WBJEE Physics Sample Paper-12

Duration: 60 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **40** Multiple Choice Questions divided into **3 Sections**.
- **Section A (Q1–Q30):** Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- **Section B (Q31–Q35):** Each correct answer carries **+2 marks**. Incorrect answer: **−0.5** marks. Only **one** correct option.
- **Section C (Q36–Q40):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section–A — 30 Questions × 1 Mark Each
(Negative Marking: −0.25) [Single Correct]

Q1. A physical quantity X depends on three observables a, b, and c according to the relation:

$$X = \frac{a^2 b^3}{\sqrt{c}}$$

If the maximum percentage errors in the measurement of a, b, and c are 1%, 2%, and 4% respectively, the maximum percentage error in the measurement of X is:

(A) 6%

(B) 10%

(C) 12%

(D) 14%

Q2. The velocity v of a transverse wave in a stretched string depends on the tension T applied to the string and its mass per unit length μ. Using dimensional analysis, the expression for v is found to be proportional to:

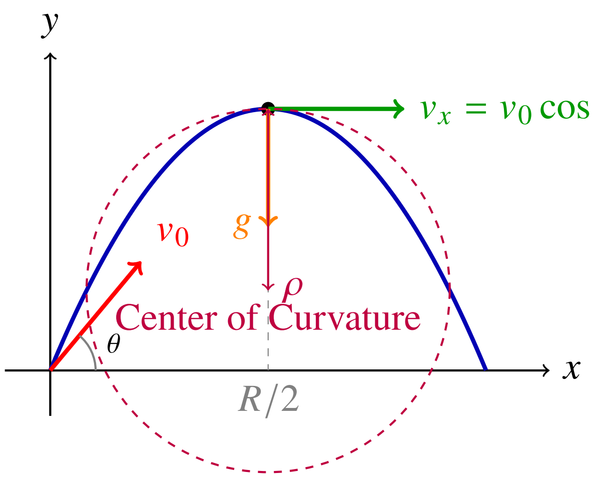
(A) $\sqrt{\frac{T}{\mu}}$

(B) $\frac{T}{\mu}$

(C) $\sqrt{T\mu}$

(D) $\frac{\mu}{T}$

Q3. A projectile is fired from the ground with an initial velocity v_0 at an angle θ with the horizontal. The radius of curvature of its parabolic trajectory at the highest point of its motion is:



(A) $\frac{v_0^2 \sin^2\theta}{g}$

(B) $\frac{v_0^2 \cos^2\theta}{g}$

(C) $\frac{v_0^2}{g}$

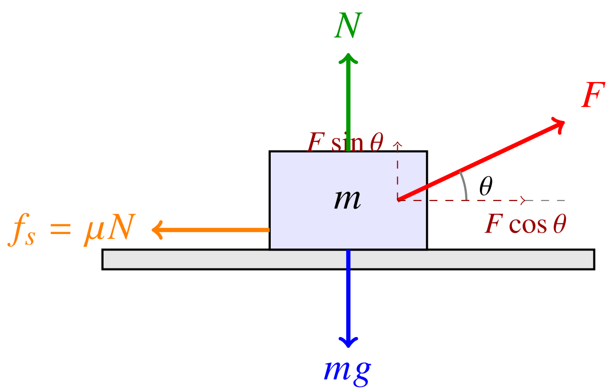
(D) $\frac{v_0^2 \cos^2\theta}{g \sin\theta}$

Q4. A particle moves along a straight line such that its displacement x at time t satisfies the equation $x^2 = t^2 + 1$. The acceleration of the particle at any instant is proportional to:

(A) $\frac{1}{x}$

(B) $\frac{1}{x^2}$
(C) $\frac{1}{x^3}$
(D) $-\frac{1}{x^2}$

Q5. A block of mass m is placed on a rough horizontal surface with a coefficient of static friction μ. A pulling force F is applied at an angle θ with the horizontal. The minimum magnitude of force F required to just slide the block along the surface is:



(A) $\frac{\mu mg}{\sqrt{1+\mu^2}}$
(B) $\frac{\mu mg}{\sqrt{1-\mu^2}}$
(C) μmg
(D) $\frac{mg}{\sqrt{1+\mu^2}}$

Q6. A car of mass m is negotiating a leveled unbanked circular turn of radius R. If the coefficient of static friction between the tires and the road surface is μ, the maximum safe speed with which the car can take the turn without skidding is:

(A) $\sqrt{\frac{\mu g}{R}}$
(B) $\sqrt{\mu g R}$
(C) $\mu g R$
(D) $\sqrt{\frac{g}{\mu R}}$

Q7. A block of mass m moving with a velocity v collides head-on elastically with another block of mass $2m$ which is initially at rest. The fraction of the total initial kinetic energy transferred by the lighter block to the heavier block is:

(A) $\frac{1}{9}$

(B) $\frac{4}{9}$

(C) $\frac{8}{9}$

(D) $\frac{2}{3}$

Q8. A uniform solid sphere and a uniform solid cylinder, both having the same mass and radius, are released from rest from the top of an inclined plane. If both roll down the plane without slipping, the ratio of their linear accelerations $\frac{a_{\text{sphere}}}{a_{\text{cylinder}}}$ is:

(A) 14 : 15

(B) 15 : 14

(C) 10 : 9

(D) 21 : 25

Q9. The acceleration due to gravity at a height h above the Earth's surface is found to be equal to the acceleration due to gravity at a depth d inside the Earth. If both h and d are much smaller than the radius of the Earth R ($h, d \ll R$), then the relation between h and d is:

(A) $d = h$

(B) $d = 2h$

(C) $h = 2d$

(D) $d = \frac{h}{2}$

Q10. A metal wire of length L and radius r is fixed at one end and stretched by suspending a load F from its other end, producing an elongation ΔL. If another wire made of the same material but with length $2L$ and radius $2r$ is stretched by the same load F, the elongation produced will be:

(A) ΔL

(B) $2\Delta L$

(C) $\frac{\Delta L}{2}$

(D) $\frac{\Delta L}{4}$

Q11. A large spherical liquid drop of radius R breaks up into 8 identical smaller spherical droplets. If the surface tension of the liquid is T, the total work done against surface tension in this process is:

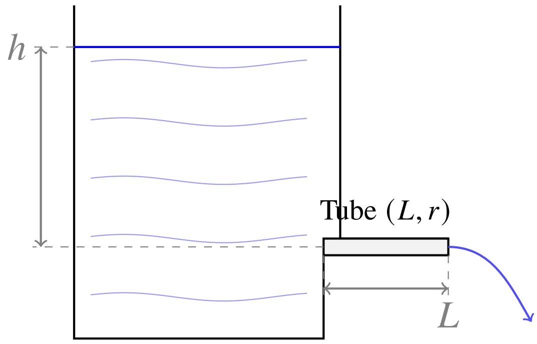
(A) $4\pi R^2 T$

(B) $8\pi R^2 T$

(C) $2\pi R^2 T$

(D) $12\pi R^2 T$

Q12. A large open water tank has a small hole on its vertical wall at a depth h below the free surface of water. To control the discharge, a narrow horizontal capillary tube of length L and internal radius r is fitted tightly into the hole. The steady-state volumetric flow rate of water issuing from the capillary tube is:



(A) $\frac{\pi \rho g h r^4}{8 \eta L}$

(B) $\frac{\pi \rho g h r^4}{4 \eta L}$

(C) $\pi r^2 \sqrt{2gh}$

(D) $\frac{8 \eta L \pi \rho g h}{r^4}$

Q13. One mole of an ideal monoatomic gas ($C_V = \frac{3}{2}R$) undergoes a thermodynamic process in which its pressure P varies with volume V as $P = kV^2$, where k is a positive constant. The molar heat capacity C of the gas during this specific process is:

(A) $\frac{11}{6}R$

(B) $\frac{5}{3}R$

(C) $\frac{7}{4}R$

(D) $\frac{11}{3}R$

Q14. The root-mean-square (rms) speed of hydrogen gas (H_2) molecules at an absolute temperature T is v. If the temperature is raised to $2T$ and the hydrogen molecules dissociate completely into individual atoms (H), the new rms speed of the hydrogen atoms will be:

(A) v

(B) $\sqrt{2}v$

(C) $2v$

(D) $4v$

Q15. A black body at an absolute temperature T radiates energy at a rate of E Watts. If its absolute temperature is reduced to $\frac{T}{2}$, the rate of energy radiation from the body becomes:

(A) $\frac{E}{2}$

(B) $\frac{E}{4}$

(C) $\frac{E}{8}$

(D) $\frac{E}{16}$

Q16. A particle is executing simple harmonic motion (SHM) along the x-axis. If its maximum velocity is v_{max} and its maximum acceleration is a_{max}, the amplitude of its oscillation is given by:

(A) $\frac{v_{\text{max}}^2}{a_{\text{max}}}$

(B) $\frac{a_{\text{max}}^2}{v_{\text{max}}}$

(C) $\frac{v_{\text{max}}}{a_{\text{max}}}$

(D) $\frac{v_{\text{max}}^2}{a_{\text{max}}^2}$

Q17. A whistle emitting a sound wave of frequency 600 Hz approaches a stationary listener with a constant speed of 30 m/s. Given that the speed of sound in still air is 330 m/s, the apparent frequency of the sound heard by the listener is:

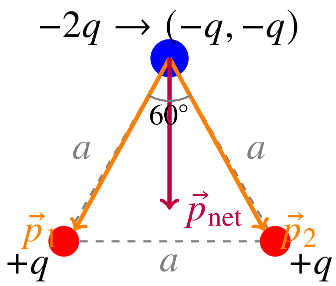
(A) 550 Hz

(B) 660 Hz

(C) 600 Hz

(D) 700 Hz

Q18. Three point charges $+q$, $+q$, and $-2q$ are placed at the three vertices of an equilateral triangle of side length a. The net electric dipole moment of this system of charges has a magnitude equal to:



(A) qa

(B) $\sqrt{2}qa$

(C) $\sqrt{3}qa$

(D) $2qa$

Q19. A parallel-plate air capacitor is fully charged by a battery and then disconnected from it. If the distance between the two plates is now doubled using insulating handles, which of the following happens to its capacitance C and the energy stored U?

