 = \frac{6 \times 5 \times 4!}{(2 \times 1) \times 4!}$$

Cancel out 4!:

$$^6C_2 = \frac{6 \times 5}{2 \times 1}$$

$$^6C_2 = \frac{30}{2}$$

$$^6C_2 = 15$$

The value of ${}^6C_2$ is 15.
Final Answer: $\boxed{15}$

Answer: (C)

Go Back to Question 47

Q48.

Solution

Concept: An Arithmetic Progression (AP) is a sequence of numbers such that the difference between consecutive terms is constant. This constant difference is called the common difference, denoted by d. The n-th term of an AP is given by the formula:

$$a_n = a_1 + (n - 1)d$$

where a_1 is the first term and d is the common difference.

Solution: The given Arithmetic Progression (AP) is $3, 7, 11, \ldots$.
Step 1: Identify the first term (a_1). The first term is $a_1 = 3$.
Step 2: Find the common difference (d). Subtract any term from its succeeding term: $d = 7 - 3 = 4$ $d = 11 - 7 = 4$ The common difference is $d = 4$.
Step 3: Determine the term number (n). We need to find the 10th term, so $n = 10$.
Step 4: Use the formula for the n-th term of an AP.

$$a_n = a_1 + (n - 1)d$$

Substitute the values $a_1 = 3$, $n = 10$, and $d = 4$:

$$a_{10} = 3 + (10 - 1)4$$

$$a_{10} = 3 + (9)4$$

$$a_{10} = 3 + 36$$

$$a_{10} = 39$$

The 10th term of the AP is 39.
Final Answer: $\boxed{39}$

Answer: (C)

Go Back to Question 48

Q49.

Solution

Concept: The Binomial Theorem states that for any non-negative integer n, the expansion of $(x + y)^n$ is given by:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

where $\binom{n}{k}$ is the binomial coefficient, calculated as $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

In this problem, we have $(1 + x)^4$. So, x in the formula is 1, y in the formula is x, and $n = 4$. The general term in the expansion of $(1 + x)^4$ is:

$$T_{k+1} = \binom{4}{k}(1)^{4-k}(x)^k = \binom{4}{k}x^k$$

We need to find the coefficient of x. The term with x corresponds to x^1, which means $k = 1$.

Solution: We need to find the coefficient of x in the expansion of $(1 + x)^4$.
Step 1: Use the Binomial Theorem for the expansion of $(1 + x)^4$. The general term is $\binom{4}{k}x^k$.
Step 2: Identify the term that contains x^1. We need the term where $k = 1$.
Step 3: Calculate the binomial coefficient for $k = 1$ and $n = 4$. The binomial coefficient is $\binom{4}{1}$.

$$\binom{4}{1} = \frac{4!}{1!(4-1)!} = \frac{4!}{1!3!}$$

$$\binom{4}{1} = \frac{4 \times 3 \times 2 \times 1}{(1)(3 \times 2 \times 1)}$$

$$\binom{4}{1} = \frac{4}{1} = 4$$

Step 4: The coefficient of x is the calculated binomial coefficient. The coefficient of x in the expansion of $(1 + x)^4$ is 4.
Let's expand the first few terms to verify: $(1+x)^4 = \binom{4}{0}1^4x^0 + \binom{4}{1}1^3x^1 + \ldots = 1\times1\times1 + 4\times1\times x + \ldots$ $= 1 + 4x + \ldots$ The coefficient of x is indeed 4.
Final Answer: $\boxed{4}$

Answer: (D)

Go Back to Question 49

Q50.

Solution

Concept: The limit $\lim_{x\to 0} \dfrac{\sin x}{x} = 1$ is a fundamental trigonometric limit. It can be proven using geometric arguments (squeeze theorem) or Taylor series expansion.

