WBJEE Physics Sample Paper-15

Duration: 60 Minutes **Maximum Marks: 50**

Instructions

- This paper contains **40** Multiple Choice Questions divided into **3 Categories**.
- **Section A (Q1–Q30):** Each correct answer carries **+1 mark**. Incorrect answer: **−0.25** marks. Only **one** correct option.
- **Section B (Q31–Q35):** Each correct answer carries **+2 marks**. Incorrect answer: **−0.5** marks. Only **one** correct option.
- **Section C (Q36–Q40):** Each correct answer carries **+2 marks**. **No negative marking**. One or **more** correct options may be correct; full marks only if all correct options are marked.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Section A - 30 Questions × 1 Mark Each
(Negative Marking: −0.25) [Single Correct]

Q1. A heavy block of mass M is hanging from a uniform string of mass m and length L. A transverse wave pulse is generated at the bottom of the string. If the wavelength of the pulse at the bottom is λ_0, then its wavelength when it reaches the top of the string is

(A) $\lambda_0\sqrt{\frac{M+m}{M}}$
(B) $\lambda_0\frac{M+m}{M}$
(C) $\lambda_0\sqrt{\frac{M}{M+m}}$
(D) λ_0

Q2. A cylindrical tank of cross-sectional area A is filled with water to a height H. A small hole of area a ($a \ll A$) is made at the bottom of the tank. The time taken for the water level to fall from H to $\frac{H}{4}$ is

(A) $\frac{A}{a}\sqrt{\frac{H}{g}}$

(B) $\frac{A}{a}\sqrt{\frac{2H}{g}}$

(C) $\frac{A}{2a}\sqrt{\frac{H}{2g}}$

(D) $\frac{A}{a}\sqrt{\frac{H}{2g}}$

Q3. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are 1.5% and 1.0% respectively, the maximum possible error in determining the density is

(A) 2.5%

(B) 3.5%

(C) 4.5%

(D) 6.0%

Q4. A particle moves along a straight line such that its displacement at any time t is given by $s = t^3 - 6t^2 + 3t + 4$ meters. The velocity of the particle when its acceleration is zero is

(A) 3 m/s

(B) −12 m/s

(C) −9 m/s

(D) 2 m/s

Q5. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$, where k is a constant. The power delivered to the particle by the forces acting on it is

(A) mk^2r^2t

(B) $mk^2r^2t^2$

(C) $\frac{1}{2}mk^2r^2t$

(D) $zero$

Q6. A satellite is revolving in a circular orbit close to the surface of a planet of mean density ρ. If G is the universal gravitational constant, the time period of the satellite depends only on ρ and is given by

(A) $\sqrt{\frac{3\pi}{G\rho}}$

(B) $\sqrt{\frac{3}{G\rho}}$

(C) $\sqrt{\frac{G\rho}{3\pi}}$

(D) $\sqrt{\frac{4\pi}{3G\rho}}$

Q7. Two identical positive point charges Q are fixed at a distance $2d$ apart in air. A particle of mass m and charge $-q$ is released from rest at a point on the perpendicular bisector of the line joining the fixed charges, at a distance x ($x \ll d$) from the midpoint. The frequency of simple harmonic motion executed by the particle is

(A) $\frac{1}{2\pi}\sqrt{\frac{Qq}{2\pi\varepsilon_0 md^3}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{Qq}{4\pi\varepsilon_0 md^3}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{2Qq}{\pi\varepsilon_0 md^3}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{Qq}{2\pi\varepsilon_0 md^2}}$

Q8. A particle of mass m moving with velocity v collides elastically head-on with another stationary particle of mass $2m$. The fraction of kinetic energy retained by the striking particle of mass m after the collision is

(A) $\frac{1}{9}$

(B) $\frac{8}{9}$

(C) $\frac{4}{9}$

(D) $\frac{1}{3}$

Q9. A solid sphere, a disc, and a hollow sphere, all of the same mass and radius, are released from rest from the top of a rough inclined plane. If they roll without slipping, which one reaches the bottom with the maximum linear velocity?

(A) Solid sphere

(B) Disc

(C) Hollow sphere

(D) All will reach with the same velocity

Q10. A horizontal pipeline carries water in a streamlined flow. At a point where the cross-sectional area is A, the velocity of water is v and the pressure is P. At another point where the cross-sectional area is $\frac{A}{2}$, the pressure is (d = density of water)

(A) $P - \frac{1}{2}dv^2$

(B) $P - \frac{3}{2}dv^2$

(C) $P - 2dv^2$

(D) $P + \frac{3}{2}dv^2$

Q11. An ideal gas undergoes a thermodynamic process wherein its pressure P and volume V are related as PV^2 = constant. If the initial temperature of the gas is T and it is expanded to twice its initial volume, its final temperature will be

(A) $2T$

(B) $\frac{T}{2}$

(C) $\sqrt{2}T$

(D) $\frac{T}{\sqrt{2}}$

Q12. A car blowing a horn of frequency 360 Hz acts as a source moving towards a vertical wall with a speed of 30 m/s. If the speed of sound in air is 330 m/s, the frequency of the beats heard by a person sitting inside the car due to the direct sound and reflected sound is

(A) 60 Hz

(B) 72 Hz

(C) 36 Hz

(D) 0 Hz

Q13. In a Young's double slit experiment, the intensity at a point on the screen where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of light used) is I. If I_0 denotes the maximum intensity of the interference pattern, then $\frac{I}{I_0}$ is equal to

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{3}{4}$

(D) $\frac{1}{4}$

Q14. The electric potential in a region of space is given by $V(x, y, z) = 2x^2y - 3yz$ volts, where coordinates are in meters. The magnitude of the electric field vector at the point $(1, -1, 2)$ is

(A) $\sqrt{21}$ V/m

(B) $\sqrt{41}$ V/m

(C) 5 V/m

(D) $\sqrt{29}$ V/m

Q15. A long, straight wire of circular cross-section of radius R carries a steady current I. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at a distance $\frac{R}{2}$ from the axis to that at a distance $2R$ from the axis is

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

Q16. A circular coil of N turns and radius R carries a current I. It is unwound and rewound into another circular coil of radius $\frac{R}{3}$, keeping the total length of the wire unchanged. If the same current I is passed through the new coil, the ratio of the magnetic moment of the new coil to that of the original coil is

(A) 3 : 1

(B) 1 : 3

(C) 1 : 9

(D) 9 : 1

Q17. A metal rod of length l rotates with a constant angular velocity ω about an axis passing through one of its ends and perpendicular to its length, in a uniform magnetic field B directed parallel to the axis of rotation. The induced electromotive force between the two ends of the rod is

(A) $Bl^2\omega$

(B) $\frac{1}{2}Bl^2\omega$

(C) $2Bl^2\omega$

(D) $\frac{1}{4}Bl^2\omega$

Q18. The electric field component of an electromagnetic wave traveling in a vacuum is given by $E_y = 60\cos(10^8 t - kx)$ V/m. The amplitude of the corresponding magnetic field component is

(A) 2×10^{-7} T

(B) 60 T

(C) 1.8×10^{10} T

(D) 5×10^{-6} T

Q19. A convex lens of focal length f in air is immersed in water (refractive index $= \frac{4}{3}$). If the refractive index of the glass of the lens is $\frac{3}{2}$, its focal length in water becomes

(A) $2f$

(B) $4f$

(C) $\frac{4}{3}f$

(D) $3f$

Q20. When a monochromatic light of wavelength λ is incident on a photosensitive metallic surface of work function ϕ, the maximum kinetic energy of the emitted photoelectrons is K. If the wavelength of the incident light is halved, the maximum kinetic energy of the new photoelectrons will be

(A) $2K$

(B) $K + \frac{hc}{\lambda}$

(C) $2K + \phi$

(D) $2K - \phi$

Q21. According to Bohr's theory of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in the n-th orbit is

(A) $1 : 1$

(B) $1 : -1$

(C) $2 : -1$

(D) $1 : -2$

Q22. In a common-emitter transistor amplifier, the audio signal voltage across the collector resistance of 2 kΩ is 2 V. If the current amplification factor (β) of the transistor is 100 and the base resistance is 1 kΩ, the input signal voltage is

(A) 10 mV

(B) 20 mV

(C) 5 mV

(D) 1 mV

Q23. If the value of acceleration due to gravity g at the surface of the earth is g_0, its value at an altitude equal to half the radius of the earth ($R/2$) is

(A) $\frac{4}{9}g_0$

(B) $\frac{2}{3}g_0$

(C) $\frac{1}{2}g_0$

(D) $\frac{9}{4}g_0$

Q24. A particle is moving in a plane such that its position coordinates (x, y) as a function of time t are given by $x = a\cos(\omega t)$ and $y = b\sin(\omega t)$, where a, b, and ω are positive constants. The trajectory of the particle is a/an