(A) C is doubled, U is halved.

(B) C is halved, U is doubled.

(C) C is halved, U remains unchanged.

(D) C is doubled, U remains unchanged.

Q20. In a potentiometer experiment, a standard cell of emf 1.25 V balances against a length of 35 cm of the potentiometer wire. When this cell is replaced by an unknown cell, the balancing length shifts to 63 cm. The emf of the unknown cell is:

(A) 2.25 V

(B) 2.50 V

(C) 2.00 V

(D) 1.75 V

Q21. Two resistors of resistances R_1 and R_2 $(R_1 > R_2)$ are connected in parallel across an ideal voltage source of constant potential difference. The ratio of the thermal energy produced per second in R_1 to that in R_2 is:

(A) $\frac{R_1}{R_2}$

(B) $\frac{R_2}{R_1}$

(C) $\frac{R_1^2}{R_2^2}$

(D) 1 : 1

Q22. A long straight cylindrical wire carries a steady current I along the positive z-axis. The magnetic field vector $\vec{B}$ at any point $(x, y, 0)$ in the xy-plane is proportional to:

(A) $\frac{x\hat{i}+y\hat{j}}{x^2+y^2}$

(B) $\frac{-y\hat{i}+x\hat{j}}{x^2+y^2}$

(C) $\frac{x\hat{i}-y\hat{j}}{\sqrt{x^2+y^2}}$

(D) $\frac{y\hat{i}-x\hat{j}}{x^2+y^2}$

Q23. A particle of mass m and charge q enters a region containing a uniform magnetic field $\vec{B}$ with a velocity vector $\vec{v}$ that makes an angle of 30° with the direction of $\vec{B}$. The particle describes a helical path. The ratio of the pitch P of the helix to its radius R is:

(A) $\pi\sqrt{3}$

(B) $2\pi\sqrt{3}$

(C) $\frac{2\pi}{\sqrt{3}}$

(D) $\frac{\pi}{\sqrt{3}}$

Q24. A flat circular coil consisting of N closely wound turns and radius r is placed in a uniform magnetic field B such that the plane of the coil is perpendicular to the magnetic field lines. If the coil is flipped over by 180° about one of its coplanar diameters in a time interval Δt, the average induced electromotive force (emf) in the coil is:

(A) zero

(B) $\frac{\pi N B r^2}{\Delta t}$

(C) $\frac{2\pi N B r^2}{\Delta t}$

(D) $\frac{4\pi N B r^2}{\Delta t}$

Q25. The electric field vector of a plane electromagnetic wave propagating in a vacuum along the x-axis is given by $E_y = 50\sin(10^8 t - kx)$ V/m. The amplitude of the oscillating magnetic field component associated with this wave is:

(A) 1.67×10^{-7} T

(B) 5.0×10^{-7} T

(C) 1.5×10^{10} T

(D) 2.5×10^{-8} T

Q26. A thin convex lens of focal length f forms a sharp real image of a real object on a screen. If the linear magnification of the image is m, the distance of the object from the optical center of the lens is:

(A) $(m+1)f$

(B) $\left(\frac{m+1}{m}\right) f$

(C) $(m-1)f$

(D) $\left(\frac{1-m}{m}\right) f$

Q27. In a standard Young's double-slit experiment, the intensity of light at a point on the screen where the optical path difference between the interfering waves is $\frac{\lambda}{6}$ (λ being the wavelength of light) is denoted by I. If I_0 represents the maximum intensity at the center of a bright fringe, the ratio $\frac{I}{I_0}$ is:

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{3}{4}$

(D) $\frac{1}{4}$

Q28. The de Broglie wavelength of an electron accelerated from rest through a potential difference of V Volts is λ. If the accelerating potential difference is increased to $4V$, the new de Broglie wavelength of the electron becomes:

(A) 2λ

(B) 4λ

(C) $\frac{\lambda}{2}$

(D) $\frac{\lambda}{4}$

Q29. A radioactive nucleus A undergoes successive disintegrations according to the following decay sequence:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\alpha} A_3$$

If the mass number and atomic number of the parent nucleus A are 180 and 72 respectively, the mass number and atomic number of the product nucleus A_3 will be:

(A) $172, 69$

(B) $172, 70$

(C) $174, 68$

(D) $176, 69$

Q30. Two logic inputs A and B are fed simultaneously into a NOR gate and an AND gate. The outputs of these two gates are then fed as inputs into a final OR gate to give a net output Y. The Boolean expression representing Y is:

(A) $A \cdot B$

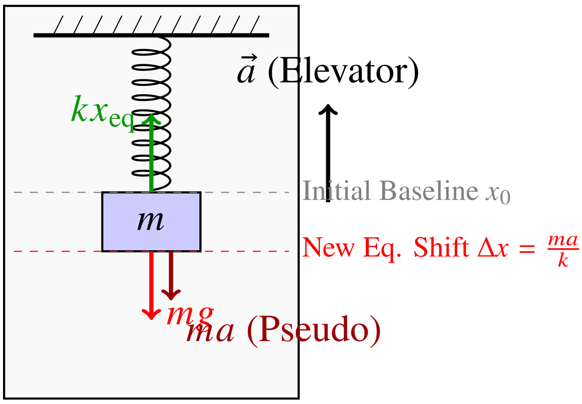
(B) $\overline{A \oplus B}$

(C) $A + B$

(D) $A \oplus B$

Section–B — 5 Questions × 2 Marks Each
(Negative Marking: −0.5) [Single Correct]

Q31. A block of mass m is suspended vertically from the ceiling of an elevator by an ideal light spring of spring constant k. Initially, the elevator is stationary and the system is in static equilibrium. Suddenly, the elevator begins to accelerate vertically upward with a constant acceleration a. The maximum extension reached by the spring from its initial unstretched length during the subsequent motion is:



(A) $\frac{m(g+a)}{k}$

(B) $\frac{m(g+2a)}{k}$

(C) $\frac{m(g+2a)}{k} + \frac{ma}{k}$

(D) $\frac{mg+2ma}{k}$

Q32. A uniform slender rod of mass M and length L is smoothly pivoted at one of its ends so that it can swing freely in a vertical plane. The rod is held horizontally and released from rest. The magnitude of the angular velocity ω of the rod at the instant it passes through its lowest vertical position is:

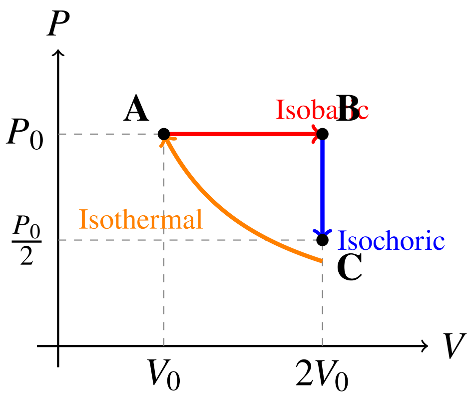
(A) $\sqrt{\frac{3g}{L}}$

(B) $\sqrt{\frac{2g}{L}}$

(C) $\sqrt{\frac{6g}{L}}$

(D) $\sqrt{\frac{g}{L}}$

Q33. One mole of an ideal monoatomic gas is taken around a closed thermodynamic cycle $ABCA$ represented on a pressure-volume (P-V) diagram. Process AB is an isobaric expansion at pressure P_0 that doubles the volume from V_0 to $2V_0$. Process BC is an isochoric cooling that reduces the pressure from P_0 to $P_0/2$. Process CA is an isothermal compression returning the gas back to its initial state (P_0, V_0). The net work done by the gas during the entire cycle is:



(A) $P_0V_0\,(1 - \ln 2)$

(B) $P_0V_0\left(\frac{1}{2} - \ln 2\right)$

(C) $P_0V_0\left(1 - \frac{1}{2}\ln 2\right)$

(D) $\frac{1}{2}P_0V_0$

Q34. A flexible string of length L is stretched and fixed firmly at both ends. It is vibrating in its fundamental standing wave mode described by the displacement equation $y(x,t) = A\sin(kx)\cos(\omega t)$, where x is measured from one fixed boundary. The transverse velocity of a string particle located at $x = \frac{L}{4}$ at the time instant $t = \frac{\pi}{2\omega}$ is:

