WBJEE Mathematics Sample Paper-9

Duration: 120 Minutes **Maximum Marks: 100**

Instructions

- This paper contains **75** Multiple Choice Questions divided into **3 Categories**.
- **Section A (Q1–Q50):** Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- **Section B (Q51–Q65):** Each correct answer carries **+2 marks**. Incorrect answer: **−0.5** marks. Only **one** correct option.
- **Section C (Q66–Q75):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 50 Questions × 1 Mark Each
(Negative Marking: −0.25) [Single Correct]

Q1. The value of $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$ is:

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

Q2. If $f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$ is continuous at $x = 3$, then k equals:

(A) 3

(B) 6

(C) 9

(D) 12

Q3. If $y = x^x$, then $\frac{dy}{dx}$ equals:

(A) x^x

(B) $x^x(1 + \ln x)$

(C) $\ln x$

(D) $1 + \ln x$

Q4. The maximum value of $4x - x^2$ is:

(A) 2

(B) 4

(C) 6

(D) 8

Q5. The value of $\int \frac{1}{x}\, dx$ is:

(A) $\ln x + C$

(B) $\frac{1}{x} + C$

(C) $x + C$

(D) $e^x + C$

Q6. The value of $\int_0^1 (2x + 3)\, dx$ is:

(A) 2

(B) 3

(C) 4

(D) 5

Q7. The solution of $\frac{dy}{dx} = 4x^3$ is:

(A) $x^4 + C$

(B) $4x + C$

(C) $x^3 + C$

(D) $4x^4 + C$

Q8. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then $\det(A)$ equals:

(A) -3

(B) -1

(C) 1

(D) 3

Q9. The value of $\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix}$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q10. If $z = 1 - i$, then $|z|$ equals:

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) $2\sqrt{2}$

Q11. The discriminant of $x^2 - 6x + 9 = 0$ is:

(A) 0

(B) 6

(C) 9

(D) 12

Q12. The sum of first 20 natural numbers is:

(A) 190

(B) 200

(C) 210

(D) 220

Q13. A die is thrown once. The probability of getting a prime number is:

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{5}{6}$

Q14. The coefficient of x^3 in $(1 + x)^5$ is:

(A) 5

(B) 10

(C) 15

(D) 20

Q15. The value of $\sin 30°$ is:

(A) 0

(B) $\frac{1}{2}$

(C) $\frac{\sqrt{2}}{2}$

(D) 1

Q16. The principal value of $\cos^{-1}(1)$ is:

(A) 0

(B) $\frac{\pi}{2}$

(C) π

(D) 2π

Q17. The equation of the line passing through $(0, 2)$ and parallel to x-axis is:

(A) $x = 2$

(B) $y = 2$

(C) $x + y = 2$

(D) $x - y = 2$

Q18. The centre of the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ is:

(A) $(-2, 3)$

(B) $(2, -3)$

(C) $(4, -6)$

(D) $(0, 0)$

Q19. The directrix of the parabola $y^2 = 4ax$ is:

(A) $x = a$

(B) $x = -a$

(C) $y = a$

(D) $y = -a$

Q20. The length of minor axis of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ is:

(A) 3

(B) 5

(C) 6

(D) 10

Q21. The eccentricity of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ is:

(A) $\frac{5}{4}$

(B) $\frac{3}{2}$

(C) $\frac{7}{4}$

(D) 2

Q22. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, then $|\vec{a}|$ equals:

(A) 2

(B) 3

(C) 4

(D) 5

Q23. The distance between $(1, 1, 1)$ and $(2, 3, 6)$ is:

(A) $\sqrt{30}$

(B) $\sqrt{35}$

(C) $\sqrt{29}$

(D) $\sqrt{26}$

Q24. The value of ${}^{7}C_3$ is:

(A) 21

(B) 28

(C) 35

(D) 42

Q25. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$, then $A - B$ is:

(A) $\{2, 3\}$

(B) $\{1\}$

(C) $\{4\}$

(D) ϕ

Q26. If $f(x) = x^2 - 1$, then $f(-2)$ equals:

(A) 3

(B) -3

(C) 5

(D) -5

Q27. The value of $\cos 45°$ is:

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{2}}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) 1

Q28. The value of $\lim_{x \to 0} \frac{\tan x}{x}$ is:

(A) 0

(B) 1

(C) 2

(D) undefined

Q29. If $y = e^x$, then $\frac{dy}{dx}$ equals:

(A) e^x

(B) xe^x

(C) 1

(D) $\ln x$

Q30. The value of $\int_0^1 x^2\, dx$ is:

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{2}{3}$

Q31. A card is drawn from a standard deck. The probability of getting a king is:

(A) $\frac{1}{13}$

(B) $\frac{1}{26}$

(C) $\frac{1}{4}$

(D) $\frac{4}{13}$

Q32. The common ratio of the GP $3, 6, 12, \ldots$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Q33. The value of i^{15} is:

(A) 1

(B) -1

(C) i

(D) $-i$

Q34. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, then A^2 equals:

(A) I

(B) $-I$

(C) A

(D) 0

Q35. If $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 5$, then $\begin{vmatrix} d & b \\ c & a \end{vmatrix}$ equals:

(A) -5

(B) 5

(C) 0

(D) 10

Q36. The angle made by the line $y = \sqrt{3}x$ with positive x-axis is:

(A) $30°$

(B) $45°$

(C) $60°$

(D) $90°$

Q37. The equation $x^2 + y^2 = 16$ represents a circle of area:

(A) 4π

(B) 8π

(C) 16π

(D) 32π

Q38. The focus of the parabola $x^2 = 8y$ is:

(A) $(0, 2)$

(B) $(2, 0)$

(C) $(0, -2)$

(D) $(-2, 0)$

Q39. The eccentricity of the ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$ is:

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{\sqrt{11}}{6}$

(D) $\frac{5}{6}$

Q40. The vertices of the hyperbola $\frac{x^2}{25} - \frac{y^2}{9} = 1$ are:

(A) $(\pm 3, 0)$

(B) $(0, \pm 5)$

(C) $(\pm 5, 0)$

(D) $(0, \pm 3)$

Q41. If $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} - \hat{j}$, then $\vec{a} \cdot \vec{b}$ equals:

(A) -1

(B) 0

(C) 1

(D) 2

Q42. The equation of yz-plane is:

(A) $x = 0$

(B) $y = 0$

(C) $z = 0$

(D) $x + y + z = 0$

Q43. The number of permutations of the word "MATH" is:

(A) 4

(B) 12

(C) 24

(D) 48

Q44. The constant term in $(x + \frac{1}{x})^4$ is:

(A) 4

(B) 6

(C) 8

(D) 16

Q45. If two coins are tossed, the probability of getting exactly one head is:

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) 1

Q46. The sum of first 5 odd numbers is:

(A) 15

(B) 20

(C) 25

(D) 30

Q47. The value of $\lim_{x\to\infty} \frac{x^2+1}{2x^2+3}$ is:

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) 2

Q48. If $y = \ln x$, then $\frac{dy}{dx}$ equals:

(A) x

(B) $\ln x$

(C) $\dfrac{1}{x}$

(D) e^x

Q49. The value of $\displaystyle\int \sec^2 x\, dx$ is:

(A) $\sec x + C$

(B) $\tan x + C$

(C) $\cot x + C$

(D) $\sin x + C$

Q50. The degree of $\left(\dfrac{d^2y}{dx^2}\right)^3 + \left(\dfrac{dy}{dx}\right)^2 + y = 0$ is:

(A) 1

(B) 2

(C) 3

(D) 4

Section–B — 15 Questions × 1 Mark Each
(Negative Marking: −0.5) [Single Correct]

Q51. If $n(A) = 20$, $n(B) = 15$, and $n(A \cap B) = 5$, then $n(A \cup B)$ is:

(A) 25

(B) 30

(C) 35

(D) 40

Q52. The domain of $f(x) = \sqrt{x-2}$ is:

(A) $x > 2$

(B) $x \geq 2$

(C) $x < 2$

(D) all real x

Q53. The value of $\tan 60^\circ$ is:

(A) 1

(B) $\sqrt{2}$

(C) $\sqrt{3}$

(D) 2

Q54. The principal value of $\tan^{-1}(0)$ is:

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Q55. The intercept made by the line $2x + 5y = 10$ on y-axis is:

(A) 2

(B) 5

(C) 10

(D) 15

Q56. The length of the diameter of the circle $x^2 + y^2 = 49$ is:

(A) 7

(B) 14

(C) 21

(D) 49

Q57. The vertex of the parabola $x^2 = -4y$ is:

(A) $(0, 0)$

(B) $(1, 0)$

(C) $(0, 1)$

(D) $(-1, 0)$

Q58. The major axis of the ellipse $\frac{x^2}{49} + \frac{y^2}{25} = 1$ lies along:

(A) x-axis

(B) y-axis

(C) line $y = x$

(D) none

Q59. The centre of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ is:

(A) $(0, 0)$

(B) $(3, 2)$

(C) $(9, 4)$

(D) $(1, 1)$

Q60. If $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = 4\hat{i} - 2\hat{j}$, then $\vec{a} + \vec{b}$ equals:

(A) $6\hat{i} + \hat{j}$

(B) $2\hat{i} + \hat{j}$

(C) $6\hat{i} + 5\hat{j}$

(D) $8\hat{i} + \hat{j}$

Q61. The distance of the point $(1, 2, 2)$ from origin is:

(A) 2

(B) 3

(C) 4

(D) 5

Q62. The value of ${}^8P_2$ is:

(A) 48

(B) 56

(C) 64

(D) 72

Q63. The 15th term of the AP $2, 5, 8, \ldots$ is:

(A) 42

(B) 44

(C) 45

(D) 47

Q64. A bag contains 3 red and 2 blue balls. The probability of drawing a blue ball is:

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{3}{5}$

(D) $\frac{4}{5}$

Q65. If $z = 2 + 2i$, then $\arg(z)$ is:

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Section C — 10 Questions × 2 Marks Each (No Negative Marking) [One or More Correct]

Q66. Which of the following statements are true for sets A and B?

(A) $A \cup B = B \cup A$

(B) $A \cap B = B \cap A$

(C) $A - B = B - A$

(D) $A \cup \phi = A$

Q67. Which of the following functions are one-one?

(A) $f(x) = x$

(B) $f(x) = x^3$

(C) $f(x) = x^2$

(D) $f(x) = e^x$

Q68. Which of the following limits exist?