(A) Circle

(B) Ellipse

(C) Parabola

(D) Straight line

Q25. A physical quantity X is defined as $X = \frac{h}{\sigma T^4}$, where h is Planck's constant, σ is Stefan-Boltzmann constant, and T is temperature. The SI unit of X is equivalent to that of

(A) Time

(B) Area × Time

(C) Mass × Length

(D) None of the fields

Q26. A block of mass m is placed on a rough horizontal surface having a coefficient of static friction μ_s. A horizontal force F is applied to the block. If $F < \mu_s mg$, the magnitude of the frictional force acting on the block by the surface is

(A) $\mu_s mg$

(B) F

(C) Zero

(D) $\sqrt{F^2 + (\mu_s mg)^2}$

Q27. A certain mass of an ideal gas expands from a state (P_1, V_1) to a state (P_2, V_2) first isothermally and then another sample of the same mass expands adiabatically from the same initial state to the same final volume V_2. If W_{iso} and W_{adia} are the works done by the gas in the respective processes, then

(A) $W_{iso} > W_{adia}$

(B) $W_{iso} < W_{adia}$

(C) $W_{iso} = W_{adia}$

(D) Relationship depends on the atomicity of the gas

Q28. Two waves represented by $y_1 = A\sin(\omega t - kx)$ and $y_2 = A\cos(\omega t - kx + \frac{\pi}{6})$ interfere with each other. The phase difference between the two waves is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{5\pi}{6}$

Q29. An alpha particle and a proton are accelerated from rest through the same potential difference V. The ratio of their de Broglie wavelengths $\frac{\lambda_\alpha}{\lambda_p}$ is

(A) $1 : \sqrt{2}$

(B) $1 : 2$

(C) $1 : 2\sqrt{2}$

(D) $2\sqrt{2} : 1$

Q30. A cell of internal resistance r is connected across a variable external resistance R. The plot of the potential difference V across the terminals of the cell as a function of R is best represented by a curve that

(A) is a straight line passing through the origin with a positive slope.

(B) increases asymptotically toward the EMF of the cell as R increases.

(C) decreases linearly to zero as R increases.

(D) remains constant and independent of R.

Section B - 5 Questions × 2 Mark Each
(Negative Marking: −0.5) [Single Correct]

Q31. A bullet of mass m moving with horizontal velocity v strikes a stationary wooden block of mass M suspended by a long massless string of length L. The bullet gets embedded in the block. The minimum value of v such that the combined block-bullet system completes a full vertical circle is

(A) $\frac{M+m}{m}\sqrt{5gL}$

(B) $\frac{m}{M+m}\sqrt{5gL}$

(C) $\frac{M+m}{m}\sqrt{2gL}$

(D) $\frac{M+m}{m}\sqrt{gL}$

Q32. A uniform thin rod of mass M and length L is bent at its midpoint to form a 90° angle (L-shape). The moment of inertia of this bent rod about an axis passing through the bend and perpendicular to the plane containing both segments is

(A) $\frac{1}{3}ML^2$

(B) $\frac{1}{12}ML^2$

(C) $\frac{1}{24}ML^2$

(D) $\frac{1}{6}ML^2$

Q33. A glass capillary tube of radius r is lowered vertically into water. The water rises up to a height h in the capillary, and the heat evolved during this process is W_1. If another glass capillary tube of radius $2r$ is lowered into water, the heat evolved during the rise of water is W_2. The ratio $\frac{W_1}{W_2}$ is

(A) 1 : 1

(B) 1 : 2

(C) 2 : 1

(D) 1 : 4

Q34. In the network shown below, the internal resistance of each cell is negligible. The steady-state charge stored in the capacitor of capacitance $C = 3\,\mu$F is given that the upper branch has a 2 V cell with resistances 2 Ω and 4 Ω, the lower branch has a 4 V cell with resistances 1 Ω and 5 Ω, and the capacitor is connected across the midpoints.

(A) 3 μC

(B) 4.5 μC

(C) 6 μC

(D) Zero

Q35. A thin glass equiconvex lens has a refractive index $n = 1.5$ and a radius of curvature R. The lens is split vertically into two identical plano-convex halves. One of the halves is silvered on its flat plane surface. An object is placed at a distance u in front of the silvered face's convex side so that a real image is formed at the position of the object itself. The value of u is

(A) R

(B) $\frac{R}{2}$

(C) $2R$

(D) $\frac{R}{3}$

Section C - 5 Questions × 2 Marks Each
(No Negative Marking) [One or More Correct]

Q36. One mole of an ideal monoatomic gas is taken through a cyclic process $A \longrightarrow B \longrightarrow C \longrightarrow A$. Process $C \longrightarrow A$ is an isothermal compression. A has coordinates (P_0, V_0), B has coordinates $(P_0, 2V_0)$, and C has coordinates $(\frac{P_0}{2}, 2V_0)$. Which of the following statements is/are correct?

(A) The work done by the gas during the process $A \longrightarrow B$ is $\frac{3}{4}P_0V_0$.

(B) The total work done in the complete cycle is $P_0V_0\,(1 - \ln 2)$.

(C) The change in internal energy during the process $B \longrightarrow C$ is $-\frac{3}{2}P_0V_0$.

(D) The process $A \longrightarrow B$ is an isobaric expansion.

Q37. A particle of mass m is attached to a spring of stiffness k and executes simple harmonic motion along the x-axis with amplitude A about the origin. At time $t = 0$, the particle is at $x = \frac{A}{2}$ and moving in the positive x-direction. If $\omega = \sqrt{\frac{k}{m}}$, which of the following options is/are correct?

(A) The total mechanical energy of the system is $\frac{1}{2}kA^2$.

(B) The position of the particle at time t is given by $x(t) = A\sin\left(\omega t + \frac{\pi}{6}\right)$.

(C) The maximum velocity magnitude achieved by the particle during its path is ωA.

(D) The kinetic energy of the particle equals its potential energy when $x = \pm\frac{A}{\sqrt{2}}$.

Q38. A parallel-plate capacitor with plate area A and plate separation d is charged to a potential difference V by a battery. The battery is then disconnected. A dielectric slab of thickness d and dielectric constant $K > 1$ is now inserted to completely fill the space between the plates. Which of the following statements is/are true?

(A) The electric field between the plates decreases by a factor of K.

(B) The energy stored in the capacitor decreases to $\frac{1}{K}$ of its initial value.

(C) The capacitance increases to K times its initial value.

(D) The charge on the plates decreases by a factor of K.

Q39. A square loop of wire of side length L and total resistance R lies in the xy-plane. A non-uniform, time-dependent magnetic field is given by $\vec{B} = B_0\left(\frac{x}{L}\right)\cos(\omega t)\,\hat{k}$ exists in the region, where B_0 and ω are constants. The loop is bounded by the lines $x = 0$, $x = L$, $y = 0$, and $y = L$. Which of the following assertions is/are correct?

(A) The total magnetic flux linking the loop at $t = 0$ is $\frac{1}{2}B_0L^2$.

(B) The maximum induced electromotive force in the loop is $\frac{1}{2}B_0L^2\omega$.

(C) The current induced in the loop flows in clockwise and counter-clockwise directions alternately over time.

(D) The induced current in the loop is independent of the resistance R.

Q40. A beam of light consisting of two wavelengths $\lambda_1 = 6000$ Å and $\lambda_2 = 4800$ Å is used to obtain interference fringes in a Young's double-slit experiment. The distance between the slits is d and the distance between the plane of the slits and the screen is D. Which of the following statements is/are correct?

(A) The third bright fringe of the wavelength λ_1 coincides with the fourth bright fringe of the wavelength λ_2.

(B) The minimum distance from the central maximum where the bright fringes of both wavelengths coincide is $\frac{12\lambda_2 D}{d}$.

(C) The structural width of a single fringe for λ_1 is greater than that for λ_2.

(D) There is no point on the screen where a bright fringe of λ_1 can overlap with a dark fringe of λ_2.

Detailed Solutions

Q1.

Solution

Concept: The speed of a transverse wave pulse along a stretched string is $v = \sqrt{T/\mu}$. Since $v = f\lambda$ and frequency f remains constant during propagation, wavelength is directly proportional to wave velocity ($\lambda \propto v$).