Solution: We need to find the value of the limit $\lim\limits_{x\to 0} \dfrac{\sin x}{x}$.
This is a standard limit in calculus. As x approaches 0, the value of $\sin x$ also approaches 0, leading to the indeterminate form $\frac{0}{0}$.
Using the known standard limit:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Alternatively, using the Taylor series expansion of $\sin x$ around $x = 0$: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots}{x}$$

$$= \lim_{x\to 0} \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots\right)$$

As $x \to 0$, all terms with x go to zero, leaving:

$$= 1$$

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 50

Q51.

Solution

Concept: A function $f(x)$ is said to be continuous at a point $x = c$ if:

$$\lim_{x \to c} f(x) = f(c)$$

Polynomial functions are continuous for every real value of x. Since they contain only powers of x with finite coefficients, they do not have breaks, jumps, or discontinuities.

Solution: The given function is:

$$f(x) = x^2$$

This is a polynomial function of degree 2.
Step 1: Recall the continuity property of polynomials.
All polynomial functions are continuous on the entire set of real numbers:

$$(-\infty, \infty)$$

Step 2: Apply this property to the given function.
Since $f(x) = x^2$ is a polynomial, it is continuous for every real value of x.
Thus, the function is continuous at:

$$x > 0, \quad x < 0, \quad \text{and } x = 0$$

Hence, $f(x) = x^2$ is continuous for all real numbers.
Final Answer: all real x

Answer: (C)

Go Back to Question 51

Q52.

Solution

Concept: Differentiation measures the rate of change of a function with respect to a variable. One of the standard derivative formulas is:

$$\frac{d}{dx}(\sin x) = \cos x$$

Solution: We are given:

$$y = \sin x$$

We need to differentiate the function with respect to x.

Step 1: Apply the standard differentiation rule for $\sin x$.

$$\frac{dy}{dx} = \frac{d}{dx}(\sin x)$$

Step 2: Differentiate.

$$\frac{dy}{dx} = \cos x$$

Therefore, the derivative of $\sin x$ is $\cos x$.

Final Answer: $\boxed{\cos x}$

Answer: (A)

Go Back to Question 52

Q53.

Solution

Concept: To find the minimum or maximum value of a function, we can find the critical points by setting the first derivative to zero. For a function $f(x)$, if $f'(x) = 0$ and $f''(x) > 0$, then the function has a local minimum at that point. For $f(x) = x^2$, the derivative is $f'(x) = 2x$. Setting $f'(x) = 0$ gives $2x = 0$, so $x = 0$. The second derivative is $f''(x) = 2$, which is positive, confirming a minimum at $x = 0$.

Solution: We need to find where the function $f(x) = x^2$ has its minimum value.
Method 1: Using Calculus Step 1: Find the first derivative of $f(x)$.

$$f'(x) = \frac{d}{dx}(x^2) = 2x$$

Step 2: Set the first derivative to zero to find critical points.

$$2x = 0$$

$$x = 0$$

Step 3: Find the second derivative.

$$f''(x) = \frac{d}{dx}(2x) = 2$$

Since $f''(0) = 2 > 0$, the function has a local minimum at $x = 0$.
Method 2: Analyzing the function $f(x) = x^2$ The function $f(x) = x^2$ represents a parabola opening upwards with its vertex at the origin. The vertex of this parabola is the point where the function attains its minimum value. The vertex of $y = x^2$ is at $(0, 0)$. For any real number x, $x^2 \geq 0$. The minimum value of x^2 is 0, which occurs when $x = 0$.
The function $f(x) = x^2$ has its minimum value at $x = 0$.
Final Answer: $\boxed{x = 0}$

Answer: (B)

Go Back to Question 53

Q54.

