Work, Energy and Power JEE Main PYQ - 1

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Work, Energy and Power

1. A body of mass 4 kg is placed on a plane at a point P having coordinate $(3, 4)$ m. Under the action of force $\vec{F} = (2\hat{i} + 3\hat{j})$ N, it moves to a new point Q having coordinates $(6, 10)$ m in 4 sec. The average power and instantaneous power at the end of 4 sec are in the ratio of: (+4, -1)

a. $6 : 13$

b. $4 : 3$

c. $1 : 2$

d. $6 : 13$

---

2. A ball of mass 100 g is projected with velocity 20 m/s at 60° with horizontal. The decrease in kinetic energy of the ball during the motion from point of projection to highest point is: (+4, -1)

a. 5 J

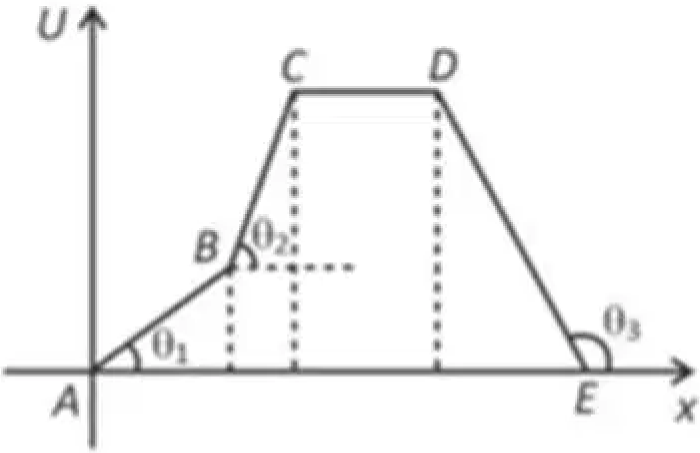
b. 15 J

c. 20 J

d. zero

---

3. A curve is given between potential energy of a particle and its position on the x-axis. (+4, -1)



Given: $\tan\theta_1 = 1$, $\tan\theta_2 = 3$, $\tan\theta_3 = -\frac{1}{2}$
If F_{AB} be the force acting on the particle during A to B, similarly F_{BC}, F_{CD}, F_{DE} are the forces during B to C, C to D, and D to E respectively. Arrange magnitudes of these forces in decreasing order.

a. $F_{BC} > F_{AB} > F_{DE} > F_{CD}$

b. $F_{AB} > F_{BC} > F_{DE} > F_{CD}$

c. $F_{AB} > F_{BC} > F_{CD} > F_{DE}$

d. $F_{BC} > F_{DE} > F_{AB} > F_{CD}$

---

4. Position x of the particle of mass 2 kg varies as function of time as $x = t^2 + t + 1$. Find out work done on the particle from $t_1 = 2$ sec to $t_2 = 3$ sec. **(+4, -1)**

a. 18 joule

b. 30 joule

c. 34 joule

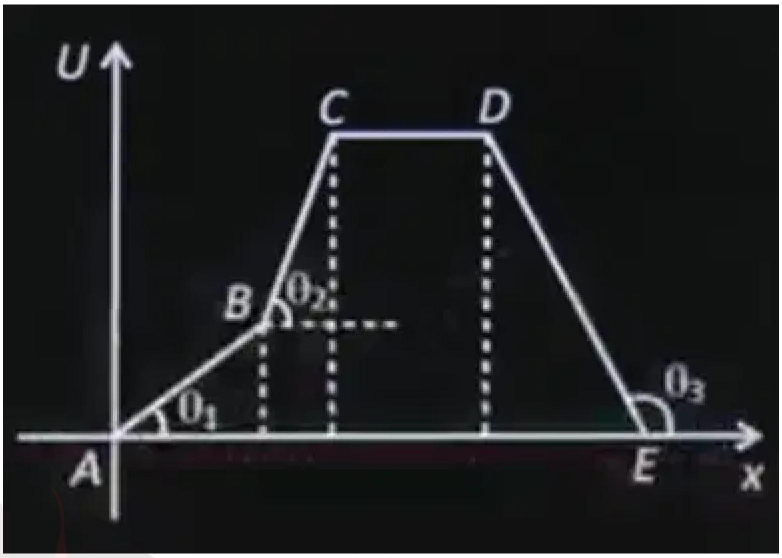
d. 24 joule

---

5. **A curve is given between the potential energy U of a particle and its position on the x-axis as shown. Given:** **(+4, -1)**

$$\tan\theta_1 = 1, \qquad \tan\theta_2 = 3, \qquad \tan\theta_3 = -\frac{1}{2}$$

