WBJEE Physics Sample Paper-13

Duration: 60 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **40** Multiple Choice Questions divided into **3 Categories**.
- **Section A (Q1–Q30):** Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- **Section B (Q31–Q35):** Each correct answer carries **+2 marks**. Incorrect answer: **−0.5** marks. Only **one** correct option.
- **Section C (Q36–Q40):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section A - 30 Questions × 1 Mark Each (Negative Marking: −0.25) [Single Correct]

Q1. A particle moves along a straight line such that its velocity v varies with displacement x as $v = \alpha\sqrt{x}$, where α is a positive constant. The average velocity of the particle during the time it travels the first distance S is:

(A) $\frac{\alpha\sqrt{S}}{2}$

(B) $\frac{2\alpha\sqrt{S}}{3}$

(C) $\alpha\sqrt{S}$

(D) $\frac{3\alpha\sqrt{S}}{4}$

Q2. If the dimension of a physical quantity is given by $M^aL^bT^c$, then which of the following is correct?

(A) Surface tension: $a = 1, b = 1, c = -2$

(B) Coefficient of viscosity: $a = 1, b = -1, c = -1$

(C) Planck's constant: $a = 1, b = 2, c = -2$

(D) Modulus of elasticity: $a = 1, b = 2, c = -1$

Q3. A block of mass m is placed on a rough horizontal surface with a coefficient of static friction μ_s. A force $F = mg$ acts on the block horizontally. If $\mu_s = 0.6$, the net force exerted by the surface on the block is:

(A) $0.6\, mg$

(B) mg

(C) $\sqrt{1.36}\, mg$

(D) $\sqrt{2}\, mg$

Q4. A potential difference $V = (100 \pm 5)$ V when applied across a resistor carries a current $I = (10 \pm 0.2)$ A. The percentage error in the measurement of resistance R is:

(A) 5%

(B) 7%

(C) 2%

(D) 3.5%

Q5. A stone is projected from the ground with a velocity v_0 at an angle θ to the horizontal. If the kinetic energy at the highest point is $\frac{3}{4}$ of its initial kinetic energy, the angle of projection θ is:

(A) 30°

(B) 45°

(C) 60°

(D) 15°

Q6. A body of mass m dropped from a height h reaches the ground with a speed $0.8\sqrt{2gh}$ due to air resistance. The work done by the air resistance is:

(A) $-0.36\, mgh$

(B) $-0.64\, mgh$

(C) $-0.20\, mgh$

(D) $-0.16\, mgh$

Q7. A light rod of length L has two point masses m_1 and m_2 attached to its two ends. The moment of inertia of the system about an axis perpendicular to the rod and passing through its centre of mass is:

(A) $(m_1 + m_2)L^2$

(B) $\frac{m_1 m_2}{m_1+m_2}L^2$

(C) $\frac{1}{2}(m_1 + m_2)L^2$

(D) $\sqrt{m_1 m_2}L^2$

Q8. At what depth below the surface of the Earth (radius R) is the acceleration due to gravity equal to the acceleration due to gravity at a height $h = R$ above the Earth's surface?

(A) $\frac{R}{4}$

(B) $\frac{R}{2}$

(C) $\frac{3R}{4}$

(D) $\frac{2R}{3}$

Q9. Two capillary tubes of radii r_1 and r_2 ($r_1 > r_2$) are dipped vertically in water. If the water rises to heights h_1 and h_2 respectively, then the correct relationship is:

(A) $h_1 r_1^2 = h_2 r_2^2$

(B) $h_1\sqrt{r_1} = h_2\sqrt{r_2}$

(C) $h_1 r_1 = h_2 r_2$

(D) $h_1/r_1 = h_2/r_2$

Q10. A modern water tank has a small hole near its bottom. If the velocity of efflux of water is v when the height of water is H, what will be the velocity of efflux when the tank is filled with a liquid of twice the density of water up to the same height H?

(A) $2v$

(B) v

(C) $\frac{v}{2}$

(D) $\sqrt{2}v$

Q11. An ideal gas undergoes a cyclic process $ABCA$ on a $P-V$ diagram, where $A(P_0, V_0)$, $B(2P_0, V_0)$, and $C(P_0, 2V_0)$. If the process paths are straight lines, the net work done by the gas per cycle is:

(A) P_0V_0

(B) $2P_0V_0$

(C) $\frac{1}{2}P_0V_0$

(D) $\frac{3}{2}P_0V_0$

Q12. The root mean square speed of hydrogen gas molecules at temperature T is v. If the temperature is doubled and the hydrogen molecules dissociate into atomic hydrogen, the new root mean square speed will be:

(A) v

(B) $2v$

(C) $\sqrt{2}v$

(D) $4v$

Q13. Two simple harmonic motions are represented by $y_1 = 5\sin(2\pi t + \pi/4)$ and $y_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$. The ratio of their amplitudes is:

(A) $1 : 2$

(B) $1 : \sqrt{2}$

(C) $1 : 1$

(D) $2 : 1$

Q14. An observer moves towards a stationary source of sound with a velocity one-fifth of the velocity of sound. The percentage increase in the apparent frequency heard by the observer is:

(A) 20%

(B) 5%

(C) 25%

(D) 10%

Q15. A thin spherical conducting shell of radius R has a charge Q. The electrostatic potential V at a distance r $(r < R)$ from its centre is:

(A) Zero

(B) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{Q(R-r)}{R^2}$

Q16. Two point charges $+q$ and $-q$ are held fixed at positions $(-a, 0)$ and $(a, 0)$ respectively. The electric field vector at the origin $(0, 0)$ is along:

(A) Positive x-axis

(B) Negative x-axis

(C) Positive y-axis

(D) Negative y-axis

Q17. In a meter bridge experiment, the balancing length from the left end is 40 cm when a known resistance of 6 Ω is connected in the right gap. The value of the unknown resistance in the left gap is:

(A) 4 Ω

(B) 9 Ω

(C) 3 Ω

(D) 5 Ω

Q18. A long straight wire of circular cross-section of radius a carries a steady current I distributed uniformly across its cross-section. The magnetic field B inside the wire at a distance r $(r < a)$ from its axis varies as:

(A) $B \propto \frac{1}{r}$

(B) $B \propto r$

(C) $B \propto r^2$

(D) $B \propto \frac{1}{r^2}$

Q19. A charged particle enters a uniform magnetic field with a velocity vector making an angle of 30° with the magnetic field lines. The path traced by the particle is a:

(A) Helix with uniform pitch

(B) Circle

(C) Straight line

(D) Helix with non-uniform pitch

Q20. A circular loop of radius R carrying current I produces a magnetic field B_0 at its centre. The magnetic field at an axial point at a distance $\sqrt{3}R$ from the centre of the loop is:

(A) $\frac{B_0}{2}$

(B) $\frac{B_0}{4}$

(C) $\frac{B_0}{8}$

(D) $\frac{B_0}{3\sqrt{3}}$

Q21. A conducting square loop of side L and resistance R moves with a uniform velocity v perpendicular to one of its sides, entering a region of uniform magnetic field B perpendicular to the plane of the loop. The induced current in the loop while it is partially inside the field is:

(A) $\frac{BLv}{R}$

(B) $\frac{2BLv}{R}$

(C) Zero

(D) $\frac{BL^2v}{R}$

Q22. The electric field vector of an electromagnetic wave propagating in free space is given by $\vec{E} = E_0 \cos(kz - \omega t)\hat{i}$. The corresponding magnetic field vector $\vec{B}$ is:

(A) $\vec{B} = \frac{E_0}{c} \cos(kz - \omega t)\hat{j}$

(B) $\vec{B} = -\frac{E_0}{c} \cos(kz - \omega t)\hat{j}$

(C) $\vec{B} = \frac{E_0}{c} \cos(kz - \omega t)\hat{k}$

(D) $\vec{B} = E_0 c \cos(kz - \omega t)\hat{j}$

Q23. A convex lens of focal length f in air is immersed completely in water ($\mu = 4/3$). If the refractive index of the glass lens is 1.5, its focal length in water becomes:

(A) $2f$

(B) $4f$

(C) $\frac{3}{2}f$

(D) $\frac{4}{3}f$

Q24. In a Young's double-slit experiment, the slit separation is doubled and the distance between the slits and the screen is halved. The fringe width becomes:

(A) Four times

(B) Double

(C) Half

(D) One-fourth

Q25. When light of wavelength λ falls on a photosensitive metallic surface, the maximum kinetic energy of the emitted photoelectrons is K. If the wavelength is reduced to $\frac{\lambda}{3}$, the maximum kinetic energy of the photoelectrons will be:

(A) Equal to $3K$

(B) Greater than $3K$

(C) Less than $3K$

(D) Equal to $\frac{K}{3}$

Q26. An electron in a hydrogen atom makes a transition from an excited state $n = 3$ to the ground state $n = 1$. The recoil speed of the hydrogen atom (mass M) due to the emission of the photon is (where R_∞ is Rydberg constant and h is Planck's constant):

(A) $\frac{8hR_\infty}{9M}$

(B) $\frac{9hR_\infty}{8M}$

(C) $\frac{3hR_\infty}{4M}$

(D) $\frac{hR_\infty}{2M}$

Q27. The boolean expression for the output Y of the logic circuit consisting of two inputs A and B processed through a NAND gate followed by a NOT gate is equivalent to a/an:

(A) OR gate

(B) AND gate

(C) NOR gate

(D) XOR gate

Q28. Two bodies of masses 1 kg and 4 kg are moving with equal linear momenta. The ratio of their kinetic energies is:

(A) 4 : 1

(B) 1 : 4

(C) 2 : 1

(D) 1 : 2

Q29. A block of mass M hanging from a vertical spring of spring constant k executes SHM with a time period T. If the block is replaced by another block of mass $2M$, the new time period of the system will be:

(A) $\sqrt{2}T$

(B) $2T$

(C) $\frac{T}{\sqrt{2}}$

(D) $4T$

Q30. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force given by $F = -\frac{k}{r^2}$, where k is a constant. The total mechanical energy of the particle (taking potential energy to be zero at infinity) is:

(A) $-\frac{k}{2r}$

(B) $\frac{k}{2r}$

(C) $-\frac{k}{r}$

(D) Zero

Section B - 5 Questions × 2 Mark Each
(Negative Marking: −0.5) [Single Correct]