Solution

Concept: The indefinite integral of a function gives its antiderivative. For trigonometric functions, the standard integration formula is:

$$\int \cos x \, dx = \sin x + C$$

where C is the constant of integration. This result follows because:

$$\frac{d}{dx}(\sin x) = \cos x$$

Solution: We need to evaluate:

$$\int \cos x \, dx$$

Step 1: Recall the standard integration formula for $\cos x$.

$$\int \cos x \, dx = \sin x + C$$

Step 2: Write the final result.
Therefore,

$$\int \cos x \, dx = \sin x + C$$

where C represents the arbitrary constant of integration.
Final Answer: $\boxed{\sin x + C}$

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept: To evaluate a definite integral $\int_a^b f(x)dx$, we find the antiderivative of $f(x)$, say $F(x)$, and then calculate $F(b) - F(a)$ (Fundamental Theorem of Calculus). The antiderivative of $\sin x$ is $-\cos x$.

Solution: We need to evaluate the definite integral $\int_0^\pi \sin x\, dx$.
Step 1: Find the antiderivative of $\sin x$. The antiderivative of $\sin x$ is $-\cos x$. Let $F(x) = -\cos x$.
Step 2: Apply the Fundamental Theorem of Calculus.

$$\int_0^\pi \sin x\, dx = F(\pi) - F(0)$$

Step 3: Evaluate $F(x)$ at the upper and lower limits. At the upper limit $x = \pi$:

$$F(\pi) = -\cos(\pi)$$

The value of $\cos(\pi)$ is -1.

$$F(\pi) = -(-1) = 1$$

At the lower limit $x = 0$:

$$F(0) = -\cos(0)$$

The value of $\cos(0)$ is 1.

$$F(0) = -(1) = -1$$

Step 4: Subtract the value at the lower limit from the value at the upper limit.

$$\int_0^\pi \sin x\, dx = F(\pi) - F(0) = 1 - (-1)$$

$$= 1 + 1$$

$$= 2$$

The value of the definite integral is 2.
Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 55

Q56.

Solution

Concept: The degree of a differential equation is the highest power of the highest order derivative present in the equation after removing radicals and fractions involving derivatives. The order and degree are not always the same.

Solution: The given differential equation is:

$$\left(\frac{dy}{dx}\right)^2 + 1 = 0$$

Step 1: Identify the highest order derivative.
The derivative present is:

$$\frac{dy}{dx}$$

This is the first derivative, so the order of the differential equation is 1.
Step 2: Find the power of the highest order derivative.
The derivative appears as:

$$\left(\frac{dy}{dx}\right)^2$$

Hence, the power of the highest order derivative is 2.
Step 3: Determine the degree.
The degree of a differential equation is the exponent of the highest order derivative. Therefore, the degree is:

$$2$$

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 56

Q57.

Solution

Concept: Probability measures the chance of an event occurring and is given by:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

For a fair coin, the two outcomes Head (H) and Tail (T) are equally likely.

Solution: We need to find the probability of getting a head when a coin is tossed once.
Step 1: Write the possible outcomes.
When a coin is tossed once, the sample space is:

$$S = \{H, T\}$$

Therefore, the total number of possible outcomes is:

$$n(S) = 2$$

Step 2: Find the favorable outcomes.
The favorable outcome for getting a head is:

$$\{H\}$$

So, the number of favorable outcomes is:

$$1$$

Step 3: Apply the probability formula.

$$P(\text{Head}) = \frac{1}{2}$$

Hence, the probability of getting a head is:

$$\frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (C)

Go Back to Question 57

Q58.

Solution

Concept: If a complex number is written as:

$$z = a + bi$$

then its conjugate is obtained by changing the sign of the imaginary part:

$$\bar{z} = a - bi$$

The real part remains unchanged.

Solution: We are given the complex number:

$$z = 3 + 2i$$

Step 1: Identify the real and imaginary parts.
Comparing with the form $a + bi$:

$$a = 3, \quad b = 2$$

Step 2: Change the sign of the imaginary part.
The imaginary term is:

$$2i$$

Changing its sign gives:

$$-2i$$

Step 3: Write the conjugate.
Therefore,

$$\overline{3 + 2i} = 3 - 2i$$

Hence, the conjugate of the given complex number is:

$$3 - 2i$$

Final Answer: $\boxed{3 - 2i}$

Answer: (A)

Go Back to Question 58

Q59.