Solution: Step 1: Tension at the bottom (T_1) is due only to the hanging mass M:

$$T_1 = Mg$$

Step 2: Wave velocity at the bottom (v_1) with linear mass density $\mu = m/L$:

$$v_1 = \sqrt{\frac{Mg}{\mu}}$$

Step 3: Tension at the top (T_2) supports both the hanging mass and the string:

$$T_2 = (M + m)g$$

Step 4: Wave velocity at the top (v_2):

$$v_2 = \sqrt{\frac{(M + m)g}{\mu}}$$

Step 5: Using $\lambda \propto v$ since frequency is constant:

$$\frac{\lambda_2}{\lambda_1} = \frac{v_2}{v_1}$$

Step 6: Substitute velocities and solve for the final wavelength λ_2 (where $\lambda_1 = \lambda_0$):

$$\frac{\lambda_2}{\lambda_0} = \frac{\sqrt{\frac{(M+m)g}{\mu}}}{\sqrt{\frac{Mg}{\mu}}} = \sqrt{\frac{M + m}{M}} \implies \lambda_2 = \lambda_0\sqrt{\frac{M + m}{M}}$$

Final Answer: $\boxed{\lambda_0\sqrt{\frac{M + m}{M}}}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept:

Torricelli's Law of Efflux and Equation of Continuity.

Solution:

By continuity, rate of decrease of water volume equals rate of efflux:

$$-A\frac{dh}{dt} = av \quad \text{where } v = \sqrt{2gh}$$

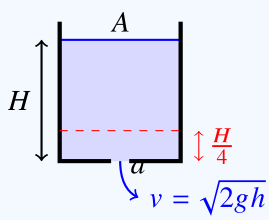
$$-\frac{dh}{\sqrt{h}} = \frac{a}{A}\sqrt{2g}\,dt$$

Integrating from $h = H$ at $t = 0$ to $h = \frac{H}{4}$ at $t = T$:

$$-\int_H^{\frac{H}{4}} h^{-1/2}\,dh = \frac{a}{A}\sqrt{2g}\int_0^T dt \implies -\left[2\sqrt{h}\right]_H^{\frac{H}{4}} = \frac{a}{A}\sqrt{2g}T$$

$$2\sqrt{H} - 2\sqrt{\frac{H}{4}} = \frac{a}{A}\sqrt{2g}T \implies \sqrt{H} = \frac{a}{A}\sqrt{2g}T$$

$$T = \frac{A}{a}\sqrt{\frac{H}{2g}}$$



Final Answer: D

Answer: (D)

Go Back to Question 1

Q3.

Solution

Concept: Density is $\rho = m/V$. For a cube of side L, $V = L^3$. To find the maximum permissible percentage error, fractional errors of the components are added to calculate the worst-case uncertainty accumulation.

Solution: Step 1: Express density in terms of mass and length:

$$\rho = mL^{-3}$$

Step 2: Take the natural logarithm on both sides:

$$\ln(\rho) = \ln(m) - 3\ln(L)$$

Step 3: Take differentials and sum absolute values to find maximum fractional error:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta L}{L}$$

Step 4: Convert to percentage error form:

$$\left(\frac{\Delta\rho}{\rho} \times 100\right) = \left(\frac{\Delta m}{m} \times 100\right) + 3\left(\frac{\Delta L}{L} \times 100\right)$$

Step 5: Substitute given errors (mass = 1.5%, length = 1.0%):

$$\text{Percentage Error in } \rho = 1.5\% + 3 \times (1.0\%)$$

Step 6: Compute the final maximum error:

$$\text{Percentage Error in } \rho = 1.5\% + 3.0\% = 4.5\%$$

Final Answer: 4.5%

Answer: (C)

Go Back to Question 3

Q4.

Solution

Concept: Velocity is the first derivative of displacement ($v = \frac{ds}{dt}$), and acceleration is the first derivative of velocity ($a = \frac{dv}{dt}$). To find the velocity when acceleration is zero, find t where $a(t) = 0$ and substitute it into $v(t)$.

Solution: Step 1: Differentiate displacement $s(t) = t^3 - 6t^2 + 3t + 4$ to find velocity $v(t)$:

$$v(t) = \frac{ds}{dt} = 3t^2 - 12t + 3$$

Step 2: Differentiate velocity $v(t)$ to find acceleration $a(t)$:

$$a(t) = \frac{dv}{dt} = 6t - 12$$

Step 3: Find the time when acceleration is zero:

$$6t - 12 = 0 \implies t = 2 \text{ seconds}$$

Step 4: Substitute $t = 2$ s into the velocity equation:

$$v(2) = 3(2)^2 - 12(2) + 3$$

Step 5: Simplify the arithmetic:

$$v(2) = 12 - 24 + 3 = -9 \text{ m/s}$$

Final Answer: $\boxed{-9 \text{ m/s}}$

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept: By the Work-Energy Theorem, power delivered to a particle equals the time rate of change of its kinetic energy ($P = \frac{dK}{dt}$). In circular motion, only the tangential force component changes the kinetic energy and performs work.

Solution: Step 1: Identify the given centripetal acceleration expression:

$$a_c = k^2rt^2$$

Step 2: Equate to standard centripetal acceleration $a_c = \frac{v^2}{r}$ to find v^2:

$$\frac{v^2}{r} = k^2rt^2 \implies v^2 = k^2r^2t^2$$

Step 3: Solve for linear speed v:

$$v = krt$$

Step 4: Write the expression for kinetic energy K:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mk^2r^2t^2$$

Step 5: Relate power to the time derivative of kinetic energy:

$$P = \frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mk^2r^2t^2\right)$$

Step 6: Differentiate with respect to time t:

$$P = \frac{1}{2}mk^2r^2(2t) = mk^2r^2t$$

Final Answer: $\boxed{mk^2r^2t}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept: For a satellite in a circular orbit close to a planet's surface, the orbit radius matches the planet's radius (R). The gravitational force provides the necessary centripetal force. The orbital time period is $T = 2\pi R/v$.

Solution: Step 1: Balance gravitational and centripetal forces for planet mass M and satellite mass m:

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

Step 2: Solve for the orbital velocity v:

$$v = \sqrt{\frac{GM}{R}}$$

Step 3: Express the time period T using the orbital path distance:

$$T = \frac{2\pi R}{v} = 2\pi\sqrt{\frac{R^3}{GM}}$$

Step 4: Relate the mass M to its uniform density ρ and volume $V = \frac{4}{3}\pi R^3$:

$$M = \rho\left(\frac{4}{3}\pi R^3\right)$$

Step 5: Substitute M into the time period equation:

$$T = 2\pi\sqrt{\frac{R^3}{G\cdot\rho\left(\frac{4}{3}\pi R^3\right)}}$$

Step 6: Cancel out R^3 and simplify terms:

$$T = 2\pi\sqrt{\frac{3}{4\pi G\rho}} = \sqrt{\frac{3\pi}{G\rho}}$$

Final Answer: $\boxed{\sqrt{\frac{3\pi}{G\rho}}}$

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept: A negative charge $-q$ on the perpendicular bisector of two identical positive charges experiences a restoring force toward the center. For a small displacement $x \ll d$, this force is proportional to displacement ($F \propto -x$), resulting in Simple Harmonic Motion (SHM).

Solution: Step 1: Find the distance r from either charge Q at $(\pm d, 0)$ to $-q$ at $(0, x)$:

$$r = \sqrt{d^2 + x^2}$$

Step 2: Write Coulomb's force magnitude from one charge Q:

$$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{d^2 + x^2}$$

Step 3: Sum the vertical components (horizontal components cancel by symmetry):

$$F_{net} = -2F_1 \cos\theta = -2F_1 \left(\frac{x}{\sqrt{d^2 + x^2}} \right)$$

Step 4: Combine expressions to get the net restoring force:

$$F_{net} = -\frac{2Qqx}{4\pi\varepsilon_0 (d^2 + x^2)^{3/2}}$$

Step 5: Apply the approximation $x \ll d$, so $(d^2 + x^2)^{3/2} \approx d^3$:

$$F_{net} \approx -\frac{Qq}{2\pi\varepsilon_0 d^3} x$$

Step 6: Match with $F = -m\omega^2 x$ to find angular frequency ω and linear frequency $f = \frac{\omega}{2\pi}$:

$$\omega = \sqrt{\frac{Qq}{2\pi\varepsilon_0 m d^3}} \implies f = \frac{1}{2\pi} \sqrt{\frac{Qq}{2\pi\varepsilon_0 m d^3}}$$

Final Answer: $\boxed{\frac{1}{2\pi} \sqrt{\frac{Qq}{2\pi\varepsilon_0 m d^3}}}$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: In a perfectly elastic head-on collision, linear momentum and kinetic energy are conserved. The fraction of kinetic energy retained by the striking mass is the ratio of its final kinetic energy to its initial kinetic energy (K_f/K_i).