Q31. A Uniform solid cylinder of mass M and radius R rolls down an inclined plane of inclination θ without slipping. The acceleration of the centre of mass of the cylinder is:

(A) $g \sin\theta$

(B) $\frac{2}{3}g \sin\theta$

(C) $\frac{1}{2}g \sin\theta$

(D) $\frac{3}{5}g \sin\theta$

Q32. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If the wire is stretched by an amount x, the work done in stretching the wire is:

(A) $\frac{YAx^2}{2L}$

(B) $\frac{YAx^2}{L}$

(C) $\frac{YAx}{2L^2}$

(D) $\frac{2YAx^2}{L}$

Q33. Two cells of emfs E_1 and E_2 ($E_1 > E_2$) and internal resistances r_1 and r_2 respectively are connected in parallel combination such that their positive terminals are connected together. The equivalent emf E_{eq} of the combination is:

(A) $\frac{E_1 r_1 + E_2 r_2}{r_1 + r_2}$

(B) $\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

(C) $E_1 + E_2$

(D) $\frac{E_1 + E_2}{2}$

Q34. In an alternating current circuit containing an inductance L and a resistance R connected in series to an AC source of angular frequency ω, the phase angle ϕ by which the voltage leads the current is given by:

(A) $\tan\phi = \frac{R}{\omega L}$

(B) $\tan\phi = \frac{\omega L}{R}$

(C) $\sin\phi = \frac{\omega L}{R}$

(D) $\cos\phi = \frac{\omega L}{R}$

Q35. A glass prism has a refracting angle A. The refractive index of the material of the prism is $\cot(A/2)$. The angle of minimum deviation δ_m for this prism is:

(A) $180° - 2A$

(B) $180° - A$

(C) $90° - A$

(D) $2A$

Section C - 5 Questions × 2 Marks Each
(No Negative Marking) [One or More Correct]

Q36. A thermodynamic system undergoes a process in which the internal energy decreases by 300 J while the system performs 200 J of work on its surroundings. Which of the following statements are correct?

(A) The heat supplied to the system is -100 J.

(B) The process is exothermic.

(C) The temperature of the system must have decreased if it is an ideal gas.

(D) The process is adiabatic.

Q37. A parallel-plate capacitor is charged by a battery and then disconnected from it. A dielectric slab is now inserted to completely fill the space between the plates. Which of the following quantities will change?

(A) The charge stored in the capacitor

(B) The potential difference across the plates

(C) The energy stored in the capacitor

(D) The capacitance of the capacitor

Q38. A rectangular loop carrying a steady current I is placed near a long straight wire carrying a steady current I_0 in the same plane. Which of the following statements are true regarding the forces acting on the loop?

(A) The net magnetic force on the loop is non-zero.

(B) The net torque acting on the loop is zero.

(C) The loop will experience an attractive force if the current adjacent to the wire flows in the same direction.

(D) The net magnetic force on the loop is completely independent of the distance from the wire.

Q39. Radioactive nuclei of a sample A have a half-life T_A, and those of sample B have a half-life T_B. Initially, both samples contain the same number of active nuclei. Which of the following statements are true?

(A) If $T_A > T_B$, sample B decays faster initially.

(B) The activity of sample A will always be higher than B if $T_A < T_B$.

(C) The decay constant λ is larger for the sample with the smaller half-life.

(D) After one mean life, exactly 50% of the initial sample remains undecayed.

Q40. A uniform magnetic field $\vec{B}(t)$ exists perpendicular to the plane of a stationary circular loop of radius r and resistance R. The magnitude of the magnetic field varies with time as $B(t) = B_0 + \alpha t^2$. Which of the following options are correct?

(A) The induced electromotive force in the loop has a magnitude of $2\pi r^2 \alpha t$.

(B) The induced current increases linearly with time t.

(C) The total charge flowing through the loop up to time t depends quadratically on time.

(D) The electric field induced along the perimeter of the wire is uniform in magnitude and equal to $r\alpha t$.

Detailed Solutions

Q1.

Solution

Concept: The instantaneous velocity is given by $v = \frac{dx}{dt}$. By separating variables and integrating, we find displacement as a function of time. Average velocity is defined as the total displacement divided by the total time taken.

Solution: Step 1: Write the given velocity function:

$$v = \alpha\sqrt{x}$$

Step 2: Express velocity as $\frac{dx}{dt}$ and separate variables:

$$\frac{dx}{dt} = \alpha\sqrt{x} \implies x^{-\frac{1}{2}}\,dx = \alpha\,dt$$

Step 3: Integrate both sides from rest ($x = 0$ at $t = 0$) to find the total time T to cover distance S:

$$\int_0^S x^{-\frac{1}{2}}\,dx = \int_0^T \alpha\,dt$$

$$2\sqrt{S} = \alpha T \implies T = \frac{2\sqrt{S}}{\alpha}$$

Step 4: Calculate the average velocity v_{avg} by dividing total displacement S by total time T:

$$v_{\text{avg}} = \frac{S}{T} = \frac{S}{\left(\frac{2\sqrt{S}}{\alpha}\right)} = \frac{\alpha\sqrt{S}}{2}$$

Final Answer: $\boxed{\frac{\alpha\sqrt{S}}{2}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: Dimensional analysis allows us to verify physical equations by matching the exponents of fundamental dimensions: Mass (M), Length (L), and Time (T). The general representation of any derived physical quantity's dimension is expressed as $M^a L^b T^c$, where a, b, and c are the characteristic dimensions of mass, length, and time respectively.

Solution: Step 1: Analyze option (A), Surface tension. Surface tension is defined as force per unit length:

$$\text{Surface Tension} = \frac{\text{Force}}{\text{Length}} = \frac{M^1 L^1 T^{-2}}{L^1} = M^1 L^0 T^{-2}$$

Comparing with $M^a L^b T^c$, we find $a = 1, b = 0, c = -2$. The option specifies $b = 1$, so option (A) is incorrect.

Step 2: Analyze option (B), Coefficient of viscosity. According to Newton's law of viscous flow, the viscous force is given by $F = \eta A \frac{dv}{dx}$, where η is the coefficient of viscosity, A is the area, and $\frac{dv}{dx}$ is the velocity gradient:

$$\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}$$

$$\text{Dimensions of } \eta = \frac{M^1 L^1 T^{-2}}{(L^2) \cdot (T^{-1})} = \frac{M^1 L^1 T^{-2}}{L^2 T^{-1}} = M^1 L^{-1} T^{-1}$$

Comparing with $M^a L^b T^c$, we find $a = 1, b = -1, c = -1$. This perfectly matches the parameters given in option (B).

Step 3: Analyze option (C), Planck's constant (h). Using the energy-frequency relationship $E = h\nu$:

$$h = \frac{E}{\nu} = \frac{M^1 L^2 T^{-2}}{T^{-1}} = M^1 L^2 T^{-1}$$

Comparing with $M^a L^b T^c$, we find $a = 1, b = 2, c = -1$. The option specifies $c = -2$, so option (C) is incorrect.

Step 4: Analyze option (D), Modulus of elasticity. Modulus of elasticity is defined as the ratio of stress to strain. Since strain is dimensionless:

$$\text{Modulus of Elasticity} = \frac{\text{Stress}}{\text{Strain}} = \frac{\text{Force}}{\text{Area}} = \frac{M^1 L^1 T^{-2}}{L^2} = M^1 L^{-1} T^{-2}$$

Comparing with $M^a L^b T^c$, we find $a = 1, b = -1, c = -2$. The option specifies $b = 2, c = -1$, so option (D) is incorrect.

Final Answer: Coefficient of viscosity: $a = 1, b = -1, c = -1$

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: When a body is placed on a rough horizontal surface and subjected to a horizontal pulling force, static friction opposes the tendency of motion. The maximum value of this frictional force is the limiting friction $f_{\max} = \mu_s N$. If the applied force is less than or equal to $f_{\max}$, the static friction exactly balances the applied force. The total net force exerted by the rough surface on the block is the vector sum of the normal contact force and the friction force.

Solution: Step 1: Determine the normal reaction force N acting on the block in the vertical direction. Since there is no vertical acceleration, the normal force balances the weight:

$$N = mg$$

Step 2: Calculate the maximum limiting static frictional force $f_{\max}$ that the horizontal surface can provide to prevent slipping:

$$f_{\max} = \mu_s N = 0.6 \cdot mg = 0.6\, mg$$

Step 3: Compare the applied horizontal force F with the maximum limiting static friction $f_{\max}$. The problem states that the horizontal force is:

$$F = mg$$

Since the applied horizontal force ($F = 1\, mg$) is strictly greater than the maximum available static friction force ($f_{\max} = 0.6\, mg$), the block will overcome static friction and break away into motion. Step 4: Because the block is in motion and no kinetic friction coefficient is provided separately, the friction force operating on the moving block reaches its full saturation value, which equals the limiting frictional value:

$$f = f_{\max} = 0.6\, mg$$

Step 5: Identify the two perpendicular forces acting on the block from the surface: the vertical normal reaction force N and the horizontal frictional force f. The total contact force R exerted by the surface on the block is their vector resultant:

$$R = \sqrt{N^2 + f^2}$$

$$R = \sqrt{(mg)^2 + (0.6\, mg)^2}$$

$$R = \sqrt{1 \cdot (mg)^2 + 0.36 \cdot (mg)^2}$$

$$R = \sqrt{1.36 \cdot (mg)^2} = \sqrt{1.36}\, mg$$

Final Answer: $\boxed{\sqrt{1.36}\, mg}$

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: According to Ohm's Law, the resistance of a conductor is given by $R = \frac{V}{I}$. When calculating the maximum permissible error in a quantity derived from multiplication or division, the fractional errors of each independent measured quantity add up directly. Thus, the relative error in resistance is the sum of the relative errors in voltage and current.