If F_{AB} is the force acting on the particle during motion from A to B, similarly F_{BC}, F_{CD} and F_{DE} are the forces during B to C, C to D and D to E respectively, arrange the magnitudes of these forces in decreasing order.



a. $F_{BC} > F_{AB} > F_{CD} > F_{DE}$

b. $F_{BC} > F_{AB} > F_{DE} > F_{CD}$

c. $F_{AB} > F_{BC} > F_{DE} > F_{CD}$

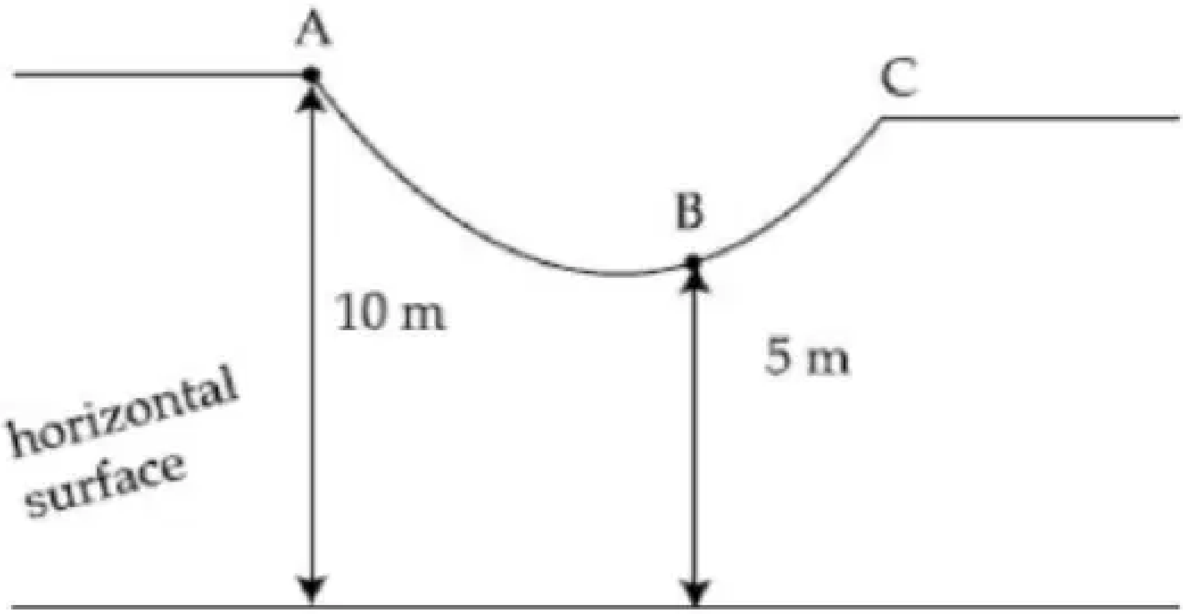
d. $F_{BC} > F_{DE} > F_{AB} > F_{CD}$

---

6. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies (K.E.)A : (K.E.)B will be A/1, so the value of A will be ______ (+4, -1)

---

7. As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take g = 10 m/s²) The value of 'x' to the nearest integer is __________. [Note: Usually $h_A = 10$m and $h_B = 5$m in this problem] (+4, -1)