Solution

Concept: The transpose of a matrix is obtained by interchanging its rows and columns. If A is a matrix, its transpose is denoted by A^T or A'. The element at row i and column j of A becomes the element at row j and column i of A^T.

Solution: We are given the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

To find the transpose of A, we interchange the rows and columns. The first row of A is $[1, 2]$. This becomes the first column of A^T. The second row of A is $[3, 4]$. This becomes the second column of A^T.

So, the transpose of A is:

$$A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

The elements on the main diagonal remain in their positions. The element in the first row, second column (2) moves to the second row, first column. The element in the second row, first column (3) moves to the first row, second column.

Final Answer: $\boxed{\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}}$

Answer: (A)

Go Back to Question 59

Q60.

Solution

Concept: The determinant of a 2×2 matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is calculated as $ad - bc$.

For a diagonal matrix (where all non-diagonal elements are zero), such as $\begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$, the determinant is simply the product of the diagonal elements: $a \times d$.

Solution: We need to find the determinant of the matrix $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$. This is a diagonal matrix.

Step 1: Identify the diagonal elements. The diagonal elements are 2 and 3. The non-diagonal elements are 0.

Step 2: Calculate the determinant. For a diagonal matrix, the determinant is the product of the diagonal elements.

$$\det = 2 \times 3$$

$$\det = 6$$

Alternatively, using the general formula $ad - bc$: Here, $a = 2$, $b = 0$, $c = 0$, $d = 3$.

$$\det = (2)(3) - (0)(0)$$

$$\det = 6 - 0$$

$$\det = 6$$

The determinant of the matrix is 6.

Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 60

Q61.

Solution

Concept: The union of two sets, written as $A \cup B$, is the set containing all elements that are present in set A, set B, or in both sets. While forming the union, repeated elements are written only once because a set contains only distinct elements.

Solution: We are given the sets:

$$A = \{1, 2, 3, 4\}$$

and

$$B = \{3, 4, 5, 6\}$$

We need to find:

$$A \cup B$$

Step 1: Write all elements of set A.

$$1, 2, 3, 4$$

Step 2: Now check the elements of set B. The elements of B are:

$$3, 4, 5, 6$$

Here, 3 and 4 are already present in set A, so they are not repeated. The new elements obtained from set B are:

$$5, 6$$

Step 3: Combine all distinct elements.

$$A \cup B = \{1, 2, 3, 4, 5, 6\}$$

Thus, the union contains every unique element from both sets.

Final Answer: $\boxed{\{1, 2, 3, 4, 5, 6\}}$

Answer: (C)

Go Back to Question 61

Q62.

Solution

Concept: A function gives a unique output value for every input value. To evaluate a function at a particular number, substitute that number in place of the variable and simplify the expression.

Solution: The given function is:

$$f(x) = x^2 + 1$$

We need to find the value of:

$$f(2)$$

Step 1: Substitute $x = 2$ into the function.

$$f(2) = (2)^2 + 1$$

Step 2: Evaluate the square term.

$$(2)^2 = 4$$

So,

$$f(2) = 4 + 1$$

Step 3: Simplify further.

$$f(2) = 5$$

Therefore, when the input is 2, the function gives the output 5.

Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 62

Q63.

Solution

Concept: The value of $\cos 60^\circ$ is a standard trigonometric ratio. In a 30°-60°-90° triangle, the sides are in the ratio:

$$1 : \sqrt{3} : 2$$

The cosine of an angle is defined as:

$$\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

Solution: We need to find:

$$\cos 60^\circ$$

Consider a right-angled triangle with angles 30°, 60°, and 90°.
In this triangle:

- The side adjacent to 60° is proportional to 1.
- The hypotenuse is proportional to 2.

Using the cosine formula,

$$\cos 60^\circ = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

Substituting the values,

$$\cos 60^\circ = \frac{1}{2}$$

Hence, the standard value of $\cos 60^\circ$ is:

$$\frac{1}{2}$$

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (B)

Go Back to Question 63

Q64.