Solution: Step 1: State the functional relationship for resistance based on the measured physical variables voltage V and current I:

$$R = \frac{V}{I}$$

Step 2: Apply the fractional error combination rules for quantities related via division. Taking the natural logarithm on both sides and differentiating gives the maximum fractional error formula:

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

Step 3: Extract the absolute values and measured values along with their corresponding uncertainties from the data given in the problem statement:

$$V = 100 \text{ V}, \quad \Delta V = 5 \text{ V}$$

$$I = 10 \text{ A}, \quad \Delta I = 0.2 \text{ A}$$

Step 4: Substitute these numeric values into the fractional error equation to determine the fractional uncertainty in the resistance measurement:

$$\frac{\Delta R}{R} = \frac{5}{100} + \frac{0.2}{10}$$

Step 5: Convert the fractional expressions to a common denominator or decimal form to combine them easily:

$$\frac{5}{100} = 0.05$$

$$\frac{0.2}{10} = \frac{2}{100} = 0.02$$

$$\frac{\Delta R}{R} = 0.05 + 0.02 = 0.07$$

Step 6: Compute the percentage error by multiplying the fractional error of the resistance by 100%:

$$\text{Percentage Error} = \left(\frac{\Delta R}{R}\right) \times 100\%$$

$$\text{Percentage Error} = 0.07 \times 100\% = 7\%$$

Final Answer: $\boxed{7\%}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: In projectile motion, the horizontal velocity component remains constant ($v_x = v_0 \cos\theta$), while the vertical component becomes zero at the highest point. Thus, kinetic energy at maximum height depends solely on the horizontal velocity.

Solution: Step 1: Write the initial kinetic energy K_0 at launch:

$$K_0 = \frac{1}{2}mv_0^2$$

Step 2: Find the velocity v_{top} and kinetic energy K_{top} at the highest point:

$$v_{top} = v_0 \cos\theta$$

$$K_{top} = \frac{1}{2}m(v_0 \cos\theta)^2 = K_0 \cos^2\theta$$

Step 3: Apply the given condition $K_{top} = \frac{3}{4}K_0$ and solve for θ:

$$K_0 \cos^2\theta = \frac{3}{4}K_0 \implies \cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \frac{\sqrt{3}}{2} \implies \theta = 30^\circ$$

Final Answer: $\boxed{30^\circ}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: According to the Work-Energy Theorem, the net work done by all the forces acting on a body is equal to the change in its kinetic energy. When a body falls through the atmosphere, both the gravitational force and the resistive air drag do work on the object.

Solution: Step 1: Write down the expression for the Work-Energy Theorem applied to the falling mass:

$$W_{\text{net}} = \Delta K = K_{\text{final}} - K_{\text{initial}}$$

Step 2: identify the forces contributing to the total work done. Here, gravity and air resistance are the only forces performing work on the stone:

$$W_{\text{gravity}} + W_{\text{air}} = K_{\text{final}} - K_{\text{initial}}$$

Step 3: Calculate the work done by gravity as the body falls vertically downward through a displacement h. Since the gravitational force mg acts in the same direction as the displacement, the work done is positive:

$$W_{\text{gravity}} = mgh$$

Step 4: Express the initial and final kinetic energies. The body is dropped from rest, meaning its initial velocity is zero. The final velocity when reaching the ground is given as $v = 0.8\sqrt{2gh}$:

$$K_{\text{initial}} = 0$$

$$K_{\text{final}} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(0.8\sqrt{2gh}\right)^2$$

Step 5: Simplify the expression for the final kinetic energy:

$$K_{\text{final}} = \frac{1}{2}m\,(0.64 \cdot 2gh) = 0.64\,mgh$$

Step 6: Substitute W_{gravity}, K_{initial}, and K_{final} back into the Work-Energy equation:

$$mgh + W_{\text{air}} = 0.64\,mgh - 0$$

Step 7: Isolate and solve for the work done by air resistance W_{air}:

$$W_{\text{air}} = 0.64\,mgh - mgh$$

$$W_{\text{air}} = -0.36\,mgh$$

The negative sign correctly indicates that the air resistive force acts in the direction opposite to the particle's displacement.

Final Answer: $\boxed{-0.36\,mgh}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: The center of mass of a two-particle system lies along the line connecting them, at distances inversely proportional to their respective masses. The total moment of inertia of a system of discrete point masses about a specified axis is given by the sum of the quantities $m_i r_i^2$, where r_i represents the perpendicular distance of each individual mass from that axis.

Solution: Step 1: Set up a coordinate system along the light rod. Let the mass m_1 be positioned at the origin ($x_1 = 0$), which means the mass m_2 is located at a distance equal to the length of the rod ($x_2 = L$).

Step 2: Find the position of the center of mass (x_{cm}) from the origin using the standard distribution formula:

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m_1(0) + m_2(L)}{m_1 + m_2} = \frac{m_2 L}{m_1 + m_2}$$

Step 3: Determine the individual perpendicular distances r_1 and r_2 of the masses m_1 and m_2 from the rotational axis passing through the center of mass:

$$r_1 = x_{\text{cm}} = \frac{m_2 L}{m_1 + m_2}$$

$$r_2 = L - x_{\text{cm}} = L - \frac{m_2 L}{m_1 + m_2} = \frac{m_1 L}{m_1 + m_2}$$

Step 4: Express the total moment of inertia I about this central perpendicular axis as the sum of individual point-mass moments:

$$I = m_1 r_1^2 + m_2 r_2^2$$

Step 5: Substitute the distance expressions found in Step 3 into the moment of inertia formula:

$$I = m_1 \left(\frac{m_2 L}{m_1 + m_2} \right)^2 + m_2 \left(\frac{m_1 L}{m_1 + m_2} \right)^2$$

$$I = \frac{m_1 m_2^2 L^2}{(m_1 + m_2)^2} + \frac{m_2 m_1^2 L^2}{(m_1 + m_2)^2}$$

Step 6: Factor out the common algebraic terms in the numerator to simplify the fraction:

$$I = \frac{m_1 m_2 (m_2 + m_1) L^2}{(m_1 + m_2)^2}$$

$$I = \frac{m_1 m_2 (m_1 + m_2) L^2}{(m_1 + m_2)^2} = \frac{m_1 m_2}{m_1 + m_2} L^2$$

This can also be written in terms of the reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$ of the system as $I = \mu L^2$.

Final Answer: $\boxed{\frac{m_1 m_2}{m_1 + m_2} L^2}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept: The acceleration due to gravity changes with both position above and below the surface of the Earth. At a height h above the surface, the effective acceleration is governed by the inverse-square law, while at a depth d below the surface, it decreases linearly because only the mass within the interior sphere contributes to the net gravitational pull.

Solution: Step 1: Recall the absolute formula for the acceleration due to gravity at a height h above the Earth's surface of radius R:

$$g_h = g\left(\frac{R}{R+h}\right)^2$$

where g is the acceleration at the surface.
Step 2: Substitute the given condition $h = R$ into the altitude equation:

$$g_h = g\left(\frac{R}{R+R}\right)^2 = g\left(\frac{R}{2R}\right)^2 = \frac{g}{4}$$

Step 3: Recall the exact formula for the acceleration due to gravity at a depth d below the Earth's surface:

$$g_d = g\left(1 - \frac{d}{R}\right)$$

Step 4: According to the problem statement, the gravitational acceleration values at depth d and height $h = R$ are perfectly equal to each other ($g_d = g_h$). Equate the expressions from Step 2 and Step 3:

$$g\left(1 - \frac{d}{R}\right) = \frac{g}{4}$$

Step 5: Cancel out the common acceleration factor g from both sides of the equation:

$$1 - \frac{d}{R} = \frac{1}{4}$$

Step 6: Rearrange the algebraic terms to solve explicitly for the depth variable d:

$$\frac{d}{R} = 1 - \frac{1}{4}$$

$$\frac{d}{R} = \frac{3}{4}$$

$$d = \frac{3R}{4}$$

Final Answer: $\boxed{\frac{3R}{4}}$

Answer: (C)

Go Back to Question 8

Q9.

Solution

Concept: The ascent or descent of a liquid in a narrow capillary tube dipped inside a fluid reservoir is governed by Jurin's Law. This phenomenon occurs due to the balance between the upward component of the surface tension force acting along the contact perimeter and the downward weight of the elevated liquid column.

Solution: Step 1: Write down the equilibrium formula for the capillary height h of a liquid column. The height depends on surface tension T, contact angle θ, liquid density ρ, capillary tube radius r, and gravitational acceleration g:

$$h = \frac{2T\cos\theta}{r\rho g}$$

Step 2: Identify which parameters remain constant in this experimental scenario. Since both tubes are dipped into the same pool of water under identical ambient atmospheric conditions, the parameters T, θ, ρ, and g are fixed constants.

Step 3: Establish the mathematical proportionality between the height h of the fluid rise and the internal radius r of the capillary tube from the formula:

$$h \propto \frac{1}{r}$$

Step 4: Rewrite this inverse proportionality as a constant product relation for any two tubes operating under these identical parameters:

$$h \cdot r = \text{Constant}$$

Step 5: Apply this constant relation to the two specific capillary tubes of radii r_1 and r_2 with corresponding fluid heights h_1 and h_2:

$$h_1 r_1 = h_2 r_2$$

This confirms that a capillary tube with a narrower radius will experience a higher fluid level column rise compared to a wider tube.

Final Answer: $\boxed{h_1 r_1 = h_2 r_2}$

Answer: (C)

Go Back to Question 9

Q10.

Solution

Concept: According to Torricelli's Law of Efflux, which is derived directly from Bernoulli's Principle of energy conservation for fluid flow, the speed with which an ideal fluid exits a small orifice near the base of an open container depends on the depth of the fluid above the opening.

Solution: Step 1: Apply Bernoulli's equation between the top free surface of the liquid (point 1) and the exit cross-section of the small hole (point 2). Let the atmospheric pressure be P_0.

$$P_0 + \rho g H + \frac{1}{2}\rho v_1^2 = P_0 + 0 + \frac{1}{2}\rho v^2$$

Step 2: Since the cross-sectional area of the water storage tank is much larger than the area of the small discharge hole, the velocity of the upper fluid level dropping is negligible ($v_1 \approx 0$). Simplifying the equation gives:

$$\rho g H = \frac{1}{2}\rho v^2$$

Step 3: Cancel out the fluid density parameter ρ from both sides of the equation:

$$gH = \frac{1}{2}v^2$$

Step 4: Solve for the efflux velocity v:

$$v = \sqrt{2gH}$$

Step 5: Analyze the final mathematical expression for v. Notice that the velocity depends only on the local acceleration due to gravity g and the depth of the fluid H. It does not contain the fluid density variable ρ.