8. A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to : (+4, -1)

a. $t^3/2$

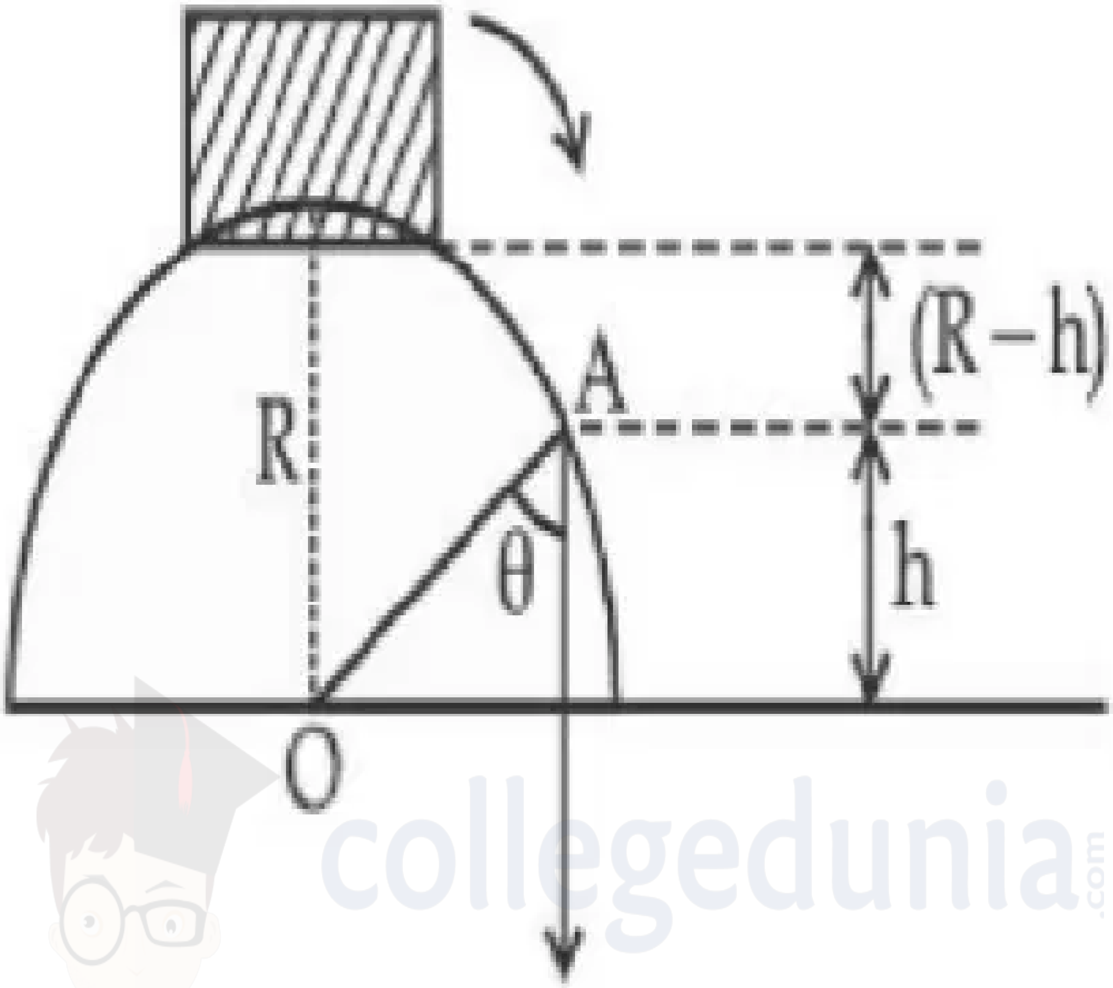
b. $t^1/2$

c. $t^2/3$

d. t

9. A pendulum bob has a speed of 3 m/s at its lowest position. The pendulum is 50 cm long. The speed of bob, when the length makes an angle of 60° to the vertical will be $(g = 10\ \mathrm{m/s^2})$ ________ m/s. (+4, -1)

10. A small block slides down from the top of hemisphere of radius R=3 m as shown in the figure. The height 'h' at which the block will lose contact with the surface of the sphere is ________ m. (Assume there is no friction between the block and the hemisphere) (+4, -1)



