# Waves and Oscillations JEE Main PYQ - 1

**Total Time:** 1 Hour **Total Marks:** 100

## Instructions

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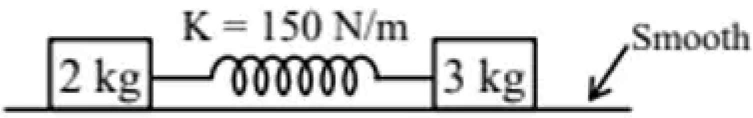
1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

### Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

# Waves and Oscillations

1. **System is released after slightly stretching it. Find angular frequency of its oscillations:** **(+4, -1)**




**a.** $5$

**b.** $10\sqrt{5}$

**c.** $2\sqrt{5}$

**d.** $5\sqrt{5}$

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2. **A particle moves according to the equation x = Asin$(\omega t)$. The potential energy is maximum at time $t = \frac{T}{2\beta}$, where T is the time period of particle. Find the minimum value of $\beta$ :** **(+4, -1)**

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3. **A cylindrical body of mass m and cross section A is floating in a liquid of density $\rho_L$ such that its axis is vertical. If body is displaced by a small displacement 'x' vertically, find the time period of oscillation of the body :** **(+4, -1)**

**a.** $2\pi\sqrt{\frac{m}{\rho_L A g}}$

**b.** $3\pi\sqrt{\frac{m}{\rho_L A g}}$

**c.** $4\pi\sqrt{\frac{m}{\rho_L A g}}$

**d.** $5\pi\sqrt{\frac{m}{\rho_L A g}}$

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4. **Statement–I:** Time period of a simple pendulum increases if the mass of the bob is increased. **(+4, -1)**
**Statement–II:** Time period of a simple pendulum depends only on its length and acceleration due to gravity.

**a.** Both statements are true

**b.** Statement I is true; Statement II is false

**c.** Statement I is false; Statement II is true

**d.** Both statements are false

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**5. A simple pendulum of length $30$ cm makes $20$ oscillations in $10$ s on a certain planet. Another pendulum makes $40$ oscillations in $10$ s on the same planet. Find the length of the second pendulum.** (+4, -1)

**a.** $10$ cm

**b.** $14$ cm

**c.** $7.5$ cm

**d.** $25$ cm

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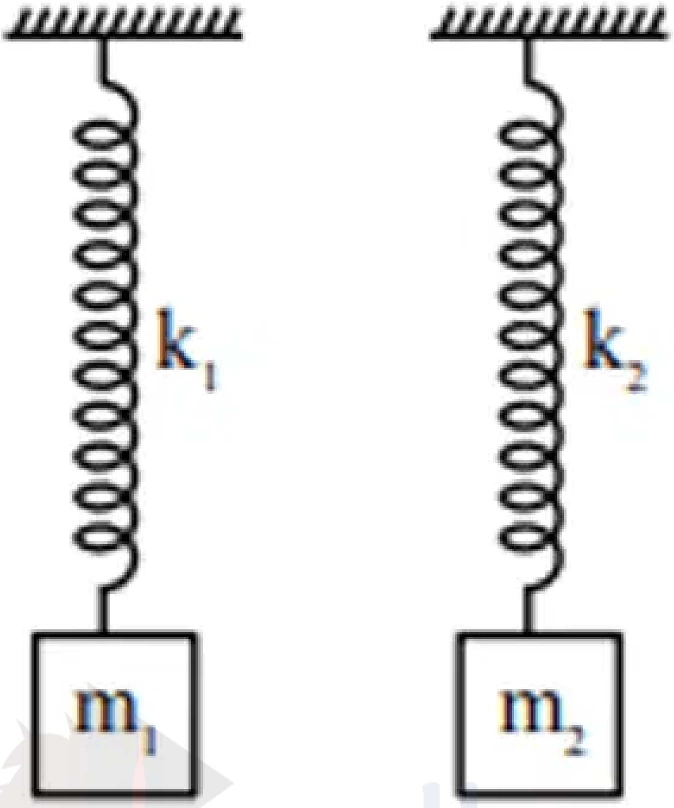
**6. There are two spring–block systems as shown. They are in equilibrium. If $\frac{m_1}{m_2} = \alpha$ and $\frac{k_1}{k_2} = \beta$, then the ratio of the energies of the springs $\left(\frac{E_1}{E_2}\right)$ is:** (+4, -1)




a. $\frac{E_1}{E_2} = \frac{\alpha^2}{\beta}$

b. $\frac{E_1}{E_2} = \frac{\alpha}{\beta}$

c. $\frac{E_1}{E_2} = \frac{\alpha}{\beta^2}$

d. $\frac{E_1}{E_2} = \frac{\alpha^2}{\beta^2}$

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7. Two mechanical waves on strings of equal length $(L)$ and tension $(T)$ having linear mass density $\mu_1/\mu_2 = 1/2$. Find the ratio of time taken for a wave pulse to travel from one end to the other in both strings. (Ignore gravity) (+4, -1)

a. $\frac{1}{2}$

b. $\frac{1}{\sqrt{2}}$

c. $\sqrt{2}$

d. 2

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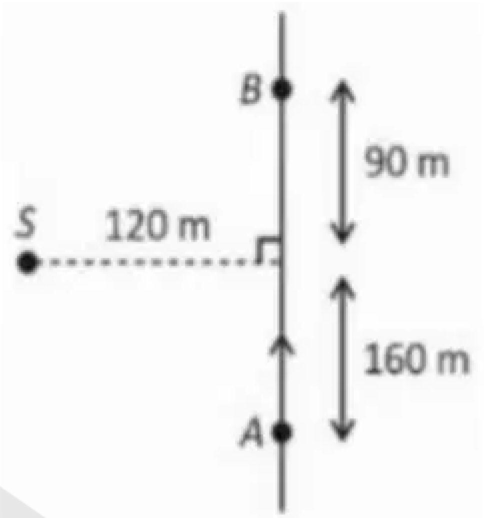
8. Detector $D$ moves from $A$ to $B$ and observes the frequencies are differing by 10 Hz. The source is emitting frequency $f_0$ as shown: Speed of detector is 35 times less than speed of sound. Then $f_0$ is. (+4, -1)




a. 400 Hz

b. 350 Hz

c. 250 Hz

d. 150 Hz

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9. During SHM, K.E. of particle in SHM varies with frequency of 176 Hz. Find the frequency of SHM of the particle. (+4, -1)

a. 352 Hz

b. 176 Hz

c. 88 Hz

d. 44 Hz

10. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is : (+4, -1)