Step 6: Evaluate the outcome when the tank is filled to the exact same level H with another liquid having twice the density (2ρ). Since density cancels out during the derivation, the velocity of efflux remains completely unchanged.

$$v_{new} = v = \sqrt{2gH}$$

Final Answer: $\boxed{v}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept:
Work done in a cyclic process on a $P - V$ diagram.

Solution:

Step 1: The net work done by an ideal gas during a cyclic process is represented by the area enclosed by the closed loop on the $P - V$ diagram.

$$W_{\text{net}} = \text{Area enclosed by the cycle}$$

Step 2: Identify the positions of the vertices on the $P - V$ diagram, where pressure P is on the vertical axis and volume V is on the horizontal axis:

- $A(P_0, V_0) \implies (V_0, P_0)$
- $B(2P_0, V_0) \implies (V_0, 2P_0)$
- $C(P_0, 2V_0) \implies (2V_0, P_0)$

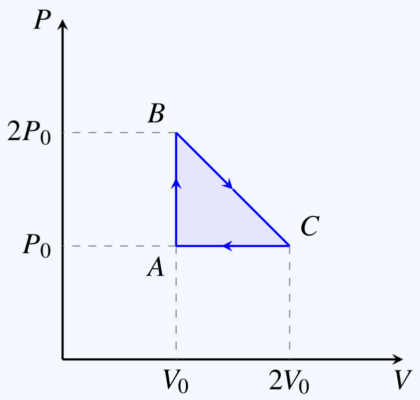
Step 3: Since the process paths are straight lines, the cycle forms a right-angled triangle ABC with the right angle at vertex A:

- Length of the base (along the isobaric path CA) = $2V_0 - V_0 = V_0$
- Length of the height (along the isochoric path AB) = $2P_0 - P_0 = P_0$

Step 4: Calculate the area of this right-angled triangle:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times V_0 \times P_0 = \frac{1}{2}P_0V_0$$

Since the cyclic process $ABCA$ proceeds in a clockwise direction on the $P - V$ diagram, the net work done by the gas is positive. Therefore, $W_{\text{net}} = \frac{1}{2}P_0V_0$.



Final Answer: $\boxed{\frac{1}{2}P_0V_0}$

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept: The root mean square (v_{rms}) speed of molecules in an ideal gas depends directly on the absolute temperature T and inversely on the molar mass M of the gas particles. When diatomic molecules dissociate into single individual atoms, the molar mass of the functional gas particles changes accordingly.

Solution: Step 1: Recall the standard kinetic theory formula for the root mean square velocity of gas molecules:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

Step 2: Set up the initial state for the hydrogen gas molecules (H_2). Let the initial absolute temperature be $T_1 = T$ and the initial molecular weight of diatomic hydrogen be $M_1 = M_0$:

$$v = \sqrt{\frac{3RT}{M_0}}$$

Step 3: Define the properties of the final state. The absolute temperature is doubled, so $T_2 = 2T$. Additionally, the hydrogen molecules completely dissociate into separate single atomic forms ($H_2 \rightarrow 2H$). This means the new active mass of a single particle is exactly halved, making the new effective molar mass $M_2 = \frac{M_0}{2}$.

Step 4: Write down the expression for the new root mean square velocity v' using the updated state variables T_2 and M_2:

$$v' = \sqrt{\frac{3R(2T)}{\left(\frac{M_0}{2}\right)}}$$

Step 5: Simplify the terms inside the square root by rearranging the fraction:

$$v' = \sqrt{\frac{3R \cdot 2T \cdot 2}{M_0}} = \sqrt{\frac{3RT \cdot 4}{M_0}}$$

Step 6: Pull the perfect square factor out of the radical sign to express v' in terms of the initial speed v:

$$v' = 2 \cdot \sqrt{\frac{3RT}{M_0}}$$

$$v' = 2v$$

Final Answer: $\boxed{2v}$

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: The standard equation of a simple harmonic motion is given by $y = A\sin(\omega t + \phi)$. When a motion is composed of a linear combination of a sine function and a cosine function with identical frequencies, it can be combined into a single harmonic wave using trigonometric vector addition rules.

Solution: Step 1: Identify the amplitude A_1 of the first simple harmonic motion equation directly from its given form $y_1 = 5\sin(2\pi t + \pi/4)$:

$$A_1 = 5$$

Step 2: Examine the second given equation for harmonic motion:

$$y_2 = 5\sqrt{2}(\sin 2\pi t + \cos 2\pi t)$$

Step 3: Expand the expression to view the individual component coefficients clearly:

$$y_2 = 5\sqrt{2}\sin 2\pi t + 5\sqrt{2}\cos 2\pi t$$

Step 4: Find the resultant amplitude A_2 of a combined wave of the form $y = a\sin\omega t + b\cos\omega t$. The two components are orthogonal ($\pi/2$ out of phase), so their amplitudes combine as follows:

$$A_2 = \sqrt{a^2 + b^2}$$

where $a = 5\sqrt{2}$ and $b = 5\sqrt{2}$.
Step 5: Substitute the numerical values into the vector summation equation to compute A_2:

$$A_2 = \sqrt{(5\sqrt{2})^2 + (5\sqrt{2})^2}$$

$$A_2 = \sqrt{(25 \cdot 2) + (25 \cdot 2)} = \sqrt{50 + 50} = \sqrt{100} = 10$$

Step 6: Compute the ratio of the first amplitude to the second calculated amplitude:

$$\text{Ratio} = \frac{A_1}{A_2} = \frac{5}{10} = \frac{1}{2} = 1 : 2$$

Final Answer: $\boxed{1 : 2}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: The Doppler Effect describes the shift in the apparent frequency of a wave when there is relative motion between the source and the observer. When an observer moves towards a stationary sound source, they intercept more wave crests per unit time, causing the apparent frequency to increase.

Solution: Step 1: Write down the general equation for the apparent frequency f' heard by an observer moving towards a stationary sound source:

$$f' = f\left(\frac{v_s + v_o}{v_s}\right)$$

where f is the true frequency emitted by the source, v_s is the speed of sound in the medium, and v_o is the speed of the observer.
Step 2: Substitute the given condition into the formula, where the observer's velocity is one-fifth of the velocity of sound ($v_o = \frac{v_s}{5}$):

$$f' = f\left(\frac{v_s + \frac{v_s}{5}}{v_s}\right)$$

Step 3: Simplify the expression within the brackets:

$$f' = f\left(\frac{\frac{6v_s}{5}}{v_s}\right) = \frac{6}{5}f = 1.2f$$

Step 4: Calculate the absolute increase in frequency Δf:

$$\Delta f = f' - f = 1.2f - f = 0.2f$$

Step 5: Compute the percentage increase in the apparent frequency relative to the initial true sound frequency:

$$\text{Percentage Increase} = \left(\frac{\Delta f}{f}\right) \times 100\%$$

$$\text{Percentage Increase} = \left(\frac{0.2f}{f}\right) \times 100\% = 0.2 \times 100\% = 20\%$$

Final Answer: $\boxed{20\%}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: By applying Gauss's Law to a hollow spherical conducting shell of radius R containing a total net charge Q, we find that the internal electric field at any point inside the cavity ($r < R$) is exactly zero. Because the electric field is zero everywhere inside, no work is done moving a charge within the shell, which means the electrostatic potential remains perfectly uniform throughout the interior and equals its value at the outer surface.

Solution: Step 1: Determine the electric field inside the shell. For any Gaussian sphere of radius $r < R$, the enclosed charge is zero:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = 0 \implies E_{\text{internal}} = 0$$

Step 2: Relate the electric field to the spatial gradient of the electrostatic potential:

$$E = -\frac{dV}{dr}$$

Step 3: Since the internal electric field is zero at all points inside the conducting shell, set the potential derivative to zero:

$$-\frac{dV}{dr} = 0 \implies V = \text{Constant for all } r \leq R$$

Step 4: Determine the value of this constant potential by evaluating the potential at the boundary surface ($r = R$). For a spherical charge distribution, the potential at its surface is given by:

$$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Step 5: Conclude that since the potential is constant everywhere inside up to the surface, the potential at any interior distance $r < R$ must be exactly equal to the surface potential:

$$V_{\text{internal}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Final Answer: $\boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}}$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept:

Principle of superposition of electric fields and direction of electric fields due to point charges.

Solution:

Step 1: Understand the nature of the electric field from individual point charges. An electric field points radially away from a positive charge $(+q)$ and radially toward a negative charge $(-q)$.

Step 2: Identify the positions of the given charges on the coordinate system:

- Charge $q_1 = +q$ is located at point $A(-a, 0)$ on the negative x-axis.
- Charge $q_2 = -q$ is located at point $B(a, 0)$ on the positive x-axis.

Step 3: Determine the electric field vector contributed by each charge at the origin $O(0, 0)$:

- The electric field $\vec{E}_1$ due to the positive charge at $A(-a, 0)$ points away from it, which means it points along the $+x$ direction at the origin:

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\hat{i}$$

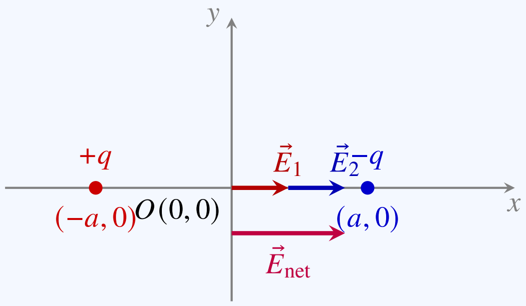
- The electric field $\vec{E}_2$ due to the negative charge at $B(a, 0)$ points toward it, which also means it points along the $+x$ direction at the origin:

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{a^2}\hat{i}$$

Step 4: Use the principle of superposition to find the net electric field $\vec{E}_{\text{net}}$ at the origin:

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}\hat{i}$$

Since the unit vector is $\hat{i}$, the net electric field vector at the origin points along the positive x-axis.



Final Answer: Positive x-axis

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept:

Working principle of a meter bridge, which is based on the balanced Wheatstone bridge condition.

Solution:

Step 1: For a meter bridge of total wire length 100 cm, the condition for balance when an unknown resistance X is in the left gap and a known resistance R is in the right gap is given by:

$$\frac{X}{R} = \frac{l}{100 - l}$$

where l is the balancing length measured from the left end.