11. An automobile of mass 'm' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by: (+4, -1)

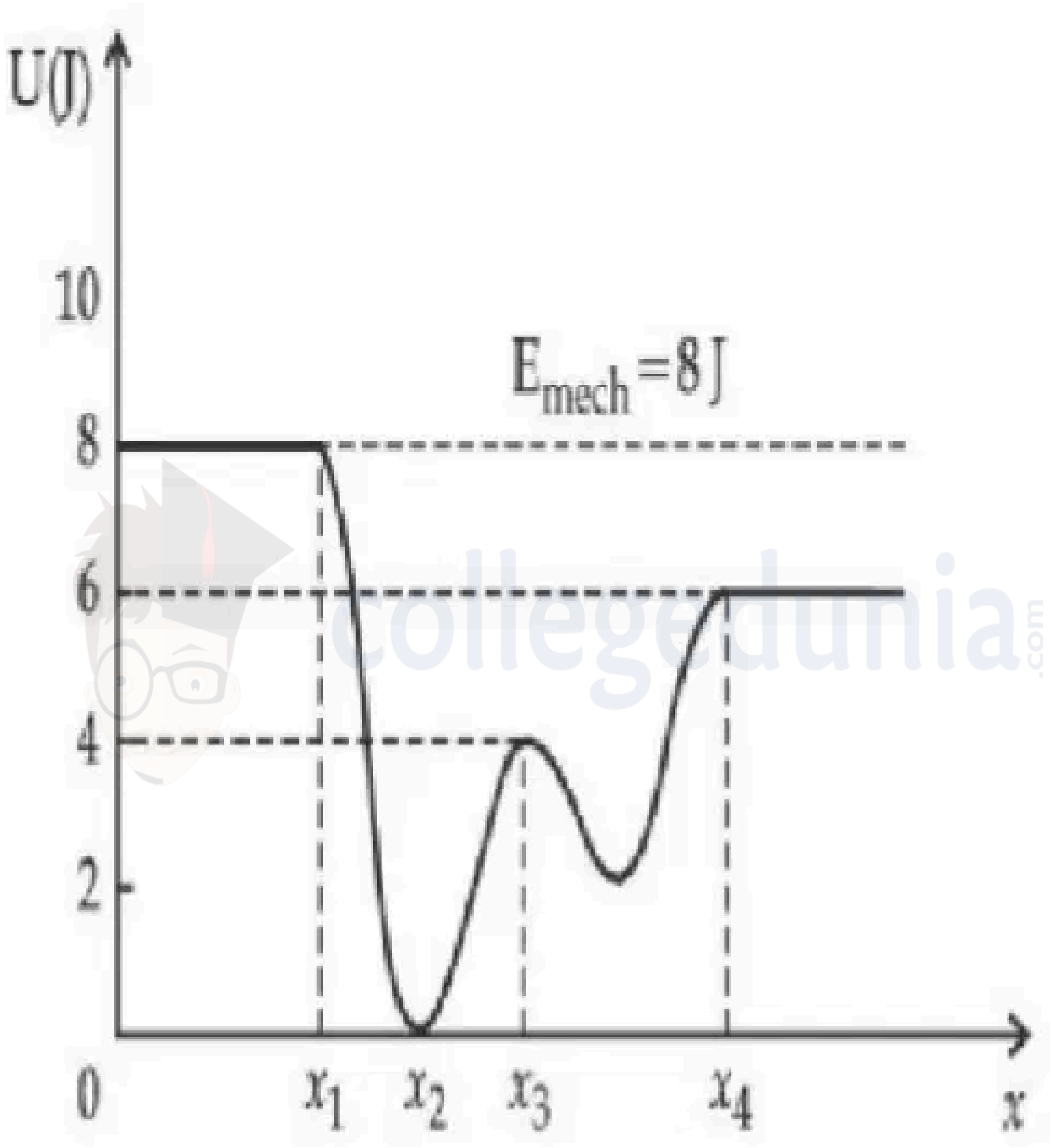
a. $\left(\frac{9P}{8m}\right)^{1/3}t^{2}$

b. $\left(\frac{8P}{9m}\right)^{1/2}t^{2/3}$

c. $\left(\frac{8P}{9m}\right)^{1/2}t^{3/2}$

d. $\left(\frac{9m}{8P}\right)^{1/2}t^{3/2}$

12. Given below is the plot of a potential energy function U(x) for a system, in which a particle is in one dimensional motion, while a conservative force F(x) acts on it. Suppose that $E_{mech} = 8$ J, the incorrect statement for this system is : (+4, -1)



[where K.E. = kinetic energy]

a. at $x = x_2$, K.E. is greatest and the particle is moving at the fastest speed.

b. at $x < x_1$, K.E. is smallest and the particle is moving at the slowest speed.

c. at $x > x_4$, K.E. is constant throughout the region.

d. at $x = x_3$, K.E. = 4 J.