(A) $\lim_{x \to 0} \sin x$

(B) $\lim_{x \to 0} \frac{1}{x}$

(C) $\lim_{x \to 2} (x^2 + 1)$

(D) $\lim_{x \to 0} |x|$

Q69. Which of the following derivatives are correct?

(A) $\frac{d}{dx}(\sin x) = \cos x$

(B) $\frac{d}{dx}(\cos x) = -\sin x$

(C) $\frac{d}{dx}(e^x) = e^x$

(D) $\frac{d}{dx}(\ln x) = x$

Q70. For the function $f(x) = x^3 - 3x$, which of the following statements are true?

(A) $f'(x) = 3x^2 - 3$

(B) Critical points occur at $x = \pm 1$

(C) The function has a local maximum at $x = 1$

(D) The function has a local minimum at $x = 1$

Q71. Which of the following integrals are correct?

(A) $\int \cos x \, dx = \sin x + C$

(B) $\int e^x \, dx = e^x + C$

(C) $\int \frac{1}{x} \, dx = \ln |x| + C$

(D) $\int \sin x \, dx = \cos x + C$

Q72. Which of the following matrices are singular?

(A) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

Q73. Which of the following are valid probability values?

(A) 0

(B) $\frac{3}{4}$

(C) 1

(D) $\frac{5}{4}$

Q74. Which of the following points lie on the circle $x^2 + y^2 = 25$?

(A) $(3, 4)$

(B) $(4, 3)$

(C) $(5, 0)$

(D) $(2, 2)$

Q75. Which of the following statements about vectors are true?

(A) $\hat{i} \cdot \hat{j} = 0$

(B) $\hat{i} \times \hat{i} = 0$

(C) $|\hat{i}| = 1$

(D) $\hat{i} + \hat{j}$ is a unit vector

Detailed Solutions

Q1.

Solution

Concept: The limit

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$$

is of the indeterminate form $\frac{0}{0}$.
Using the Taylor expansion:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

Solution: We need to evaluate:

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2}$$

Step 1: Substitute the Taylor expansion of e^x.

$$= \lim_{x\to 0} \frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) - 1 - x}{x^2}$$

Step 2: Simplify the numerator.

$$= \lim_{x\to 0} \frac{\frac{x^2}{2!} + \frac{x^3}{3!} + \cdots}{x^2}$$

Step 3: Divide by x^2.

$$= \lim_{x\to 0} \left(\frac{1}{2} + \frac{x}{3!} + \cdots\right)$$

Step 4: Evaluate the limit.
As $x \to 0$, all remaining terms vanish.

$$= \frac{1}{2}$$

Final Answer:

$$\boxed{\frac{1}{2}}$$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: For a function to be continuous at a point $x = c$, the following condition must hold: $\lim_{x\to c} f(x) = f(c)$.
For a piecewise function, we need to evaluate the limit of the function as x approaches the point from the non-equal side and set it equal to the function's value at that point.

Solution: The function is defined as $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$. For the function to be continuous at $x = 3$, we must have $\lim_{x\to 3} f(x) = f(3)$.
Step 1: Evaluate $f(3)$. From the definition, $f(3) = k$.
Step 2: Evaluate the limit of $f(x)$ as x approaches 3. For $x \neq 3$, $f(x) = \frac{x^2-9}{x-3}$.

$$\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3}$$

The expression $\frac{x^2-9}{x-3}$ is an indeterminate form $\frac{0}{0}$ as $x \to 3$. We can factor the numerator: $x^2 - 9 = (x-3)(x+3)$.

$$\lim_{x\to 3} \frac{(x-3)(x+3)}{x-3}$$

Since $x \to 3$, $x \neq 3$, so we can cancel the $(x-3)$ term:

$$= \lim_{x\to 3}(x+3)$$

Now, substitute $x = 3$:

$$= 3 + 3 = 6$$

Step 3: Apply the continuity condition. For continuity at $x = 3$, we must have $\lim_{x\to 3} f(x) = f(3)$.

$$6 = k$$

Therefore, k must be 6 for the function to be continuous at $x = 3$.
Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: To differentiate a function of the form $y = x^x$ (or more generally $y = [f(x)]^{g(x)}$), we use logarithmic differentiation. This involves taking the natural logarithm of both sides, differentiating implicitly, and then solving for $\frac{dy}{dx}$.

Solution: We need to find the derivative of $y = x^x$.
Step 1: Take the natural logarithm of both sides.

$$\ln y = \ln(x^x)$$

Using the logarithm property $\ln(a^b) = b \ln a$:

$$\ln y = x \ln x$$

Step 2: Differentiate both sides with respect to x. Differentiate the left side using implicit differentiation and the right side using the product rule. Derivative of $\ln y$ with respect to x is $\frac{1}{y}\frac{dy}{dx}$. Derivative of $x \ln x$ using the product rule $(uv)' = u'v + uv'$: Let $u = x$ and $v = \ln x$. Then $u' = 1$ and $v' = \frac{1}{x}$. Derivative of $x \ln x = (1)(\ln x) + (x)(\frac{1}{x}) = \ln x + 1$.
So, the differentiated equation is:

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$$

Step 3: Solve for $\frac{dy}{dx}$. Multiply both sides by y:

$$\frac{dy}{dx} = y(\ln x + 1)$$

Step 4: Substitute back $y = x^x$.

$$\frac{dy}{dx} = x^x(\ln x + 1)$$

The derivative of $y = x^x$ is $x^x(1 + \ln x)$.
Final Answer: $\boxed{x^x(1 + \ln x)}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: A quadratic function $f(x) = ax^2 + bx + c$ attains its maximum or minimum value at the vertex of the parabola.
If $a < 0$, the parabola opens downward and the function has a maximum value. The x-coordinate of the vertex is:

$$x = \frac{-b}{2a}$$

Solution: We need to find the maximum value of:

$$f(x) = 4x - x^2$$

Rewriting:

$$f(x) = -x^2 + 4x$$

Here,

$$a = -1, \qquad b = 4$$

Since $a < 0$, the function has a maximum value. Step 1: Find the x-coordinate of the vertex.

$$x = \frac{-b}{2a}$$

$$x = \frac{-4}{2(-1)}$$

$$x = 2$$

Step 2: Find the maximum value.

$$f(2) = 4(2) - (2)^2$$

$$f(2) = 8 - 4$$

$$f(2) = 4$$

Therefore, the maximum value of the function is:

$$4$$

Final Answer:

$$\boxed{4}$$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: The indefinite integral of $\frac{1}{x}$ is given by the formula:

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

where C is the constant of integration. The absolute value $|x|$ is used because the domain of $\ln x$ is $x > 0$, but the function $\frac{1}{x}$ is defined for all $x \neq 0$.

Solution: We need to find the indefinite integral of $\frac{1}{x}$.

Step 1: Apply the integration rule for $\frac{1}{x}$.

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

Here, C is the constant of integration.
The value of $\int \frac{1}{x}\,dx$ is $\ln|x| + C$. The option provided is $\ln x + C$, which implies $x > 0$. Assuming the domain is restricted to positive x, this is correct.
Final Answer: $\boxed{\ln x + C}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: To evaluate a definite integral $\int_a^b f(x)dx$, we first find the antiderivative of $f(x)$, say $F(x)$, and then calculate $F(b) - F(a)$ (Fundamental Theorem of Calculus). The power rule for integration is $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.

Solution: We need to evaluate the definite integral $\int_0^1 (2x+3)\,dx$.
Step 1: Find the antiderivative of $f(x) = 2x + 3$. We integrate each term using the power rule:

$$\int (2x+3)\,dx = \int 2x\,dx + \int 3\,dx$$

$$= 2\int x^1\,dx + 3\int 1\,dx$$

$$= 2\left(\frac{x^{1+1}}{1+1}\right) + 3\left(\frac{x^{0+1}}{0+1}\right) + C$$

$$= 2\left(\frac{x^2}{2}\right) + 3(x) + C$$

$$= x^2 + 3x + C$$

Let $F(x) = x^2 + 3x$.
Step 2: Apply the Fundamental Theorem of Calculus.

$$\int_0^1 (2x+3)\,dx = F(1) - F(0)$$

Step 3: Evaluate $F(x)$ at the upper and lower limits. At the upper limit $x = 1$:

$$F(1) = (1)^2 + 3(1) = 1 + 3 = 4$$

At the lower limit $x = 0$:

$$F(0) = (0)^2 + 3(0) = 0 + 0 = 0$$

Step 4: Subtract the value at the lower limit from the value at the upper limit.

$$\int_0^1 (2x+3)\,dx = 4 - 0 = 4$$

The value of the definite integral is 4.
Final Answer: $\boxed{4}$

Answer: (C)

Go Back to Question 6

Q7.

Solution

Concept: The solution to a differential equation is found by integrating both sides with respect to their respective variables. The power rule for integration is $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.

Solution: The given differential equation is $\frac{dy}{dx} = 4x^3$.
Step 1: Separate the variables. Multiply both sides by dx:

$$dy = 4x^3\, dx$$

Step 2: Integrate both sides. Integrate the left side with respect to y and the right side with respect to x:

$$\int dy = \int 4x^3\, dx$$

Step 3: Perform the integration. The integral of dy is y. The integral of $4x^3$ is found using the power rule:

$$\int 4x^3\, dx = 4\int x^3\, dx = 4\left(\frac{x^{3+1}}{3+1}\right) + C$$

$$= 4\left(\frac{x^4}{4}\right) + C$$

$$= x^4 + C$$

Here, C is the constant of integration.
Step 4: Combine the results.

$$y = x^4 + C$$

The solution to the differential equation is $y = x^4 + C$.
Final Answer: $\boxed{x^4 + C}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: The determinant of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by the formula $\det(A) = ad - bc$.

Solution: We are given the matrix $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Here, $a = 1$, $b = 2$, $c = 2$, and $d = 1$.

Step 1: Apply the formula for the determinant of a 2×2 matrix.

$$\det(A) = ad - bc$$

Step 2: Substitute the values from the matrix into the formula.

$$\det(A) = (1)(1) - (2)(2)$$

Step 3: Perform the multiplication and subtraction.

$$\det(A) = 1 - 4$$

$$\det(A) = -3$$

The determinant of the matrix A is -3.

Final Answer: $\boxed{-3}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: The determinant of a 2×2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is calculated as $ad - bc$.

Solution: We need to calculate the value of the determinant $\begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix}$. Here, $a = 2$, $b = 5$, $c = 1$, and $d = 3$.