Step 2: Identify the given values from the problem statement:

- Balancing length from the left end, $l = 40\,\text{cm}$
- Remaining length of the wire, $100 - l = 100 - 40 = 60\,\text{cm}$
- Known resistance in the right gap, $R = 6\,\Omega$

Step 3: Substitute the given values into the balancing condition to solve for the unknown resistance X:

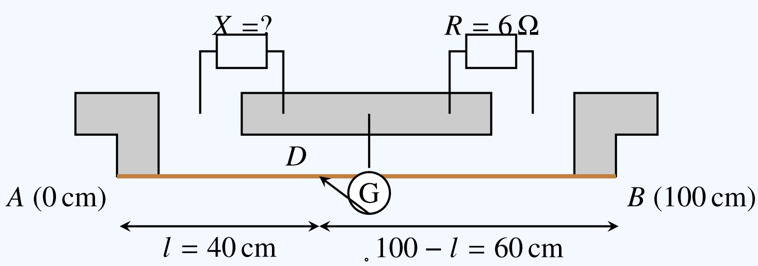
$$\frac{X}{6} = \frac{40}{60}$$

Step 4: Simplify the fraction and solve for X:

$$\frac{X}{6} = \frac{2}{3}$$

$$X = 6 \times \frac{2}{3} = 4\,\Omega$$

Thus, the value of the unknown resistance in the left gap is $4\,\Omega$.



Final Answer: $\boxed{4\,\Omega}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept: Ampere's Circuital Law states that the line integral of the magnetic field vector around any closed loop is equal to μ_0 times the total net current passing through the surface enclosed by that loop. For a long cylindrical wire carrying a uniform current density, the current enclosed within an internal radius increases quadratically with distance.

Solution: Step 1: Define a circular Amperian loop of radius r ($r < a$) inside the wire, concentric with the cylinder's central longitudinal axis.
Step 2: Calculate the line integral of the magnetic field B along this closed symmetric loop:

$$\oint \vec{B} \cdot d\vec{l} = B \cdot (2\pi r)$$

Step 3: Determine the current density J across the entire cross-sectional area of the wire of radius a:

$$J = \frac{I}{\pi a^2}$$

Step 4: Calculate the total current I_{enclosed} passing through the surface area bounded by our internal Amperian loop of radius r:

$$I_{\text{enclosed}} = J \cdot (\pi r^2) = \left(\frac{I}{\pi a^2}\right) \cdot (\pi r^2) = I\frac{r^2}{a^2}$$

Step 5: Apply Ampere's Circuital Law by equating the results from Step 2 and Step 4:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

$$B \cdot (2\pi r) = \mu_0 I \frac{r^2}{a^2}$$

Step 6: Solve explicitly for the internal magnetic field magnitude B:

$$B = \frac{\mu_0 I r^2}{2\pi r a^2} = \left(\frac{\mu_0 I}{2\pi a^2}\right) \cdot r$$

Step 7: Identify the relationship between B and r. Since μ_0, I, and a are all fixed parameters, the magnetic field is directly proportional to the distance:

$$B \propto r$$

Final Answer: $\boxed{B \propto r}$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept:

Motion of a charged particle in a uniform magnetic field when entering at an arbitrary angle.

Solution:

Step 1: Resolve the velocity vector $\vec{v}$ of the particle into two components relative to the uniform magnetic field $\vec{B}$:

- Component parallel to the magnetic field: $v_{\parallel} = v\cos(30°)$
- Component perpendicular to the magnetic field: $v_{\perp} = v\sin(30°)$

Step 2: Analyze the effect of each velocity component on the motion of the charged particle:

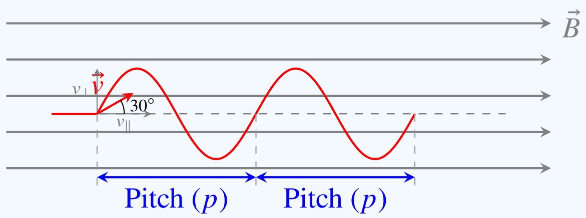
- The perpendicular component $v_{\perp}$ experiences a magnetic Lorentz force ($\vec{F} = q(\vec{v}_{\perp} \times \vec{B})$) which provides the necessary centripetal force to make the particle move in a circle.
- The parallel component $v_{\parallel}$ experiences no magnetic force ($\vec{F} = q(\vec{v}_{\parallel} \times \vec{B}) = 0$) because the angle is $0°$. Thus, it maintains a constant linear velocity along the direction of the magnetic field.

Step 3: Combine these two independent motions. The simultaneous circular motion in the perpendicular plane and uniform linear motion along the field lines results in a helical path.

Step 4: Determine if the pitch is uniform. The pitch p is the linear distance traveled along the magnetic field line in one time period T:

$$p = v_{\parallel} \times T = (v\cos 30°) \times \left(\frac{2\pi m}{qB}\right)$$

Since $v_{\parallel}$, m, q, and B are all constant throughout the motion, the distance covered per revolution remains constant. Thus, the path is a helix with uniform pitch.



Final Answer: Helix with uniform pitch

Answer: (A)

Go Back to Question 19

Q20.

Solution

Concept:
Magnetic field along the axis of a current-carrying circular loop.
Solution:
Step 1: Write down the formula for the magnetic field B at an axial point at a distance x from the centre of a circular loop of radius R carrying current I:

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

Step 2: Express the magnetic field B_0 at the centre of the loop by setting $x = 0$:

$$B_0 = \frac{\mu_0 I R^2}{2(R^2)^{3/2}} = \frac{\mu_0 I}{2R}$$

Step 3: Substitute the given axial distance $x = \sqrt{3}R$ into the general formula to find the field B at that specific point:

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + (\sqrt{3}R)^2\right)^{3/2}}$$

$$B = \frac{\mu_0 I R^2}{2\left(R^2 + 3R^2\right)^{3/2}} = \frac{\mu_0 I R^2}{2\left(4R^2\right)^{3/2}}$$

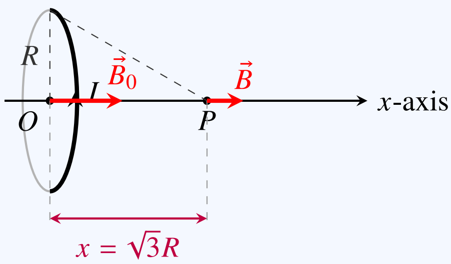
Step 4: Simplify the denominator:

$$(4R^2)^{3/2} = (2R)^3 = 8R^3$$

$$B = \frac{\mu_0 I R^2}{2 \times 8R^3} = \frac{\mu_0 I}{16R}$$

Step 5: Establish the relationship between B and B_0:

$$B = \frac{1}{8}\left(\frac{\mu_0 I}{2R}\right) = \frac{B_0}{8}$$



Final Answer: $\boxed{\frac{B_0}{8}}$

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept: According to Faraday's Law of Electromagnetic Induction, a change in the magnetic flux linked with a conducting circuit induces an electromotive force (emf) given by $e = -\frac{d\Phi}{dt}$. When a rigid loop enters a uniform magnetic field, the area enclosed within the field increases linearly over time, creating a motional emf across the leading segment of the loop.

Solution: Step 1: Write down the formula for the magnetic flux Φ passing through the square loop at any instant when a length x of the loop has entered the magnetic field region:

$$\Phi = B \cdot A_{\text{enclosed}} = B \cdot (L \cdot x)$$

Step 2: Differentiate the magnetic flux expression with respect to time to calculate the magnitude of the induced electromotive force (e):

$$e = \frac{d\Phi}{dt} = \frac{d}{dt}(BLx) = BL\frac{dx}{dt}$$

Step 3: Replace the rate of change of position $\frac{dx}{dt}$ with the uniform translation velocity v of the loop:

$$e = BLv$$

Step 4: Use Ohm's Law to calculate the induced current I flowing through the circuit, given that the total electrical resistance of the loop is R:

$$I = \frac{e}{R}$$

Step 5: Substitute the induced electromotive force from Step 3 into the Ohm's law equation:

$$I = \frac{BLv}{R}$$

This current flows in a direction determined by Lenz's Law to oppose the increase in inward magnetic flux.

Final Answer: $\boxed{\frac{BLv}{R}}$

Answer: (A)

Go Back to Question 21

Q22.

Solution

Concept: An electromagnetic wave consists of oscillating electric and magnetic field vectors that are perpendicular to each other and also perpendicular to the direction of wave propagation. The directional relationship is governed by the vector cross product, where the unit vector of propagation $\hat{k}_{prop}$ is equal to the cross product of the electric field unit vector $\hat{e}$ and the magnetic field unit vector $\hat{b}$ ($\hat{k}_{prop} = \hat{e} \times \hat{b}$).

Solution: Step 1: Identify the direction of propagation of the electromagnetic wave from the phase term of the given electric field equation $\vec{E} = E_0 \cos(kz - \omega t)\hat{i}$. The phase contains kz, which indicates that the wave is propagating along the positive z-axis:

$$\hat{k}_{prop} = \hat{k}$$

Step 2: Identify the polarization direction of the electric field vector from its unit vector component:

$$\hat{e} = \hat{i}$$

Step 3: Set up the cross product relation to find the unknown unit vector direction $\hat{b}$ of the magnetic field:

$$\hat{e} \times \hat{b} = \hat{k}_{prop}$$

$$\hat{i} \times \hat{b} = \hat{k}$$

Step 4: Recall the cyclic cross product rules for standard Cartesian unit vectors ($\hat{i} \times \hat{j} = \hat{k}$). Comparing this with the expression in Step 3 shows that the magnetic field vector must point along the positive y-axis:

$$\hat{b} = \hat{j}$$

Step 5: Relate the peak amplitude B_0 of the magnetic field to the peak amplitude E_0 of the electric field using the speed of light c in free space:

$$B_0 = \frac{E_0}{c}$$

Step 6: Combine the amplitude, phase, and directional unit vector to write the complete vector expression for the magnetic field $\vec{B}$:

$$\vec{B} = \frac{E_0}{c} \cos(kz - \omega t)\hat{j}$$

Final Answer: $\boxed{\vec{B} = \frac{E_0}{c} \cos(kz - \omega t)\hat{j}}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: The focal length of a thin spherical lens is determined by the Lens Maker's Formula. The refractive power of the lens depends on the relative refractive index of the lens material with respect to the surrounding medium. When a lens is transferred from air to a liquid medium, this relative refractive index changes, which alters its focal length.