---

13. A ball of mass 4 kg, moving with a velocity of 10 ms^{-1}, collides with a spring of length 8 m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is __________. (+4, -1)

---

14. The potential energy (U) of a diatomic molecule is $U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = _________. (+4, -1)

---

15. Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is n : 2. The value of n will be ________ . (+4, -1)

---

16. A block moving horizontally on a smooth surface with a speed of $40\,ms^{-1}$ splits into two equal parts. If one of the parts moves at $60\,ms^{-1}$ in the same direction, then the fractional change in the kinetic energy will be $x : 4$ where $x = \ldots\ldots\ldots$ (+4, -1)

---

17. Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angles 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is ________. (+4, -1)

---

18. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is _________.
(Take $g = 10\,\mathrm{m/s^2}$) (+4, -1)

---

19. A force $\mathbf{F} = (2 + 3x)\hat{i})$ acts on a particle in the x-direction where F is in newton and x is in meter. The work done by this force during a displacement from $x = 0$ to $x = 4\,\mathrm{m}$ is _____ J. (+4, -1)

---

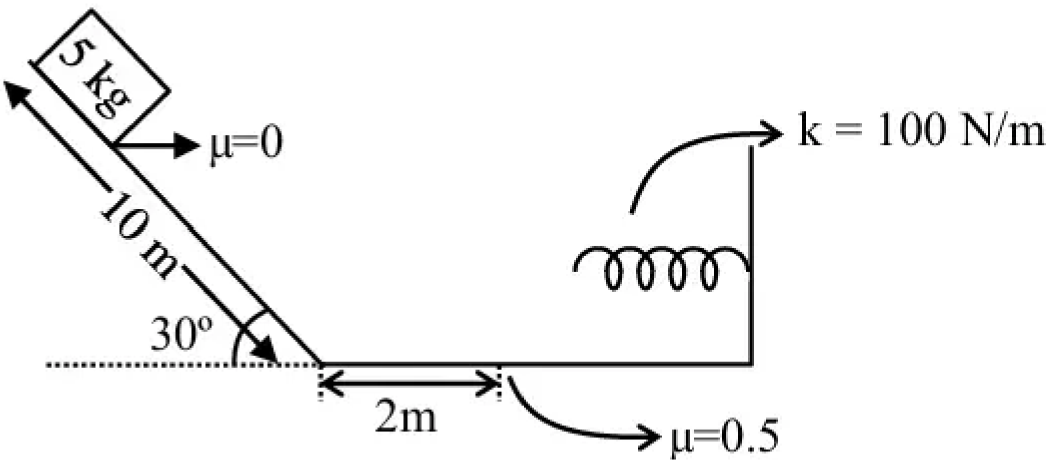
20. A player caught a cricket ball of mass 150 g moving at a speed of 20 m/s. If the catching process is completed in 0.1 s, the magnitude of force exerted by the ball on the hand of the player is: (+4, -1)

a. 150 N

b. 3 N

c. 30 N

d. 300 N



21. (+4, -1)

A block is simply released from the top of an inclined plane as shown in the figure above. The maximum compression in the spring when the block hits the spring is :