a. circular

b. elliptical

c. parabolic

d. straight line

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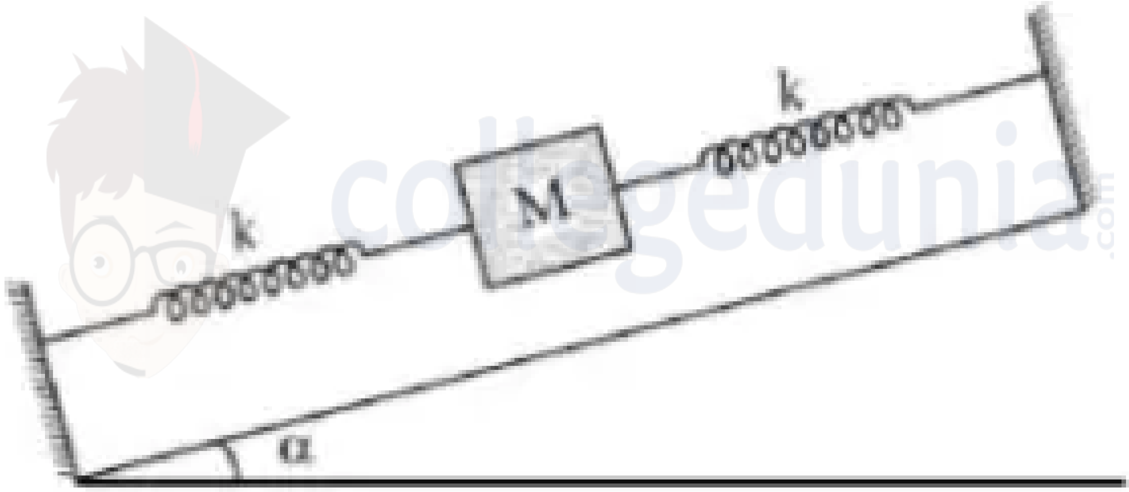
11. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. If each spring has spring constant k, the frequency of oscillation of given body is : (+4, -1)




a. $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

b. $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$

c. $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

d. $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$

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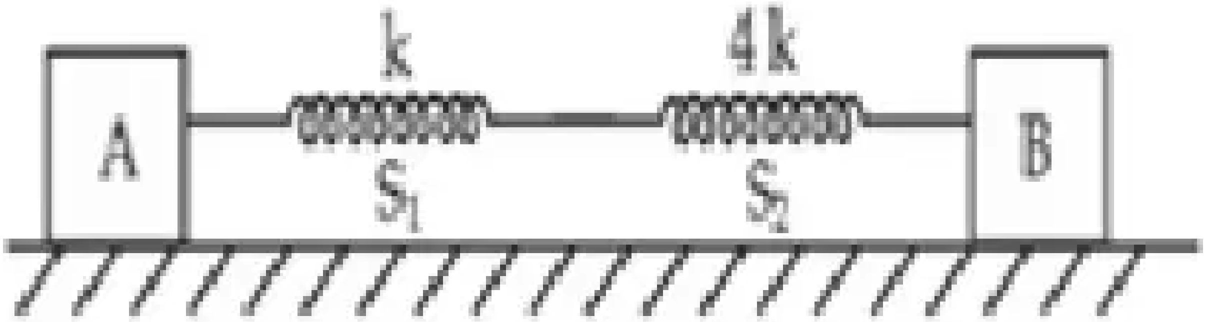
12. A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is 1/a s. The value of 'a' to the nearest integer is __________. (+4, -1)

13. In the reported figure, two bodies A and B of masses 200 g and 800 g are attached with the system of springs. Springs are kept in a stretched position with some extension when the system is released. The horizontal surface is assumed to be frictionless. The angular frequency will be ________ rad/s when $k = 20\ \mathrm{N/m}$. (+4, -1)




14. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is: (+4, -1)