(A) $-\frac{A\omega}{\sqrt{2}}$

(B) zero

(C) $\frac{A\omega}{2}$

(D) $-A\omega$

Q35. Consider an infinite, regular two-dimensional square grid mesh of uniform wires. The electrical resistance of the wire segment between any two adjacent intersecting nodes is exactly R. If an ideal ohmmeter is connected across any two immediately adjacent nodes A and B, the equivalent electrical resistance measured will be:

(A) R

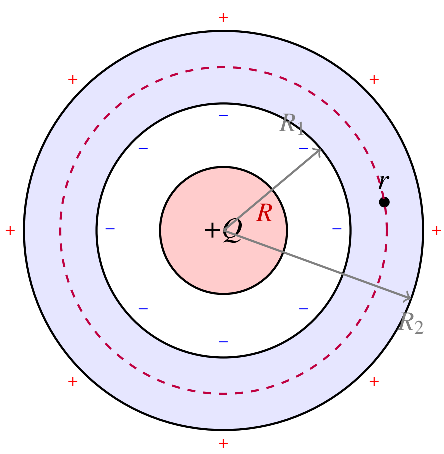
(B) $\frac{R}{2}$

(C) $\frac{R}{4}$

(D) $2R$

Section–C — 5 Questions × 2 Marks Each (No Negative Marking) [One or More Correct]

Q36. A solid isolated conducting sphere of radius R is given a net charge $+Q$. It is coaxially surrounded by a thick, uncharged, concentric conducting spherical shell of inner radius R_1 and outer radius R_2. The electrostatic potential $V(r)$ at a radial distance r from the common center O in the region $R_1 < r < R_2$ is:



(A) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R_1}$

(C) $\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r} - \frac{Q}{R_1} + \frac{Q}{R_2}\right]$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R_2}$

Q37. A uniform static magnetic field $\vec{B} = B_0\hat{k}$ is established throughout a region of space. A particle of mass m and positive charge $+q$ is launched from the origin with an initial velocity vector $\vec{v} = v_x\hat{i} + v_z\hat{k}$ (where $v_x > 0$ and $v_z > 0$). Which of the following statements is/are true regarding the subsequent motion of the particle?

(A) The trajectory of the particle is a helix with a constant pitch.

(B) The time period of one complete helical turn is completely independent of both v_x and v_z.

(C) The central longitudinal axis of the helical path is parallel to the z-axis.

(D) The kinetic energy of the particle changes periodically due to the magnetic force.

Q38. A series LCR circuit is driven by an alternating voltage source of adjustable frequency given by $V = V_0 \sin(\omega t)$. Let $\omega_0 = \frac{1}{\sqrt{LC}}$ denote the natural resonance angular frequency of the circuit. As the source frequency ω is varied, which of the following statements is/are correct?

(A) At resonance ($\omega = \omega_0$), the total electrical impedance of the circuit is minimum and is exactly equal to R.

(B) For operating frequencies $\omega > \omega_0$, the circuit is inductive, and the alternating current lags behind the source voltage.

(C) For operating frequencies $\omega < \omega_0$, the circuit is capacitive, and the alternating current leads the source voltage.

(D) At resonance, the power factor of the circuit reaches its maximum possible value of unity.

Q39. A ray of monochromatic light traveling inside an optically denser medium of refractive index n_1 is incident on a plane boundary separating it from an optically rarer medium of refractive index n_2 ($n_1 > n_2$). If θ represents the angle of incidence at the interface, Total Internal Reflection (TIR) will occur if:

(A) $\theta > \sin^{-1}\left(\frac{n_2}{n_1}\right)$

(B) $\theta < \sin^{-1}\left(\frac{n_2}{n_1}\right)$

(C) The critical angle θ_c for this interface satisfies $\sin\theta_c = \frac{n_2}{n_1}$

(D) The incident light is directed from the rarer medium into the denser medium

Q40. In a photoelectric effect experiment, the stopping potential V_s required to reduce the photocurrent to zero is measured for different frequencies ν of the incident light on a given metallic surface. If a linear graph is plotted with stopping potential V_s on the vertical y-axis and frequency ν on the horizontal x-axis, which of the following statements is/are true?

(A) The slope of the straight line is equal to $\frac{h}{e}$, where h is Planck's constant and e is the electronic charge magnitude.

(B) The intercept on the frequency x-axis corresponds exactly to the threshold frequency ν_0 of the metal.

(C) The intercept on the potential y-axis is equal to $-\frac{\phi}{e}$, where ϕ is the work function of the metal.

(D) The slope of the straight line increases if a metal with a higher work function is chosen.

Detailed Solutions

Q1.

Solution

Concept:
The calculation of maximum fractional and percentage errors in a derived physical quantity relies on the propagation of errors. For a generalized algebraic function, the relative error in the calculated result is the sum of the relative errors of its constituent independent variables, each multiplied by the absolute value of its respective power exponent.

Solution:

(a) Given the functional relationship for the physical quantity X:

$$X = \frac{a^2 b^3}{\sqrt{c}} = a^2 b^3 c^{-1/2}$$

(b) Taking the natural logarithm on both sides of the equation yields:

$$\ln X = 2\ln a + 3\ln b - \frac{1}{2}\ln c$$

(c) Differentiating both sides to find the fractional change and taking the maximum absolute value for worst-case error accumulation gives:

$$\frac{\Delta X}{X} = 2\left(\frac{\Delta a}{a}\right) + 3\left(\frac{\Delta b}{b}\right) + \frac{1}{2}\left(\frac{\Delta c}{c}\right)$$

(d) To express this in terms of the maximum percentage error, multiply each individual term by 100%:

$$\frac{\Delta X}{X} \times 100\% = 2\left(\frac{\Delta a}{a} \times 100\%\right) + 3\left(\frac{\Delta b}{b} \times 100\%\right) + \frac{1}{2}\left(\frac{\Delta c}{c} \times 100\%\right)$$

(e) Substitute the given percentage errors, which are 1% for a, 2% for b, and 4% for c:

$$\text{Maximum \% Error in } X = 2(1\%) + 3(2\%) + \frac{1}{2}(4\%)$$

$$\text{Maximum \% Error in } X = 2\% + 6\% + 2\% = 10\%$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept:
Dimensional analysis finds relationships between variables by equating fundamental dimensions (M, L, T) on both sides of an equation.
Solution:

(a) Express velocity v in terms of tension T and mass per unit length μ:

$$v = kT^a \mu^b$$

(b) Substitute the corresponding dimensional formulas:

$$[M^0 L^1 T^{-1}] = [M^1 L^1 T^{-2}]^a [M^1 L^{-1} T^0]^b$$

$$[M^0 L^1 T^{-1}] = [M^{a+b} L^{a-b} T^{-2a}]$$

(c) Equate exponents for matching base units:

$$\text{For } M : a + b = 0 \implies b = -a$$

$$\text{For } T : -2a = -1 \implies a = \frac{1}{2}$$

(d) Solve for the powers to get $a = \frac{1}{2}$ and $b = -\frac{1}{2}$.

(e) Substitute back to obtain the final proportional relationship:

$$v \propto T^{1/2} \mu^{-1/2} \implies v \propto \sqrt{\frac{T}{\mu}}$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept:
The radius of curvature R_c of any curved planar path describes how sharply the trajectory curves at a given point. It is mathematically defined as the ratio of the square of the particle's instantaneous speed to the component of acceleration acting perpendicular to the instantaneous direction of motion.

Solution:

(a) A projectile launched with initial velocity v_0 at an angle θ experiences a constant downward gravitational acceleration g.

(b) At the highest point of its parabolic flight path, the vertical component of its velocity becomes zero ($v_y = 0$).

(c) The horizontal component of its velocity remains unaltered throughout the journey because there is no horizontal force component. Therefore, the speed at the peak is:

$$v_{\text{top}} = v_0 \cos\theta$$

(d) At this topmost point, the velocity vector points entirely in the horizontal direction.

(e) The acceleration due to gravity g points straight downwards, making it exactly perpendicular to the horizontal velocity vector. Thus, the normal acceleration is:

$$a_n = g$$

(f) Substitute these variables into the structural formula for the local radius of curvature:

$$R_c = \frac{v_{\text{top}}^2}{a_n} = \frac{(v_0 \cos\theta)^2}{g} = \frac{v_0^2 \cos^2\theta}{g}$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept:
In rectilinear motion, velocity is $v = \frac{dx}{dt}$ and acceleration is $a = \frac{dv}{dt}$. Implicit differentiation links these coupled variables.
Solution:

(a) Given displacement relation:

$$x^2 = t^2 + 1$$

(b) Differentiate with respect to time t:

$$2xv = 2t \implies xv = t \implies v = \frac{t}{x}$$

(c) Differentiate $xv = t$ again using the product rule:

$$x\frac{dv}{dt} + v\frac{dx}{dt} = 1 \implies xa + v^2 = 1$$

(d) Substitute $v = \frac{t}{x}$ into the acceleration equation:

$$xa = 1 - \frac{t^2}{x^2} = \frac{x^2 - t^2}{x^2}$$

(e) Since $x^2 - t^2 = 1$, substitute it to find the relation:

$$xa = \frac{1}{x^2} \implies a = \frac{1}{x^3} \implies a \propto \frac{1}{x^3}$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept:
Sliding begins when the horizontal pulling component overcomes static friction. Minimizing this force involves optimizing the angle to reduce the normal force via vertical lifting.
Solution:

(a) Resolve force F at angle θ and balance vertical forces for normal reaction N:

$$N = mg - F\sin\theta$$

(b) Set the horizontal pulling component equal to maximum static friction:

$$F\cos\theta = \mu N = \mu(mg - F\sin\theta)$$

(c) Rearrange the terms to express F as a function of angle θ:

$$F = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$

(d) Maximize the denominator by differentiating with respect to θ, giving $\tan\theta = \mu$.