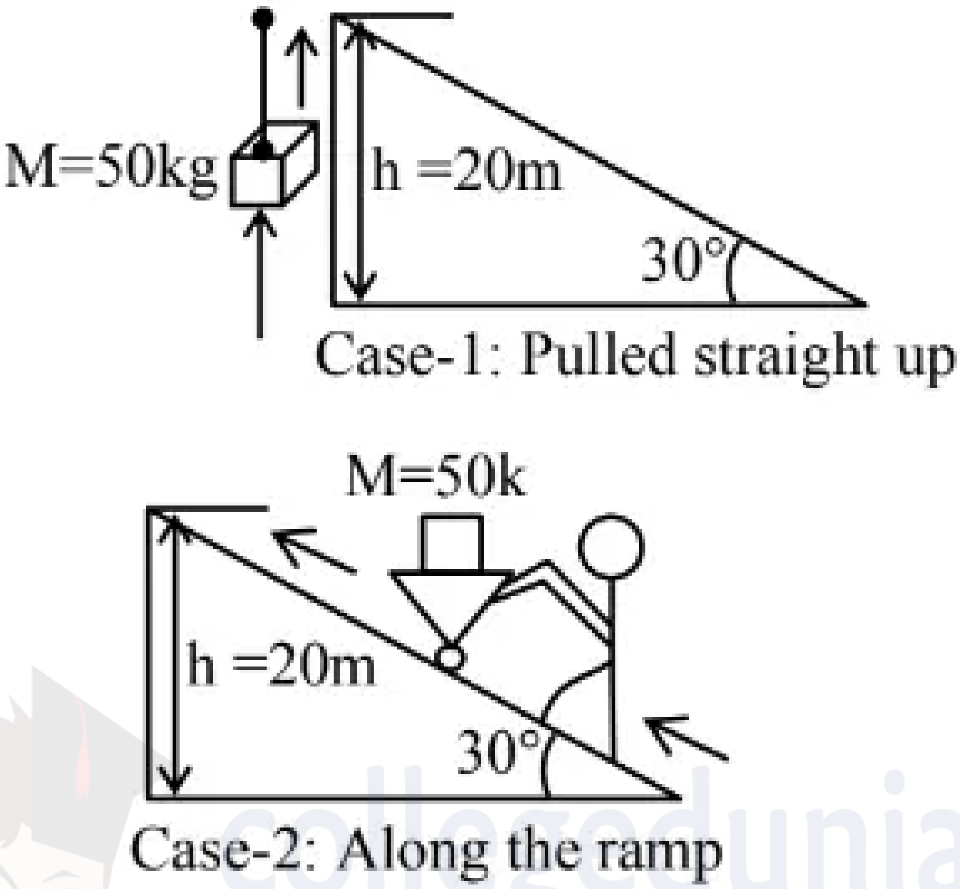
a. $\sqrt{6}\,\mathrm{m}$

b. $2\,\mathrm{m}$

c. $1\,\mathrm{m}$

d. $\sqrt{5}\,\mathrm{m}$

22. A body of mass 50 kg is lifted to a height of 20 m from the ground in the two different ways as shown in the figures. The ratio of work done against the gravity in both the respective cases, will be: (+4, -1)



a. 1:1

b. 2:1

c. $\sqrt{3} : 2$

d. 1:2

---

23. A common example of alpha decay is (+4, -1)

$$^{238}_{92}U \longrightarrow {}^{234}_{90}Th + {}_2He^4 + Q$$

Given :

$^{238}_{92}U = 238.05060\,u$,

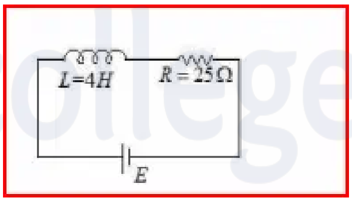
$^{234}_{90}Th = 234.04360\,u$,

$^{4}_{2}He = 4.00260\,u$ and

$1u = 931.5\,\frac{MeV}{c^2}$

---

24. In the given figure, an inductor and a resistor are connected in series with a battery of emf E volt. $\frac{E^a}{2b}$ j/s represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\frac{b}{a}$ will be ______. **(+4, -1)**



---

25. Given below are two statements: **(+4, -1)**

Statement I : The diamagnetic property depends on temperature.

Statement II : The included magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.