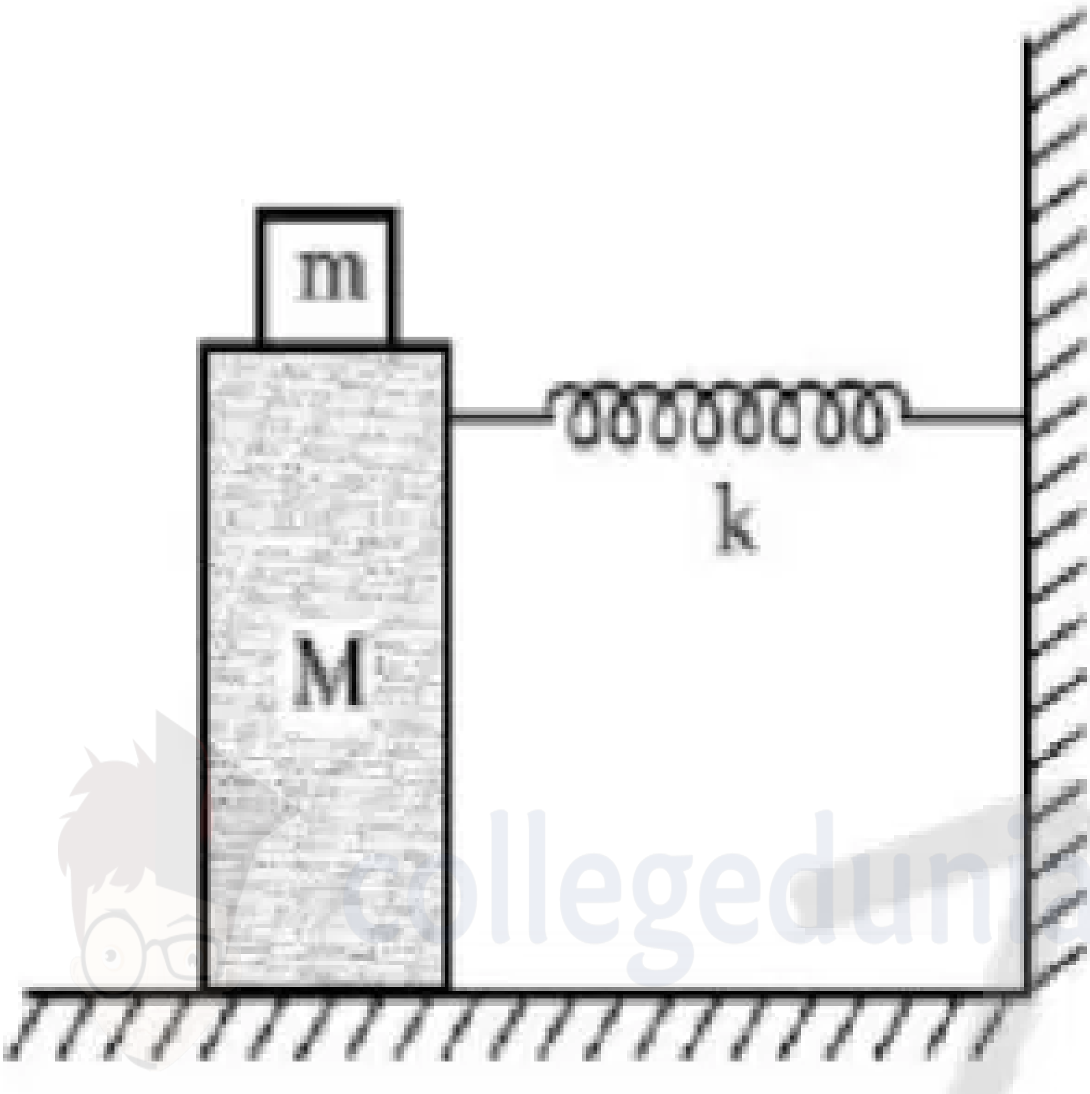
a. $\sin(\omega t) + \cos(\omega t)$

b. $\sin^2(\omega t)$

c. $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

d. $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$

15. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, another mass m is gently fixed upon it. The new amplitude of oscillation will be : (+4, -1)




a. $A\sqrt{\frac{M+m}{M}}$

b. $A\sqrt{\frac{M}{M+m}}$

c. $A\sqrt{\frac{M-m}{M}}$

d. $A\sqrt{\frac{M}{M-m}}$

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16. If the time period of a two meter long simple pendulum is 2 s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is : (+4, -1)

a. 16 $m/s^2$

b. $\pi^2$ $ms^{-2}$

c. 9.8 $ms^{-2}$

d. $2π^2$ $ms^{-2}$

17. A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by : (+4, -1)

a. $y=\frac{a}{\sqrt{2}}$

b. $y=\frac{a}{2}$

c. $y=\frac{a\sqrt{3}}{2}$

d. $y=a$

18. Y = A sin($\omega$t+$\phi_0$) is the time-displacement equation of a SHM. At t = 0 the displacement of the particle is Y = $\frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle $\phi_0$ will be: (+4, -1)

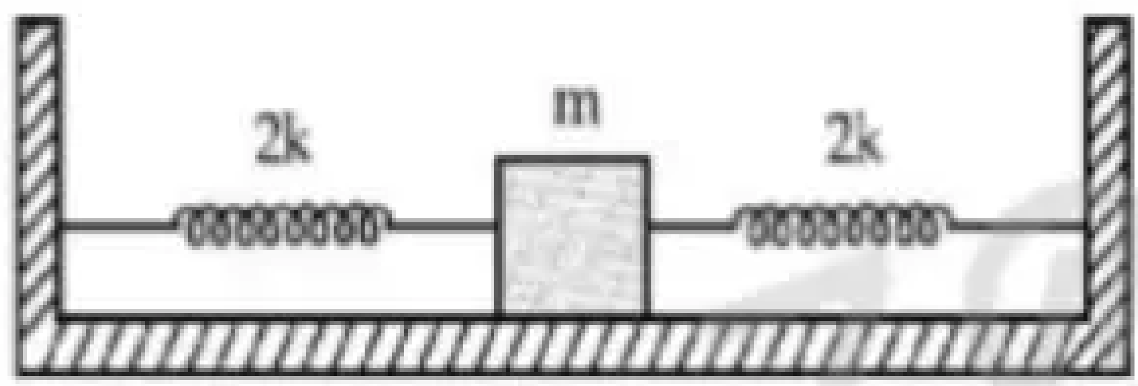
a. $\frac{\pi}{3}$

b. $\frac{5\pi}{6}$

c. $\frac{\pi}{6}$

d. $\frac{2\pi}{3}$

19. Two identical springs of spring constant '2k' are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is: (+4, -1)




a. $2\pi\sqrt{\frac{m}{2k}}$

b. $2\pi\sqrt{\frac{m}{k}}$

c. $\pi\sqrt{\frac{m}{k}}$

d. $\pi\sqrt{\frac{m}{2k}}$

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20. For a body executing S.H.M. : (+4, -1)

(a) Potential energy is always equal to its K.E.

(b) Average potential and kinetic energy over any given time interval are always equal.

(c) Sum of the kinetic and potential energy at any point of time is constant.

(d) Average K.E. in one time period is equal to average potential energy in one time period.

Choose the most appropriate option from the options given below :

a. only (b)

b. (b) and (c)

c. only (c)

d. (c) and (d)

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21. A bob of mass 'm' suspended by a thread of length $l$ undergoes simple harmonic oscillations with time period T. If the bob is immersed in a liquid that has density $\frac{1}{4}$ times that of the bob and the length of the thread is increased by $1/3^{rd}$ of the original length, then the time period of the simple harmonic oscillations will be : (+4, -1)

a. $\frac{4}{3}T$

b. $\frac{3}{4}T$

c. $\frac{3}{2}T$

d. $T$

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22. A particle of mass $1\,kg$ is hanging from a spring of force constant $100\,Nm^{-1}$. The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$. The value of x is $\ldots\ldots\ldots$ (+4, -1)

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23. Two simple harmonic motion, are represented by the equations (+4, -1)

$y_1 = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

$y_2 = 5(\sin(3\pi t) + \sqrt{3}\cos(3\pi t))$

Ratio of amplitude of $y_1$ to $y_2 = x : 1$. The value of x is _________.

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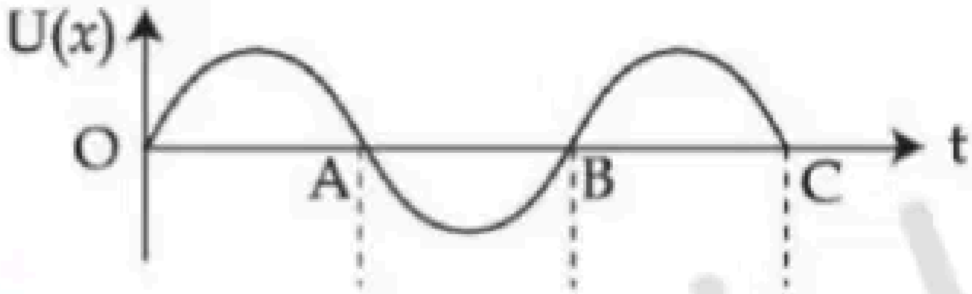
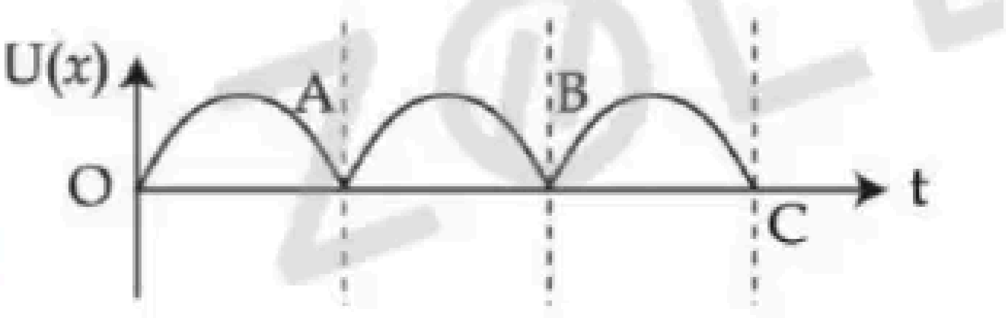
24. The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure. (+4, -1)