Step 1: Apply the formula for the determinant of a 2×2 matrix.

$$\det = ad - bc$$

Step 2: Substitute the given values into the formula.

$$\det = (2)(3) - (5)(1)$$

Step 3: Perform the calculations.

$$\det = 6 - 5$$

$$\det = 1$$

The value of the determinant is 1.

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept: For a complex number $z = a + bi$, the modulus (or magnitude) of z is denoted by $|z|$ and is calculated using the formula:

$$|z| = \sqrt{a^2 + b^2}$$

where a is the real part and b is the imaginary part.

Solution: We are given the complex number $z = 1 - i$. This can be written as $z = 1 + (-1)i$. Here, the real part is $a = 1$ and the imaginary part is $b = -1$.
Step 1: Identify the real and imaginary parts. Real part, $a = 1$. Imaginary part, $b = -1$.
Step 2: Apply the formula for the modulus of a complex number.

$$|z| = \sqrt{a^2 + b^2}$$

Step 3: Substitute the values of a and b.

$$|z| = \sqrt{(1)^2 + (-1)^2}$$

Step 4: Perform the calculations.

$$|z| = \sqrt{1 + 1}$$

$$|z| = \sqrt{2}$$

The modulus of the complex number $z = 1 - i$ is $\sqrt{2}$.
Final Answer: $\boxed{\sqrt{2}}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: The discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by the formula $\Delta = b^2 - 4ac$. The value of the discriminant determines the nature of the roots:
- If $\Delta > 0$, there are two distinct real roots.
- If $\Delta = 0$, there is exactly one real root (a repeated root).
- If $\Delta < 0$, there are two complex conjugate roots.

Solution: We need to find the discriminant of the quadratic equation $x^2 - 6x + 9 = 0$. Here, $a = 1$, $b = -6$, and $c = 9$.
Step 1: Apply the discriminant formula.

$$\Delta = b^2 - 4ac$$

Step 2: Substitute the values of a, b, and c.

$$\Delta = (-6)^2 - 4(1)(9)$$

Step 3: Perform the calculations.

$$\Delta = 36 - 36$$

$$\Delta = 0$$

The discriminant of the quadratic equation $x^2 - 6x + 9 = 0$ is 0.
Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept: The sum of the first n natural numbers (1, 2, 3, ..., n) is given by the formula:

$$S_n = \frac{n(n+1)}{2}$$

Solution: We need to find the sum of the first 20 natural numbers. Here, $n = 20$.

Step 1: Use the formula for the sum of the first n natural numbers.

$$S_{20} = \frac{20(20+1)}{2}$$

Step 2: Perform the calculations.

$$S_{20} = \frac{20(21)}{2}$$

$$S_{20} = \frac{420}{2}$$

$$S_{20} = 210$$

The sum of the first 20 natural numbers is 210.

Final Answer: $\boxed{210}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: Probability is the ratio of favorable outcomes to the total possible outcomes. When a standard six-sided die is thrown once, the possible outcomes are $\{1, 2, 3, 4, 5, 6\}$. A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

Solution: We need to find the probability of getting a prime number when a die is thrown once.
Step 1: Determine the total number of possible outcomes. When a standard six-sided die is thrown, the possible outcomes are $\{1, 2, 3, 4, 5, 6\}$. So, the total number of possible outcomes is 6.
Step 2: Identify the prime numbers among the outcomes. The numbers from 1 to 6 are: - 1 is not prime. - 2 is prime (divisors are 1, 2). - 3 is prime (divisors are 1, 3). - 4 is not prime (divisors are 1, 2, 4). - 5 is prime (divisors are 1, 5). - 6 is not prime (divisors are 1, 2, 3, 6). The prime numbers in the set $\{1, 2, 3, 4, 5, 6\}$ are $\{2, 3, 5\}$.
Step 3: Determine the number of favorable outcomes. The number of prime numbers is 3.
Step 4: Calculate the probability.

$$P(\text{getting a prime number}) = \frac{\text{Number of prime numbers}}{\text{Total number of outcomes}}$$

$$P(\text{getting a prime number}) = \frac{3}{6}$$

Step 5: Simplify the fraction.

$$\frac{3}{6} = \frac{1}{2}$$

The probability of getting a prime number when a die is thrown once is $\frac{1}{2}$.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: The Binomial Theorem states that for any non-negative integer n, the expansion of $(x+y)^n$ is given by:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

where $\binom{n}{k}$ is the binomial coefficient, calculated as $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.
In this problem, we are expanding $(1+x)^5$. This means x in the formula is 1, and y in the formula is x. The power is $n = 5$.
The general term in the expansion of $(1+x)^5$ is:

$$T_{k+1} = \binom{n}{k} x^{n-k} y^k = \binom{5}{k} (1)^{5-k} (x)^k = \binom{5}{k} x^k$$

We are looking for the coefficient of x^3. This corresponds to the term where $k = 3$.

Solution: We need to find the coefficient of x^3 in the expansion of $(1+x)^5$.
Step 1: Use the Binomial Theorem for the expansion of $(1+x)^5$. The general term is $\binom{5}{k} x^k$.
Step 2: Identify the term that contains x^3. We need to find the value of k such that $x^k = x^3$. This means $k = 3$.
Step 3: Calculate the binomial coefficient for $k = 3$ and $n = 5$. The binomial coefficient is $\binom{5}{3}$.

$$\binom{5}{3} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!}$$

$$\binom{5}{3} = \frac{5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(2 \times 1)}$$

$$\binom{5}{3} = \frac{5 \times 4}{2 \times 1}$$

$$\binom{5}{3} = \frac{20}{2}$$

$$\binom{5}{3} = 10$$

Step 4: The coefficient of x^3 is the calculated binomial coefficient. The coefficient of x^3 in the expansion of $(1+x)^5$ is 10.
Final Answer: $\boxed{10}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: The sine of an angle in a right triangle is defined as:

$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

The angle 30° is a standard angle, and its trigonometric values are obtained from a 30°-60°-90° triangle.
In such a triangle:

$$\text{Side opposite } 30^\circ = \frac{1}{2} \times \text{Hypotenuse}$$

Solution: We need to find:

$$\sin 30^\circ$$

Step 1: Consider a 30°-60°-90° triangle.
Let the hypotenuse be:

$$2k$$

Then the side opposite the angle 30° becomes:

$$k$$

Step 2: Apply the definition of sine.

$$\sin 30^\circ = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

Substituting the values:

$$\sin 30^\circ = \frac{k}{2k}$$

Step 3: Simplify.

$$\sin 30^\circ = \frac{1}{2}$$

Therefore, the value of $\sin 30^\circ$ is:

$$\frac{1}{2}$$

Final Answer:

$$\boxed{\frac{1}{2}}$$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept: The inverse cosine function $\cos^{-1}(x)$ gives the angle whose cosine is x.
The principal value range of $\cos^{-1}(x)$ is:

$$[0, \pi]$$

This means the answer must lie between 0 and π.

Solution: We need to find:

$$\cos^{-1}(1)$$

Step 1: Recall the angle whose cosine is 1.
We know:

$$\cos 0 = 1$$

Step 2: Check the principal value range.
Since:

$$0 \in [0, \pi]$$

the angle satisfies the required range.
Therefore,

$$\cos^{-1}(1) = 0$$

Final Answer:

$$\boxed{0}$$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: A line parallel to the x-axis has the same y-coordinate at every point.
Hence, the equation of a line parallel to the x-axis is always of the form:

$$y = c$$

where c is a constant.

Solution: We need to find the equation of the line passing through:

$$(0, 2)$$

and parallel to the x-axis.
Step 1: Use the property of lines parallel to the x-axis.
Since the line is parallel to the x-axis, its y-coordinate remains constant.
Thus, the equation will be:

$$y = c$$

Step 2: Use the given point.
The point $(0, 2)$ lies on the line.
So the constant value of y is:

$$2$$

Therefore, the equation becomes:

$$y = 2$$

Step 3: Verify.
Any point on this line has coordinates of the form:

$$(x, 2)$$

which confirms that the y-coordinate is always 2.
Final Answer:

$$\boxed{y = 2}$$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: The general equation of a circle is:

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Its centre is:

$$(-g, -f)$$

Solution: The given equation is:

$$x^2 + y^2 + 4x - 6y + 9 = 0$$

Step 1: Compare with the general form.

$$2g = 4 \Rightarrow g = 2$$

$$2f = -6 \Rightarrow f = -3$$

Step 2: Find the centre.

$$\text{Centre} = (-g, -f)$$

$$= (-2, -(-3))$$

$$= (-2, 3)$$

Therefore, the centre of the circle is:

$$(-2, 3)$$

Final Answer:

$$\boxed{(-2, 3)}$$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: The standard equation of a parabola opening towards the right is:

$$y^2 = 4ax$$

For this parabola:

- Vertex is $(0, 0)$
- Focus is $(a, 0)$
- Directrix is $x = -a$

Solution: We are given the parabola:

$$y^2 = 4ax$$

Step 1: Identify the standard form.
The equation already matches the standard form:

$$y^2 = 4ax$$

So the parabola opens towards the right.
Step 2: Recall the equation of the directrix.
For a parabola of the form:

$$y^2 = 4ax$$

the directrix is:

$$x = -a$$

Therefore, the directrix of the parabola is:

$$x = -a$$

Final Answer:

$$\boxed{x = -a}$$

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: The standard equation of an ellipse centred at the origin is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where:

$$a > b$$

The lengths of the axes are:

$$\text{Major axis} = 2a$$

$$\text{Minor axis} = 2b$$

Solution: The equation of the ellipse is:

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Step 1: Identify a^2 and b^2. Comparing with the standard form:

$$a^2 = 25, \qquad b^2 = 9$$

Step 2: Find the value of b.

$$b = \sqrt{9}$$

$$b = 3$$

Step 3: Calculate the length of the minor axis.

$$\text{Minor axis} = 2b$$

$$= 2 \times 3$$

$$= 6$$

Therefore, the length of the minor axis is:

$$6$$

Final Answer:

$$\boxed{6}$$

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept: The eccentricity e of a hyperbola in the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by the formula $e = \sqrt{1 + \frac{b^2}{a^2}}$. For a hyperbola with the transverse axis along the y-axis, $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, the formula for eccentricity is the same: $e = \sqrt{1 + \frac{b^2}{a^2}}$.