Solution

Concept: The inverse tangent function $\tan^{-1}(x)$ gives the angle whose tangent value is x. The principal value of $\tan^{-1}(x)$ always lies in the interval:

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

excluding the endpoints.

Solution: We need to evaluate:

$$\tan^{-1}(1)$$

This means we must find an angle θ such that:

$$\tan\theta = 1$$

We know from standard trigonometric values that:

$$\tan\left(\frac{\pi}{4}\right) = 1$$

Now check whether this angle lies in the principal value interval:

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Since

$$\frac{\pi}{4}$$

lies within this interval, it is the required principal value.
Therefore,

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 64

Q65.

Solution

Concept: To find the slope of a line given by the equation $Ax + By + C = 0$, we can rearrange it into the slope-intercept form $y = mx + c$, where m is the slope. The slope is the coefficient of x when the equation is in this form. Alternatively, the slope can be directly found as $m = -\frac{A}{B}$.

Solution: The equation of the line is given as $2x + 3y = 6$.
Method 1: Rearranging into slope-intercept form Step 1: Isolate the y term.

$$3y = -2x + 6$$

Step 2: Solve for y by dividing by the coefficient of y.

$$y = \frac{-2x + 6}{3}$$

$$y = -\frac{2}{3}x + \frac{6}{3}$$

$$y = -\frac{2}{3}x + 2$$

The equation is now in the form $y = mx + c$, where m is the slope. The slope $m = -\frac{2}{3}$.
Method 2: Using the formula $m = -\frac{A}{B}$ The equation is in the form $Ax + By + C = 0$ (or $Ax + By = -C$). Here, $A = 2$, $B = 3$, and $C = -6$ (if we consider $2x + 3y - 6 = 0$). The slope is $m = -\frac{A}{B} = -\frac{2}{3}$.
The slope of the line $2x + 3y = 6$ is $-\frac{2}{3}$.

Final Answer: $\boxed{-\frac{2}{3}}$

Answer: (B)

Go Back to Question 65

Q66.

Solution

Concept: This question tests the evaluation of limits as x approaches 0. We need to determine which expressions approach 0.

Solution: Step 1: Evaluate limit (A): $\lim_{x\to 0} x\sin x$.
As $x \to 0$, $x \to 0$. As $x \to 0$, $\sin x \to 0$.
The limit is of the form 0×0, which is 0.
This limit is 0.

Step 2: Evaluate limit (B): $\lim_{x\to 0} \frac{\sin x}{x}$.
This is a standard limit whose value is 1.
This limit is not 0.

Step 3: Evaluate limit (C): $\lim_{x\to 0} x^2 \cos x$.
As $x \to 0$, $x^2 \to 0$. As $x \to 0$, $\cos x \to \cos 0 = 1$.
The limit is of the form 0×1, which is 0.
This limit is 0.

Step 4: Evaluate limit (D): $\lim_{x\to 0} (1 - \cos x)$.
As $x \to 0$, $\cos x \to \cos 0 = 1$.
The limit is $(1 - 1) = 0$.
This limit is 0.

Step 5: Identify the limits equal to 0.
Limits (A), (C), and (D) are equal to 0.

Final Answer:

$$\boxed{\lim_{x\to 0} x\sin x, \quad \lim_{x\to 0} x^2\cos x, \quad \lim_{x\to 0} (1-\cos x)}$$

Answer: (A,C,D)

Go Back to Question 66

Q67.

Solution

Concept: A function is increasing if its derivative is positive. We check the derivative of each function.

Solution: Step 1: Condition for increasing function.
A function $f(x)$ is increasing if $f'(x) > 0$ for all x.

Step 2: Analyze derivatives.
- (A) $f(x) = e^x \implies f'(x) = e^x$. $e^x > 0$ for all x. Increasing.
- (B) $f(x) = x^3 \implies f'(x) = 3x^2$. $f'(x) \geq 0$. $f'(0) = 0$, but it's strictly increasing overall. Increasing.
- (C) $f(x) = -x \implies f'(x) = -1$. $-1 < 0$ for all x. Decreasing. Not increasing.
- (D) $f(x) = x^2 \implies f'(x) = 2x$. $f'(x) > 0$ only for $x > 0$. Decreasing for $x < 0$. Not increasing for all x.