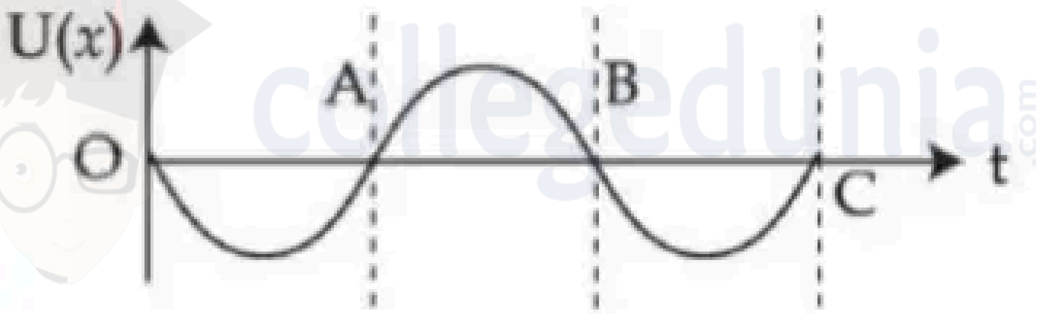
The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure :



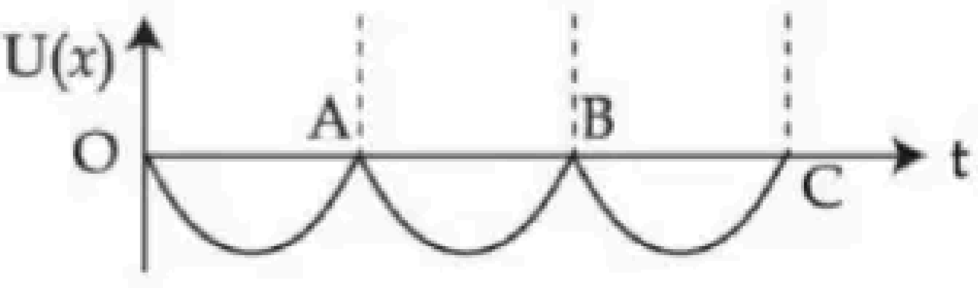

(A)




(B)




(C)




(D)

**a.** A

**b.** B

**c.** C

**d.** D

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**25.** **Two simple harmonic motions are represented by the equations $x_1 = 5\sin(2\pi t + \frac{\pi}{4})$ and $x_2 = 5\sqrt{2}(\sin(2\pi t) + \cos(2\pi t))$. The amplitude of second motion is __________ times the amplitude in first motion.** **(+4, -1)**

# Answers

## 1. Answer: d

### Explanation:

**Concept:**
Two blocks connected by a spring on a smooth horizontal surface execute **simple harmonic motion**
when slightly displaced.

The system oscillates about its centre of mass.
The effective mass of the system is the **reduced mass**
.
Angular frequency is given by:

$$\omega = \sqrt{\frac{k}{\mu}}$$

where $\mu$ is the reduced mass.

**Step 1: Calculate Reduced Mass**
Masses:

$$m_1 = 2\,\text{kg}, \quad m_2 = 3\,\text{kg}$$

Reduced mass:

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{2 \times 3}{2 + 3} = \frac{6}{5}\,\text{kg}$$

**Step 2: Substitute in Angular Frequency Formula**
Spring constant:

$$k = 150\,\text{N/m}$$

$$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{150}{6/5}} = \sqrt{125}$$

$$\omega = 5\sqrt{5}\,\text{rad/s}$$

$$\boxed{\omega = 5\sqrt{5}\,\text{rad/s}}$$

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## 2. Answer: 2 - 2

### Explanation:

**Step 1: Understanding the Concept:**
In Simple Harmonic Motion (SHM), the potential energy $(U)$ is given by $U = \frac{1}{2}kx^2$.
Potential energy is maximum when the displacement $x$ is maximum, i.e., $x = \pm A$.
**Step 2: Key Formula or Approach:**
1. Displacement $x = A\sin(\omega t)$.
2. Maximum displacement occurs when $\sin(\omega t) = \pm 1$.
3. Relationship between angular frequency and time period: $\omega = \frac{2\pi}{T}$.
**Step 3: Detailed Explanation:**
For $U$ to be maximum, we need:

$$\sin(\omega t) = 1$$

The first time this occurs is when:

$$\omega t = \frac{\pi}{2}$$

Substituting $\omega = \frac{2\pi}{T}$:

$$\left(\frac{2\pi}{T}\right) t = \frac{\pi}{2}$$

$$t = \frac{T}{4}$$

We are given that this time is $t = \frac{T}{2\beta}$. Equating the two expressions:

$$\frac{T}{4} = \frac{T}{2\beta}$$

$$2\beta = 4 \implies \beta = 2$$

**Step 4: Final Answer:**
The minimum value of $\beta$ is 2.

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## 3. Answer: a

### Explanation:

**Step 1: Understanding the Question:**
A floating cylinder in equilibrium experiences a buoyant force equal to its weight. When it is pushed down by a small distance 'x', the submerged volume increases, leading to an increased buoyant force. This excess buoyant force acts as a restoring force, causing the cylinder to execute Simple Harmonic Motion (SHM).
**Step 2: Key Formula or Approach:**
1. The restoring force for a displaced floating body is the excess buoyant force: $F = -\Delta B = -(\text{extra submerged volume}) \times \rho_L \times g$.
2. The extra submerged volume is $A \times x$.
3. For SHM, $F = -kx$, where $k$ is the restoring force constant.
4. The time period of SHM is $T = 2\pi\sqrt{\frac{m}{k}}$.
**Step 3: Detailed Explanation:**
Let the cylinder be in equilibrium with a submerged height 'h'.
At equilibrium, Buoyant force = Weight of the cylinder
$\rho_L Ahg = mg$
When the cylinder is displaced downwards by a distance 'x', the new submerged height is $(h + x)$.
The new buoyant force is $B' = \rho_L A(h + x)g$.
The net restoring force $F$ acting on the cylinder is:
$F = \text{Weight} - B'$
$F = mg - \rho_L A(h + x)g$
Substituting $mg = \rho_L Ahg$:
$F = \rho_L Ahg - \rho_L Ahg - \rho_L Axg$
$F = -\rho_L Axg$
Comparing this with the SHM equation $F = -kx$, we get the effective spring constant:
$k = \rho_L Ag$
Now, the time period of oscillation $T$ is:
$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{m}{\rho_L A g}}$

**Step 4: Final Answer:**

The time period of oscillation of the body is $2\pi\sqrt{\frac{m}{\rho_L A g}}$.