Solution: Step 1: Set initial variables: $m_1 = m$, $u_1 = v$, $m_2 = 2m$, and target velocity $u_2 = 0$.
Step 2: Use the standard elastic collision velocity formula for the first mass:

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2$$

Step 3: Substitute the mass and velocity values:

$$v_1 = \left(\frac{m - 2m}{m + 2m}\right) v = -\frac{1}{3} v$$

Step 4: Write the initial kinetic energy of the mass m:

$$K_i = \frac{1}{2} m v^2$$

Step 5: Compute the post-collision kinetic energy using v_1:

$$K_f = \frac{1}{2} m \left(-\frac{1}{3} v\right)^2 = \frac{1}{9} \left(\frac{1}{2} m v^2\right) = \frac{1}{9} K_i$$

Step 6: Calculate the fraction of kinetic energy retained:

$$\text{Fraction} = \frac{K_f}{K_i} = \frac{1}{9}$$

Final Answer: $\boxed{\frac{1}{9}}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept:
Rolling without slipping on an inclined plane and Conservation of Mechanical Energy.

Solution:
Step 1: When a rigid body of mass M and radius R rolls down an inclined plane of height h from rest without slipping, its total potential energy at the top converts into both translational and rotational kinetic energy at the bottom.

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

Step 2: For rolling without slipping, the angular velocity is linked to the linear velocity by $\omega = \frac{v}{R}$. Substituting this and expressing the moment of inertia as $I = MK^2$ (where K is the radius of gyration):

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(MK^2)\left(\frac{v}{R}\right)^2$$

$$Mgh = \frac{1}{2}Mv^2\left(1 + \frac{K^2}{R^2}\right)$$

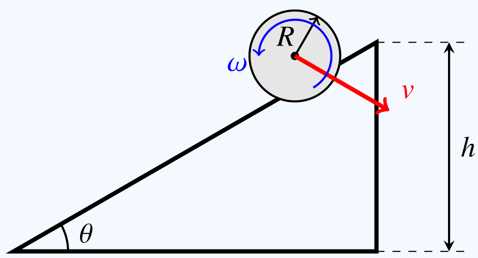
Step 3: Solving for the final linear velocity v at the bottom of the incline gives:

$$v = \sqrt{\frac{2gh}{1 + \frac{K^2}{R^2}}} = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$$

Step 4: From the formula, the linear velocity v is inversely proportional to $\left(1 + \frac{I}{MR^2}\right)$. This means the object with the **smallest** moment of inertia I will achieve the **maximum** linear velocity.

Step 5: Let's compare the moments of inertia for the given shapes of the same mass M and radius R: * Solid sphere: $I_{\text{solid sphere}} = \frac{2}{5}MR^2 = 0.4\,MR^2$ * Disc: $I_{\text{disc}} = \frac{1}{2}MR^2 = 0.5\,MR^2$ * Hollow sphere: $I_{\text{hollow sphere}} = \frac{2}{3}MR^2 \approx 0.67\,MR^2$

Step 6: Since the solid sphere has the least moment of inertia ($0.4\,MR^2$), it will have the smallest value of $\left(1 + \frac{I}{MR^2}\right)$, resulting in the maximum linear velocity at the bottom.



Rolling down a rough incline

Final Answer: A

Answer: (A)

Go Back to Question 1

Q10.

Solution

Concept: For steady, horizontal, incompressible fluid flow, Bernoulli's equation states $P + \frac{1}{2}dv^2 =$ constant. The continuity equation requires that the volume flow rate remains constant ($A_1v_1 = A_2v_2$) across changing cross-sections.

Solution: Step 1: Let the initial parameters be $A_1 = A$, $v_1 = v$, $P_1 = P$. Let the constricted parameters be $A_2 = \frac{A}{2}$ and pressure be P_2.
Step 2: Use the continuity equation to calculate the velocity v_2 at the constriction:

$$Av = \left(\frac{A}{2}\right)v_2 \implies v_2 = 2v$$

Step 3: Set up Bernoulli's equation for horizontal flow:

$$P_1 + \frac{1}{2}dv_1^2 = P_2 + \frac{1}{2}dv_2^2$$

Step 4: Substitute the initial values and v_2 into the relation:

$$P + \frac{1}{2}dv^2 = P_2 + \frac{1}{2}d(2v)^2$$

Step 5: Expand the squared velocity term:

$$P + \frac{1}{2}dv^2 = P_2 + 2dv^2$$

Step 6: Isolate and solve for the final pressure P_2:

$$P_2 = P + \frac{1}{2}dv^2 - 2dv^2 = P - \frac{3}{2}dv^2$$

Final Answer: $\boxed{P - \frac{3}{2}dv^2}$

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: An ideal gas satisfies $PV = nRT$. For a polytropic process PV^n = constant, substituting the ideal gas law isolates the relationship between temperature and volume as TV^{n-1} = constant.

Solution: Step 1: Write the given process equation:

$$PV^2 = \text{constant}$$

Step 2: Express pressure P using the ideal gas law:

$$P = \frac{nRT}{V}$$

Step 3: Substitute P into the process equation:

$$\left(\frac{nRT}{V}\right)V^2 = \text{constant}$$

Step 4: Simplify by grouping constants n and R:

$$T \cdot V = \text{constant}$$

Step 5: Apply this conservation law between the initial state ($T_1 = T, V_1 = V$) and final state ($V_2 = 2V$):

$$T \cdot V = T_2 \cdot (2V)$$

Step 6: Solve for the final absolute temperature T_2:

$$T_2 = \frac{T}{2}$$

Final Answer: $\boxed{\frac{T}{2}}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept: The Doppler Effect formula is $f' = f_0\left(\frac{v \pm v_o}{v \mp v_s}\right)$. For a vehicle approaching a stationary wall, the wall first acts as a stationary observer receiving a shifted frequency, and then acts as a stationary source reflecting it back to the moving driver.

Solution: Step 1: Identify given parameters: $f_0 = 360$ Hz, $v_s = v_o = 30$ m/s, and $v = 330$ m/s.
Step 2: Calculate the frequency f' received by the stationary wall:

$$f' = 360\left(\frac{330}{330-30}\right) = 360\left(\frac{330}{300}\right) = 396 \text{ Hz}$$

Step 3: The stationary wall reflects the sound, maintaining the frequency at 396 Hz.
Step 4: Calculate the frequency f'' heard by the moving driver:

$$f'' = 396\left(\frac{330+30}{330}\right) = 396\left(\frac{360}{330}\right) = 432 \text{ Hz}$$

Step 5: Define the beat frequency as the absolute difference between the frequencies:

$$f_{\text{beat}} = f'' - f_0$$

Step 6: Compute the final beat frequency value:

$$f_{\text{beat}} = 432 \text{ Hz} - 360 \text{ Hz} = 72 \text{ Hz}$$

Final Answer: 72 Hz

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept: In Young's Double Slit Experiment (YDSE), the resultant intensity is given by $I = I_0 \cos^2\left(\frac{\phi}{2}\right)$, where I_0 is the maximum intensity. The phase difference ϕ is linked to the path difference Δx by $\phi = \frac{2\pi}{\lambda}\Delta x$.

Solution: Step 1: Note the path difference at the specified screen position:

$$\Delta x = \frac{\lambda}{6}$$

Step 2: Convert the path difference into an angular phase difference ϕ:

$$\phi = \frac{2\pi}{\lambda}\left(\frac{\lambda}{6}\right) = \frac{\pi}{3}$$

Step 3: State the double-slit intensity formulation:

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right)$$

Step 4: Substitute $\phi = \frac{\pi}{3}$ into the intensity equation:

$$I = I_0 \cos^2\left(\frac{\pi}{6}\right)$$

Step 5: Evaluate the trigonometric term using $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$:

$$\cos^2\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$$

Step 6: Write out the final intensity ratio:

$$I = I_0\left(\frac{3}{4}\right) \implies \frac{I}{I_0} = \frac{3}{4}$$

Final Answer: $\boxed{\frac{3}{4}}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept: The electric field vector $\vec{E}$ is the negative gradient of the potential V, computed via partial derivatives: $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, and $E_z = -\frac{\partial V}{\partial z}$. The scalar magnitude is found using the Euclidean norm formula $|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2}$.

Solution: Step 1: Note the given potential function:

$$V(x, y, z) = 2x^2y - 3yz$$

Step 2: Determine the E_x component by differentiating with respect to x:

$$E_x = -\frac{\partial V}{\partial x} = -4xy$$

Step 3: Determine the E_y component by differentiating with respect to y:

$$E_y = -\frac{\partial V}{\partial y} = -2x^2 + 3z$$

Step 4: Determine the E_z component by differentiating with respect to z:

$$E_z = -\frac{\partial V}{\partial z} = 3y$$

Step 5: Evaluate each component at the coordinates $(1, -1, 2)$:

$$E_x = -4(1)(-1) = 4, \quad E_y = -2(1)^2 + 3(2) = 4, \quad E_z = 3(-1) = -3$$

Step 6: Construct the vector and calculate its magnitude:

$$|\vec{E}| = \sqrt{4^2 + 4^2 + (-3)^2} = \sqrt{16 + 16 + 9} = \sqrt{41} \text{ V/m}$$

Final Answer: $\boxed{\sqrt{41} \text{ V/m}}$

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: By Ampere's Law, the internal magnetic field of a long wire ($r \leq R$) grows linearly: $B_{\text{internal}} = \frac{\mu_0 I r}{2\pi R^2}$. Outside the wire ($r \geq R$), the field decays inversely with distance: $B_{\text{external}} = \frac{\mu_0 I}{2\pi r}$.