Solution: The equation of the hyperbola is given as $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

Step 1: Identify a^2 and b^2 from the equation. Comparing with the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, we have: $a^2 = 16 \implies a = \sqrt{16} = 4$ $b^2 = 9 \implies b = \sqrt{9} = 3$

Step 2: Use the formula for eccentricity of a hyperbola. The eccentricity e is given by:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

Step 3: Substitute the values of a^2 and b^2 into the formula.

$$e = \sqrt{1 + \frac{9}{16}}$$

Step 4: Perform the addition inside the square root.

$$e = \sqrt{\frac{16}{16} + \frac{9}{16}}$$

$$e = \sqrt{\frac{16 + 9}{16}}$$

$$e = \sqrt{\frac{25}{16}}$$

Step 5: Calculate the square root.

$$e = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$$

The eccentricity of the hyperbola is $\frac{5}{4}$.

Final Answer: $\boxed{\frac{5}{4}}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: The magnitude (or modulus) of a vector $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ is given by the formula:

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Solution: We are given the vector $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$. This can be written as $\vec{a} = 2\hat{i} + (-1)\hat{j} + 2\hat{k}$.
Step 1: Identify the components of the vector. The components are $a_1 = 2$, $a_2 = -1$, and $a_3 = 2$.
Step 2: Apply the formula for the magnitude of a vector.

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$|\vec{a}| = \sqrt{(2)^2 + (-1)^2 + (2)^2}$$

Step 3: Perform the calculations.

$$|\vec{a}| = \sqrt{4 + 1 + 4}$$

$$|\vec{a}| = \sqrt{9}$$

Step 4: Calculate the square root.

$$|\vec{a}| = 3$$

The magnitude of the vector $\vec{a}$ is 3.
Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept: The distance between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) in three-dimensional space is given by the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Solution: We need to find the distance between the points $P_1 = (1, 1, 1)$ and $P_2 = (2, 3, 6)$.
Step 1: Identify the coordinates of the two points. $(x_1, y_1, z_1) = (1, 1, 1)$ $(x_2, y_2, z_2) = (2, 3, 6)$
Step 2: Apply the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Substitute the coordinates:

$$d = \sqrt{(2 - 1)^2 + (3 - 1)^2 + (6 - 1)^2}$$

Step 3: Calculate the differences.

$$d = \sqrt{(1)^2 + (2)^2 + (5)^2}$$

Step 4: Square the differences.

$$d = \sqrt{1 + 4 + 25}$$

Step 5: Add the squared differences.

$$d = \sqrt{30}$$

The distance between the points $(1, 1, 1)$ and $(2, 3, 6)$ is $\sqrt{30}$.
Final Answer: $\boxed{\sqrt{30}}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: The number of combinations of n distinct objects taken r at a time, denoted by ${}^nC_r$ or $C(n, r)$ or $\binom{n}{r}$, is given by the formula:

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

This formula is used when the order of selection does not matter.

Solution: We need to calculate the value of ${}^7C_3$. Here, $n = 7$ (the total number of objects) and $r = 3$ (the number of objects to choose).
Step 1: Write down the formula for combinations.

$$ {}^nC_r = \frac{n!}{r!(n-r)!} $$

Step 2: Substitute the given values of n and r.

$$ {}^7C_3 = \frac{7!}{3!(7-3)!} $$

Step 3: Simplify the expression.

$$ {}^7C_3 = \frac{7!}{3!4!} $$

Step 4: Expand the factorials. $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ $3! = 3 \times 2 \times 1$ $4! = 4 \times 3 \times 2 \times 1$
Step 5: Calculate the value.

$$ {}^7C_3 = \frac{7 \times 6 \times 5 \times 4!}{(3 \times 2 \times 1) \times 4!} $$

Cancel out 4!:

$$ {}^7C_3 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} $$

$$ {}^7C_3 = \frac{7 \times 6 \times 5}{6} $$

Cancel out 6:

$$ {}^7C_3 = 7 \times 5 $$

$$ {}^7C_3 = 35 $$

The value of ${}^7C_3$ is 35.
Final Answer: $\boxed{35}$

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept: The set difference $A - B$ (or $A \setminus B$) is the set of elements that are in set A but not in set B.

Solution: We are given two sets: $A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$
We need to find $A - B$, which means we need to find the elements that are in set A but not in set B.
Step 1: List the elements of set A. Elements of A are 1, 2, and 3.
Step 2: Identify elements from set A that are also in set B. The elements 2 and 3 are present in both set A and set B.
Step 3: Remove the common elements from set A. Removing 2 and 3 from set A leaves only the element 1.

$$A - B = \{1\}$$

The set difference $A - B$ is $\{1\}$.
Final Answer: $\boxed{\{1\}}$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: To evaluate a function $f(x)$ at a specific value, substitute that value for x in the function's expression and simplify.

Solution: We are given the function $f(x) = x^2 - 1$. We need to find the value of $f(-2)$.
Step 1: Substitute the input value into the function's expression. Replace every instance of x in the expression $x^2 - 1$ with the value -2.

$$f(-2) = (-2)^2 - 1$$

Step 2: Perform the calculation.

$$f(-2) = 4 - 1$$

$$f(-2) = 3$$

The value of $f(-2)$ is 3.
Final Answer: $\boxed{3}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: The cosine of 45° is a standard trigonometric value. In a 45-45-90 right-angled triangle (an isosceles right-angled triangle), the two non-hypotenuse sides are equal. The cosine of an angle is the ratio of the adjacent side to the hypotenuse.

Solution: We need to find the value of $\cos 45°$.
Consider an isosceles right-angled triangle. Let the two equal sides (adjacent and opposite to 45° angles) have a length of 1 unit. Using the Pythagorean theorem, the hypotenuse h is: $h^2 = 1^2 + 1^2 = 1 + 1 = 2$ $h = \sqrt{2}$
Using the definition of cosine:

$$\cos 45° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{1}{\sqrt{2}}$$

To rationalize the denominator, multiply the numerator and denominator by $\sqrt{2}$:

$$\cos 45° = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Alternatively, using the unit circle, the point corresponding to 45° has coordinates $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$. The cosine value is the x-coordinate.
The value of $\cos 45°$ is $\frac{\sqrt{2}}{2}$.

Final Answer: $\boxed{\frac{\sqrt{2}}{2}}$

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: The limit $\lim_{x\to 0} \dfrac{\tan x}{x} = 1$ is a fundamental trigonometric limit. It can be proven using geometric arguments (squeeze theorem) or by using the fact that $\tan x = \frac{\sin x}{\cos x}$, so $\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \lim_{x\to 0} \frac{1}{\cos x} = 1 \cdot \frac{1}{\cos 0} = 1 \cdot \frac{1}{1} = 1$.

Solution: We need to find the value of the limit $\lim\limits_{x\to 0} \dfrac{\tan x}{x}$.
This is a standard limit in calculus. As x approaches 0, both the numerator ($\tan x$) and the denominator (x) approach 0, leading to the indeterminate form $\frac{0}{0}$.
Using the known standard limit:

$$\lim_{x\to 0} \frac{\tan x}{x} = 1$$

Alternatively, using L'Hôpital's Rule: Since the limit is of the form $\frac{0}{0}$, we can differentiate the numerator and the denominator. Derivative of numerator ($\tan x$) is $\sec^2 x$. Derivative of denominator (x) is 1.

$$\lim_{x\to 0} \frac{\sec^2 x}{1}$$

Substitute $x = 0$:

$$\sec^2(0) = \left(\frac{1}{\cos 0}\right)^2 = \left(\frac{1}{1}\right)^2 = 1^2 = 1$$

The value of the limit is 1.
Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 28

Q29.

Solution

Concept: Differentiation measures the rate of change of a function with respect to a variable. The exponential function e^x has a unique property:

$$\frac{d}{dx}(e^x) = e^x$$

This means the derivative of e^x is the function itself.

Solution: We are given:

$$y = e^x$$

We need to find:

$$\frac{dy}{dx}$$

Step 1: Apply the standard differentiation rule for the exponential function.

$$\frac{d}{dx}(e^x) = e^x$$

Therefore,

$$\frac{dy}{dx} = \frac{d}{dx}(e^x)$$

$$\frac{dy}{dx} = e^x$$

Step 2: State the derivative.
Hence, the derivative of $y = e^x$ is:

$$e^x$$

Final Answer:

$$\boxed{e^x}$$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: To evaluate a definite integral $\int_a^b f(x)dx$, we find the antiderivative of $f(x)$, say $F(x)$, and then calculate $F(b) - F(a)$ (Fundamental Theorem of Calculus). The power rule for integration is $\int x^n dx = \frac{x^{n+1}}{n+1} + C$.

Solution: We need to evaluate the definite integral $\displaystyle\int_0^1 x^2\, dx$.

Step 1: Find the antiderivative of $f(x) = x^2$. Using the power rule with $n = 2$:

$$\int x^2\, dx = \frac{x^{2+1}}{2+1} + C = \frac{x^3}{3} + C$$

Let $F(x) = \dfrac{x^3}{3}$.

Step 2: Apply the Fundamental Theorem of Calculus.

$$\int_0^1 x^2\, dx = F(1) - F(0)$$

Step 3: Evaluate $F(x)$ at the upper and lower limits. At the upper limit $x = 1$:

$$F(1) = \frac{(1)^3}{3} = \frac{1}{3}$$

At the lower limit $x = 0$:

$$F(0) = \frac{(0)^3}{3} = 0$$

Step 4: Subtract the value at the lower limit from the value at the upper limit.

$$\int_0^1 x^2\, dx = \frac{1}{3} - 0 = \frac{1}{3}$$

The value of the definite integral is $\dfrac{1}{3}$.

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept: Probability is defined as the ratio of the number of favorable outcomes to the total number of possible outcomes. A standard deck of 52 cards contains 4 suits (hearts, diamonds, clubs, spades), and each suit has 13 cards. There are 4 Kings in a standard deck of 52 cards (one King for each suit).

Solution: We want to find the probability of drawing a king from a standard deck of 52 cards.
Step 1: Determine the total number of possible outcomes. The total number of cards in a standard deck is 52. So, the total number of possible outcomes when drawing one card is 52.
Step 2: Determine the number of favorable outcomes. The favorable outcome is drawing a king. In a standard deck of 52 cards, there are 4 Kings (King of Hearts, King of Diamonds, King of Clubs, King of Spades). So, the number of favorable outcomes is 4.
Step 3: Calculate the probability.

$$P(\text{getting a King}) = \frac{\text{Number of Kings}}{\text{Total number of cards}}$$

$$P(\text{getting a King}) = \frac{4}{52}$$

Step 4: Simplify the fraction. Divide both the numerator and the denominator by their greatest common divisor, which is 4.

$$\frac{4}{52} = \frac{4 \div 4}{52 \div 4} = \frac{1}{13}$$

The probability of getting a king is $\frac{1}{13}$.