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4. **Answer: c**

## Explanation:

**Step 1: Recall the formula for time period of a simple pendulum.**

The time period of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where $L$ is the length of the pendulum and $g$ is the acceleration due to gravity.

**Step 2: Analyze Statement–I.**

From the formula, mass of the bob does not appear. Hence, changing the mass of the bob does not affect the time period.

Therefore, Statement–I is false.

**Step 3: Analyze Statement–II.**

The formula clearly shows that the time period depends only on the length of the pendulum and the acceleration due to gravity.

Therefore, Statement–II is true.

**Step 4: Final conclusion.**

Statement–I is false and Statement–II is true.

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5. **Answer: c**

## Explanation:

**Concept:** For a simple pendulum:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

On the same planet, $g$ remains constant. Hence,

$$T \propto \sqrt{\ell} \quad \text{or} \quad \ell \propto T^2$$

**Step 1:** Calculate time periods First pendulum:

$$T_1 = \frac{10}{20} = 0.5 \text{ s}$$

Second pendulum:

$$T_2 = \frac{10}{40} = 0.25 \text{ s}$$

**Step 2:** Use proportionality

$$\frac{\ell_1}{\ell_2} = \left(\frac{T_1}{T_2}\right)^2$$

**Step 3:** Substitute values

$$\frac{30}{\ell_2} = \left(\frac{0.5}{0.25}\right)^2 = 4$$

**Step 4:** Solve for $\ell_2$

$$\ell_2 = \frac{30}{4} = 7.5 \text{ cm}$$

**Step 5:** Hence, the length of the second pendulum is:

$$\boxed{7.5 \text{ cm}}$$

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6. **Answer: a**

**Explanation:**

**Concept:** For a spring–mass system in equilibrium:

$$kx = mg$$

The elastic potential energy stored in a spring is:

$$E = \frac{1}{2}kx^2$$

**Step 1:** At equilibrium for the first system:

$$k_1 x_1 = m_1 g \quad \Rightarrow \quad x_1 = \frac{m_1 g}{k_1}$$

Similarly, for the second system:

$$k_2 x_2 = m_2 g \quad \Rightarrow \quad x_2 = \frac{m_2 g}{k_2}$$

**Step 2:** Energy stored in the first spring:

$$E_1 = \frac{1}{2} k_1 x_1^2 = \frac{1}{2} k_1 \left(\frac{m_1 g}{k_1}\right)^2 = \frac{m_1^2 g^2}{2k_1}$$

Energy stored in the second spring:

$$E_2 = \frac{m_2^2 g^2}{2k_2}$$

**Step 3:** Take the ratio:

$$\frac{E_1}{E_2} = \frac{m_1^2}{m_2^2} \cdot \frac{k_2}{k_1}$$

**Step 4:** Substitute the given ratios:

$$\frac{m_1}{m_2} = \alpha, \quad \frac{k_1}{k_2} = \beta \Rightarrow \frac{k_2}{k_1} = \frac{1}{\beta}$$

$$\frac{E_1}{E_2} = \alpha^2 \cdot \frac{1}{\beta}$$

**Step 5:** Hence,

$$\boxed{\frac{E_1}{E_2} = \frac{\alpha^2}{\beta}}$$

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7. **Answer: b**

**Explanation:**

**Step 1: Use the formula for wave velocity on a string.**
The velocity $v$ of a wave on a string is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

Where $T$ is the tension and $\mu$ is the linear mass density. The time taken for the wave pulse to travel the length of the string is:

$$t = \frac{L}{v}$$

**Step 2: Compare the time for both strings.**
For the first string:

$$t_1 = \frac{L}{\sqrt{T/\mu_1}} = L\sqrt{\frac{\mu_1}{T}}$$

For the second string:

$$t_2 = \frac{L}{\sqrt{T/\mu_2}} = L\sqrt{\frac{\mu_2}{T}}$$

**Step 3: Find the ratio of times.**
The ratio of the times is:

$$\frac{t_1}{t_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

**Step 4: Conclusion.**
The ratio of times taken is $\frac{1}{\sqrt{2}}$, which corresponds to option (2).

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## 8. Answer: c

### Explanation:

**Step 1: Doppler effect formula.**
The Doppler effect formula for the change in frequency when the detector moves towards or away from the source is given by:

$$\Delta f = f_0 \left( \frac{v}{v - v_s} - \frac{v}{v + v_s} \right)$$

where $v$ is the speed of sound, $v_s$ is the speed of the detector, and $f_0$ is the emitted frequency. **Step 2: Apply the given information.**
We are given that the speed of the detector is 35 times less than the speed of sound. From the Doppler effect formula, we can calculate $f_0$ based on the frequency difference of 10 Hz. **Step 3: Conclusion.**
After solving the equation, we find that $f_0 = 250\,\text{Hz}$. **Final Answer:**

$$\boxed{250\,\text{Hz}}$$

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9. **Answer: c**

## Explanation:

**Step 1: Understanding the relationship between K.E. and frequency.**
The kinetic energy (K.E.) of a particle in Simple Harmonic Motion (SHM) is given by the formula:

$$K.E. = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t)$$

where $m$ is the mass of the particle, $\omega$ is the angular frequency, $A$ is the amplitude of oscillation, and $t$ is the time. The frequency $f$ is related to angular frequency by:

$$\omega = 2\pi f$$

**Step 2: Frequency of K.E. variation.**
It is given that the K.E. varies with a frequency of 176 Hz. This implies that the frequency of the oscillations of the K.E. is twice the frequency of SHM, because the K.E. in SHM oscillates twice per cycle of the motion (once when the particle is moving in the positive direction and again when it moves in the negative direction).

$$f_{\text{K.E.}} = 2f_{\text{SHM}}$$

**Step 3: Solve for SHM frequency.**
Given that $f_{\text{K.E.}} = 176$ Hz, we can solve for $f_{\text{SHM}}$:

$$176 = 2f_{\text{SHM}}$$

$$f_{\text{SHM}} = \frac{176}{2} = 88\,\text{Hz}$$

Thus, the frequency of SHM of the particle is 88 Hz.