Solution: Step 1: Identify evaluation points: $r_1 = \frac{R}{2}$ (internal) and $r_2 = 2R$ (external).
Step 2: Apply the internal magnetic field equation at $r_1 = \frac{R}{2}$:

$$B_1 = \frac{\mu_0 I \left(\frac{R}{2}\right)}{2\pi R^2}$$

Step 3: Simplify the expression for B_1:

$$B_1 = \frac{\mu_0 I}{4\pi R}$$

Step 4: Apply the external magnetic field equation at $r_2 = 2R$:

$$B_2 = \frac{\mu_0 I}{2\pi(2R)}$$

Step 5: Simplify the expression for B_2:

$$B_2 = \frac{\mu_0 I}{4\pi R}$$

Step 6: Calculate the ratio of the two fields:

$$\frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{4\pi R}}{\frac{\mu_0 I}{4\pi R}} = \frac{1}{1}$$

Final Answer: $\boxed{1 : 1}$

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept: The magnetic dipole moment of a loop is $M = NIA$. When a wire of length L is rewound from N_1 turns of radius R_1 to N_2 turns of radius R_2, the parameters are bound by the length conservation constraint $L = N \cdot (2\pi R)$.

Solution: Step 1: Write the initial wire length for N_1 turns and radius $R_1 = R$:

$$L = N_1(2\pi R)$$

Step 2: Write the wire length for N_2 turns and radius $R_2 = \frac{R}{3}$:

$$L = N_2\left(2\pi\frac{R}{3}\right)$$

Step 3: Equate both expressions to find N_2 in terms of N_1:

$$N_1(2\pi R) = N_2\left(2\pi\frac{R}{3}\right) \implies N_2 = 3N_1$$

Step 4: Express the initial magnetic dipole moment M_1:

$$M_1 = N_1 I(\pi R^2)$$

Step 5: Express the final magnetic dipole moment M_2:

$$M_2 = N_2 I(\pi R_2^2) = (3N_1)I\left[\pi\left(\frac{R}{3}\right)^2\right] = 3N_1 I\left(\frac{\pi R^2}{9}\right)$$

Step 6: Simplify to find the ratio M_2/M_1:

$$M_2 = \frac{1}{3}N_1 I(\pi R^2) = \frac{1}{3}M_1 \implies \frac{M_2}{M_1} = \frac{1}{3}$$

Final Answer: $\boxed{1 : 3}$

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept: A conducting rod rotating in a magnetic field experiences a motional EMF. Because the linear velocity varies continuously along its length ($v = \omega x$), the total induced EMF is found by integrating infinitesimal EMF elements ($d\varepsilon = Bvdx$) along the rod.

Solution: Step 1: Consider a small element dx at a distance x from the pivot.
Step 2: State its instantaneous velocity:

$$v = \omega x$$

Step 3: Write the elemental motional EMF $d\varepsilon$:

$$d\varepsilon = Bvdx = B\omega xdx$$

Step 4: Set up the definite integral across the total length l:

$$\varepsilon = \int_0^l B\omega xdx$$

Step 5: Pull out constants and apply the integration power rule:

$$\varepsilon = B\omega \left[\frac{x^2}{2}\right]_0^l$$

Step 6: Evaluate across limits to find the total EMF:

$$\varepsilon = \frac{1}{2}Bl^2\omega$$

Final Answer: $\boxed{\frac{1}{2}Bl^2\omega}$

Answer: (B)

Go Back to Question 17

Q18.

Solution

Concept: In an electromagnetic wave propagating through a vacuum, the peak amplitudes of the electric field (E_0) and magnetic field (B_0) are coupled by the speed of light constant (c) according to the relationship $E_0 = cB_0$.

Solution: Step 1: Identify the peak electric field amplitude E_0 from the given function:

$$E_0 = 60 \text{ V/m}$$

Step 2: State the standard constant for the speed of light in a vacuum:

$$c = 3 \times 10^8 \text{ m/s}$$

Step 3: Arrange the amplitude ratio equation to isolate B_0:

$$B_0 = \frac{E_0}{c}$$

Step 4: Substitute the values into the equation:

$$B_0 = \frac{60}{3 \times 10^8}$$

Step 5: Perform the division:

$$B_0 = 20 \times 10^{-8} \text{ T}$$

Step 6: Convert to standard scientific notation:

$$B_0 = 2 \times 10^{-7} \text{ T}$$

Final Answer: $\boxed{2 \times 10^{-7} \text{ T}}$

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept: The Lens Maker's Formula relates focal length to the relative refractive index of the lens and surrounding medium: $\frac{1}{f} = \left(\frac{n_{\text{lens}}}{n_{\text{medium}}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$. Writing this for air and water environments allows the geometry terms to cancel out.

Solution: Step 1: Write the Lens Maker's Formula for the lens in air ($n_{\text{air}} = 1$):

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Step 2: Substitute $n_g = \frac{3}{2}$ to find the geometric constant profile:

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \implies \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2}{f}$$

Step 3: Write the formula for the lens immersed in water ($n_w = \frac{4}{3}$):

$$\frac{1}{f_w} = \left(\frac{n_g}{n_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Step 4: Substitute n_g and n_w values into the immersion equation:

$$\frac{1}{f_w} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{8}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Step 5: Substitute the geometric expression ($\frac{2}{f}$) from Step 2:

$$\frac{1}{f_w} = \frac{1}{8}\left(\frac{2}{f}\right) = \frac{1}{4f}$$

Step 6: Invert both sides to find the new focal length in water:

$$f_w = 4f$$

Final Answer: $\boxed{4f}$

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept: Einstein's Photoelectric Equation dictates that incident photon energy is split between the surface work function (ϕ) and electron kinetic energy: $K_{\max} = \frac{hc}{\lambda} - \phi$. Evaluating this relation across two wavelengths determines the final kinetic energy.

Solution: Step 1: Write the energy balance for the initial state with wavelength λ and kinetic energy K:

$$K = \frac{hc}{\lambda} - \phi \implies \frac{hc}{\lambda} = K + \phi$$

Step 2: Set up the equation for the second state with wavelength $\lambda' = \frac{\lambda}{2}$:

$$K' = \frac{hc}{\lambda'} - \phi = \frac{hc}{\lambda/2} - \phi$$

Step 3: Simplify the expression:

$$K' = 2\left(\frac{hc}{\lambda}\right) - \phi$$

Step 4: Substitute the isolated $\frac{hc}{\lambda}$ term from Step 1:

$$K' = 2(K + \phi) - \phi$$

Step 5: Expand the distributed multiplication:

$$K' = 2K + 2\phi - \phi$$

Step 6: Combine like terms to isolate the final maximum kinetic energy:

$$K' = 2K + \phi$$

Final Answer: $\boxed{2K + \phi}$

Answer: (C)

Go Back to Question 20

Q21.

Solution

Concept: In Bohr's atomic model, an electron in a stable circular orbit maintains a balance between Coulomb attraction and centripetal force. Its total mechanical energy E is the sum of its kinetic energy K and potential energy U, which satisfy the proportional relationships $K = -E$ and $U = 2E$.

Solution: Step 1: Set up the electrostatic force balance equation for the orbiting electron:

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

Step 2: Isolate the mass-velocity squared term mv^2:

$$mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Step 3: Write out the electron's kinetic energy K:

$$K = \frac{1}{2}mv^2 = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$$

Step 4: Write out the electrostatic potential energy U:

$$U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Step 5: Compute the total mechanical energy $E = K + U$:

$$E = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}$$

Step 6: Determine the numerical ratio of kinetic energy to total energy (K/E):

$$\frac{K}{E} = \frac{\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}}{-\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}} = -1 \implies 1 : -1$$

Final Answer: $\boxed{1 : -1}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: In a common-emitter (CE) transistor configuration, the AC voltage gain is $A_v = V_{\text{out}}/V_{\text{in}}$. This gain can also be formulated using the current amplification factor β and resistance values as $A_v = \beta(R_C/R_B)$.