Final Answer: $\boxed{\frac{1}{13}}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: A Geometric Progression (GP) is a sequence in which every term after the first is obtained by multiplying the previous term by a fixed non-zero number called the common ratio. If the terms are:

$$a,\ ar,\ ar^2,\ ar^3,\ldots$$

then:

- a is the first term
- r is the common ratio

The common ratio is found using:

$$r = \frac{\text{next term}}{\text{previous term}}$$

Solution: The given GP is:

$$3,\ 6,\ 12,\ldots$$

Step 1: Identify the first term.
The first term is:

$$a = 3$$

Step 2: Find the common ratio using consecutive terms.

$$r = \frac{6}{3}$$

$$r = 2$$

Step 3: Verify the ratio using the next pair of terms.

$$r = \frac{12}{6}$$

$$r = 2$$

Since the ratio is the same, the sequence is a GP with common ratio:

$$2$$

Final Answer:

$$\boxed{2}$$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: The imaginary unit i is defined as:

$$i = \sqrt{-1}$$

The powers of i follow a repeating cycle of four:

$$i^1 = i, \qquad i^2 = -1, \qquad i^3 = -i, \qquad i^4 = 1$$

This pattern repeats for all higher powers.

Solution: We need to find:

$$i^{15}$$

Step 1: Divide the exponent by 4.

$$15 \div 4 = 3 \text{ remainder } 3$$

Thus,

$$15 = 4 \times 3 + 3$$

Step 2: Rewrite the power.

$$i^{15} = i^{4\times3+3}$$

$$i^{15} = (i^4)^3 \cdot i^3$$

Step 3: Use the value of i^4.
Since:

$$i^4 = 1$$

we get:

$$i^{15} = 1^3 \cdot i^3 = i^3$$

Step 4: Use the value of i^3.

$$i^3 = -i$$

Therefore,

$$i^{15} = -i$$

Final Answer:

$$\boxed{-i}$$

Answer: (D)

Go Back to Question 33

Q34.

Solution

Concept: The matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ has specific properties. Its square, A^2, can be calculated by matrix multiplication. The identity matrix I is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and the zero matrix 0 is $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Solution: We are given the matrix $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. We need to find A^2.

$$A^2 = A \times A$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Step 1: Perform matrix multiplication. Element in row 1, column 1: $(0\times0)+(1\times-1) = 0-1 = -1$. Element in row 1, column 2: $(0 \times 1) + (1 \times 0) = 0 + 0 = 0$. Element in row 2, column 1: $(-1\times0) + (0\times-1) = 0+0 = 0$. Element in row 2, column 2: $(-1\times1) + (0\times0) = -1+0 = -1$. So, $A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

Step 2: Compare the result with the given options. The matrix $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ can be written as $-1 \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, which is $-I$.

Final Answer: $\boxed{-I}$

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept: One of the properties of determinants is that if two rows (or columns) of a matrix are interchanged, the determinant of the resulting matrix is the negative of the determinant of the original matrix.

Solution: We are given that $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 5$. We need to find the value of $\begin{vmatrix} d & b \\ c & a \end{vmatrix}$.

Step 1: Observe the relationship between the two determinants. The first determinant is $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$.

The second determinant is $\begin{vmatrix} d & b \\ c & a \end{vmatrix}$.

Let's examine how the second matrix is obtained from the first. If we consider the columns, the first column of the first determinant is $\begin{bmatrix} a \\ c \end{bmatrix}$, and the first column of the second determinant is $\begin{bmatrix} d \\ c \end{bmatrix}$. This doesn't seem like a simple row/column interchange.

Let's re-examine the options and the question. The structure of the second determinant $\begin{vmatrix} d & b \\ c & a \end{vmatrix}$ suggests a relationship with the first. Consider the matrix $M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Its determinant is $\det(M) = ad - bc = 5$. Consider the matrix $N = \begin{bmatrix} d & b \\ c & a \end{bmatrix}$. We need to find $\det(N)$. This is not obtained by a single row or column swap.

Let's consider the transpose. The transpose of M is $M^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$. $\det(M^T) = ad - cb = ad - bc = 5$. The determinant of the second matrix is $d \times a - b \times c = ad - bc = 5$.

Therefore, $\begin{vmatrix} d & b \\ c & a \end{vmatrix} = ad - bc$. Since $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc = 5$, the value of the second determinant is also 5.

Final Answer: $\boxed{5}$

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept: The intersection of two sets, denoted by $A \cap B$, contains all elements that are common to both sets. If no common element exists, the intersection is the empty set ϕ.

Solution: We are given:

$$A = \{1, 2, 3\}$$

$$B = \{3, 4, 5\}$$

We need to find:

$$A \cap B$$

Step 1: Write the elements of both sets.
Set A contains:

$$1,\ 2,\ 3$$

Set B contains:

$$3,\ 4,\ 5$$

Step 2: Identify the common elements.
The element present in both sets is:

$$3$$

Step 3: Write the intersection set.

$$A \cap B = \{3\}$$

Therefore, the intersection of sets A and B is:

$$\{3\}$$

Final Answer:

$$\boxed{\{3\}}$$

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept: To find the value of a function at a given input, substitute the value of the variable into the function expression.

Solution: We are given:

$$f(x) = 2x + 1$$

We need to find:

$$f(3)$$

Step 1: Substitute $x = 3$ into the function.

$$f(3) = 2(3) + 1$$

Step 2: Simplify the expression.

$$f(3) = 6 + 1$$

$$f(3) = 7$$

Therefore, the value of the function at $x = 3$ is:

$$7$$

Final Answer:

$$\boxed{7}$$

Answer: (C)

Go Back to Question 37

Q38.

Solution

Concept: The tangent function is defined as:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

The value of $\tan 45^\circ$ is a standard trigonometric value.

Solution: We need to find:

$$\tan 45^\circ$$

Step 1: Recall the standard trigonometric values.

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$
$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

Step 2: Use the tangent identity.

$$\tan 45^\circ = \frac{\sin 45^\circ}{\cos 45^\circ}$$

Substituting the values:

$$\tan 45^\circ = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$$

$$\tan 45^\circ = 1$$

Therefore, the value of $\tan 45^\circ$ is:

$$1$$

Final Answer:

$$\boxed{1}$$

Answer: (B)

Go Back to Question 38

Q39.

Solution

Concept: The inverse cosine function $\cos^{-1}(x)$ gives the angle whose cosine is x. The principal value range of $\cos^{-1}(x)$ is:

$$[0, \pi]$$

Solution: We need to find:

$$\cos^{-1}(0)$$

Step 1: Find the angle whose cosine is 0.
We know:

$$\cos\frac{\pi}{2} = 0$$

Step 2: Check the principal value range.
Since:

$$\frac{\pi}{2} \in [0, \pi]$$

it is the required principal value.
Therefore,

$$\cos^{-1}(0) = \frac{\pi}{2}$$

Final Answer:

$$\boxed{\frac{\pi}{2}}$$

Answer: (B)

Go Back to Question 39

Q40.

Solution

Concept: The Cartesian coordinate system consists of two perpendicular number lines:

- the horizontal axis called the x-axis
- the vertical axis called the y-axis

Every point in the plane is represented by coordinates (x, y). For all points lying on the x-axis, the y-coordinate is always zero.

Solution: We need to determine the equation of the x-axis.
Step 1: Recall the property of points on the x-axis.
Any point on the x-axis has the form:

$$(x, 0)$$

This means the y-coordinate of every point on the x-axis is zero.
Step 2: Write the equation.
Since the value of y is always zero, the equation representing the x-axis is:

$$y = 0$$

Thus, every point satisfying $y = 0$ lies on the x-axis.
Final Answer:

$$\boxed{y = 0}$$

Answer: (B)

Go Back to Question 40

Q41.

Solution

Concept: The dot product (or scalar product) of two vectors $\vec{a} = a_1\hat{i} + a_2\hat{j}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j}$ is given by $\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2$. The dot product is a scalar quantity.

Solution: We are given two vectors: $\vec{a} = \hat{i} + \hat{j} = 1\hat{i} + 1\hat{j}$ $\vec{b} = \hat{i} - \hat{j} = 1\hat{i} + (-1)\hat{j}$
We need to find the dot product $\vec{a} \cdot \vec{b}$.
Step 1: Identify the components of the vectors. For $\vec{a}$, $a_1 = 1$, $a_2 = 1$. For $\vec{b}$, $b_1 = 1$, $b_2 = -1$.
Step 2: Apply the dot product formula.

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2$$

$$\vec{a} \cdot \vec{b} = (1)(1) + (1)(-1)$$

Step 3: Perform the calculations.

$$\vec{a} \cdot \vec{b} = 1 + (-1)$$

$$\vec{a} \cdot \vec{b} = 1 - 1$$

$$\vec{a} \cdot \vec{b} = 0$$

The dot product of $\vec{a}$ and $\vec{b}$ is 0. This also indicates that the vectors are orthogonal.
Final Answer: $\boxed{0}$

Answer: (B)

Go Back to Question 41

Q42.

Solution

Concept: In three-dimensional Cartesian geometry, the coordinate planes are formed by setting one coordinate equal to zero.

- xy-plane: $z = 0$
- yz-plane: $x = 0$
- xz-plane: $y = 0$

The yz-plane contains the y-axis and z-axis.

Solution: We need to find the equation of the yz-plane.
Step 1: Understand the yz-plane.
Any point in three-dimensional space is represented as:

$$(x, y, z)$$

The yz-plane consists of all points lying along the y-direction and z-direction.
Step 2: Identify the coordinate that remains zero.
Since the yz-plane does not extend along the x-direction, the x-coordinate of every point on this plane is zero.
Thus, every point on the yz-plane is of the form:

$$(0, y, z)$$

Step 3: Write the equation of the plane.
Therefore, the equation representing the yz-plane is:

$$x = 0$$

Final Answer:

$$\boxed{x = 0}$$

Answer: (A)

Go Back to Question 42

Q43.