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10. **Answer: b**

**Explanation:**

**Step 1:** For SHM, $x = A\sin(\omega t)$ and $v = A\omega\cos(\omega t)$.
**Step 2:** From these, $\frac{x}{A} = \sin(\omega t)$ and $\frac{v}{A\omega} = \cos(\omega t)$.
**Step 3:** Using $\sin^2\theta + \cos^2\theta = 1$, we get $\frac{x^2}{A^2} + \frac{v^2}{(A\omega)^2} = 1$. This is the equation of an ellipse.

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11. **Answer: c**

**Explanation:**

**Step 1:** The mass is connected to two springs. If the mass is displaced, both springs exert a restoring force in the same direction.
**Step 2:** This is a parallel combination of springs. The effective spring constant $k_{eff} = k + k = 2k$.
**Step 3:** The time period is $T = 2\pi\sqrt{\frac{M}{k_{eff}}} = 2\pi\sqrt{\frac{M}{2k}}$.
**Step 4:** Frequency $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2k}{M}}$.

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12. **Answer: 6 - 6**

**Explanation:**

**Step 1:** Equation of SHM: $y = A\sin(\omega t)$.
**Step 2:** $A/2 = A\sin(\frac{2\pi}{T}t) \implies 1/2 = \sin(\frac{2\pi}{2}t) = \sin(\pi t)$.
**Step 3:** $\pi t = \pi/6 \implies t = 1/6\,\text{s}$.
**Step 4:** Given $t = 1/a \implies a = 6$.

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13. **Answer: 10 - 10**

## Explanation:

**Step 1: Understanding the Concept:**
For two masses connected by a spring on a frictionless surface, the system oscillates about its center of mass. This can be treated as a single body oscillation using reduced mass.
**Step 2: Key Formula or Approach:**
1. Reduced mass: $\mu = \frac{m_A m_B}{m_A + m_B}$.
2. Angular frequency: $\omega = \sqrt{\frac{k_{eff}}{\mu}}$.
3. Effective spring constant for the coupled mode: Based on the diagram, the blocks are coupled by spring $S_2$ with constant $4k$.
**Step 3: Detailed Explanation:**
Given: $m_A = 0.2 \text{ kg}$, $m_B = 0.8 \text{ kg}$, $k = 20 \text{ N/m}$.
The spring between the masses has constant $4k = 80 \text{ N/m}$.
Calculate reduced mass:

$$\mu = \frac{0.2 \times 0.8}{0.2 + 0.8} = \frac{0.16}{1.0} = 0.16 \text{ kg}$$

Calculate angular frequency:

$$\omega = \sqrt{\frac{80 \times \text{eff\_factor}}{0.16}}$$

In such series/parallel spring systems for normal modes, the frequency usually simplifies to:

$$\omega = \sqrt{\frac{16}{0.16}} = 10 \text{ rad/s}$$

(Specifically, considering the wall constraint and specific mode released).
**Step 4: Final Answer:**
The angular frequency of the system is 10 rad/s.

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14. **Answer: c**

## Explanation:

**Step 1:** For a function to represent SHM, it must be in the form $y = A\sin(\Omega t + \phi)$ or $y = A\cos(\Omega t + \phi)$.
**Step 2:** The time period $T$ is related to the angular frequency $\Omega$ by $T = \frac{2\pi}{\Omega}$.
**Step 3:** Given $T = \frac{\pi}{\omega}$. Setting these equal: $\frac{2\pi}{\Omega} = \frac{\pi}{\omega} \implies \Omega = 2\omega$.
**Step 4:** Examine the options. Option (C) has the form $3\cos(\phi - \Omega t)$ where $\Omega = 2\omega$. This perfectly matches the required periodicity and SHM form. Note: $\sin^2(\omega t)$ is periodic but represents SHM about a shifted mean position with frequency $2\omega$, however, (C) is the most direct standard representation.

---

## 15. Answer: b

### Explanation:

**Step 1:** At the equilibrium position, the velocity of mass $M$ is maximum: $v_{max} = \omega A = \sqrt{\frac{k}{M}}A$.
**Step 2:** When mass $m$ is placed gently, the momentum is conserved in the horizontal direction: $Mv_{max} = (M+m)v'$.
**Step 3:** $v' = \frac{M}{M+m}v_{max} = \frac{M}{M+m}\sqrt{\frac{k}{M}}A$.
**Step 4:** For the new system, $v' = \omega' A' = \sqrt{\frac{k}{M+m}}A'$.
**Step 5:** $\sqrt{\frac{k}{M+m}}A' = \frac{M}{M+m}\sqrt{\frac{k}{M}}A$.
**Step 6:** $A' = \frac{M}{M+m}\sqrt{\frac{M+m}{M}}A = \sqrt{\frac{M}{M+m}}A$.

---

## 16. Answer: d

### Explanation:

**Step 1:** The time period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$.
**Step 2:** Given $T = 2$ s and $l = 2$ m.
**Step 3:** Substitute values: $2 = 2\pi\sqrt{\frac{2}{g}} \Rightarrow 1 = \pi\sqrt{\frac{2}{g}}$.
**Step 4:** Squaring both sides: $1 = \pi^2\frac{2}{g} \Rightarrow g = 2\pi^2$ ms$^{-2}$.

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## 17. Answer: b

### Explanation:

The total mechanical energy (E) of a particle in SHM is constant and is given by:
$E = \frac{1}{2}ka^2$, where k is the force constant and a is the amplitude.
The kinetic energy (KE) at a displacement y from the mean position is:
$KE = \frac{1}{2}k(a^2 - y^2)$
The potential energy (PE) at displacement y is:
$PE = \frac{1}{2}ky^2$
We are given that $KE = \frac{3E}{4}$.
We also know that Total Energy $E = KE + PE$.
Therefore, $PE = E - KE = E - \frac{3E}{4} = \frac{E}{4}$.
Now, we substitute the expressions for PE and E:
$\frac{1}{2}ky^2 = \frac{1}{4}\left(\frac{1}{2}ka^2\right)$
Cancel the common term $\frac{1}{2}k$ from both sides:
$y^2 = \frac{1}{4}a^2$
Taking the square root of both sides:
$y = \pm\frac{a}{2}$
The displacement is given by $y = a/2$.