Step 3: Identify increasing functions.
Functions (A) and (B) are increasing for all real x.

Final Answer:

$$\boxed{e^x, \quad x^3}$$

Answer: (A,B)

Go Back to Question 67

Q68.

Solution

Concept: For a quadratic function $f(x) = ax^2 + bx + c$:

$$x_{\text{vertex}} = -\frac{b}{2a}$$

If $a > 0$, the parabola opens upwards and the vertex gives the minimum value.

Solution: Step 1: Given:

$$f(x) = x^2 - 4x + 3$$

Here,

$$a = 1, \quad b = -4$$

Step 2: Check statement (A). Since $a = 1 > 0$, the parabola opens upwards. Hence, (A) is true.
Step 3: Find the vertex.

$$x = -\frac{b}{2a} = -\frac{-4}{2(1)} = 2$$

$$f(2) = 2^2 - 4(2) + 3 = 4 - 8 + 3 = -1$$

So, the vertex is:

$$(2, -1)$$

Hence, (B) is true.
Step 4: Check statement (C). Since the parabola opens upwards, the minimum value is:

$$-1$$

Hence, (C) is true.
Step 5: Check statement (D). Axis of symmetry:

$$x = 2$$

So, $x = -2$ is false. Therefore, statements (A), (B), and (C) are correct.
Final Answer:

$$\boxed{\begin{array}{c}\text{The graph opens upwards}\\ \text{Vertex lies at } (2,-1)\\ \text{Minimum value is } -1\end{array}}$$

Answer: (A,B,C)

Go Back to Question 68

Q69.

Solution

Concept: This question asks for integrals that evaluate to $\ln|x| + C$. We need to check the derivative of $\ln|x|$, which is $\frac{1}{x}$.

Solution: Step 1: Recall the derivative of $\ln|x|$.
The derivative of $\ln|x|$ is $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$.
Therefore, $\int \frac{1}{x}\,dx = \ln|x| + C$.

Step 2: Analyze integral (A): $\displaystyle\int \frac{1}{x}\,dx$.
This integral is directly equal to $\ln|x| + C$. True.

Step 3: Analyze integral (B): $\displaystyle\int \frac{\sec^2 x}{\tan x}\,dx$.
Let $u = \tan x$. Then $du = \sec^2 x\,dx$.
The integral becomes $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C = \ln|\tan x| + C$.
This is not $\ln|x| + C$. False.

Step 4: Analyze integral (C): $\displaystyle\int \frac{2}{x}\,dx$.
This integral can be written as $2\displaystyle\int \frac{1}{x}\,dx = 2(\ln|x| + C) = 2\ln|x| + C$.
Since $2\ln|x| = \ln(x^2)$, this is $\ln(x^2) + C$, not $\ln|x| + C$. False.

Step 5: Analyze integral (D): $\displaystyle\int \frac{1}{|x|}\,dx$.
The integral of $\frac{1}{|x|}$ is $\ln|x| + C$ for $x > 0$ and $-\ln|x| + C$ for $x < 0$. More precisely, the integral is $\text{sgn}(x)\ln|x| + C$. It is not simply $\ln|x| + C$ over all intervals. However, in some contexts, the absolute value might be handled differently, but generally, this is not considered equal to $\ln|x| + C$.

Let's re-evaluate (D). The derivative of $\ln|x|$ is $\frac{1}{x}$. The integrand is $\frac{1}{|x|}$.
If $x > 0$, $\frac{1}{|x|} = \frac{1}{x}$, integral is $\ln x + C = \ln|x| + C$.
If $x < 0$, $\frac{1}{|x|} = \frac{1}{-x}$, integral is $\int \frac{1}{-x}dx = -\ln|-x| + C = -\ln(-x) + C$. Since $|x| = -x$ for $x < 0$, this is $-\ln|x| + C$.
So, $\int \frac{1}{|x|}dx$ is not universally $\ln|x| + C$.