Solution: Step 1: List the provided parameters: $V_{\text{out}} = 2$ V, $R_C = 2$ kΩ, $\beta = 100$, and $R_B = 1$ kΩ.
Step 2: State the primary voltage definition for A_v:

$$A_v = \frac{V_{\text{out}}}{V_{\text{in}}}$$

Step 3: State the structural parameter relation for A_v:

$$A_v = \beta \times \frac{R_C}{R_B}$$

Step 4: Equate both definitions to construct an expression for the input voltage V_{in}:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \beta\left(\frac{R_C}{R_B}\right)$$

Step 5: Substitute the operational values into the equation:

$$\frac{2}{V_{\text{in}}} = 100 \times \left(\frac{2000}{1000}\right) = 200$$

Step 6: Solve explicitly for V_{in}:

$$V_{\text{in}} = \frac{2}{200} = 0.01 \text{ V} = 10 \text{ mV}$$

Final Answer: $\boxed{10 \text{ mV}}$

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept: The acceleration due to gravity at the earth's surface is $g_0 = \frac{GM}{R^2}$. At an altitude h above the surface, the distance from the center is $(R + h)$, altering the value according to the inverse-square law relation $g_h = g_0 \left(\frac{R}{R+h}\right)^2$.

Solution: Step 1: Identify the given altitude parameter:

$$h = \frac{R}{2}$$

Step 2: Note the height-dependent gravitational variation formula:

$$g_h = \frac{GM}{(R+h)^2}$$

Step 3: Substitute the value of h into the relation:

$$g_h = \frac{GM}{\left(R + \frac{R}{2}\right)^2}$$

Step 4: Simplify the denominator term:

$$R + \frac{R}{2} = \frac{3R}{2} \implies \left(\frac{3R}{2}\right)^2 = \frac{9R^2}{4}$$

Step 5: Insert the simplified denominator back into the equation:

$$g_h = \frac{GM}{\frac{9R^2}{4}} = \frac{4}{9}\left(\frac{GM}{R^2}\right)$$

Step 6: Express the final value in terms of surface gravity g_0:

$$g_h = \frac{4}{9} g_0$$

Final Answer: $\boxed{\frac{4}{9} g_0}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept:

Equation of trajectory of a particle in two-dimensional motion.

Solution:

The position coordinates of the particle at any time t are given by:

$$x = a\cos(\omega t) \implies \frac{x}{a} = \cos(\omega t)$$

$$y = b\sin(\omega t) \implies \frac{y}{b} = \sin(\omega t)$$

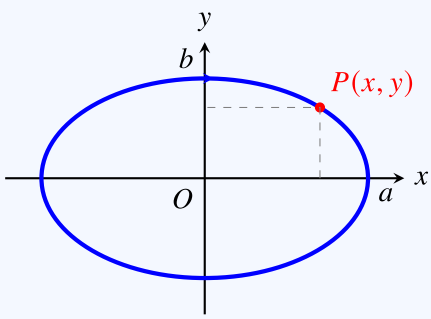
To find the equation of the trajectory, we eliminate the time parameter t by squaring and adding both equations:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = \cos^2(\omega t) + \sin^2(\omega t)$$

Using the standard trigonometric identity $\cos^2\theta + \sin^2\theta = 1$:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

This is the standard equation of an ellipse centered at the origin with semi-major axis a and semi-minor axis b.



Final Answer: B

Answer: (B)

Go Back to Question 1

Q25.

Solution

Concept: The base identity of a complex variable X can be determined via dimensional analysis. By identifying the units of Planck's constant h ($E = hf$) and the Stefan-Boltzmann constant σ ($E_{\text{flux}} = \sigma T^4$), we can calculate the net unit structure of the formula.

Solution: Step 1: State the definition of the target quantity X:

$$X = \frac{h}{\sigma T^4}$$

Step 2: Extract the metrics of Planck's constant h:

$$\text{Unit of } h = \frac{\text{Energy}}{\text{Frequency}} = \text{J} \cdot \text{s}$$

Step 3: Extract the metrics of the Stefan-Boltzmann constant σ:

$$\text{Unit of } \sigma = \frac{\text{Energy}}{\text{Area} \times \text{Time} \times \text{Temperature}^4} = \frac{\text{J}}{\text{m}^2 \cdot \text{s} \cdot \text{K}^4}$$

Step 4: Substitute the individual components into the definition of X:

$$\text{Unit of } X = \frac{\text{J} \cdot \text{s}}{\left(\frac{\text{J}}{\text{m}^2 \cdot \text{s} \cdot \text{K}^4}\right) \cdot \text{K}^4}$$

Step 5: Cancel out common variables (J and K^4):

$$\text{Unit of } X = \frac{\text{s}}{\left(\frac{1}{\text{m}^2 \cdot \text{s}}\right)}$$

Step 6: Simplify the expression to find the final metric composition:

$$\text{Unit of } X = \text{m}^2 \cdot \text{s}^2 = \text{Area} \times \text{Time}^2$$

Final Answer: None of the fields

Answer: (D)

Go Back to Question 25

Q26.

Solution

Concept: Static friction is a self-adjusting force that matches the applied external force exactly up to a limiting maximum value ($f_{\text{limiting}} = \mu_s N = \mu_s mg$). If the pulling force is below this threshold, the object remains at rest, and the frictional force equals the applied force.

Solution: Step 1: Calculate the normal force balancing the weight of the block:

$$N = mg$$

Step 2: State the formula for the maximum limiting static friction:

$$f_{\text{limiting}} = \mu_s mg$$

Step 3: Evaluate the given pulling force condition:

$$F < \mu_s mg \implies F < f_{\text{limiting}}$$

Step 4: Conclude that since the applied force is less than limiting friction, the block remains stationary.
Step 5: Apply Newton's first law for horizontal static equilibrium:

$$F - f_s = 0$$

Step 6: Solve for the static frictional force magnitude:

$$f_s = F$$

Final Answer: $\boxed{F}$

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept:

Work done during thermodynamic processes represented on an Indicator Diagram (P-V graph).

Solution:

Step 1: On a P-V diagram, the work done by a gas during expansion is equal to the area under the process curve bounded by the volume axis.

$$W = \int P\,dV = \text{Area under } P\text{-}V \text{ curve}$$

Step 2: The slope of an adiabatic curve is steeper than the slope of an isothermal curve from a common initial point. Mathematically:

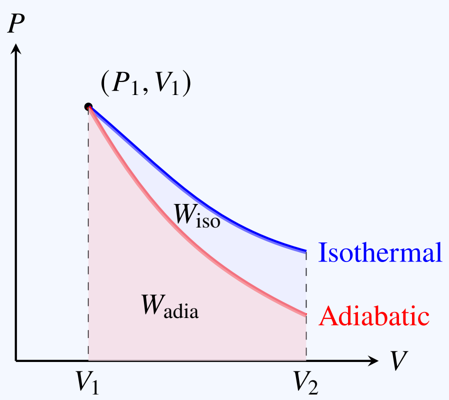
$$\left(\frac{dP}{dV}\right)_{\text{adiabatic}} = -\gamma\frac{P}{V} \quad \text{and} \quad \left(\frac{dP}{dV}\right)_{\text{isothermal}} = -\frac{P}{V}$$

Since the adiabatic index $\gamma > 1$, the adiabatic curve falls more rapidly than the isothermal curve during expansion.

Step 3: When expanding from the same initial volume V_1 to the same final volume V_2, the isothermal curve always lies above the adiabatic curve.

Step 4: Consequently, the area under the isothermal curve is larger than the area under the adiabatic curve. Therefore, the work done in the isothermal process is greater than that in the adiabatic process:

$$W_{\text{iso}} > W_{\text{adia}}$$



Final Answer: A

Answer: (A)

Go Back to Question 1

Q28.

Solution

Concept: To find the true phase difference between two wave functions, both must be expressed using the same trigonometric function with positive amplitudes. The identity $\cos(\theta) = \sin\left(\theta + \frac{\pi}{2}\right)$ can convert a cosine expression into a sine format.

Solution: Step 1: State the first wave equation in standard sine form:

$$y_1 = A\sin(\omega t - kx)$$

Step 2: Note the second wave equation given in cosine form:

$$y_2 = A\cos\left(\omega t - kx + \frac{\pi}{6}\right)$$

Step 3: Convert y_2 into a sine function using the trigonometric phase identity:

$$y_2 = A\sin\left(\omega t - kx + \frac{\pi}{6} + \frac{\pi}{2}\right)$$

Step 4: Simplify the combined constant phase angles:

$$\frac{\pi}{6} + \frac{\pi}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$$

Step 5: Write out the fully formatted sine equation for the second wave:

$$y_2 = A\sin\left(\omega t - kx + \frac{2\pi}{3}\right)$$

Step 6: Calculate the phase difference $\Delta\phi$ by subtracting the phase constants:

$$\Delta\phi = \left(\omega t - kx + \frac{2\pi}{3}\right) - (\omega t - kx) = \frac{2\pi}{3}$$

Final Answer: $\boxed{\frac{2\pi}{3}}$

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept: The de Broglie wavelength of a particle is $\lambda = \frac{h}{p}$. For a particle of mass m and charge q accelerated from rest through a potential difference V, kinetic energy is $K = qV$. Since momentum is $p = \sqrt{2mK}$, the wavelength is expressed as $\lambda = \frac{h}{\sqrt{2mqV}}$.