In the light of given statement, choose the correct answer from the options given below:

a. Both Statement I and Statement II are true.

b. Both Statement I and Statement II are false.

c. Statement I is correct but Statement II is false.

d. Statement I is incorrect but Statement II is true

---

26. An engine operating between the boiling and freezing points of water will have **(+4, -1)**

1. efficiency more than 27%
2. efficiency less than the efficiency a Carnot engine operating between the same two temperatures.
3. efficiency equal to 27%
4. efficiency less than 27%

a. B and C only

b. B and D only

c. A and B only

d. B, C and D only

---

27. The amplitude of 15sin(1000πt) is modulated by 10sin(4πt) signal. The amplitude modulated signal contains frequency (ies) of **(+4, -1)**

(A) 500 Hz
(B) 2 Hz
(C) 250 Hz
(D) 498 Hz (E) 502 Hz

Choose the correct answer from the options given below:

a. A, D and E only

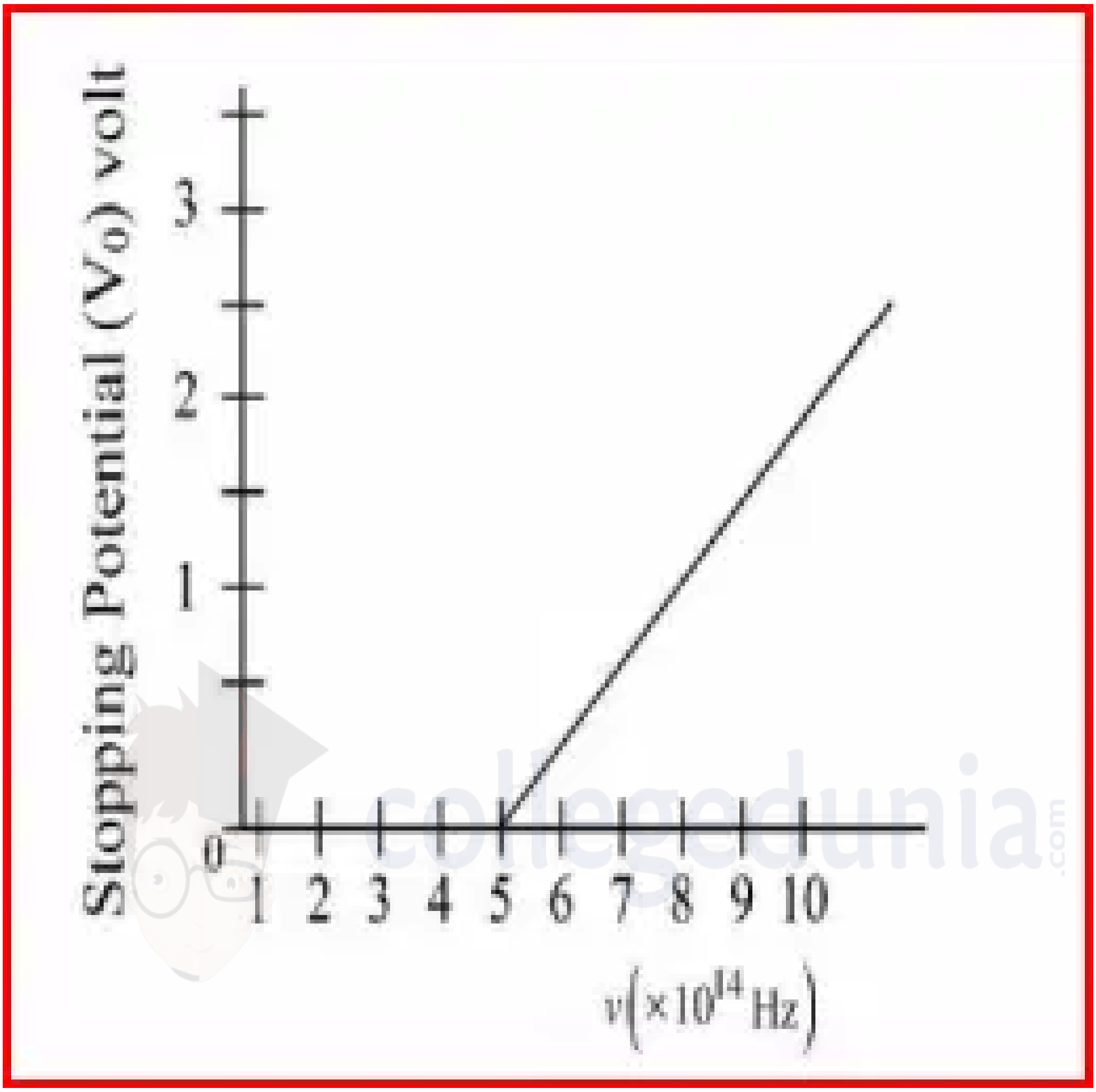
b. A and B only

c. A and B only

d. A and D only

---

28. The variation of stopping potential (V0) as a function of the frequency (v) of the incident light for a metal is shown in figure. The work function of the surface is **(+4, -1)**