Step 6: Identify the integrals equal to $\ln|x| + C$.
Only integral (A) is definitively equal to $\ln|x| + C$.

Final Answer:

$$\boxed{\int \frac{1}{x}\,dx}$$

Answer: (A)

Go Back to Question 69

Q70.

Solution

Concept: A diagonal matrix is a square matrix where all the elements outside the main diagonal are zero.

Solution: Step 1: Definition of a diagonal matrix.
A diagonal matrix is a square matrix where $a_{ij} = 0$ for all $i \neq j$.

Step 2: Analyze matrix (A): $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.
The elements outside the main diagonal ($a_{12} = 0, a_{21} = 0$) are zero. This is a diagonal matrix.

Step 3: Analyze matrix (B): $\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$.
The elements outside the main diagonal ($a_{12} = 0, a_{21} = 0$) are zero. This is a diagonal matrix (specifically, a scalar matrix).

Step 4: Analyze matrix (C): $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.
The element $a_{12} = 2$ is non-zero. This is an upper triangular matrix, not a diagonal matrix.

Step 5: Analyze matrix (D): $\begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$.
The element $a_{21} = 1$ is non-zero. This is a lower triangular matrix, not a diagonal matrix.

Step 6: Identify the diagonal matrices.
Matrices (A) and (B) are diagonal matrices.

Final Answer:

$$\boxed{\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}}$$

Answer: (A,B)

Go Back to Question 70

Q71.

Solution

Concept: Important properties of determinants:

- Interchanging two rows changes the sign of the determinant.
- If two rows are equal, determinant becomes zero.
- Determinant of the identity matrix is 1.
- Determinant of a triangular matrix is the product of diagonal elements.

Solution: Step 1: Check statement (A). Interchanging two rows changes the sign of the determinant. Hence, (A) is true.
Step 2: Check statement (B). If two rows are equal, the determinant is zero. Hence, (B) is true.
Step 3: Check statement (C). The determinant of the identity matrix is:

$$|I| = 1$$

So the statement is false.
Step 4: Check statement (D). For a triangular matrix, determinant equals the product of diagonal entries. Hence, (D) is true. Therefore, statements (A), (B), and (D) are correct.
Final Answer:

Interchanging two rows changes the sign of determinant

If two rows are equal, determinant is zero

Determinant of a triangular matrix is product of diagonal elements

Answer: (A,B,D)
Go Back to Question 71

Q72.

Solution

Concept: A complex number:

$$z = a + bi$$

is purely imaginary if:

$$a = 0 \quad \text{and} \quad b \neq 0$$

Solution: Step 1: Check (A):

$$3i = 0 + 3i$$

Real part is 0. Hence, purely imaginary.
Step 2: Check (B):

$$-5i = 0 - 5i$$

Real part is 0. Hence, purely imaginary.
Step 3: Check (C):

$$2 + i$$

Real part is $2 \neq 0$. Hence, not purely imaginary.
Step 4: Check (D):

$$-i = 0 - 1i$$

Real part is 0. Hence, purely imaginary.
Therefore, options (A), (B), and (D) are purely imaginary numbers.
Final Answer:

$$\boxed{3i, \quad -5i, \quad -i}$$

Answer: (A,B,D)

Go Back to Question 72

Q73.

Solution

Concept: A standard deck contains 52 cards. Each rank (King, Queen, Ace, etc.) appears 4 times, once in each suit.