(e) Substitute the maximum denominator value $\sqrt{1+\mu^2}$ back into the equation:

$$F_{\min} = \frac{\mu mg}{\sqrt{1+\mu^2}}$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:
When a vehicle travels along a flat, unbanked circular road, the required centripetal force keeping it in its circular track is provided entirely by the static friction between the tires and the road surface. Skidding occurs if the required centripetal force exceeds the maximum static friction limit.

Solution:

(a) Consider the forces acting on the vehicle along the vertical axis. Since there is no vertical acceleration, the normal force matches the weight:

$$N = mg$$

(b) The maximum static friction force that the road can exert on the tires is given by:

$$f_{\max} = \mu N = \mu mg$$

(c) For a car of mass m navigating a turn of radius R at a speed v, the required centripetal force directed toward the center of the turn is:

$$F_c = \frac{mv^2}{R}$$

(d) To ensure safe travel without sliding outward, this centripetal force must be less than or equal to the maximum available static friction:

$$\frac{mv^2}{R} \leq \mu mg \implies v^2 \leq \mu gR \implies v \leq \sqrt{\mu gR}$$

(e) The maximum safe threshold velocity is the upper boundary of this inequality:

$$v_{\max} = \sqrt{\mu gR}$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:
In a perfectly elastic collision, both total linear momentum and total kinetic energy are conserved. The fraction of energy transferred during a head-on impact depends on the mass ratio of the colliding bodies.

Solution:

(a) Let $m_1 = m$ be the moving mass with initial velocity $u_1 = v$, and $m_2 = 2m$ be the stationary mass ($u_2 = 0$).

(b) The initial kinetic energy of the system is entirely contained within the lighter block:

$$K_i = \frac{1}{2}mv^2$$

(c) For a perfectly elastic collision, the post-collision velocity v_2 of the second mass can be calculated using the standard formula:

$$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)u_2}{m_1 + m_2}$$

(d) Substitute the given values into this expression:

$$v_2 = \frac{2(m)(v)}{m + 2m} + 0 = \frac{2mv}{3m} = \frac{2}{3}v$$

(e) The kinetic energy gained by the heavier block after the impact is:

$$K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2m)\left(\frac{2}{3}v\right)^2 = m\left(\frac{4}{9}v^2\right) = \frac{4}{9}\left(\frac{1}{2}mv^2\right) \times 2 = \frac{8}{9}\left(\frac{1}{2}mv^2\right)$$

(f) Compute the fraction of energy transferred:

$$\text{Fraction} = \frac{K_2}{K_i} = \frac{\frac{8}{9}(\frac{1}{2}mv^2)}{\frac{1}{2}mv^2} = \frac{8}{9}$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept:
When a symmetric body rolls down an inclined plane without slipping, its potential energy is converted into both translational and rotational kinetic energy. The linear acceleration depends on its distribution of mass, which is characterized by its moment of inertia.
Solution:

(a) The linear acceleration a of a body rolling without slipping down an incline of angle θ is given by:

$$a = \frac{g \sin\theta}{1 + \frac{I}{MR^2}}$$

where I is the moment of inertia about the central rotational axis.

(b) For a uniform solid sphere, the moment of inertia is:

$$I_{\text{sphere}} = \frac{2}{5}MR^2 \implies \frac{I_{\text{sphere}}}{MR^2} = \frac{2}{5}$$

(c) Calculate the linear acceleration of the sphere:

$$a_{\text{sphere}} = \frac{g \sin\theta}{1 + \frac{2}{5}} = \frac{g \sin\theta}{\frac{7}{5}} = \frac{5}{7} g \sin\theta$$

(d) For a uniform solid cylinder, the moment of inertia is:

$$I_{\text{cylinder}} = \frac{1}{2}MR^2 \implies \frac{I_{\text{cylinder}}}{MR^2} = \frac{1}{2}$$

(e) Calculate the linear acceleration of the cylinder:

$$a_{\text{cylinder}} = \frac{g \sin\theta}{1 + \frac{1}{2}} = \frac{g \sin\theta}{\frac{3}{2}} = \frac{2}{3} g \sin\theta$$

(f) Determine the ratio of their accelerations:

$$\frac{a_{\text{sphere}}}{a_{\text{cylinder}}} = \frac{\frac{5}{7} g \sin\theta}{\frac{2}{3} g \sin\theta} = \frac{5}{7} \times \frac{3}{2} = \frac{15}{14}$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept:
The acceleration due to gravity changes with both altitude above the Earth's surface and depth below it. For small changes compared to the Earth's total radius, simplified linear approximations can be used.

Solution:

(a) The exact expression for gravity at a height h above the surface is $g_h = g\left(1+\frac{h}{R}\right)^{-2}$. Under the condition $h \ll R$, we apply the binomial expansion approximation:

$$g_h \approx g\left(1 - \frac{2h}{R}\right)$$

(b) The exact expression for gravity at a depth d below the surface, assuming a uniform planetary mass density, scales linearly with distance from the center:

$$g_d = g\left(1 - \frac{d}{R}\right)$$

(c) The problem states that these two gravitational acceleration values are equal:

$$g_h = g_d$$

(d) Substitute the two expressions into this equality:

$$g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$$

(e) Simplify the equation by canceling common terms on both sides:

$$1 - \frac{2h}{R} = 1 - \frac{d}{R}$$

$$\frac{2h}{R} = \frac{d}{R} \implies d = 2h$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept:
Hooke's Law within elastic limits states that the elongation produced in a structural wire is proportional to the applied tensile load and its original length, and inversely proportional to its cross-sectional area and Young's Modulus of the material.
Solution:

(a) Young's Modulus Y is defined as the ratio of tensile stress to tensile strain:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

(b) Rearranging this formula allows us to solve for the structural elongation ΔL:

$$\Delta L = \frac{FL}{A\Delta L} \implies \Delta L = \frac{FL}{\pi r^2 Y}$$

(c) For the first wire, the baseline elongation is:

$$\Delta L_1 = \frac{FL}{\pi r^2 Y} = \Delta L$$

(d) The second wire is made of the same material, meaning its Young's Modulus Y is identical. It is subjected to the same load F, but its geometric properties are changed to $L_2 = 2L$ and $r_2 = 2r$.

(e) Express the elongation of the second wire using these new parameters:

$$\Delta L_2 = \frac{FL_2}{\pi r_2^2 Y} = \frac{F(2L)}{\pi(2r)^2 Y}$$

$$\Delta L_2 = \frac{2FL}{4\pi r^2 Y} = \frac{1}{2}\left(\frac{FL}{\pi r^2 Y}\right)$$

(f) Substitute the baseline elongation expression into this result:

$$\Delta L_2 = \frac{\Delta L}{2}$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept:
When a liquid drop breaks into smaller droplets, its total mass and volume remain constant, but its total surface area increases. Work must be done against surface tension forces to create this additional surface area.

Solution:

(a) Let R be the radius of the large drop and r be the radius of each of the 8 smaller droplets.

(b) Equate the initial volume to the total final volume to find the relationship between the radii:

$$\frac{4}{3}\pi R^3 = 8 \times \left(\frac{4}{3}\pi r^3\right) \implies R^3 = 8r^3 \implies R = 2r \implies r = \frac{R}{2}$$

(c) Calculate the initial surface area of the single large drop:

$$A_i = 4\pi R^2$$

(d) Calculate the combined total surface area of the 8 smaller droplets:

$$A_f = 8 \times (4\pi r^2) = 8 \times 4\pi \left(\frac{R}{2}\right)^2 = 8 \times 4\pi \frac{R^2}{4} = 8\pi R^2$$

(e) Determine the net increase in surface area:

$$\Delta A = A_f - A_i = 8\pi R^2 - 4\pi R^2 = 4\pi R^2$$

(f) The work done against surface tension is the product of the surface tension and the change in surface area:

$$W = T \cdot \Delta A = 4\pi R^2 T$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 11

Q12.

Solution

Concept:
Poiseuille's Law determines the steady volumetric flow rate of a viscous fluid through a narrow cylindrical capillary tube. The flow rate is proportional to the pressure difference across the tube and the fourth power of its internal radius.

Solution:

(a) According to Poiseuille's equation, the volume flow rate Q through a capillary tube of length L and radius r is:

$$Q = \frac{\pi \Delta P r^4}{8\eta L}$$

where ΔP is the net hydrostatic pressure difference across the tube length.