Solution: Step 1: State the comprehensive de Broglie wavelength formula for an accelerated charge:

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Step 2: Formulate the wavelength for a proton ($m_p = m, q_p = e$):

$$\lambda_p = \frac{h}{\sqrt{2meV}}$$

Step 3: Formulate the wavelength for an alpha particle ($m_\alpha = 4m, q_\alpha = 2e$):

$$\lambda_\alpha = \frac{h}{\sqrt{2(4m)(2e)V}} = \frac{h}{\sqrt{16meV}}$$

Step 4: Set up the direct ratio of the wavelengths using a single radical:

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\sqrt{2m_p q_p V}}{\sqrt{2m_\alpha q_\alpha V}}$$

Step 5: Substitute the mass and charge relationships into the ratio:

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m \cdot e}{4m \cdot 2e}} = \sqrt{\frac{1}{8}} = \frac{1}{2\sqrt{2}}$$

Step 6: Express the final comparison as a simplified ratio:

$$\frac{\lambda_\alpha}{\lambda_p} = 1 : 2\sqrt{2}$$

Final Answer: $\boxed{1 : 2\sqrt{2}}$

Answer: (C)

Go Back to Question 29

Q30.

Solution

Concept:

Terminal potential difference of a cell as a function of external load resistance.

Solution:

Step 1: The current I flowing through a circuit containing a cell of electromotive force (EMF) E, internal resistance r, and an external variable resistance R is given by Ohm's law:

$$I = \frac{E}{R + r}$$

Step 2: The terminal potential difference V across the cell is the voltage drop across the external resistance R:

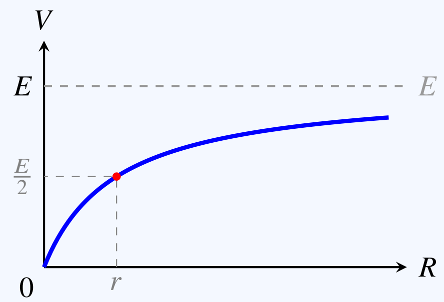
$$V = IR = \left(\frac{E}{R + r}\right) R$$

Step 3: To analyze the dependence of V on R, rewrite the expression by dividing the numerator and denominator by R:

$$V = \frac{E}{1 + \frac{r}{R}}$$

Step 4: Analyze the behavior of V at boundary conditions: * When $R = 0$ (short circuit): $V = \frac{E}{1+\infty} = 0$. The curve starts from the origin. * When $R = r$: $V = \frac{E}{1+1} = \frac{E}{2}$. * When $R \to \infty$ (open circuit): $\frac{r}{R} \to 0$, so $V \to E$.

Step 5: This shows that as R increases from zero, V increases non-linearly and approaches the value E asymptotically.



Final Answer: B

Answer: (B)

Go Back to Question 1

Q31.

Solution

Concept: This system undergoes an inelastic collision followed by vertical circular motion. Momentum conservation determines the initial velocity of the combined mass. For the system to successfully complete a full vertical loop of radius L, its minimum velocity at the lowest point must equal $\sqrt{5gL}$.

Solution: Step 1: Apply linear momentum conservation along the horizontal axis:

$$mv = (M + m)V_1 \implies V_1 = \frac{mv}{M + m}$$

Step 2: State the critical velocity requirement for full vertical circular motion:

$$V_{\text{min}} = \sqrt{5gL}$$

Step 3: Equate the post-collision velocity V_1 to the minimum critical velocity:

$$\frac{mv}{M + m} = \sqrt{5gL}$$

Step 4: Rearrange the algebraic equation to isolate the bullet velocity v:

$$v = \left(\frac{M + m}{m}\right)\sqrt{5gL}$$

Final Answer: $\boxed{\frac{M + m}{m}\sqrt{5gL}}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: The total moment of inertia of a composite system is the sum of its individual components. A thin rod bent at its midpoint splits into two symmetric segments. Each segment acts as a rod of mass $m' = \frac{M}{2}$ and length $l' = \frac{L}{2}$ rotating about one of its ends.

Solution: Step 1: Write down the mass and length properties of each independent half:

$$m' = \frac{M}{2}, \quad l' = \frac{L}{2}$$

Step 2: Recall the moment of inertia formula for a rod rotating about its endpoint:

$$I_{\text{end}} = \frac{1}{3}ml^2$$

Step 3: Apply this formula to find the inertia I_1 of a single segment:

$$I_1 = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2$$

Step 4: Simplify the expression for I_1:

$$I_1 = \frac{ML^2}{24}$$

Step 5: Set up the total system sum for both symmetric segments:

$$I_{\text{total}} = 2 \times I_1$$

Step 6: Compute the final total moment of inertia:

$$I_{\text{total}} = 2 \times \left(\frac{ML^2}{24}\right) = \frac{1}{12}ML^2$$

Final Answer: $\boxed{\frac{1}{12}ML^2}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: By Jurin's Law, capillary rise height is $h = \frac{2T\cos\theta}{r\rho g}$, meaning $h \cdot r$ = constant. The work done by surface tension forces is converted into stored gravitational potential energy and liberated heat energy (W), where conservation of energy dictates $W = \frac{1}{2}\pi\rho g h^2 r^2 \propto (h \cdot r)^2$.

Solution: Step 1: State Jurin's inverse relation between tube radius and height:

$$h \cdot r = \text{constant}$$

Step 2: Note the total work done by surface tension:

$$W_{\text{surface}} = (2\pi r T\cos\theta) \cdot h = \pi\rho g h^2 r^2$$

Step 3: Note the gained gravitational potential energy of the fluid column:

$$U = mg\left(\frac{h}{2}\right) = \frac{1}{2}\pi\rho g h^2 r^2$$

Step 4: Express the heat energy evolved W as the difference:

$$W = W_{\text{surface}} - U = \frac{1}{2}\pi\rho g h^2 r^2$$

Step 5: Relate the heat expression to the radius-height product:

$$W \propto (h \cdot r)^2 \implies W \propto \text{constant}$$

Step 6: Conclude that the heat evolved is independent of individual radius changes, giving a 1 : 1 ratio.

Final Answer: 1 : 1

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept: In a DC circuit at steady-state, a capacitor acts as an open circuit ($I_C = 0$). The potential difference V_C across its nodes is found by analyzing the current loops in the remaining resistor network, which allows calculating the stored charge via $Q = CV_C$.

Solution: Step 1: Treat the capacitor branch as an open circuit, splitting the network into independent upper and lower loops.
Step 2: Compute the steady-state current I_1 in the upper loop:

$$I_1 = \frac{2\text{ V}}{2\,\Omega + 4\,\Omega} = \frac{1}{3}\text{ A}$$

Step 3: Find the electrical potential at the top node of the capacitor:

$$V_{\text{top}} = 2 - I_1(2) = 2 - \frac{2}{3} = \frac{4}{3}\text{ V}$$

Step 4: Compute the steady-state current I_2 in the lower loop:

$$I_2 = \frac{4\text{ V}}{1\,\Omega + 5\,\Omega} = \frac{2}{3}\text{ A}$$

Step 5: Find the electrical potential at the bottom node of the capacitor:

$$V_{\text{bottom}} = 4 - I_2(1) = 4 - \frac{2}{3} = \frac{10}{3}\text{ V}$$

Step 6: Calculate the potential difference V_C and the stored charge Q:

$$V_C = \frac{10}{3} - \frac{4}{3} = 2\text{ V} \implies Q = (3\,\mu\text{F}) \times 2\text{ V} = 6\,\mu\text{C}$$

Final Answer: $\boxed{6\,\mu\text{C}}$

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept: When a plano-convex lens is silvered on its flat side, it acts as an equivalent mirror with a power of $\frac{1}{F} = \frac{2}{f_L} + \frac{1}{f_M}$. For the system to form a real image that overlaps with the object itself, the object must be placed at the center of curvature of this equivalent mirror system ($u = 2F$).