Solution: Step 1: Write down the Lens Maker's Formula for the convex lens when it is placed in air (where the refractive index of air is $\mu_{\text{air}} = 1$):

$$\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where $\mu_g = 1.5 = \frac{3}{2}$ is the refractive index of the glass material.

Step 2: Substitute the numeric value of μ_g into the equation to find an expression for the geometric factor:

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2}{f}$$

Step 3: Write down the Lens Maker's Formula for the same lens when it is completely immersed in water ($\mu_w = \frac{4}{3}$):

$$\frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Step 4: Calculate the relative refractive index of the glass lens with respect to water ($\frac{\mu_g}{\mu_w}$):

$$\frac{\mu_g}{\mu_w} = \frac{\left(\frac{3}{2}\right)}{\left(\frac{4}{3}\right)} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

Step 5: Substitute this relative refractive index value into the focal length equation from Step 3:

$$\frac{1}{f_w} = \left(\frac{9}{8} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Step 6: Substitute the geometric curvature relation $\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2}{f}$ derived in Step 2 into this equation:

$$\frac{1}{f_w} = \frac{1}{8} \times \left(\frac{2}{f}\right) = \frac{1}{4f}$$

Step 7: Take the reciprocal of both sides to find the final focal length of the lens in water:

$$f_w = 4f$$

Final Answer: $\boxed{4f}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept: In Young's Double Slit Experiment, the linear distance between two consecutive bright or dark interference fringes on the screen is called the fringe width (β). The fringe width depends directly on the wavelength of the light source and the distance between the slit plane and the screen, and inversely on the separation distance between the two coherent slits.

Solution: Step 1: Write down the standard formula for the fringe width β in a baseline double-slit configuration:

$$\beta = \frac{\lambda D}{d}$$

where λ is the wavelength of light, D is the distance from the slits to the screen, and d is the distance separating the two slits.
Step 2: Identify the modifications made to the geometric parameters as stated in the problem:

$$\text{New slit separation, } d' = 2d$$

$$\text{New screen distance, } D' = \frac{D}{2}$$

The light source remains the same, so the wavelength λ is constant.
Step 3: Write down the expression for the new modified fringe width β' using these adjusted parameters:

$$\beta' = \frac{\lambda D'}{d'}$$

Step 4: Substitute the expressions for D' and d' from Step 2 into this formula:

$$\beta' = \frac{\lambda \left(\frac{D}{2}\right)}{2d}$$

Step 5: Simplify the fraction by collecting the numerical factors together:

$$\beta' = \frac{\lambda D}{4d} = \frac{1}{4} \cdot \left(\frac{\lambda D}{d}\right)$$

Step 6: Substitute the initial fringe width expression β back into the equation:

$$\beta' = \frac{\beta}{4}$$

This shows that the interference fringes become four times closer together, making the pattern more tightly packed on the observation screen.
Final Answer: One-fourth

Answer: (D)

Go Back to Question 24

Q25.

Solution

Concept: The photoelectric effect is governed by Einstein's Photoelectric Equation, which is based on the principle of conservation of energy. The total energy of an incident photon ($E = \frac{hc}{\lambda}$) is spent in two ways: a fixed minimum amount is used as the work function (ϕ_0) to release the electron from the metal surface, and the remaining energy is converted into the maximum kinetic energy (K) of the emitted photoelectron.

Solution: Step 1: Write down Einstein's photoelectric equation for the first case with incident wavelength λ and maximum kinetic energy K:

$$K = \frac{hc}{\lambda} - \phi_0$$

Step 2: Rearrange this equation to express the initial photon energy in terms of kinetic energy and the work function:

$$\frac{hc}{\lambda} = K + \phi_0$$

Step 3: Write down the photoelectric equation for the second case where the incident wavelength is reduced to $\lambda' = \frac{\lambda}{3}$:

$$K' = \frac{hc}{\lambda'} - \phi_0 = \frac{hc}{\left(\frac{\lambda}{3}\right)} - \phi_0$$

$$K' = 3\left(\frac{hc}{\lambda}\right) - \phi_0$$

Step 4: Substitute the expression for $\frac{hc}{\lambda}$ from Step 2 into this equation:

$$K' = 3(K + \phi_0) - \phi_0$$

Step 5: Expand the bracket and simplify the expression:

$$K' = 3K + 3\phi_0 - \phi_0$$

$$K' = 3K + 2\phi_0$$

Step 6: Analyze the final expression for K'. Since the work function ϕ_0 of a metal is a positive quantity ($\phi_0 > 0$), the term $2\phi_0$ is always greater than zero. Therefore, we can conclude that:

$$K' > 3K$$

Final Answer: Greater than $3K$

Answer: (B)

Go Back to Question 25

Q26.

Solution

Concept: When an atom undergoes a transition from a higher energy state to a lower energy state, it emits a photon. According to the law of conservation of linear momentum, if the atom is initially at rest, it must experience a recoil velocity in the direction opposite to the emitted photon's momentum. The momentum of a photon of wavelength λ is given by $p = \frac{h}{\lambda}$.

Solution: Step 1: Use the Rydberg formula to calculate the wave number $\frac{1}{\lambda}$ of the photon emitted during the atomic transition from $n_{\text{initial}} = 3$ to $n_{\text{final}} = 1$:

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2} \right)$$

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{1^2} - \frac{1}{3^2} \right) = R_\infty \left(1 - \frac{1}{9} \right) = \frac{8}{9} R_\infty$$

Step 2: Calculate the linear momentum p_{photon} of the emitted photon using its de Broglie relation:

$$p_{\text{photon}} = \frac{h}{\lambda} = h \cdot \left(\frac{8}{9} R_\infty \right) = \frac{8hR_\infty}{9}$$

Step 3: Apply the conservation of linear momentum to the entire system. Since the hydrogen atom is initially stationary, the total initial momentum is zero. Therefore, the magnitude of the atom's recoil momentum must exactly equal the photon's momentum:

$$p_{\text{atom}} = p_{\text{photon}}$$

$$M v_{\text{recoil}} = \frac{8hR_\infty}{9}$$

Step 4: Solve explicitly for the recoil speed v_{recoil} of the hydrogen atom by dividing by its mass M:

$$v_{\text{recoil}} = \frac{8hR_\infty}{9M}$$

Final Answer: $\boxed{\frac{8hR_\infty}{9M}}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: Logic gates process binary inputs according to Boolean algebra logic. A NAND gate performing on inputs A and B produces a boolean output given by $Y_1 = \overline{A \cdot B}$. Passing this intermediate signal directly through a NOT gate inverts the boolean state, returning the logical complement of the expression.

Solution: Step 1: Write down the characteristic Boolean expression for the output of a standard two-input NAND gate given inputs A and B:

$$Y_{\text{NAND}} = \overline{A \cdot B}$$

Step 2: Track the signal propagation into the next stage of the logic circuit. The output of the NAND gate acts directly as the input to a subsequent NOT gate.
Step 3: Apply the functional rule for a NOT gate, which acts as a logical inverter. The final output expression Y is the complement of its input:

$$Y = \overline{Y_{\text{NAND}}}$$

Step 4: Substitute the expression from Step 1 into this equation:

$$Y = \overline{\left(\overline{A \cdot B}\right)}$$

Step 5: Apply the Boolean algebra law of double negation, which states that complementing a boolean expression twice returns the original uncomplemented expression ($\overline{\overline{X}} = X$):

$$Y = A \cdot B$$

Step 6: Identify the standard logic gate corresponding to this final Boolean product formula. The expression $Y = A \cdot B$ is the exact mathematical definition of an AND gate.
Final Answer: AND gate

Answer: (B)

Go Back to Question 27

Q28.

Solution

Concept: The kinetic energy K of a moving body can be expressed either in terms of its linear speed ($K = \frac{1}{2}mv^2$) or in terms of its linear momentum magnitude p using the relation $K = \frac{p^2}{2m}$. When comparing two bodies with identical linear momenta, their kinetic energies are inversely proportional to their respective masses.

Solution: Step 1: State the general formula relating the kinetic energy K of an object to its linear momentum p and mass m:

$$K = \frac{p^2}{2m}$$

Step 2: Apply this expression to create separate equations for the two individual moving bodies described in the problem statement:

$$K_1 = \frac{p_1^2}{2m_1} \quad \text{and} \quad K_2 = \frac{p_2^2}{2m_2}$$

Step 3: Incorporate the given constraint that both bodies move with exactly equal linear momenta ($p_1 = p_2 = p$):

$$K_1 = \frac{p^2}{2m_1} \quad \text{and} \quad K_2 = \frac{p^2}{2m_2}$$

Step 4: Set up the ratio of the first kinetic energy to the second kinetic energy to compare them:

$$\frac{K_1}{K_2} = \frac{\left(\frac{p^2}{2m_1}\right)}{\left(\frac{p^2}{2m_2}\right)}$$

Step 5: Simplify the complex fraction by cancelling out the common factor $\frac{p^2}{2}$:

$$\frac{K_1}{K_2} = \frac{m_2}{m_1}$$

Step 6: Substitute the given mass values $m_1 = 1$ kg and $m_2 = 4$ kg into this inverse mass ratio:

$$\frac{K_1}{K_2} = \frac{4}{1} = 4 : 1$$

This indicates that the lighter object possesses a larger share of kinetic energy when sharing equal momentum with a heavier object.

Final Answer: $\boxed{4 : 1}$

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept: The time period T of a mass-spring system executing simple harmonic motion (SHM) along a vertical axis depends entirely on the inertia of the hanging mass and the stiffness constant of the spring. It is independent of external factors like local gravitational acceleration or the amplitude of oscillation.