(b) The inner end of the tube is located at a depth h below the free surface of the water tank. The hydrostatic gauge pressure at this depth is:

$$P_{\text{inside}} = \rho g h$$

(c) The outer end of the horizontal capillary tube discharges into the open atmosphere, where the gauge pressure is zero:

$$P_{\text{outside}} = 0$$

(d) The net pressure driving the fluid through the tube is the difference between these two pressures:

$$\Delta P = P_{\text{inside}} - P_{\text{outside}} = \rho g h$$

(e) Substitute this pressure difference back into Poiseuille's equation:

$$Q = \frac{\pi(\rho g h) r^4}{8\eta L} = \frac{\pi \rho g h r^4}{8\eta L}$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept:
The molar heat capacity C of an ideal gas during a customized thermodynamic process can be found using the First Law of Thermodynamics along with the specific relationship linking pressure and volume.

Solution:

(a) The first law of thermodynamics states that $dQ = dU + dW$. For one mole of gas ($n = 1$), this can be written as:

$$CdT = C_V dT + PdV \implies C = C_V + P\frac{dV}{dT}$$

(b) We are given the process equation $P = kV^2$. Using the ideal gas law for one mole ($PV = RT$), we can substitute P:

$$(kV^2)V = RT \implies kV^3 = RT$$

(c) Differentiate this equation with respect to temperature T to find how volume changes with temperature:

$$\frac{d}{dT}(kV^3) = \frac{d}{dT}(RT) \implies 3kV^2\frac{dV}{dT} = R \implies \frac{dV}{dT} = \frac{R}{3kV^2}$$

(d) Substitute this derivative back into the molar heat capacity equation:

$$C = C_V + P\left(\frac{R}{3kV^2}\right)$$

(e) Replace P with kV^2:

$$C = C_V + (kV^2)\left(\frac{R}{3kV^2}\right) = C_V + \frac{R}{3}$$

(f) Substitute the given value for a monoatomic gas, $C_V = \frac{3}{2}R$:

$$C = \frac{3}{2}R + \frac{1}{3}R = \left(\frac{9+2}{6}\right)R = \frac{11}{6}R$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept:
The root-mean-square (rms) speed of gas molecules depends on the absolute temperature of the gas and the molar mass of the individual particles.

Solution:

(a) The root-mean-square speed of a gas particle is given by the formula:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where M is the molar mass of the particles and T is the absolute temperature.

(b) Initially, for hydrogen gas molecules (H_2), let the molar mass of a hydrogen atom be M_0. Thus, the molar mass of diatomic hydrogen is $M_{\text{initial}} = 2M_0$. The initial rms speed is:

$$v = \sqrt{\frac{3RT}{2M_0}}$$

(c) The gas then undergoes a change where the temperature is doubled to $2T$, and the molecules completely dissociate into single atoms (H). This reduces the molar mass of the particles to:

$$M_{\text{final}} = M_0$$

(d) Calculate the new rms speed v' with these updated values:

$$v' = \sqrt{\frac{3R(2T)}{M_0}} = \sqrt{\frac{6RT}{M_0}}$$

(e) To find the relationship between the old and new speeds, divide v' by v:

$$\frac{v'}{v} = \frac{\sqrt{\frac{6RT}{M_0}}}{\sqrt{\frac{3RT}{2M_0}}} = \sqrt{\frac{6}{3/2}} = \sqrt{6 \times \frac{2}{3}} = \sqrt{4} = 2$$

$$v' = 2v$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 14

Q15.

Solution

Concept:

The Stefan-Boltzmann Law states that the total energy radiated per unit surface area of a black body per unit time is directly proportional to the fourth power of its absolute temperature.

Solution:

(a) According to the Stefan-Boltzmann law, the total radiant power E emitted by a black body of surface area A at an absolute temperature T is given by:

$$E = \sigma A T^4$$

where σ is the Stefan-Boltzmann constant.

(b) Initially, at temperature $T_1 = T$, the emitted power is:

$$E_1 = \sigma A T^4 = E$$

(c) The absolute temperature is then reduced to half of its initial value, so $T_2 = \frac{T}{2}$. Assuming the surface area remains unchanged, the new radiant power is:

$$E_2 = \sigma A T_2^4 = \sigma A \left(\frac{T}{2}\right)^4$$

(d) Expand the power term in the denominator:

$$E_2 = \sigma A \left(\frac{T^4}{16}\right) = \frac{1}{16}(\sigma A T^4)$$

(e) Substitute the initial power E into this expression:

$$E_2 = \frac{E}{16}$$

Final Answer: The correct choice is (D).

Answer: (D)

Go Back to Question 15

Q16.

Solution

Concept:
In Simple Harmonic Motion (SHM), displacement, velocity, and acceleration vary sinusoidal over time. The maximum values of velocity and acceleration occur at specific points in the oscillation cycle and depend directly on the amplitude and the angular frequency.

Solution:

(a) Let the amplitude of the oscillation be A and the angular frequency be ω.

(b) The maximum velocity $v_{\max}$ of a particle in SHM occurs as it passes through the central equilibrium position, and its value is:

$$v_{\max} = \omega A$$

(c) The maximum acceleration $a_{\max}$ occurs at the extreme turnaround points where displacement is at its maximum, and its value is:

$$a_{\max} = \omega^2 A$$

(d) To eliminate the angular frequency ω and find an expression for amplitude A, square the equation for maximum velocity:

$$v_{\max}^2 = \omega^2 A^2$$

(e) Divide this squared velocity by the maximum acceleration equation:

$$\frac{v_{\max}^2}{a_{\max}} = \frac{\omega^2 A^2}{\omega^2 A} = A$$

(f) Rearranging this gives the final formula for the amplitude of oscillation:

$$A = \frac{v_{\max}^2}{a_{\max}}$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept:
The Doppler Effect describes the change in the observed frequency of a wave when there is relative motion between the source emitting the wave and the observer receiving it. When a sound source moves toward a stationary observer, the sound waves are compressed, causing the observer to hear a higher frequency.
Solution:

(a) The general formula for the apparent frequency f' heard due to the Doppler effect is:

$$f' = f_0 \left(\frac{v \pm v_o}{v \mp v_s} \right)$$

where v is the speed of sound in air, v_o is the speed of the observer, v_s is the speed of the source, and f_0 is the emitted frequency.

(b) Identify the given parameters from the problem:

$$\text{Emitted frequency } f_0 = 600 \text{ Hz}$$

$$\text{Speed of sound in air } v = 330 \text{ m/s}$$

$$\text{Speed of the stationary listener } v_o = 0$$

$$\text{Speed of the approaching source } v_s = 30 \text{ m/s}$$

(c) Since the source is approaching the stationary listener, choose the negative sign in the denominator to increase the observed frequency:

$$f' = f_0 \left(\frac{v}{v - v_s} \right)$$

(d) Substitute the values into this formula:

$$f' = 600 \left(\frac{330}{330 - 30} \right) = 600 \left(\frac{330}{330} \right) = 600 \left(\frac{330}{300} \right)$$

$$f' = 600 \times 1.1 = 660 \text{ Hz}$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept:
An electric dipole consists of two equal and opposite point charges separated by a distance. A complex system of multiple charges can be analyzed by breaking it down into a combination of individual electric dipoles and finding their vector sum.

Solution:

(a) The charge $-2q$ located at one vertex can be split into two separate charges of $-q$ and $-q$ residing at the same point.

(b) This allows us to treat the system as two separate dipoles. Each dipole consists of a $+q$ charge at one vertex and a $-q$ charge at the base vertex.

(c) The magnitude of the dipole moment vector for each pair is the product of the charge and the separation distance a:

$$p_1 = qa \quad \text{and} \quad p_2 = qa$$

(d) Dipole vectors point from the negative charge toward the positive charge. In this equilateral triangle configuration, the two vectors open outward from the base vertex with an angle of $60°$ between them.

(e) Use the vector addition formula to find the net total dipole moment p_{net}:

$$p_{\text{net}} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos(60°)}$$

(f) Substitute $p_1 = p_2 = qa$ and $\cos(60°) = 0.5$ into the equation:

$$p_{\text{net}} = \sqrt{(qa)^2 + (qa)^2 + 2(qa)(qa)(0.5)}$$

$$p_{\text{net}} = \sqrt{(qa)^2 + (qa)^2 + (qa)^2} = \sqrt{3(qa)^2} = \sqrt{3}qa$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 18

Q19.

Solution

Concept:
The capacitance of a parallel-plate capacitor depends on its geometry. When a charged capacitor is disconnected from its charging battery, the electric charge on its plates becomes trapped and must remain constant due to the conservation of charge.

Solution:

(a) The initial capacitance of a parallel-plate air capacitor with plate separation d is:

$$C = \frac{\epsilon_0 A}{d}$$

(b) When the separation distance between plates is doubled ($d' = 2d$), the new capacitance C' becomes:

$$C' = \frac{\epsilon_0 A}{2d} = \frac{C}{2}$$

Thus, the capacitance is cut in half.

(c) Since the capacitor was disconnected from the battery before making this adjustment, the stored charge Q cannot escape and remains constant.