Solution: Step 1: Apply the Lens Maker's Formula for the lens before silvering ($R_1 = R, R_2 = \infty$):

$$\frac{1}{f_L} = (1.5 - 1)\left(\frac{1}{R} - 0\right) = \frac{1}{2R} \implies f_L = 2R$$

Step 2: Note the focal length of the flat silvered mirror surface:

$$\frac{1}{f_M} = \frac{1}{\infty} = 0$$

Step 3: Compute the equivalent focal length F of the silvered system:

$$\frac{1}{F} = 2\left(\frac{1}{2R}\right) + 0 = \frac{1}{R} \implies F = R$$

Step 4: Note that paths retrace completely when the object is located at the center of curvature.
Step 5: State the object position condition for self-coincidence:

$$u = 2F$$

Step 6: Substitute the value of F to find the required distance:

$$u = 2R$$

Final Answer: $\boxed{2R}$

Answer: (C)

Go Back to Question 35

Q36.

Solution

Concept: For an ideal gas, work done is $W = \int PdV$ and internal energy is $U = \frac{3}{2}PV$. In a cycle, total work is the sum of work done in each individual stage.

Solution: Step 1: Analyze process $A \to B$: State $A(P_0, V_0)$ to state $B(P_0, 2V_0)$ is at constant pressure, so it is an isobaric expansion. Statement (D) is correct.
Step 2: Calculate work for $A \to B$:

$$W_{A\to B} = P_0(2V_0 - V_0) = P_0V_0 \neq \frac{3}{4}P_0V_0 \implies \text{(A) is incorrect.}$$

Step 3: Analyze process $B \to C$: Constant volume $2V_0$ means $W_{B\to C} = 0$. The change in internal energy is:

$$\Delta U_{B\to C} = \frac{3}{2}P_CV_C - \frac{3}{2}P_BV_B = \frac{3}{2}\left(\frac{P_0}{2}\right)(2V_0) - \frac{3}{2}P_0(2V_0) = -\frac{3}{2}P_0V_0$$

Thus, statement (C) is correct.
Step 4: Analyze process $C \to A$: Given as isothermal compression ($P_CV_C = P_AV_A = P_0V_0$). Work is:

$$W_{C\to A} = P_0V_0 \ln\left(\frac{V_0}{2V_0}\right) = -P_0V_0 \ln 2$$

Total work is $W_{\text{total}} = P_0V_0 + 0 - P_0V_0 \ln 2 = P_0V_0(1 - \ln 2)$. Statement (B) is correct.
Step 5: Conclude that statements (B), (C), and (D) are true.
Final Answer: $\boxed{B, C, D}$

Answer: (B, C, D)

Go Back to Question 36

Q37.

Solution

Concept: A particle in SHM is modeled by $x(t) = A\sin(\omega t + \phi)$ and $v(t) = \omega A\cos(\omega t + \phi)$. Total energy is conserved at $E = \frac{1}{2}kA^2$, where kinetic energy is $K = \frac{1}{2}k(A^2 - x^2)$ and potential energy is $U = \frac{1}{2}kx^2$.

Solution: Step 1: State the conserved mechanical energy $E = \frac{1}{2}kA^2$. Statement (A) is correct.
Step 2: Apply initial conditions $t = 0, x = \frac{A}{2} \implies \sin\phi = \frac{1}{2} \implies \phi = \frac{\pi}{6}$ or $\frac{5\pi}{6}$. Since $v(0) = \omega A\cos\phi > 0$, $\phi = \frac{\pi}{6}$ is chosen. Thus $x(t) = A\sin(\omega t + \frac{\pi}{6})$. Statement (B) is correct.
Step 3: State maximum velocity when $\cos\phi = 1$:

$$v_{\max} = \omega A \implies \text{(C) is correct.}$$

Step 4: Find position where $K = U$:

$$\frac{1}{2}k(A^2 - x^2) = \frac{1}{2}kx^2 \implies A^2 = 2x^2 \implies x = \pm\frac{A}{\sqrt{2}} \implies \text{(D) is correct.}$$

Step 5: Conclude that all four statements are correct.
Final Answer: $\boxed{A, B, C, D}$

Answer: (A, B, C, D)

Go Back to Question 37

Q38.

Solution

Concept: Disconnecting a charged capacitor isolates its charge ($Q = Q_0$). Inserting a dielectric slab increases capacitance to $C = KC_0$, which dynamically scales the internal electric field ($E = \frac{E_0}{K}$) and stored energy ($U = \frac{U_0}{K}$).

Solution: Step 1: Since the battery is disconnected, the charge is isolated and must remain constant ($Q = Q_0$). Statement (D) is incorrect.

Step 2: Evaluate capacitance with the dielectric:

$$C = KC_0 \implies \text{(C) is correct.}$$

Step 3: Evaluate the new internal electric field:

$$E = \frac{Q}{K\epsilon_0 A} = \frac{E_0}{K} \implies \text{(A) is correct.}$$

Step 4: Evaluate the final electrostatic energy:

$$U = \frac{Q^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K} \implies \text{(B) is correct.}$$

Step 5: Statements (A), (B), and (C) are verified to be correct.

Final Answer: $\boxed{A, B, C}$

Answer: (A, B, C)

Go Back to Question 38

Q39.

Solution

Concept: By Faraday's Law, a varying magnetic flux induces an emf given by $\mathcal{E} = -\frac{d\Phi}{dt}$, where $\Phi = \int \vec{B} \cdot d\vec{A}$. Induced current is $I = \frac{\mathcal{E}}{R}$, which depends directly on the loop resistance.

Solution: Step 1: Integrate flux over the loop boundary from $x = 0$ to $x = L$ using $dA = L\,dx$:

$$\Phi(t) = \int_0^L \left[B_0\left(\frac{x}{L}\right)\cos(\omega t)\right] L\,dx = \frac{1}{2}B_0L^2\cos(\omega t) \implies \Phi(0) = \frac{1}{2}B_0L^2 \implies \text{(A) is correct.}$$

Step 2: Differentiate flux to determine the induced emf:

$$\mathcal{E} = -\frac{d\Phi}{dt} = \frac{1}{2}B_0L^2\omega\sin(\omega t) \implies \mathcal{E}_{\text{max}} = \frac{1}{2}B_0L^2\omega \implies \text{(B) is correct.}$$

Step 3: Analyze the current expression $I(t) = \frac{B_0L^2\omega}{2R}\sin(\omega t)$. The alternating $\sin(\omega t)$ function means the current direction reverses periodically. Statement (C) is correct.

Step 4: Since $I = \mathcal{E}/R$, current is inversely proportional to resistance. Statement (D) is incorrect.

Step 5: Conclude that statements (A), (B), and (C) are correct.

Final Answer: $\boxed{A, B, C}$

Answer: (A, B, C)

Go Back to Question 39

Q40.

Solution

Concept: In YDSE, bright fringe positions are $y_n = \frac{n\lambda D}{d}$ and fringe width is $\beta = \frac{\lambda D}{d}$. Overlapping bright fringes require $n_1\lambda_1 = n_2\lambda_2$, while a bright fringe overlaps a dark fringe if $n_1\lambda_1 = \frac{(2n_2-1)\lambda_2}{2}$.

Solution: Step 1: Equate bright fringe positions for coincidence:

$$\frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{4800}{6000} = \frac{4}{5} \implies \text{4th of } \lambda_1 \text{ matches 5th of } \lambda_2. \implies \text{(A) is incorrect.}$$

Step 2: Find the minimum overlap position using the smallest integers $n_1 = 4, n_2 = 5$:

$$y_{\min} = \frac{4\lambda_1 D}{d} = \frac{5\lambda_2 D}{d} \neq \frac{12\lambda_2 D}{d} \implies \text{(B) is incorrect.}$$

Step 3: Compare fringe widths using $\beta \propto \lambda$. Since $\lambda_1 = 6000$ Å $> \lambda_2 = 4800$ Å, then $\beta_1 > \beta_2$. Statement (C) is correct.

Step 4: Check if a bright fringe of λ_1 overlaps a dark fringe of λ_2:

$$n_1\lambda_1 = \frac{(2n_2-1)\lambda_2}{2} \implies \frac{5n_1}{2} = 2n_2-1 \implies \text{For } n_1 = 2, n_2 = 3 \text{ (valid integer)} \implies \text{(D) is incorrect.}$$

Step 5: Conclude that only statement (C) is correct.

Final Answer: $\boxed{C}$

Answer: (C)

Go Back to Question 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	D	3	C	4	C	5	A
6	A	7	A	8	A	9	A	10	B
11	B	12	B	13	C	14	B	15	A
16	B	17	B	18	A	19	B	20	C
21	B	22	A	23	A	24	B	25	D
26	B	27	A	28	C	29	C	30	B
31	A	32	B	33	A	34	C	35	C
36	B, C, D	37	A, B, C, D	38	A, B, C	39	A, B, C	40	C

Note: Section C (Q36–Q40): One or more correct options may be correct. Full marks only if all correct options are marked. Partial marking is not applicable.