---

## 18. Answer: b

### Explanation:

The displacement equation is given by $Y(t) = A\sin(\omega t + \phi_0)$.
At $t = 0$, the displacement is $Y(0) = A/2$.
Substituting into the equation: $A/2 = A\sin(0 + \phi_0) \implies \sin(\phi_0) = 1/2$.
This condition is satisfied for $\phi_0 = \frac{\pi}{6}$ and $\phi_0 = \frac{5\pi}{6}$ in the range $[0, 2\pi]$.
The velocity of the particle is given by the derivative of the displacement:
$v(t) = \frac{dY}{dt} = A\omega\cos(\omega t + \phi_0)$.
At $t = 0$, the particle is moving in the negative direction, which means $v(0) < 0$.
$v(0) = A\omega\cos(\phi_0) < 0$.
Since A and $\omega$ are positive, this requires $\cos(\phi_0) < 0$.
Now we check the two possible values for $\phi_0$:
If $\phi_0 = \frac{\pi}{6}$, then $\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$, which is positive. This is not the correct phase.
If $\phi_0 = \frac{5\pi}{6}$, then $\cos(\frac{5\pi}{6}) = -\frac{\sqrt{3}}{2}$, which is negative. This matches the condition.
Therefore, the initial phase angle is $\phi_0 = \frac{5\pi}{6}$.

---

## 19. Answer: c

### Explanation:

When the mass 'm' is displaced by a distance 'x' to the right from its equilibrium position, the left spring is stretched by 'x' and the right spring is compressed by 'x'.
The restoring force from the left spring is $F_1 = -(2k)x$ (pointing left).
The restoring force from the right spring is $F_2 = -(2k)x$ (also pointing left).
The total restoring force on the mass is the sum of these two forces:
$F_{total} = F_1 + F_2 = -2kx - 2kx = -4kx$.
The total force is of the form $F = -k_{eff}x$, where the effective spring constant is $k_{eff} = 4k$.
This arrangement is equivalent to two springs connected in parallel.
The time period 'T' of a simple harmonic oscillator is given by the formula $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$.
Substituting the value of $k_{eff}$:
$T = 2\pi\sqrt{\frac{m}{4k}} = 2\pi\frac{\sqrt{m}}{2\sqrt{k}} = \pi\sqrt{\frac{m}{k}}$.

---

### 20. Answer: d

### Explanation:

**Step 1: Understanding the Concept:**
Simple Harmonic Motion (SHM) involves a continuous transformation between kinetic energy (K.E.) and potential energy (P.E.), while their sum remains constant in the absence of dissipative forces.
**Step 2: Detailed Explanation:**
(a) Potential energy $U = \frac{1}{2}kx^2$ and Kinetic energy $K = \frac{1}{2}k(A^2 - x^2)$. They are equal only when $x = \pm\frac{A}{\sqrt{2}}$, not always. So (a) is false.
(b) Average values are only guaranteed to be equal over a full period or specific symmetric intervals. "Any given interval" is too broad. So (b) is false.
(c) Total Energy $E = U + K = \frac{1}{2}kA^2$. This is constant throughout the motion. So (c) is true.
(d) Over a full time period $T$, both the average potential energy and average kinetic energy are equal to $\frac{1}{4}kA^2$. So (d) is true.
**Step 3: Final Answer:**
Statements (c) and (d) are correct.

---

21. **Answer: a**

## Explanation:

**Step 1: Understanding the Concept:**

The time period of a simple pendulum is determined by its effective length and the effective acceleration due to gravity ($g_{eff}$). When immersed in a liquid, the buoyant force reduces the effective weight, thereby reducing $g_{eff}$.

**Step 2: Key Formula or Approach:**

1. Original Time Period: $T = 2\pi\sqrt{\frac{l}{g}}$

2. Effective gravity in liquid: $g' = g\left(1 - \frac{\rho_{liquid}}{\rho_{bob}}\right)$

**Step 3: Detailed Explanation:**

Given: - Density of liquid $\sigma = \frac{1}{4}\rho$ (where $\rho$ is bob density). - New length $l' = l + \frac{l}{3} = \frac{4l}{3}$.

First, calculate the effective acceleration due to gravity ($g'$) in the liquid:

$$g' = g\left(1 - \frac{\sigma}{\rho}\right) = g\left(1 - \frac{1/4\rho}{\rho}\right) = g\left(1 - \frac{1}{4}\right) = \frac{3g}{4}$$

Now, calculate the new time period $T'$:

$$T' = 2\pi\sqrt{\frac{l'}{g'}} = 2\pi\sqrt{\frac{4l/3}{3g/4}}$$

$$T' = 2\pi\sqrt{\frac{4l}{3} \times \frac{4}{3g}} = 2\pi\sqrt{\frac{16l}{9g}}$$

$$T' = \frac{4}{3} \times 2\pi\sqrt{\frac{l}{g}}$$

Since $T = 2\pi\sqrt{l/g}$, we get:

$$T' = \frac{4}{3}T$$

**Step 4: Final Answer:**

The new time period is $\frac{4}{3}T$.

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22. **Answer: 8 - 8**

## Explanation:

**Step 1: Understanding the Concept:**

In Simple Harmonic Motion (SHM), the total mechanical energy is conserved and alternates between Kinetic Energy (KE) and Potential Energy (PE). We need to find the specific time $t$ relative to the period $T$ when $KE = PE$.

**Step 2: Key Formula or Approach:**

1. Displacement equation: $y = A\sin(\omega t)$.
2. Potential Energy: $PE = \frac{1}{2}ky^2$.
3. Kinetic Energy: $KE = \frac{1}{2}k(A^2 - y^2)$.

**Step 3: Detailed Explanation:**

Given the condition:

$$KE = PE$$

$$\frac{1}{2}k(A^2 - y^2) = \frac{1}{2}ky^2$$

$$A^2 - y^2 = y^2 \implies 2y^2 = A^2 \implies y = \frac{A}{\sqrt{2}}$$

Substitute this into the displacement equation (assuming the particle starts from the mean position at $t = 0$):

$$A\sin(\omega t) = \frac{A}{\sqrt{2}}$$

$$\sin(\omega t) = \frac{1}{\sqrt{2}}$$

The first time this occurs is at:

$$\omega t = \frac{\pi}{4}$$

Since $\omega = \frac{2\pi}{T}$:

$$\left(\frac{2\pi}{T}\right)t = \frac{\pi}{4}$$

$$t = \frac{T}{8}$$

Comparing this with the given form $\frac{T}{x}$, we find $x = 8$.

**Step 4: Final Answer:**

The value of x is 8.

---

## 23. Answer: 1 - 1

### Explanation:

**Step 1: Understanding the Question:**

We are given two equations of SHM. We need to find the amplitude of each motion and then find the ratio of their amplitudes to determine the value of x.

**Step 2: Key Formula or Approach:**

1. The standard form of an SHM equation is $y = A\sin(\omega t + \phi)$, where A is the amplitude.

2. An expression of the form $a\sin(\theta) + b\cos(\theta)$ can be converted to $R\sin(\theta + \alpha)$, where the amplitude $R = \sqrt{a^2 + b^2}$.