(d) The electrostatic potential energy stored in a capacitor can be expressed in terms of its charge and capacitance as:

$$U = \frac{Q^2}{2C}$$

(e) Calculate the new stored energy U' using the updated capacitance C':

$$U' = \frac{Q^2}{2C'} = \frac{Q^2}{2(C/2)} = 2\left(\frac{Q^2}{2C}\right) = 2U$$

Thus, the stored potential energy doubles.

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept:
A potentiometer works by establishing a uniform potential drop across a long wire. When a cell is balanced against a specific length of this wire, no current flows through the cell, and its electromotive force (emf) is directly proportional to the balancing length.
Solution:

(a) Let ϕ represent the potential gradient, which is the uniform voltage drop per unit length along the potentiometer wire.

(b) The balancing condition for a cell with an electromotive force E balancing at a length l is:

$$E = \phi \cdot l$$

(c) Write the balancing equations for both the standard cell and the unknown cell:

$$E_1 = \phi \cdot l_1 \quad \text{and} \quad E_2 = \phi \cdot l_2$$

(d) Divide the second equation by the first to eliminate the unknown potential gradient ϕ:

$$\frac{E_2}{E_1} = \frac{l_2}{l_1} \implies E_2 = E_1 \left(\frac{l_2}{l_1}\right)$$

(e) Substitute the given values into this ratio ($E_1 = 1.25$ V, $l_1 = 35$ cm, and $l_2 = 63$ cm):

$$E_2 = 1.25 \times \left(\frac{63}{35}\right)$$

(f) Simplify the fraction by dividing both terms by their common factor of 7:

$$\frac{63}{35} = \frac{9}{5} = 1.8$$

$$E_2 = 1.25 \times 1.8 = 2.25 \text{ V}$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:
When electrical components are connected in a parallel configuration across an ideal voltage source, the potential difference V across each component is identical. The rate of electrical thermal energy generation (power dissipation) can be determined using a power formula that highlights this constant voltage.

Solution:

(a) The rate of thermal energy produced per second in a resistor is equal to the electrical power dissipation P.

(b) Because the resistors are connected in parallel, they both experience the same voltage V. Therefore, we use the power formula:

$$P = \frac{V^2}{R}$$

(c) Write the expressions for power dissipation in both resistors R_1 and R_2:

$$P_1 = \frac{V^2}{R_1} \quad \text{and} \quad P_2 = \frac{V^2}{R_2}$$

(d) Determine the ratio of the power dissipated by R_1 to that dissipated by R_2:

$$\frac{P_1}{P_2} = \frac{V^2/R_1}{V^2/R_2} = \frac{R_2}{R_1}$$

(e) This shows that for a constant voltage, the rate of heat generation is inversely proportional to the resistance. Since $R_1 > R_2$, the larger resistor generates less heat per second.

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:
The Biot-Savart Law and Ampere's Law show that the magnetic field produced by a long, straight current-carrying wire wraps around the wire in concentric circles. The direction of this field can be found using the right-hand rule, and its vector form can be derived using cross products.
Solution:

(a) The wire lies along the z-axis with current flowing in the $\hat{k}$ direction. For any point $(x, y, 0)$ in the xy-plane, the position vector relative to the wire is:

$$\vec{r} = x\hat{i} + y\hat{j}$$

(b) The magnitude of the displacement from the wire is $r = \sqrt{x^2 + y^2}$. The magnitude of the magnetic field at this distance is:

$$B = \frac{\mu_0 I}{2\pi r} \propto \frac{1}{\sqrt{x^2 + y^2}}$$

(c) According to the vector form of Ampere's Law, the direction of the magnetic field is given by the cross product of the current direction unit vector ($\hat{k}$) and the radial position unit vector ($\hat{r}$):

$$\hat{\theta} = \hat{k} \times \hat{r} = \hat{k} \times \left(\frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}} \right)$$

(d) Evaluate the cross products ($\hat{k} \times \hat{i} = \hat{j}$ and $\hat{k} \times \hat{j} = -\hat{i}$):

$$\hat{\theta} = \frac{x\hat{j} - y\hat{i}}{\sqrt{x^2 + y^2}} = \frac{-y\hat{i} + x\hat{j}}{\sqrt{x^2 + y^2}}$$

(e) Combine the magnitude and direction to find the full magnetic field vector $\vec{B}$:

$$\vec{B} = B \cdot \hat{\theta} \propto \frac{1}{\sqrt{x^2 + y^2}} \cdot \left(\frac{-y\hat{i} + x\hat{j}}{\sqrt{x^2 + y^2}} \right) = \frac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept:
When a charged particle enters a uniform magnetic field at an angle, its velocity can be split into two components. The component parallel to the field lines maintains constant linear motion, while the perpendicular component drives uniform circular motion, resulting in a helical path.

Solution:

(a) Split the initial velocity v into components relative to the magnetic field lines:

$$v_{\|} = v\cos(30^\circ) = v\frac{\sqrt{3}}{2}$$

$$v_{\perp} = v\sin(30^\circ) = v\frac{1}{2}$$

(b) The radius R of the circular loop depends on the perpendicular velocity component:

$$R = \frac{mv_{\perp}}{qB} = \frac{mv}{2qB}$$

(c) The time period T required to complete one full revolution depends on the total circumference and the perpendicular speed:

$$T = \frac{2\pi m}{qB}$$

(d) The pitch P of the helix is the linear distance traveled along the direction of the magnetic field during one full rotational period:

$$P = v_{\|} \cdot T = \left(v\frac{\sqrt{3}}{2}\right)\left(\frac{2\pi m}{qB}\right) = \frac{\pi m v\sqrt{3}}{qB}$$

(e) Calculate the ratio of the helical pitch to the radius:

$$\frac{P}{R} = \frac{\frac{\pi m v\sqrt{3}}{qB}}{\frac{mv}{2qB}} = \frac{\pi\sqrt{3}}{1/2} = 2\pi\sqrt{3}$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept:
Faraday's Law of Induction states that a change in the magnetic flux passing through a conductive coil induces an electromotive force (emf) proportional to the rate of that change. Flipping a coil changes the orientation of its area vector, altering the magnetic flux.
Solution:

(a) The initial magnetic flux Φ_i passing through a flat coil with N turns, area $A = \pi r^2$, and aligned perpendicular to a magnetic field B is:

$$\Phi_i = NBA\cos(0^\circ) = NB(\pi r^2)$$

(b) The coil is flipped by 180°, reversing the direction of its area vector relative to the magnetic field lines. The final magnetic flux Φ_f is:

$$\Phi_f = NBA\cos(180^\circ) = -NB(\pi r^2)$$

(c) Calculate the net change in magnetic flux ($\Delta\Phi$) over this interval:

$$\Delta\Phi = \Phi_f - \Phi_i = -NB(\pi r^2) - NB(\pi r^2) = -2\pi NBr^2$$

(d) According to Faraday's Law, the magnitude of the average induced electromotive force is the change in flux divided by the time interval Δt:

$$|\mathcal{E}| = \left|\frac{\Delta\Phi}{\Delta t}\right| = \frac{2\pi NBr^2}{\Delta t}$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept:

In a plane electromagnetic wave traveling through a vacuum, the oscillating electric and magnetic fields are perpendicular to each other and to the direction of wave propagation. The amplitudes of these two fields are directly related by the speed of light in a vacuum.

Solution:

(a) The given equation for the electric field component is:

$$E_y = 50\sin(10^8 t - kx) \text{ V/m}$$

From this expression, extract the peak amplitude of the electric field:

$$E_0 = 50 \text{ V/m}$$

(b) The fundamental constant linking the peak electric field E_0 and the peak magnetic field B_0 in an electromagnetic wave is the speed of light c:

$$c = \frac{E_0}{B_0} \implies B_0 = \frac{E_0}{c}$$

(c) The speed of light in a vacuum is a standard physical constant:

$$c \approx 3 \times 10^8 \text{ m/s}$$

(d) Substitute these values into the relationship to calculate B_0:

$$B_0 = \frac{50}{3 \times 10^8} = \frac{50}{3} \times 10^{-8} \approx 16.67 \times 10^{-8} \text{ T} = 1.67 \times 10^{-7} \text{ T}$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:
The thin lens formula maps the quantitative relationship between the object distance u, image distance v, and focal length f. Linear magnification m is defined as the ratio of image distance to object distance, and its sign depends on the nature of the image.

Solution:

(a) According to standard Cartesian sign conventions for a convex lens forming a real image:

$$\text{Object distance } = -u \quad (u > 0)$$

$$\text{Image distance } = +v \quad (v > 0)$$

$$\text{Focal length } = +f$$

(b) The linear magnification for a real image is inverted, meaning m is negative:

$$-m = \frac{v}{-u} \implies v = mu$$

(c) Write down the standard thin lens formula:

$$\frac{1}{v} - \frac{1}{-u} = \frac{1}{f} \implies \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

(d) Substitute the expression $v = mu$ into this lens formula:

$$\frac{1}{mu} + \frac{1}{u} = \frac{1}{f}$$

(e) Factor out the common term $\frac{1}{u}$ from the left side:

$$\frac{1}{u}\left(\frac{1}{m} + 1\right) = \frac{1}{f} \implies \frac{1}{u}\left(\frac{1+m}{m}\right) = \frac{1}{f}$$

(f) Invert the equation to solve directly for the object distance u:

$$u = \left(\frac{m+1}{m}\right) f$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept:
In a Young's Double-Slit Experiment, the intensity distribution across the interference fringes depends on the phase difference between the two overlapping light waves. This phase difference is directly related to the optical path difference.