Solution

Concept: The number of permutations of n distinct objects is:

$$n!$$

A permutation means arranging objects in a specific order.

Solution: The word "MATH" contains 4 distinct letters:

$$M,\ A,\ T,\ H$$

Step 1: Identify the number of letters.

$$n = 4$$

Step 2: Use the permutation formula.

$$\text{Number of permutations} = 4!$$

Step 3: Calculate the factorial.

$$4! = 4 \times 3 \times 2 \times 1$$

$$4! = 24$$

Therefore, the number of permutations of the letters of the word "MATH" is:

$$24$$

Final Answer:

$$\boxed{24}$$

Answer: (C)

Go Back to Question 43

Q44.

Solution

Concept: The Binomial Theorem states that the expansion of $(x+y)^n$ is given by $\sum_{k=0}^{n}\binom{n}{k}x^{n-k}y^k$. The constant term is the term that does not contain any variable, which means the power of the variable is zero.

Solution: We need to find the constant term in the expansion of $(x+\frac{1}{x})^4$. Here, $n=4$. The general term is $T_{k+1}=\binom{n}{k}x^{n-k}y^k$. In this case, x in the formula is x, and y in the formula is $\frac{1}{x}$.

$$T_{k+1}=\binom{4}{k}(x)^{4-k}\left(\frac{1}{x}\right)^k$$

$$T_{k+1}=\binom{4}{k}x^{4-k}x^{-k}$$

$$T_{k+1}=\binom{4}{k}x^{4-k-k}$$

$$T_{k+1}=\binom{4}{k}x^{4-2k}$$

Step 1: Find the value of k for which the term is constant. A term is constant if the power of x is zero. So, we set the exponent of x to 0:

$$4-2k=0$$

$$2k=4$$

$$k=2$$

Step 2: Calculate the binomial coefficient for $k=2$ and $n=4$. The constant term is given by $\binom{4}{2}$.

$$\binom{4}{2}=\frac{4!}{2!(4-2)!}=\frac{4!}{2!2!}$$

$$\binom{4}{2}=\frac{4\times3\times2\times1}{(2\times1)(2\times1)}$$

$$\binom{4}{2}=\frac{4\times3}{2\times1}=\frac{12}{2}=6$$

The constant term in the expansion is 6.

Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 44

Q45.

Solution

Concept: When two coins are tossed, there are $2 \times 2 = 4$ possible outcomes. These outcomes are: HH (Head, Head), HT (Head, Tail), TH (Tail, Head), TT (Tail, Tail). Probability is the ratio of favorable outcomes to the total possible outcomes.

Solution: We need to find the probability of getting exactly one head when two coins are tossed.
Step 1: Determine the total number of possible outcomes. When two coins are tossed, the possible outcomes are: HH, HT, TH, TT There are 4 possible outcomes.
Step 2: Identify the favorable outcomes (exactly one head). The outcomes with exactly one head are HT and TH. There are 2 favorable outcomes.
Step 3: Calculate the probability.

$$P(\text{exactly one head}) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

$$P(\text{exactly one head}) = \frac{2}{4}$$

Step 4: Simplify the fraction.

$$\frac{2}{4} = \frac{1}{2}$$

The probability of getting exactly one head when two coins are tossed is $\frac{1}{2}$.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (B)

Go Back to Question 45

Q46.

Solution

Concept: The sum of the first n odd natural numbers is:

$$n^2$$

Solution: We need to find the sum of the first 5 odd numbers.

Step 1: Identify the value of n.

$$n = 5$$

Step 2: Apply the formula.

$$\text{Sum} = n^2$$

$$\text{Sum} = 5^2$$

$$\text{Sum} = 25$$

Therefore, the sum of the first 5 odd numbers is:

$$25$$

Final Answer:

$$\boxed{25}$$

Answer: (C)

Go Back to Question 46

Q47.

Solution

Concept: To evaluate the limit of a rational function as $x \to \infty$, we divide both the numerator and the denominator by the highest power of x in the denominator. If the degrees of the numerator and denominator are equal, the limit is the ratio of the leading coefficients.

Solution: We need to evaluate the limit $\lim_{x\to\infty} \frac{x^2+1}{2x^2+3}$.
Step 1: Identify the highest power of x in the denominator. The highest power of x in the denominator $(2x^2+3)$ is x^2.
Step 2: Divide both the numerator and the denominator by x^2.

$$\lim_{x\to\infty} \frac{x^2+1}{2x^2+3} = \lim_{x\to\infty} \frac{\frac{x^2}{x^2}+\frac{1}{x^2}}{\frac{2x^2}{x^2}+\frac{3}{x^2}}$$

Step 3: Simplify the expression.

$$= \lim_{x\to\infty} \frac{1+\frac{1}{x^2}}{2+\frac{3}{x^2}}$$

Step 4: Evaluate the limit as $x \to \infty$. As $x \to \infty$, $\frac{1}{x^2} \to 0$ and $\frac{3}{x^2} \to 0$.

$$= \frac{1+0}{2+0}$$

$$= \frac{1}{2}$$

The value of the limit is $\frac{1}{2}$.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (B)

Go Back to Question 47

Q48.

Solution

Concept: Differentiation measures the rate of change of a function with respect to a variable. One of the standard differentiation formulas is:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Thus, the derivative of the natural logarithmic function is the reciprocal of x.

Solution: We are given:

$$y = \ln x$$

We need to find:

$$\frac{dy}{dx}$$

Step 1: Apply the standard differentiation rule.

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Therefore,

$$\frac{dy}{dx} = \frac{d}{dx}(\ln x)$$

$$\frac{dy}{dx} = \frac{1}{x}$$

Step 2: State the derivative.
Hence, the derivative of $y = \ln x$ is:

$$\frac{1}{x}$$

Final Answer:

$$\boxed{\frac{1}{x}}$$

Answer: (C)

Go Back to Question 48

Q49.

Solution

Concept: Integration is the reverse process of differentiation. Since:

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

therefore:

$$\int \sec^2 x\, dx = \tan x + C$$

where C is the constant of integration.

Solution: We need to evaluate:

$$\int \sec^2 x\, dx$$

Step 1: Recall the standard integration formula.

$$\int \sec^2 x\, dx = \tan x + C$$

Step 2: Write the result.
Thus,

$$\int \sec^2 x\, dx = \tan x + C$$

where C is an arbitrary constant.
Final Answer:

$$\boxed{\tan x + C}$$

Answer: (B)

Go Back to Question 49

Q50.

Solution

Concept: The degree of a differential equation is the power of the highest order derivative after removing radicals and fractions involving derivatives.

Solution: Given:

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + y = 0$$

Step 1: Identify the highest order derivative.
The highest order derivative is:

$$\frac{d^2y}{dx^2}$$

Step 2: Find its power.
It appears as:

$$\left(\frac{d^2y}{dx^2}\right)^3$$

So, its power is 3.
Therefore, the degree of the differential equation is:

$$3$$

Final Answer:

$$\boxed{3}$$

Answer: (C)

Go Back to Question 50

Q51.

Solution

Concept: For two sets A and B, the number of elements in their union is given by the Principle of Inclusion-Exclusion:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

The common elements are subtracted once because they are counted twice while adding $n(A)$ and $n(B)$.

Solution: We are given:

$$n(A) = 20$$

$$n(B) = 15$$

$$n(A \cap B) = 5$$

We need to find:

$$n(A \cup B)$$

Step 1: Apply the inclusion-exclusion formula.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Step 2: Substitute the given values.

$$n(A \cup B) = 20 + 15 - 5$$

Step 3: Simplify.

$$n(A \cup B) = 35 - 5$$

$$n(A \cup B) = 30$$

Step 4: State the result.
Therefore, the number of elements in $A \cup B$ is:

$$30$$

Final Answer:

$$\boxed{30}$$

Answer: (B)

Go Back to Question 51

Q52.

Solution

Concept: For a square root function:

$$\sqrt{g(x)}$$

the expression inside the square root must be non-negative:

$$g(x) \geq 0$$

Solution: Given:

$$f(x) = \sqrt{x-2}$$

Step 1: Set the expression inside the square root greater than or equal to zero.

$$x - 2 \geq 0$$

Step 2: Solve the inequality.
Adding 2 to both sides:

$$x \geq 2$$

Thus, the function is defined for all real values of x such that:

$$x \geq 2$$

Final Answer:

$$\boxed{x \geq 2}$$

Answer: (B)

Go Back to Question 52

Q53.

Solution

Concept: The tangent of $60°$ is a standard trigonometric value. In a 30-60-90 right-angled triangle, the side opposite the $60°$ angle is $\sqrt{3}$ times the side opposite the $30°$ angle. The tangent of an angle is the ratio of the opposite side to the adjacent side.

Solution: We need to find the value of $\tan 60°$.
Consider a 30-60-90 triangle. Let the side opposite the $30°$ angle be k. Then the side opposite the $60°$ angle is $k\sqrt{3}$. The hypotenuse is $2k$.
For the angle $60°$: The opposite side is $k\sqrt{3}$. The adjacent side is k.
Using the definition of tangent:

$$\tan 60° = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{k\sqrt{3}}{k}$$

$$\tan 60° = \sqrt{3}$$

Alternatively, using the unit circle, the point corresponding to $60°$ has coordinates $(\frac{1}{2}, \frac{\sqrt{3}}{2})$. The tangent value is the ratio of the y-coordinate to the x-coordinate:

$$\tan 60° = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

The value of $\tan 60°$ is $\sqrt{3}$.
Final Answer: $\boxed{\sqrt{3}}$

Answer: (C)

Go Back to Question 53

Q54.

Solution

Concept: The inverse tangent function gives the angle whose tangent is a given number.

$$\tan^{-1}(x) = \theta \quad \text{if} \quad \tan\theta = x$$

The principal value of $\tan^{-1} x$ lies in:

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Solution: We need to find:

$$\tan^{-1}(0)$$

Step 1: Find the angle whose tangent is 0.
We know:

$$\tan 0 = 0$$

Step 2: Check the principal value range.
Since:

$$0 \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

it is the required principal value.
Therefore,

$$\tan^{-1}(0) = 0$$

Final Answer:

$$\boxed{0}$$

Answer: (A)

Go Back to Question 54

Q55.

Solution

Concept: The intercept made by a line on an axis is the coordinate of the point where the line crosses that axis. For the y-axis, this is the y-coordinate when $x = 0$. We can find this by rearranging the line's equation into the slope-intercept form $y = mx+c$, where c is the y-intercept.