Solution: Step 1: Recall the standard time period formula for a mass-spring harmonic oscillator:

$$T = 2\pi\sqrt{\frac{M}{k}}$$

where M is the mass suspended from the spring and k is the spring stiffness constant.
Step 2: Set up the initial baseline state where mass M oscillates with time period T:

$$T = 2\pi\sqrt{\frac{M}{k}}$$

Step 3: Define the expression for the updated time period T' when the original block is detached and replaced with a heavier block of mass $M' = 2M$, while keeping the same spring (k remains unchanged):

$$T' = 2\pi\sqrt{\frac{2M}{k}}$$

Step 4: Factor out the numerical constant from inside the radical sign to separate the initial formula structure:

$$T' = 2\pi \cdot \sqrt{2} \cdot \sqrt{\frac{M}{k}} = \sqrt{2} \cdot \left(2\pi\sqrt{\frac{M}{k}}\right)$$

Step 5: Substitute the baseline time period T from Step 2 into this new equation:

$$T' = \sqrt{2}T$$

This shows that increasing the system's inertia increases the time period of oscillation by a factor of $\sqrt{2}$.

Final Answer: $\boxed{\sqrt{2}T}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: For a particle executing uniform circular motion under a central force field, the required centripetal acceleration is provided entirely by that radial force ($F_c = -\frac{mv^2}{r}$). The total mechanical energy E of the system is the sum of its kinetic energy K and its potential energy U, which is derived by integrating the conservative force field relation $F = -\frac{dU}{dr}$.

Solution: Step 1: Relate the given radial force magnitude to the centripetal force equation to find the kinetic energy of the particle:

$$\frac{mv^2}{r} = \frac{k}{r^2}$$

Step 2: Rearrange terms to solve for mv^2:

$$mv^2 = \frac{k}{r}$$

Step 3: Calculate the kinetic energy K of the particle:

$$K = \frac{1}{2}mv^2 = \frac{k}{2r}$$

Step 4: Find the potential energy function $U(r)$ by integrating the conservative force with respect to position, setting $U(\infty) = 0$:

$$F = -\frac{dU}{dr} \implies dU = -F\,dr$$

$$U(r) = -\int_{\infty}^{r} \left(-\frac{k}{r^2}\right) dr = k\int_{\infty}^{r} r^{-2}\,dr$$

$$U(r) = k\left[-\frac{1}{r}\right]_{\infty}^{r} = -\frac{k}{r}$$

Step 5: Sum the kinetic energy and potential energy components together to find the total mechanical energy E:

$$E = K + U$$

$$E = \frac{k}{2r} + \left(-\frac{k}{r}\right)$$

Step 6: Combine the two fractions using a common denominator:

$$E = \frac{k - 2k}{2r} = -\frac{k}{2r}$$

The negative net mechanical energy indicates that the particle is in a stable, bound orbit within the central force field.

Final Answer: $\boxed{-\frac{k}{2r}}$

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept:
Rolling motion of a rigid body down an inclined plane without slipping.
Solution:
Step 1: Write down the general formula for the linear acceleration a of the center of mass of a body rolling down an inclined plane of inclination θ:

$$a = \frac{g\sin\theta}{1 + \frac{I_{cm}}{MR^2}}$$

where I_{cm} is the moment of inertia of the body about its central axis, M is its mass, and R is its radius.
Step 2: Identify the expression for the moment of inertia for the given body. For a uniform solid cylinder about its geometric axis:

$$I_{cm} = \frac{1}{2}MR^2$$

Step 3: Substitute the value of I_{cm} into the acceleration formula:

$$a = \frac{g\sin\theta}{1 + \frac{\frac{1}{2}MR^2}{MR^2}}$$

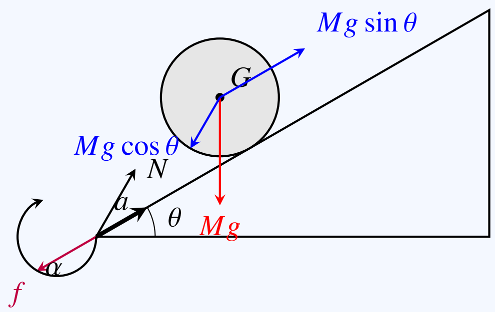
$$a = \frac{g\sin\theta}{1 + \frac{1}{2}}$$

Step 4: Simplify the expression in the denominator:

$$1 + \frac{1}{2} = \frac{3}{2}$$

$$a = \frac{g\sin\theta}{\frac{3}{2}} = \frac{2}{3}g\sin\theta$$

Thus, the linear acceleration of the center of mass of the cylinder is $\frac{2}{3}g\sin\theta$.



Final Answer: $\boxed{\frac{2}{3}g\sin\theta}$

Answer: (B)

Go Back to Question 31

Q32.

Solution

Concept: When an elastic metal wire is stretched within its proportionality limit, it behaves like a mechanical spring that obeys Hooke's Law. The stretching force increases linearly with deformation. The work done by an external agent to stretch the wire is stored entirely inside the material as elastic potential energy, which can be found by integrating the force over the elongation distance.

Solution: Step 1: Recall the definition of Young's Modulus Y, which relates longitudinal stress to longitudinal strain:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)}$$

where F is the tension force, A is the cross-sectional area, L is the original length, and ΔL is the variable extension.

Step 2: Rearrange the equation to express the pulling force F as a function of an arbitrary intermediate elongation variable y:

$$F(y) = \frac{YA}{L} \cdot y$$

Step 3: Set up the definite integral to calculate the total work done W in stretching the wire from its initial undeformed state ($y = 0$) to its final extension ($y = x$):

$$W = \int_0^x F(y)\, dy$$

$$W = \int_0^x \left(\frac{YA}{L} \cdot y\right) dy$$

Step 4: Pull the constant material parameters outside the integration symbol:

$$W = \frac{YA}{L} \int_0^x y\, dy$$

Step 5: Perform the integration with respect to the variable y:

$$W = \frac{YA}{L} \left[\frac{y^2}{2}\right]_0^x$$

Step 6: Evaluate the definite integral limits to find the final expression for work done:

$$W = \frac{YA}{L} \left(\frac{x^2}{2} - 0\right) = \frac{YAx^2}{2L}$$

This equation can also be interpreted as $W = \frac{1}{2} \times \text{Load} \times \text{Extension}$, where the maximum load reached is $F_{\text{max}} = \frac{YAx}{L}$.

Final Answer: $\boxed{\frac{YAx^2}{2L}}$

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept: When multiple electrical cells are connected together in a parallel combination with matching polarities, the net open-circuit potential difference across the common terminals is called the equivalent electromotive force (E_{eq}). This equivalent value can be derived by applying Kirchhoff's Loop Rules or using Millman's Theorem for parallel active networks.

Solution: Step 1: Consider two cells with parameters (E_1, r_1) and (E_2, r_2) connected in a parallel configuration across an external load, such that their positive terminals meet at a single node and their negative terminals meet at another node.
Step 2: Use the standard parallel source transformation formula for active circuits, which states that the sum of the individual short-circuit currents equals the total short-circuit current of the equivalent system:

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

where r_{eq} is the equivalent internal resistance of the parallel combination.
Step 3: Recall the formula for the equivalent resistance of two individual resistors connected in a parallel combination:

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \implies r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

Step 4: Simplify the right-hand side of the source current equation by finding a common denominator:

$$\frac{E_1}{r_1} + \frac{E_2}{r_2} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2}$$

Step 5: Substitute this simplified current expression back into the terminal balance equation:

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2}$$

Step 6: Isolate the equivalent electromotive force variable E_{eq} by multiplying through by r_{eq}:

$$E_{eq} = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 r_2}\right) \cdot r_{eq}$$

Step 7: Substitute the value of r_{eq} from Step 3 into the expression and cancel out the common terms:

$$E_{eq} = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 r_2}\right) \cdot \left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

Final Answer: $\boxed{\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}}$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: In a series $L - R$ alternating current circuit, the pure resistance causes current and voltage to remain in phase, while the inductor causes the voltage to lead the current by a phase angle of 90°. The combination of these components produces a total electrical impedance vector Z in the complex plane, where the total voltage leads the net circuit current by a phase angle ϕ between 0° and 90°.

Solution: Step 1: Write down the expressions for the alternating voltage drops across each circuit component in terms of the instantaneous loop current I. The voltage drop across the resistor is $V_R = I \cdot R$ and the voltage drop across the inductor is $V_L = I \cdot X_L$.

Step 2: Express the inductive reactance X_L in terms of the operating angular frequency ω and the inductance value L:

$$X_L = \omega L$$

Step 3: Construct the vector phasor diagram for the series combination. The resistor voltage phasor V_R is aligned along the horizontal current reference axis, while the inductor voltage phasor V_L points along the vertical axis, leading by +90°.

Step 4: Determine the geometry of the voltage right-angled triangle formed by these phasors. The horizontal leg represents V_R, the vertical leg represents V_L, and the hypotenuse represents the total applied source voltage V.

Step 5: Use the definition of the tangent of an angle in a right-angled triangle to find the phase angle ϕ:

$$\tan\phi = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{V_L}{V_R}$$

Step 6: Substitute the component voltage relations from Step 1 and Step 2 into this trigonometric equation:

$$\tan\phi = \frac{I \cdot X_L}{I \cdot R} = \frac{X_L}{R}$$

$$\tan\phi = \frac{\omega L}{R}$$

Final Answer: $\boxed{\tan\phi = \frac{\omega L}{R}}$

Answer: (B)

Go Back to Question 34

Q35.

Solution

Concept: At the position of minimum deviation (δ_m), the refractive index μ of a prism with angle A is given by the prism formula.

Solution: Step 1: State the standard prism formula:

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

Step 2: Substitute the given value $\mu = \cot(A/2)$ and express cotangent as cosine over sine:

$$\frac{\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} \implies \cos\left(\frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right)$$

Step 3: Convert the cosine term to a sine term using $\cos\theta = \sin(90^\circ - \theta)$:

$$\sin\left(90^\circ - \frac{A}{2}\right) = \sin\left(\frac{A+\delta_m}{2}\right)$$

Step 4: Equate the arguments and solve for δ_m:

$$90^\circ - \frac{A}{2} = \frac{A+\delta_m}{2} \implies 180^\circ - A = A + \delta_m$$

$$\delta_m = 180^\circ - 2A$$

Final Answer: $\boxed{180^\circ - 2A}$

Answer: (A)

Go Back to Question 35

Q36.

Solution

Concept: The First Law of Thermodynamics establishes the principle of conservation of energy for thermal systems, expressed mathematically as $\Delta Q = \Delta U + \Delta W$. Here, ΔQ represents the net heat energy exchanged with the surroundings, ΔU is the net change in internal energy, and ΔW is the work performed by the system. For an ideal gas, internal energy is a function of absolute temperature alone ($\Delta U \propto \Delta T$).