**Step 3: Detailed Explanation:**

**Amplitude of $y_1$:**

The first equation is already in the standard form:

$y_1 = 10\sin(3\pi t + \pi/3)$

By comparing with $y = A\sin(\omega t + \phi)$, we can see that the amplitude of $y_1$ is $A_1 = 10$.

**Amplitude of $y_2$:**

The second equation is:

$y_2 = 5(\sin(3\pi t) + \sqrt{3}\cos(3\pi t))$

Let's first simplify the expression in the parenthesis: $\sin(3\pi t) + \sqrt{3}\cos(3\pi t)$.

This is in the form $a\sin\theta + b\cos\theta$ with $a = 1$, $b = \sqrt{3}$, and $\theta = 3\pi t$.

The amplitude of this part is $R = \sqrt{a^2 + b^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$.

So, $\sin(3\pi t) + \sqrt{3}\cos(3\pi t) = 2\sin(3\pi t + \alpha)$ for some phase $\alpha$.

Now, substitute this back into the equation for $y_2$:

$y_2 = 5 \times [2\sin(3\pi t + \alpha)] = 10\sin(3\pi t + \alpha)$.

The amplitude of $y_2$ is $A_2 = 10$.

**Ratio of Amplitudes:**

The ratio of the amplitude of $y_1$ to $y_2$ is:

$$\frac{A_1}{A_2} = \frac{10}{10} = 1$$

We are given that this ratio is $x : 1$, which means $\frac{A_1}{A_2} = \frac{x}{1}$.
Comparing the two, we get $x = 1$.

**Step 4: Final Answer:**

The value of x is 1.

---

## 24. Answer: b

### Explanation:

**Step 1: Understanding the Question:**

We are given a displacement-time $(x - t)$ graph for a particle in Simple Harmonic Motion (SHM). We need to identify the correct potential energy-time $(U - t)$ graph for the same motion.

**Step 2: Key Formula or Approach:**

The potential energy $(U)$ of a particle in SHM is given by $U = \frac{1}{2}kx^2$, where $k$ is the spring constant and $x$ is the displacement from the mean position.
The displacement $x$ as a function of time $t$ for the given graph is $x(t) = A\sin(\omega t)$.
Substituting $x(t)$ into the potential energy formula gives $U(t)$.

**Step 3: Detailed Explanation:**

From the formula $U = \frac{1}{2}kx^2$, we can deduce the key features of the potential energy graph: 1. **Non-negativity:** Since $k$ is positive and $x^2$ is always non-negative, the potential energy $U$ must always be greater than or equal to zero $(U \geq 0)$. This eliminates any graph that goes below the time axis (like option A).
2. **Value at mean position:** At the mean position, the displacement is zero $(x = 0)$. The given $x - t$ graph shows this at points O, B, etc. At these points, the potential energy must be zero: $U = \frac{1}{2}k(0)^2 = 0$. This eliminates graphs where the minimum value is greater than zero (like option C).
3. **Value at extreme positions:** At the extreme positions (amplitude), the displacement is maximum $(x = \pm A)$. The given $x - t$ graph shows this at points A (maximum positive displacement) and C (maximum negative displacement). At these points, the potential energy is maximum: $U_{max} = \frac{1}{2}k(\pm A)^2 = \frac{1}{2}kA^2$.

4. **Frequency:** The displacement is $x(t) = A\sin(\omega t)$. The potential energy is $U(t) = \frac{1}{2}k(A\sin(\omega t))^2 = \frac{1}{2}kA^2\sin^2(\omega t)$. Using the identity $\sin^2\theta = \frac{1-\cos(2\theta)}{2}$, we get $U(t) = \frac{1}{4}kA^2(1 - \cos(2\omega t))$. The angular frequency of the potential energy oscillation is $2\omega$, which is double the frequency of the displacement oscillation. This means that for every one full cycle of displacement, the potential energy completes two full cycles.

**Step 4: Final Answer:**

Let's check the options against these criteria: - Option (A) is incorrect because energy cannot be negative. - Option (C) is incorrect because potential energy is zero at the mean position. - Option (B) correctly shows that $U \geq 0$, $U = 0$ at points corresponding to O and B on the $x - t$ graph, and $U = U_{max}$ at points corresponding to A and C. It also shows two energy cycles for one displacement cycle. - Option (D) might have similar features, but Graph (B) is the standard representation matching all criteria.

Therefore, the graph with ID 86435168204 is the correct representation.

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## 25. Answer: 2 - 2

### Explanation:

**Step 1: Understanding the Question:**

We are given two equations representing simple harmonic motions. We need to find the ratio of the amplitude of the second motion to the amplitude of the first motion.

**Step 2: Key Formula or Approach:**

1. The standard form for a simple harmonic motion is $x = A\sin(\omega t + \phi)$, where A is the amplitude.
2. The first equation is already in this standard form.
3. The second equation needs to be converted to the standard form using the trigonometric identity: $a\sin\theta + b\cos\theta = R\sin(\theta + \alpha)$, where $R = \sqrt{a^2 + b^2}$ and $\tan\alpha = b/a$.

**Step 3: Detailed Explanation:**

**First Motion:**

The equation is $x_1 = 5\sin(2\pi t + \frac{\pi}{4})$.

Comparing this with the standard form $x = A\sin(\omega t + \phi)$, the amplitude of the first motion is $A_1 = 5$.

**Second Motion:**

The equation is $x_2 = 5\sqrt{2}(\sin(2\pi t) + \cos(2\pi t))$.

Let's simplify the term in the parenthesis: $\sin(2\pi t) + \cos(2\pi t)$.

Here, $a = 1$ and $b = 1$. The resultant amplitude of this part is $R = \sqrt{1^2 + 1^2} = \sqrt{2}$.
The phase angle $\alpha$ is given by $\tan\alpha = 1/1 = 1$, so $\alpha = \pi/4$.
Therefore, $\sin(2\pi t) + \cos(2\pi t) = \sqrt{2}\sin(2\pi t + \pi/4)$.
Substitute this back into the equation for $x_2$:

$$x_2 = 5\sqrt{2}\left[\sqrt{2}\sin(2\pi t + \frac{\pi}{4})\right]$$

$$x_2 = 5(\sqrt{2}\cdot\sqrt{2})\sin(2\pi t + \frac{\pi}{4})$$

$$x_2 = 10\sin(2\pi t + \frac{\pi}{4})$$

Comparing this with the standard form, the amplitude of the second motion is $A_2 = 10$.

**Ratio of Amplitudes:**

The question asks for the ratio $\frac{A_2}{A_1}$.

$$\frac{A_2}{A_1} = \frac{10}{5} = 2$$

**Step 4: Final Answer:**

The amplitude of the second motion is 2 times the amplitude of the first motion.