Solution: The equation of the line is given as $2x + 5y = 10$.
To find the intercept on the y-axis, we need to find the value of y when $x = 0$.
Step 1: Set $x = 0$ in the equation of the line.

$$2(0) + 5y = 10$$

Step 2: Simplify and solve for y.

$$0 + 5y = 10$$

$$5y = 10$$

$$y = \frac{10}{5}$$

$$y = 2$$

The intercept made by the line on the y-axis is 2. This means the line crosses the y-axis at the point $(0, 2)$.
Alternatively, we can convert the equation to slope-intercept form ($y = mx + c$).

$$5y = -2x + 10$$

$$y = -\frac{2}{5}x + \frac{10}{5}$$

$$y = -\frac{2}{5}x + 2$$

The y-intercept is $c = 2$.
Final Answer: $\boxed{2}$

Answer: (A)

Go Back to Question 55

Q56.

Solution

Concept: The standard equation of a circle centered at the origin is:

$$x^2 + y^2 = r^2$$

where r is the radius. The diameter of the circle is:

$$d = 2r$$

Solution: Given:

$$x^2 + y^2 = 49$$

Step 1: Compare with the standard form.

$$r^2 = 49$$

Step 2: Find the radius.

$$r = \sqrt{49} = 7$$

Step 3: Find the diameter.

$$d = 2r = 2 \times 7 = 14$$

Final Answer:

$$\boxed{14}$$

Answer: (B)

Go Back to Question 56

Q57.

Solution

Concept: The standard equation of a parabola opening downward is:

$$x^2 = -4ay$$

Its vertex is at the origin.

Solution: Given:

$$x^2 = -4y$$

Step 1: Compare with the standard form.

$$x^2 = -4ay$$

Here,

$$a = 1$$

Step 2: Identify the vertex.
For this standard form, the vertex is:

$$(0, 0)$$

Final Answer:

$$\boxed{(0, 0)}$$

Answer: (A)

Go Back to Question 57

Q58.

Solution

Concept: For an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

if $a^2 > b^2$, then the major axis lies along the x-axis.

Solution: Given:

$$\frac{x^2}{49} + \frac{y^2}{25} = 1$$

Step 1: Compare the denominators.

$$49 > 25$$

Thus,

$$a^2 = 49$$

Step 2: Identify the axis containing the larger denominator.
Since 49 is under the x^2 term, the major axis lies along the x-axis.

Final Answer:

$$\boxed{\text{x-axis}}$$

Answer: (A)

Go Back to Question 58

Q59.

Solution

Concept: The standard form of a hyperbola centered at (h, k) is:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

The center of the hyperbola is (h, k).

Solution: Given:

$$\frac{x^2}{9} - \frac{y^2}{4} = 1$$

Step 1: Compare with the standard form.

$$\frac{(x-0)^2}{9} - \frac{(y-0)^2}{4} = 1$$

Step 2: Identify the center.

$$h = 0, \qquad k = 0$$

Therefore, the center is:

$$(0, 0)$$

Final Answer:

$$\boxed{(0, 0)}$$

Answer: (A)

Go Back to Question 59

Q60.

Solution

Concept: Vectors are added by adding their corresponding components. If:

$$\vec{a} = a_1\hat{i} + a_2\hat{j}$$

and

$$\vec{b} = b_1\hat{i} + b_2\hat{j}$$

then:

$$\vec{a} + \vec{b} = (a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j}$$

Solution: Given:

$$\vec{a} = 2\hat{i} + 3\hat{j}$$

$$\vec{b} = 4\hat{i} - 2\hat{j}$$

Step 1: Add the $\hat{i}$ components.

$$2 + 4 = 6$$

Step 2: Add the $\hat{j}$ components.

$$3 + (-2) = 1$$

Step 3: Write the resultant vector.

$$\vec{a} + \vec{b} = 6\hat{i} + \hat{j}$$

Final Answer:

$$\boxed{6\hat{i} + \hat{j}}$$

Answer: (A)

Go Back to Question 60

Q61.

Solution

Concept: The distance of a point (x, y, z) from the origin $(0, 0, 0)$ in three-dimensional space is given by the formula:

$$d = \sqrt{x^2 + y^2 + z^2}$$

Solution: We need to find the distance of the point $(1, 2, 2)$ from the origin $(0, 0, 0)$.

Step 1: Identify the coordinates of the point and the origin. Point $P = (1, 2, 2)$, so $x = 1, y = 2, z = 2$. Origin $O = (0, 0, 0)$.

Step 2: Apply the distance formula from the origin.

$$d = \sqrt{x^2 + y^2 + z^2}$$

Substitute the coordinates of the point:

$$d = \sqrt{(1)^2 + (2)^2 + (2)^2}$$

Step 3: Calculate the differences (which are just the coordinates themselves since we're subtracting 0).

$$d = \sqrt{1^2 + 2^2 + 2^2}$$

Step 4: Square the coordinates.

$$d = \sqrt{1 + 4 + 4}$$

Step 5: Add the squared coordinates.

$$d = \sqrt{9}$$

Step 6: Calculate the square root.

$$d = 3$$

The distance of the point $(1, 2, 2)$ from the origin is 3.

Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 61

Q62.

Solution

Concept: The number of permutations of n distinct objects taken r at a time is:

$$^nP_r = \frac{n!}{(n-r)!}$$

Solution: We need to find:

$$^8P_2$$

Step 1: Apply the permutation formula.

$$^8P_2 = \frac{8!}{(8-2)!}$$

$$^8P_2 = \frac{8!}{6!}$$

Step 2: Simplify.

$$^8P_2 = \frac{8 \times 7 \times 6!}{6!}$$

Cancelling 6!:

$$^8P_2 = 8 \times 7$$

$$^8P_2 = 56$$

Final Answer:

$$\boxed{56}$$

Answer: (B)

Go Back to Question 62

Q63.

Solution

Concept: In an Arithmetic Progression (AP), the n^{th} term is:

$$a_n = a + (n-1)d$$

where a is the first term and d is the common difference.

Solution: The AP is:

$$2,\ 5,\ 8, \ldots$$

Step 1: Identify the values.

$$a = 2, \qquad d = 3, \qquad n = 15$$

Step 2: Use the formula.

$$a_{15} = 2 + (15-1) \times 3$$

$$a_{15} = 2 + 14 \times 3$$

$$a_{15} = 2 + 42$$

$$a_{15} = 44$$

Final Answer:

$$\boxed{44}$$

Answer: (B)

Go Back to Question 63

Q64.

Solution

Concept: Probability measures the chance of occurrence of an event. It is calculated using:

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}$$

The probability of any event always lies between 0 and 1.

Solution: A bag contains:

- 3 red balls
- 2 blue balls

We need to find the probability of drawing a blue ball. Step 1: Find the total number of balls.

$$\text{Total balls} = 3 + 2 = 5$$

Thus, the total number of possible outcomes is:

$$5$$

Step 2: Find the number of favorable outcomes. The favorable event is drawing a blue ball. Since there are 2 blue balls:

$$\text{Favorable outcomes} = 2$$

Step 3: Apply the probability formula.

$$P(\text{blue ball}) = \frac{\text{Number of blue balls}}{\text{Total number of balls}}$$

$$P(\text{blue ball}) = \frac{2}{5}$$

Step 4: State the result. Therefore, the probability of drawing a blue ball is:

$$\frac{2}{5}$$

Final Answer:

$$\boxed{\frac{2}{5}}$$

Answer: (B)

Go Back to Question 64

Q65.

Solution

Concept: or a complex number $z = a + bi$, the argument is given by:

$$\tan\theta = \frac{b}{a}$$

while considering the quadrant of the complex number.

Solution: iven:

$$z = 2 + 2i$$

Here,

$$a = 2, \qquad b = 2$$

Step 1: Find the quadrant.
Both real and imaginary parts are positive, so the complex number lies in the first quadrant.
Step 2: Find the argument.

$$\tan\theta = \frac{2}{2} = 1$$

Since:

$$\tan\frac{\pi}{4} = 1$$

we get:

$$\theta = \frac{\pi}{4}$$

Final Answer:

$$\boxed{\frac{\pi}{4}}$$

Answer: (B)

Go Back to Question 65

Q66.

Solution

Concept: Union and intersection of sets are commutative, but set difference is generally not commutative.

Solution: Step 1: Check (A):

$$A \cup B = B \cup A$$

Union is commutative. Hence, (A) is true.

Step 2: Check (B):

$$A \cap B = B \cap A$$

Intersection is also commutative. Hence, (B) is true.

Step 3: Check (C):

$$A - B = B - A$$

Set difference is not commutative in general. For example:

$$A = \{1, 2\}, \quad B = \{2, 3\}$$

Then,

$$A - B = \{1\}, \quad B - A = \{3\}$$

Since they are different, the statement is false.

Step 4: Check (D):

$$A \cup \phi = A$$

Union with the empty set does not change the set.
Hence, (D) is true.
Therefore, statements (A), (B), and (D) are correct.

Final Answer:

$$\boxed{\begin{array}{c} A \cup B = B \cup A \\ A \cap B = B \cap A \\ A \cup \phi = A \end{array}}$$

Answer: (A,B,D)

Go Back to Question 66

Q67.

Solution

Concept: A function is one-to-one if different inputs give different outputs.

Solution: Step 1: Check (A):

$$f(x) = x$$

If:

$$f(x_1) = f(x_2)$$

then:

$$x_1 = x_2$$

Hence, it is one-to-one.

Step 2: Check (B):

$$f(x) = x^3$$

If:

$$x_1^3 = x_2^3$$

then:

$$x_1 = x_2$$

Hence, it is one-to-one.

Step 3: Check (C):

$$f(x) = x^2$$

Since:

$$f(2) = 4 \quad \text{and} \quad f(-2) = 4$$

different inputs give the same output. Hence, it is not one-to-one.

Step 4: Check (D):

$$f(x) = e^x$$

The exponential function is strictly increasing, so different inputs give different outputs.
Hence, it is one-to-one.
Therefore, functions (A), (B), and (D) are one-to-one.

Final Answer:

$$\boxed{f(x) = x, \quad f(x) = x^3, \quad f(x) = e^x}$$

Answer: (A,B,D)

Go Back to Question 67

Q68.

Solution

Concept: A limit exists if the function approaches a finite value as the input approaches a certain point. We examine the behavior of each function as x approaches the given value.