Solution: Step 1: Total number of cards:

$$52$$

Step 2: Check (A): Drawing a king. There are 4 kings in a deck.

$$P(\text{king}) = \frac{4}{52} = \frac{1}{13}$$

Hence, (A) is correct.
Step 3: Check (B): Drawing a queen. There are 4 queens in a deck.

$$P(\text{queen}) = \frac{4}{52} = \frac{1}{13}$$

Hence, (B) is correct.
Step 4: Check (C): Drawing a heart. There are 13 hearts in a deck.

$$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$$

This is not equal to $\frac{1}{13}$.
Step 5: Check (D): Drawing an ace. There are 4 aces in a deck.

$$P(\text{ace}) = \frac{4}{52} = \frac{1}{13}$$

Hence, (D) is correct. Therefore, options (A), (B), and (D) have probability $\frac{1}{13}$.

Final Answer:

$$\boxed{\begin{array}{c}\text{Drawing a king}\\ \text{Drawing a queen}\\ \text{Drawing an ace}\end{array}}$$

Answer: (A,B,D)

Go Back to Question 73

Q74.

Solution

Concept: An equation of a straight line is linear in x and y. General form:

$$Ax + By + C = 0$$

Solution: Step 1: Check (A):

$$2x + 3y = 6$$

This equation is linear in x and y. Hence, it represents a straight line.

Step 2: Check (B):

$$x^2 + y^2 = 1$$

This equation contains squared terms and represents a circle, not a straight line.

Step 3: Check (C):

$$y = mx + c$$

This is the slope-intercept form of a straight line.

Step 4: Check (D):

$$x - y + 5 = 0$$

This is also a linear equation and represents a straight line.
Therefore, options (A), (C), and (D) represent straight lines.

Final Answer:

$$\boxed{\begin{array}{c} 2x + 3y = 6 \\ y = mx + c \\ x - y + 5 = 0 \end{array}}$$

Answer: (A,C,D)

Go Back to Question 74

Q75.

Solution

Concept: A periodic function repeats its values at regular intervals. The smallest such interval is called the period. We need to identify functions whose fundamental period is 2π.

Solution: Step 1: Definition of a periodic function with period P.
A function $f(x)$ is periodic with period P if $f(x+P) = f(x)$ for all x, and P is the smallest positive value for which this holds.

Step 2: Analyze function (A): $\sin x$.
The sine function repeats every 2π. $\sin(x+2\pi) = \sin x$. The fundamental period is 2π. Period is 2π.

Step 3: Analyze function (B): $\cos x$.
The cosine function also repeats every 2π. $\cos(x+2\pi) = \cos x$. The fundamental period is 2π. Period is 2π.

Step 4: Analyze function (C): $\tan x$.
The tangent function repeats every π. $\tan(x+\pi) = \tan x$. The fundamental period is π. Period is not 2π.

Step 5: Analyze function (D): $\sec x$.
The secant function is the reciprocal of cosine, $\sec x = 1/\cos x$. Since $\cos x$ has a period of 2π, $\sec x$ also has a period of 2π. $\sec(x+2\pi) = 1/\cos(x+2\pi) = 1/\cos x = \sec x$. The fundamental period is 2π. Period is 2π.

Step 6: Identify functions with period 2π.
Functions (A), (B), and (D) are periodic with a period of 2π.

Final Answer:

$$\boxed{\sin x, \quad \cos x, \quad \sec x}$$

Answer: (A,B,D)

Go Back to Question 75

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	B	5	A
6	A	7	A	8	C	9	A	10	C
11	A	12	B	13	B	14	B	15	B
16	A	17	B	18	A	19	A	20	C
21	A	22	C	23	B	24	B	25	B
26	B	27	C	28	B	29	B	30	A
31	B	32	B	33	A	34	A	35	A
36	B	37	C	38	B	39	B	40	B
41	A	42	A	43	C	44	A	45	C
46	C	47	C	48	C	49	D	50	B
51	C	52	A	53	B	54	A	55	C
56	B	57	C	58	A	59	A	60	B
61	C	62	C	63	B	64	B	65	B
66	A,C,D	67	A,B	68	A,B,C	69	A	70	A,B
71	A,B,D	72	A,B,D	73	A,B,D	74	A,C,D	75	A,B,D