Solution:

(a) The relationship linking the optical path difference Δx to the phase difference ϕ is given by:

$$\phi = \frac{2\pi}{\lambda} \cdot \Delta x$$

(b) Substitute the given path difference $\Delta x = \frac{\lambda}{6}$ into this equation:

$$\phi = \frac{2\pi}{\lambda} \cdot \left(\frac{\lambda}{6}\right) = \frac{2\pi}{6} = \frac{\pi}{3}$$

(c) The mathematical formula for the resultant intensity I at a point where two coherent waves of equal intensity interfere is:

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

where I_0 is the maximum intensity at the center of a bright fringe.

(d) Substitute the calculated phase difference $\phi = \frac{\pi}{3}$ into the intensity formula:

$$I = I_0 \cos^2\left(\frac{\pi/3}{2}\right) = I_0 \cos^2\left(\frac{\pi}{6}\right)$$

(e) The cosine of $\frac{\pi}{6}$ (or 30°) is $\frac{\sqrt{3}}{2}$. Square this value:

$$\cos^2\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

(f) Substitute this back to find the ratio of intensities:

$$I = I_0\left(\frac{3}{4}\right) \implies \frac{I}{I_0} = \frac{3}{4}$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 27

Q28.

Solution

Concept:
The de Broglie hypothesis states that a moving particle exhibits wave-like properties. The wavelength associated with an electron accelerated from rest through an electric potential difference depends on its gained kinetic energy.

Solution:

(a) The kinetic energy K gained by an electron of charge e accelerated through a potential difference V is:

$$K = eV$$

(b) The de Broglie wavelength λ is related to the particle's momentum p and kinetic energy K by:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV}}$$

This shows that the wavelength is inversely proportional to the square root of the accelerating potential:

$$\lambda \propto \frac{1}{\sqrt{V}}$$

(c) Write the initial wavelength expression for a potential V:

$$\lambda_1 = \frac{k}{\sqrt{V}} = \lambda$$

(d) The potential is then increased to $4V$. Calculate the new wavelength λ_2:

$$\lambda_2 = \frac{k}{\sqrt{4V}} = \frac{k}{2\sqrt{V}}$$

(e) Substitute the initial wavelength expression into this result:

$$\lambda_2 = \frac{\lambda}{2}$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept:

Radioactive decay alters the internal structure of an unstable nucleus. An alpha (α) decay reduces the mass number by 4 and the atomic number by 2 because a helium nucleus is emitted. A beta-minus (β^-) decay increases the atomic number by 1 while leaving the mass number unchanged because a neutron transforms into a proton.

Solution:

(a) Start with the parent nucleus A, which has a mass number $M = 180$ and an atomic number $Z = 72$:

$$^{180}_{72}A$$

(b) The first step is an alpha decay ($A \xrightarrow{\alpha} A_1$). Apply the changes to the mass and atomic numbers:

$$\text{New mass number } M_1 = 180 - 4 = 176$$

$$\text{New atomic number } Z_1 = 72 - 2 = 70$$

This gives the intermediate nucleus: $^{176}_{70}A_1$.

(c) The second step is a beta-minus decay ($A_1 \xrightarrow{\beta^-} A_2$). Apply the changes:

$$\text{New mass number } M_2 = 176 \quad \text{(unchanged)}$$

$$\text{New atomic number } Z_2 = 70 + 1 = 71$$

This gives the second intermediate nucleus: $^{176}_{71}A_2$.

(d) The final step is another alpha decay ($A_2 \xrightarrow{\alpha} A_3$). Apply the changes:

$$\text{Final mass number } M_3 = 176 - 4 = 172$$

$$\text{Final atomic number } Z_3 = 71 - 2 = 69$$

This gives the final product nucleus: $^{172}_{69}A_3$.

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept:
Combinational logic circuits process multiple binary inputs through basic logic gates to produce a single output. The overall behavior of the system can be derived by tracking how the signals change as they pass through each gate using Boolean algebra expressions.
Solution:

(a) The two logic inputs A and B are split and fed into two separate gates simultaneously.

(b) The first gate is a NOR gate. Its output expression is:

$$Y_1 = \overline{A + B}$$

(c) The second gate is an AND gate. Its output expression is:

$$Y_2 = A \cdot B$$

(d) These two intermediate outputs, Y_1 and Y_2, are then fed as inputs into a final OR gate. The total output Y is the logical sum of these two signals:

$$Y = Y_1 + Y_2 = (\overline{A + B}) + (A \cdot B)$$

(e) Use De Morgan's Laws to rewrite the first term: $\overline{A + B} = \bar{A} \cdot \bar{B}$. Substitute this back into the expression:

$$Y = (\bar{A} \cdot \bar{B}) + (A \cdot B)$$

(f) This Boolean expression matches the definition of the Exclusive-NOR (XNOR) logic function, which outputs a logical 1 only when both inputs are identical:

$$(\bar{A} \cdot \bar{B}) + (A \cdot B) = \overline{A \oplus B}$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 30

Q31.

Solution

Concept:
An upward accelerating frame introduces a downward pseudo-force ma, shifting the system's equilibrium position and initiating vertical oscillations about this new center.

Solution:

(a) Find the initial static elongation x_0 when stationary:

$$kx_0 = mg \implies x_0 = \frac{mg}{k}$$

(b) Determine the new equilibrium elongation x_{eq} under upward acceleration a:

$$kx_{eq} = m(g + a) \implies x_{eq} = \frac{m(g + a)}{k}$$

(c) Calculate the shift, which defines the oscillation amplitude A:

$$A = x_{eq} - x_0 = \frac{ma}{k}$$

(d) Compute the maximum extension reached from the initial equilibrium position:

$$x_{max} = A + \text{Amplitude} = \frac{ma}{k} + \frac{ma}{k} = \frac{2ma}{k}$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 31

Q32.

Solution

Concept:
For a rigid rod rotating about a fixed, frictionless pivot, total mechanical energy is conserved. The loss in gravitational potential energy equals the gain in rotational kinetic energy.

Solution:

(a) Identify the moment of inertia of the uniform rod pivoting about its end:

$$I = \frac{1}{3}ML^2$$

(b) Track the center of mass at its midpoint. When falling to the vertical position, it drops by $\frac{L}{2}$:

$$\Delta U = Mg\left(\frac{L}{2}\right)$$

(c) Equate this loss in potential energy to the gain in rotational kinetic energy:

$$\frac{1}{2}I\omega^2 = \Delta U \implies \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = Mg\left(\frac{L}{2}\right)$$

(d) Simplify the terms and isolate the angular velocity ω:

$$\frac{1}{6}ML^2\omega^2 = \frac{1}{2}MgL \implies \omega^2 = \frac{3g}{L} \implies \omega = \sqrt{\frac{3g}{L}}$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept:
The net mechanical work done by an ideal gas over a closed thermodynamic cycle is the algebraic sum of the work done during each individual step. Work depends on how pressure and volume change during each specific process.

Solution:

(a) Analyze process AB (isobaric expansion): The pressure remains constant at P_A. The volume doubles from V_A to $V_B = 2V_A$. The work done is:

$$W_{AB} = P_A(V_B - V_A) = P_A(2V_A - V_A) = P_A V_A$$

Using the ideal gas law for one mole ($P_A V_A = RT_0$), we get:

$$W_{AB} = RT_0$$

(b) Analyze process BC (isochoric cooling): The volume stays constant at $V_B = 2V_A$, meaning no expansion or compression occurs. Therefore, the work done during this step is zero:

$$W_{BC} = 0$$

(c) Analyze process CA (isothermal compression): The temperature remains constant at T_0, compressing the gas from an initial volume of $V_B = 2V_A$ back to its original volume V_A. The work done during an isothermal process is:

$$W_{CA} = RT_0 \ln\left(\frac{V_{\text{final}}}{V_{\text{initial}}}\right) = RT_0 \ln\left(\frac{V_A}{2V_A}\right) = RT_0 \ln\left(\frac{1}{2}\right) = -RT_0 \ln 2$$

(d) Calculate the total net work done over the entire cycle by summing the work from each step:

$$W_{\text{net}} = W_{AB} + W_{BC} + W_{CA}$$

$$W_{\text{net}} = RT_0 + 0 - RT_0 \ln 2 = RT_0(1 - \ln 2)$$

Final Answer: The correct choice is (C).

Answer: (C)

Go Back to Question 33

Q34.

Solution

Concept:
A standing wave equation describes how the displacement of particles varies with both position and time. The transverse velocity of any particle along the string is found by taking the partial derivative of this displacement equation with respect to time.