Solution: Step 1: Evaluate limit (A): $\lim_{x\to 0} \sin x$.
The sine function is continuous everywhere. As x approaches 0, $\sin x$ approaches $\sin 0$.
$\sin 0 = 0$. The limit exists and is 0.

Step 2: Evaluate limit (B): $\lim_{x\to 0} \frac{1}{x}$.
As x approaches 0 from the positive side ($x \to 0^+$), $\frac{1}{x} \to +\infty$.
As x approaches 0 from the negative side ($x \to 0^-$), $\frac{1}{x} \to -\infty$.
Since the left-hand limit and the right-hand limit are not equal (and are infinite), the limit does not exist.

Step 3: Evaluate limit (C): $\lim_{x\to 2}(x^2 + 1)$.
The function $x^2 + 1$ is a polynomial, which is continuous everywhere. As x approaches 2, the function approaches $2^2 + 1$.
$2^2 + 1 = 4 + 1 = 5$. The limit exists and is 5.

Step 4: Evaluate limit (D): $\lim_{x\to 0} |x|$.
The absolute value function $|x|$ is continuous everywhere. As x approaches 0, $|x|$ approaches $|0|$.
$|0| = 0$. The limit exists and is 0.

Step 5: Identify the limits that exist.
Limits (A), (C), and (D) exist.

Final Answer:

$$\boxed{\lim_{x\to 0} \sin x, \quad \lim_{x\to 2}(x^2 + 1), \quad \lim_{x\to 0} |x|}$$

Answer: (A,C,D)

Go Back to Question 68

Q69.

Solution

Concept: We use standard differentiation formulas to check each statement.

Solution: Step 1: Check (A):

$$\frac{d}{dx}(\sin x) = \cos x$$

This is a standard derivative formula. Hence, (A) is correct.

Step 2: Check (B):

$$\frac{d}{dx}(\cos x) = -\sin x$$

This is also a standard derivative formula. Hence, (B) is correct.

Step 3: Check (C):

$$\frac{d}{dx}(e^x) = e^x$$

The derivative of the exponential function remains the same. Hence, (C) is correct.

Step 4: Check (D):

$$\frac{d}{dx}(\ln x) = x$$

The correct derivative is:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Hence, (D) is incorrect. Therefore, statements (A), (B), and (C) are correct.

Final Answer:

$$\boxed{\begin{array}{c}\frac{d}{dx}(\sin x) = \cos x \\ \frac{d}{dx}(\cos x) = -\sin x \\ \frac{d}{dx}(e^x) = e^x\end{array}}$$

Answer: (A,B,C)

Go Back to Question 69

Q70.

Solution

Concept: Critical points of a function occur where:

$$f'(x) = 0$$

The second derivative helps identify maxima and minima.

Solution: Step 1: Given:

$$f(x) = x^3 - 3x$$

Differentiate:

$$f'(x) = 3x^2 - 3$$

Hence, (A) is correct.
Step 2: Find critical points. Set:

$$f'(x) = 0$$

$$3x^2 - 3 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

Hence, (B) is correct.
Step 3: Find the second derivative.

$$f''(x) = 6x$$

At:

$$x = 1$$

$$f''(1) = 6 > 0$$

So, the function has a local minimum at $x = 1$. Hence, (D) is correct and (C) is false. Therefore, statements (A), (B), and (D) are correct.
Final Answer:

$$\boxed{\begin{array}{c} f'(x) = 3x^2 - 3 \\ \text{Critical points occur at } x = \pm 1 \\ \text{The function has a local minimum at } x = 1 \end{array}}$$

Answer: (A,B,D)

Go Back to Question 70

Q71.

Solution

Concept: Important properties of determinants:

- Interchanging two rows changes the sign of the determinant.
- If two rows are equal, determinant becomes zero.
- Determinant of the identity matrix is 1.
- Determinant of a triangular matrix is the product of diagonal elements.

Solution: Step 1: Check (A).
Interchanging two rows changes the sign of the determinant.
Hence, (A) is true.

Step 2: Check (B).
If two rows are equal, the determinant is zero.
Hence, (B) is true.

Step 3: Check (C).
The determinant of the identity matrix is:

$$|I| = 1$$

Hence, the statement is false.

Step 4: Check (D).
For a triangular matrix, determinant equals the product of diagonal elements.
Hence, (D) is true.

Therefore, statements (A), (B), and (D) are correct.

Final Answer:

Interchanging two rows changes the sign of determinant

If two rows are equal, determinant is zero

Determinant of a triangular matrix is product of diagonal elements

Answer: (A,B,D)

Go Back to Question 71

Q72.

Solution

Concept: A singular matrix is a square matrix whose determinant is zero. We need to calculate the determinant for each matrix.

Solution: Step 1: Definition of a singular matrix.
A square matrix A is singular if its determinant, $\det(A)$, is zero.

Step 2: Analyze matrix (A): $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.
Determinant $= (1)(4) - (2)(2) = 4 - 4 = 0$.
This matrix is singular.

Step 3: Analyze matrix (B): $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
This is the identity matrix. Determinant $= (1)(1) - (0)(0) = 1 - 0 = 1$.
This matrix is not singular.

Step 4: Analyze matrix (C): $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$.
Determinant $= (2)(6) - (3)(4) = 12 - 12 = 0$.
This matrix is singular.

Step 5: Analyze matrix (D): $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$.
Determinant $= (1)(2) - (1)(1) = 2 - 1 = 1$.
This matrix is not singular.

Step 6: Identify the singular matrices.
Matrices (A) and (C) are singular.

Final Answer:

$$\boxed{\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, \quad \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}}$$

Answer: (A,C)

Go Back to Question 72

Q73.

Solution

Concept: A probability value must be between 0 and 1, inclusive. We need to check which of the given values fall within this range.

Solution: Step 1: Recall the range of valid probability values.
A probability $P(E)$ must satisfy $0 \le P(E) \le 1$.

Step 2: Analyze value (A): 0.
0 is a valid probability (representing an impossible event). Valid.

Step 3: Analyze value (B): $\frac{3}{4}$.
$\frac{3}{4} = 0.75$. Since $0 \le 0.75 \le 1$, this is a valid probability. Valid.

Step 4: Analyze value (C): 1.
1 is a valid probability (representing a certain event). Valid.

Step 5: Analyze value (D): $\frac{5}{4}$.
$\frac{5}{4} = 1.25$. Since $1.25 > 1$, this is not a valid probability. Invalid.

Step 6: Identify the valid probability values.
Values (A), (B), and (C) are valid probability values.

Final Answer:

$$\boxed{0, \quad \frac{3}{4}, \quad 1}$$

Answer: (A,B,C)

Go Back to Question 73

Q74.

Solution

Concept: A point (x, y) lies on a circle if its coordinates satisfy the circle's equation. The equation of the given circle is $x^2 + y^2 = 25$.

Solution: Step 1: Recall the equation of the circle.
The given circle has the equation $x^2 + y^2 = 25$. This is a circle centered at the origin $(0, 0)$ with a radius of 5 (since $r^2 = 25$).

Step 2: Check if point (A): $(3, 4)$ lies on the circle.
Substitute $x = 3$ and $y = 4$ into the equation:
$3^2 + 4^2 = 9 + 16 = 25$.
Since $25 = 25$, the point $(3, 4)$ lies on the circle.

Step 3: Check if point (B): $(4, 3)$ lies on the circle.
Substitute $x = 4$ and $y = 3$ into the equation:
$4^2 + 3^2 = 16 + 9 = 25$.
Since $25 = 25$, the point $(4, 3)$ lies on the circle.

Step 4: Check if point (C): $(5, 0)$ lies on the circle.
Substitute $x = 5$ and $y = 0$ into the equation:
$5^2 + 0^2 = 25 + 0 = 25$.
Since $25 = 25$, the point $(5, 0)$ lies on the circle.

Step 5: Check if point (D): $(2, 2)$ lies on the circle.
Substitute $x = 2$ and $y = 2$ into the equation:
$2^2 + 2^2 = 4 + 4 = 8$.
Since $8 \neq 25$, the point $(2, 2)$ does not lie on the circle.

Step 6: Identify the points that lie on the circle.
Points (A), (B), and (C) lie on the circle.

Final Answer:

$$\boxed{(3, 4), \quad (4, 3), \quad (5, 0)}$$

Answer: (A,B,C)

Go Back to Question 74

Q75.

Solution

Concept: A periodic function repeats its values at regular intervals. The smallest positive interval is the fundamental period. We need to find functions whose fundamental period is 2π.

Solution: Step 1: Definition of period.
A function $f(x)$ has period P if $f(x+P) = f(x)$ for all x, and P is the smallest positive number with this property.

Step 2: Analyze function (A): $\sin x$.
$\sin(x + 2\pi) = \sin x$. The fundamental period of $\sin x$ is 2π. Period is 2π.

Step 3: Analyze function (B): $\cos x$.
$\cos(x + 2\pi) = \cos x$. The fundamental period of $\cos x$ is 2π. Period is 2π.

Step 4: Analyze function (C): $\tan x$.
$\tan(x + \pi) = \tan x$. The fundamental period of $\tan x$ is π. Period is not 2π.

Step 5: Analyze function (D): $\sec x$.
$\sec x = 1/\cos x$. Since $\cos x$ has a period of 2π, $\sec x$ also has a period of 2π. $\sec(x + 2\pi) = \sec x$. The fundamental period is 2π. Period is 2π.

Step 6: Identify functions with period 2π.
Functions (A), (B), and (D) have a period of 2π.

Final Answer:

$$\boxed{\sin x, \quad \cos x, \quad \sec x}$$

Answer: (A,B,D)

Go Back to Question 75

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	B	3	B	4	B	5	A
6	C	7	A	8	A	9	A	10	B
11	A	12	C	13	B	14	B	15	B
16	A	17	B	18	A	19	B	20	C
21	A	22	B	23	A	24	C	25	B
26	A	27	B	28	B	29	A	30	A
31	A	32	B	33	D	34	B	35	B
36	B	37	C	38	B	39	B	40	B
41	B	42	A	43	C	44	B	45	B
46	C	47	B	48	C	49	B	50	C
51	B	52	B	53	C	54	A	55	A
56	B	57	A	58	A	59	A	60	A
61	B	62	B	63	B	64	B	65	B
66	A,B,D	67	A,B,D	68	A,C,D	69	A,B,C	70	A,B,D
71	A,B,D	72	A,C	73	A,B,C	74	A,B,C	75	A,B,D