Solution: Step 1: Extract the given thermodynamic parameters from the statement with their proper sign conventions. The internal energy of the system decreases, which means its sign is negative:

$$\Delta U = -300 \text{ J}$$

The system performs work on its surroundings, which represents a positive work output by convention:

$$\Delta W = +200 \text{ J}$$

Step 2: Substitute these values into the First Law of Thermodynamics equation to calculate the net heat energy exchanged (ΔQ):

$$\Delta Q = \Delta U + \Delta W$$

$$\Delta Q = -300 \text{ J} + 200 \text{ J} = -100 \text{ J}$$

Since the numerical value of ΔQ is negative, heat leaves the system, which confirms that Statement (A) is completely correct.

Step 3: Evaluate Statement (B). By definition, a thermodynamic process that releases heat into the environment ($\Delta Q < 0$) is classified as an exothermic process. Therefore, Statement (B) is also correct.

Step 4: Evaluate Statement (C). For an ideal gas system, the internal energy depends directly on its absolute temperature. A net decrease in internal energy ($\Delta U < 0$) means that the final temperature is lower than the initial temperature. Thus, Statement (C) is correct.

Step 5: Evaluate Statement (D). An adiabatic process requires zero heat exchange with the surroundings ($\Delta Q = 0$). Since we calculated that $\Delta Q = -100$ J, the process is not adiabatic, making Statement (D) incorrect.

Final Answer: A, B, C

Answer: (A, B, C)

Go Back to Question 36

Q37.

Solution

Concept: When a parallel-plate capacitor is charged by a constant voltage source and then disconnected from the source, it becomes an isolated electrical system. Because it is isolated from external circuits, the net electrical charge trapped on its conducting plates must remain constant due to the conservation of charge. Inserting a dielectric material reduces the electric field strength due to polarization, which alters the other electrical variables.

Solution: Step 1: Analyze the electric charge Q. Since the charging battery was completely disconnected before inserting the dielectric material, there is no conductive path for charge to enter or leave the plates. Therefore, the charge remains completely unchanged ($Q' = Q$). This means option (A) does not change.

Step 2: Analyze the capacitance C. Introducing a dielectric material with a dielectric constant $K > 1$ increases the capacitance by a factor of K because the material polarizes under the electric field:

$$C' = K \cdot C$$

Since $K > 1$, the capacitance changes, making option (D) a correct answer.

Step 3: Analyze the potential difference V across the capacitor plates. Using the basic definition of capacitance $V = \frac{Q}{C}$:

$$V' = \frac{Q'}{C'} = \frac{Q}{K \cdot C} = \frac{1}{K} \cdot \left(\frac{Q}{C}\right) = \frac{V}{K}$$

Since the potential difference decreases by a factor of K, it changes, making option (B) a correct answer.

Step 4: Analyze the stored electrostatic potential energy U. Using the energy formula in terms of charge and capacitance $U = \frac{Q^2}{2C}$:

$$U' = \frac{(Q')^2}{2C'} = \frac{Q^2}{2(K \cdot C)} = \frac{1}{K} \cdot \left(\frac{Q^2}{2C}\right) = \frac{U}{K}$$

Since the stored electrostatic energy decreases by a factor of K (with the energy difference spent pulling the dielectric slab into the plates), it changes, making option (C) a correct answer.

Final Answer: B, C, D

Answer: (B, C, D)

Go Back to Question 37

Q38.

Solution

Concept: The magnetic force acting on a straight segment of length L carrying a current I in a magnetic field B is given by $\vec{F} = I(\vec{L} \times \vec{B})$. Parallel conductors carrying currents in the same direction exert an attractive magnetic force on each other, while antiparallel currents repel. The magnetic field produced by a long straight wire decreases inversely with distance ($B \propto \frac{1}{r}$).

Solution: Step 1: Analyze the forces acting on the sides of the rectangular loop that are parallel to the long straight wire. Let the closer side carry current in the same direction as I_0, and the farther side carry current in the opposite direction. The closer side experiences an attractive force toward the wire, while the farther side experiences a repulsive force away from the wire.
Step 2: Evaluate Statement (A) and (C). Because the magnetic field from the straight wire decreases with distance ($B = \frac{\mu_0 I_0}{2\pi r}$), the attractive force on the closer wire segment is stronger than the repulsive force on the farther segment. This creates a net attractive force on the loop toward the straight wire. Since these opposing forces do not cancel, the net magnetic force on the loop is non-zero, making both Statement (A) and Statement (C) correct.
Step 3: Evaluate Statement (B). Let us examine the torque acting on the loop. The forces acting on the two parallel segments line up directly with each other through the center of mass, so they produce no torque. The forces acting on the top and bottom transverse wire segments are equal in magnitude and opposite in direction along the same line of action, so they cancel out and produce no torque. Therefore, the net torque acting on the loop is zero, making Statement (B) correct.
Step 4: Evaluate Statement (D). The net force on the loop depends on the magnetic field difference between the closer and farther sides. Since the field depends on position, the net force depends directly on the distance from the wire, making Statement (D) incorrect.
Final Answer: A, B, C

Answer: (A, B, C)

Go Back to Question 38

Q39.

Solution

Concept: Radioactive decay is a stochastic process that follows first-order kinetics, governed by the equation $N(t) = N_0 e^{-\lambda t}$, where λ is the decay constant. The decay constant is inversely related to the half-life ($T_{1/2} = \frac{\ln 2}{\lambda}$) and the mean life ($\tau = \frac{1}{\lambda}$). The initial activity of a sample depends on the total number of undecayed nuclei and the decay constant ($A_0 = \lambda N_0$).

Solution: Step 1: Relate the half-life of a radioactive sample to its characteristic decay constant using the standard relation:

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

This equation shows that the decay constant λ is inversely proportional to the half-life. Therefore, a sample with a shorter half-life will have a larger decay constant. This confirms Statement (C) is correct.

Step 2: Evaluate Statement (A). The initial activity A_0 of a radioactive sample is the rate of decay at $t = 0$, given by $A_0 = \lambda N_0$. The problem states that both samples start with the same number of active nuclei ($N_{0A} = N_{0B} = N_0$). If $T_A > T_B$, then from Step 1 we know that $\lambda_A < \lambda_B$. Calculating the initial activities gives:

$$A_{0B} = \lambda_B N_0 > \lambda_A N_0 = A_{0A}$$

Since sample B has a higher initial activity, it decays faster at the beginning, making Statement (A) correct.

Step 3: Evaluate Statement (B). Since sample A has a longer half-life ($T_A > T_B$), it decays more slowly, so its remaining number of active nuclei will eventually be higher than sample B. This means the activity of sample A will not always be higher than B, making Statement (B) incorrect.

Step 4: Evaluate Statement (D). By definition, the number of nuclei remaining after one mean life ($t = \tau = \frac{1}{\lambda}$) is given by:

$$N(\tau) = N_0 e^{-\lambda(\frac{1}{\lambda})} = N_0 e^{-1} = \frac{N_0}{e} \approx 0.368 N_0$$

This means approximately 36.8% of the initial nuclei remain undecayed after one mean life, not 50% (which occurs after one half-life). Therefore, Statement (D) is incorrect.

Final Answer: $\boxed{\text{A, C}}$

Answer: (A, C)

Go Back to Question 39

Q40.

Solution

Concept: According to Faraday's Law of Induction, a time-varying magnetic field induces an electromotive force (emf) inside a stationary conducting loop, given by $e = -\frac{d\Phi}{dt}$. This induced emf creates an electric field along the path of the loop, which drives an induced current determined by Ohm's Law ($I = \frac{e}{R}$).

Solution: Step 1: Write down the expression for the magnetic flux $\Phi(t)$ passing through the stationary circular loop of radius r at any time t:

$$\Phi(t) = B(t) \cdot \text{Area} = (B_0 + \alpha t^2) \cdot (\pi r^2)$$

Step 2: Differentiate the magnetic flux function with respect to time to find the magnitude of the induced electromotive force (e):

$$e = \frac{d\Phi}{dt} = \frac{d}{dt}\left[\pi r^2 (B_0 + \alpha t^2)\right] = \pi r^2 (0 + 2\alpha t) = 2\pi r^2 \alpha t$$

This matches the expression in Statement (A), making Statement (A) correct.
Step 3: Use Ohm's Law to find the induced current $I(t)$ flowing through the loop of resistance R:

$$I(t) = \frac{e}{R} = \left(\frac{2\pi r^2 \alpha}{R}\right) t$$

Since all parameters inside the brackets are constants, the induced current is directly proportional to time t, meaning it increases linearly. This confirms Statement (B) is correct.
Step 4: Evaluate Statement (C). The total charge $q(t)$ that flows through the loop up to time t is the time integral of the current:

$$q(t) = \int_0^t I(t')\, dt' = \int_0^t \left(\frac{2\pi r^2 \alpha}{R}\right) t'\, dt' = \left(\frac{\pi r^2 \alpha}{R}\right) t^2$$

Since the charge depends on t^2, it has a quadratic dependence on time, making Statement (C) correct.
Step 5: Evaluate Statement (D). The induced emf around a closed loop is equal to the line integral of the induced electric field ($e = \oint \vec{E} \cdot d\vec{l}$). Due to the circular symmetry of the loop, the induced electric field is uniform along the perimeter:

$$e = E \cdot (2\pi r) \implies 2\pi r^2 \alpha t = E \cdot (2\pi r)$$

$$E = r\alpha t$$

This matches the expression in Statement (D), making Statement (D) correct.
Final Answer: A, B, C, D

Answer: (A, B, C, D)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	B	5	A
6	A	7	B	8	C	9	C	10	B
11	C	12	B	13	A	14	A	15	C
16	A	17	A	18	B	19	A	20	C
21	A	22	A	23	B	24	D	25	B
26	A	27	B	28	A	29	A	30	A
31	B	32	A	33	B	34	B	35	A
36	A, B, C	37	B, C, D	38	A, B, C	39	A, C	40	A, B, C, D

Note: Section C (Q36–Q40): One or more correct options may be correct. Full marks only if all correct options are marked. Partial marking is not applicable.