Solution:

(a) Start with the given standing wave displacement equation:

$$y(x,t) = A\sin(kx)\cos(\omega t)$$

(b) Differentiate this equation with respect to time t, keeping position x constant, to find the transverse velocity function $v_y(x,t)$:

$$v_y(x,t) = \frac{\partial y}{\partial t} = A\sin(kx)[-\omega\sin(\omega t)] = -A\omega\sin(kx)\sin(\omega t)$$

(c) We need to find the velocity of a particle located at a position $x = \frac{L}{4}$ at a time $t = \frac{\pi}{2\omega}$.

(d) For a string of length L fixed at both ends vibrating in its fundamental mode, the wavelength λ is twice the length of the string ($\lambda = 2L$). The wave number k is:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2L} = \frac{\pi}{L}$$

(e) Substitute this value of k into the position term $\sin(kx)$ for $x = \frac{L}{4}$:

$$\sin\left(k \cdot \frac{L}{4}\right) = \sin\left(\frac{\pi}{L} \cdot \frac{L}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

(f) Substitute the time $t = \frac{\pi}{2\omega}$ into the time term $\sin(\omega t)$:

$$\sin\left(\omega \cdot \frac{\pi}{2\omega}\right) = \sin\left(\frac{\pi}{2}\right) = 1$$

(g) Combine these parts back into the velocity equation:

$$v_y = -A\omega\left(\frac{1}{\sqrt{2}}\right)(1) = -\frac{A\omega}{\sqrt{2}}$$

Final Answer: The correct choice is (A).

Answer: (A)

Go Back to Question 34

Q35.

Solution

Concept:
The equivalent resistance of an infinite grid is found using superposition and symmetry. Injecting and extracting current at adjacent nodes determines the local current split.
Solution:

(a) Inject current I into node A. By grid symmetry, it splits equally into four branches, sending current through segment AB:

$$I_{AB}^{(1)} = \frac{I}{4}$$

(b) Extract current I from node B. By symmetry, equal currents converge from four branches, contributing:

$$I_{AB}^{(2)} = \frac{I}{4}$$

(c) Superimpose both states. The total current flowing through resistor R between A and B becomes:

$$I_{AB} = \frac{I}{4} + \frac{I}{4} = \frac{I}{2}$$

(d) Apply Ohm's law to find the voltage drop and network equivalent resistance:

$$V_{AB} = I_{AB}R = \frac{IR}{2} \implies R_{\text{eq}} = \frac{V_{AB}}{I} = \frac{R}{2}$$

Final Answer: The correct choice is (B).

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept:

Electrostatic properties of conductors dictate that the electric field inside the bulk material of a conductor in equilibrium is zero. Any net charge given to an isolated conductor resides entirely on its outer surface, and charges are induced on nearby conducting surfaces to balance electric fields.

Solution:

(a) For the inner region ($R < r < R_1$), a spherical Gauss surface encloses only the net charge $+Q$ on the inner sphere. Applying Gauss's Law shows the electric field matches that of a point charge, making statement (A) correct:

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

(b) The region $R_1 < r < R_2$ lies inside the bulk material of the outer conducting shell. In electrostatic equilibrium, the internal electric field within a conductor must be zero, making statement (B) correct.

(c) For the electric field inside the shell to be zero, the total enclosed charge within a Gaussian surface embedded in the shell must be zero. This induces a charge of $-Q$ on the inner surface ($r = R_1$). Since the outer shell is uncharged, a charge of $+Q$ must move to its outer surface ($r = R_2$) to conserve charge, making statement (D) correct.

(d) For the exterior region ($r \geq R_2$), the total net enclosed charge is $(+Q) + (-Q) + (+Q) = +Q$. The electric field outside behaves as if all charge were at the center. Integrating this field from infinity to the outer surface ($r = R_2$) gives the electrostatic potential, making statement (C) correct:

$$V = \frac{Q}{4\pi\epsilon_0 R_2}$$

Final Answer: Statements (A), (B), (C), and (D) are all correct.

Answer: (A, B, C, D)

Go Back to Question 36

Q37.

Solution

Concept:
The magnetic force acting on a moving charged particle is always perpendicular to its velocity vector ($\vec{F} = q\vec{v} \times \vec{B}$). Because this force is perpendicular, it does no work on the particle, meaning its speed and kinetic energy remain constant.

Solution:

(a) The velocity vector can be split into two components relative to the magnetic field $\vec{B} = B_0\hat{k}$: a parallel component $v_z\hat{k}$ and a perpendicular component $v_x\hat{i}$.

(b) The parallel component v_z experiences no magnetic force, so it maintains a constant linear motion along the z-axis. The perpendicular component v_x drives a uniform circular motion in the xy-plane. Combining these two motions creates a helical path with a constant pitch along an axis parallel to the z-axis, making statements (A) and (C) correct.

(c) The time period T required to complete one full revolution in the xy-plane depends only on the magnetic field and the particle's mass and charge:

$$T = \frac{2\pi m}{qB_0}$$

This period is completely independent of the injection velocity components, making statement (B) correct.

(d) Since the magnetic force is always perpendicular to the velocity vector, the power delivered by the magnetic field is zero:

$$P = \vec{F} \cdot \vec{v} = 0$$

This means the particle's speed and total kinetic energy remain constant over time, making statement (D) incorrect.

Final Answer: The correct statements are (A), (B), and (C).

Answer: (A, B, C)

Go Back to Question 37

Q38.

Solution

Concept:

A series LCR circuit driven by an alternating voltage source exhibits frequency-dependent impedance. The circuit's behavior shifts between capacitive and inductive depending on whether the driving frequency is below or above the natural resonance frequency.

Solution:

(a) The total electrical impedance Z of a series LCR circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(b) At the natural resonance frequency ($\omega = \omega_0 = \frac{1}{\sqrt{LC}}$), the inductive reactance balances the capacitive reactance ($X_L = X_C$). This reduces the total impedance to its minimum possible value, which is equal to the resistance R, making statement (A) correct.

(c) For high driving frequencies ($\omega > \omega_0$), the inductive reactance becomes larger than the capacitive reactance ($X_L > X_C$). This makes the circuit inductive, causing the alternating current to lag behind the voltage, making statement (B) correct.

(d) For low driving frequencies ($\omega < \omega_0$), the capacitive reactance dominates ($X_C > X_L$). This makes the circuit capacitive, causing the current to lead the voltage, making statement (C) correct.

(e) At resonance, the phase angle ϕ between voltage and current is zero. The power factor ($\cos\phi$) reaches its maximum value of $\cos(0) = 1$, making statement (D) correct.

Final Answer: Statements (A), (B), (C), and (D) are all correct.

Answer: (A, B, C, D)

Go Back to Question 38

Q39.

Solution

Concept:
Total Internal Reflection (TIR) occurs when a light ray traveling through an optically denser medium hits a boundary with a rarer medium and reflects completely back into the denser medium. This happens when the angle of incidence exceeds a specific critical angle.

Solution:

(a) For total internal reflection to occur, two conditions must be met:

i. The light ray must be traveling inside the optically denser medium and be incident toward the interface with the optically rarer medium ($n_1 > n_2$).

ii. The angle of incidence θ must be strictly greater than the critical angle θ_c defined for that interface.

(b) According to Snell's Law, the critical angle θ_c is the angle of incidence that results in a refraction angle of 90°:

$$n_1 \sin\theta_c = n_2 \sin(90^\circ) \implies \sin\theta_c = \frac{n_2}{n_1} \implies \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

This makes statement (C) correct.

(c) Combining this critical angle value with the condition for total internal reflection ($\theta > \theta_c$) gives:

$$\theta > \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

This makes statement (A) correct, while statement (B) is incorrect.

(d) Statement (D) describes light traveling from a rarer medium to a denser medium. In this scenario, the light ray always refracts into the second medium and bends toward the normal, meaning total internal reflection is impossible. Therefore, statement (D) is incorrect.

Final Answer: The correct statements are (A) and (C).

Answer: (A, C)

Go Back to Question 39

Q40.

Solution

Concept:
Einstein's photoelectric equation linearly relates stopping potential to incident light frequency. Analyzing its slope and intercepts helps verify the properties of the resulting graph.

Solution:

(a) Express maximum kinetic energy via stopping potential V_s:

$$eV_s = h\nu - \phi \implies V_s = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e}$$

(b) Compare this to a straight line equation ($y = mx + c$):

$$\text{Slope } m = \frac{h}{e}, \quad \text{y-intercept } c = -\frac{\phi}{e}$$

(c) The slope $m = \frac{h}{e}$ relies purely on universal constants, remaining independent of the target metal. Thus, statement (A) is true and (D) is false.

(d) Set $V_s = 0$ to locate the horizontal frequency axis threshold intercept:

$$\nu_0 = \frac{\phi}{h}$$

This confirms statement (B) is true.

(e) The vertical axis intercept value equals $-\frac{\phi}{e}$, making statement (C) true.

Final Answer: The correct statements are (A), (B), and (C).

Answer: (A, B, C)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	B	4	C	5	A
6	B	7	C	8	B	9	B	10	C
11	A	12	A	13	A	14	C	15	D
16	A	17	B	18	C	19	B	20	A
21	B	22	B	23	B	24	C	25	A
26	B	27	C	28	C	29	A	30	B
31	C	32	A	33	C	34	A	35	B
36	A, B, C, D	37	A, B, C	38	A, B, C, D	39	A, C	40	A, B, C