a. 1.36 eV

b. 2.98 eV

c. 2.07 eV

d. 18.6 eV

---

29. In an experiment with Vernier callipers of least count 0.1 mm, when two jaws are joined together the zero of Vernier scale lies right to the zero of the main scale and 6th division of Vernier scale coincides with the main scale division. While measuring the diameter of a spherical bob, the zero of vernier scale lies in between 3.2 cm and 3.3 cm marks, and 4th division of vernier **(+4, -1)**

scale coincides with the main scale division. The diameter of bob is measured as

a. 3.26 cm

b. 3.25 cm

c. 3.22 cm

d. 3.18 cm

---

30. The amplitude of $15\sin(1000\pi t)$ is modulated by $10\sin(4\pi t)$ signal. The amplitude modulated signal contains frequency(ies) of **(+4, -1)**

(A) 500 Hz
(B) 2 Hz
(C) 250 Hz
(D) 498 Hz
(E) 502 Hz

Choose the correct answer from the options given below:

a. A, D and E only

b. A and B only

c. A and C only

d. A and D only

Answers

1. **Answer: a**

Explanation:

Concept:

Average power $= \dfrac{\text{Work done}}{\text{Time}}$
Instantaneous power $= \vec{F} \cdot \vec{v}$
Work done by a constant force $= \vec{F} \cdot \vec{s}$
Step 1: Displacement of the body:

$$\vec{s} = (6-3)\hat{i} + (10-4)\hat{j} = 3\hat{i} + 6\hat{j}$$

Step 2: Work done by the force:

$$W = \vec{F} \cdot \vec{s} = (2\hat{i} + 3\hat{j}) \cdot (3\hat{i} + 6\hat{j})$$

$$W = 6 + 18 = 24\,\text{J}$$

Step 3: Average power:

$$P_{\text{avg}} = \frac{W}{t} = \frac{24}{4} = 6\,\text{W}$$

Step 4: Acceleration of the body:

$$\vec{a} = \frac{\vec{F}}{m} = \frac{1}{4}(2\hat{i} + 3\hat{j}) = 0.5\hat{i} + 0.75\hat{j}$$

Step 5: Velocity at the end of 4 sec (initial velocity zero):

$$\vec{v} = \vec{a}t = 4(0.5\hat{i} + 0.75\hat{j}) = 2\hat{i} + 3\hat{j}$$

Step 6: Instantaneous power at $t = 4$ sec:

$$P_{\text{inst}} = \vec{F} \cdot \vec{v} = (2\hat{i} + 3\hat{j}) \cdot (2\hat{i} + 3\hat{j})$$

$$P_{\text{inst}} = 4 + 9 = 13\,\text{W}$$

Step 7: Required ratio:

$$P_{\text{avg}} : P_{\text{inst}} = 6 : 13$$

---

2. Answer: b

Explanation:

Concept:
In projectile motion:
The horizontal component of velocity remains constant.
The vertical component of velocity becomes zero at the highest point
Decrease in kinetic energy from projection to highest point is equal to the loss of kinetic energy associated with the vertical component of velocity.
Step 1: Given data:

$$m = 100\,\text{g} = 0.1\,\text{kg}, \quad u = 20\,\text{m/s}, \quad \theta = 60^\circ$$

Step 2: Vertical component of velocity:

$$u_y = u \sin 60^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}\,\text{m/s}$$

Step 3: Kinetic energy associated with vertical motion:

$$\Delta K = \frac{1}{2} m u_y^2$$

$$\Delta K = \frac{1}{2} \times 0.1 \times (10\sqrt{3})^2$$

$$\Delta K = 0.05 \times 300 = 15\,\text{J}$$

---

3. Answer: b

Explanation:

Step 1: Force is the negative derivative of potential energy.
The force acting on the particle is given by $F = -\frac{dU}{dx}$.
Step 2: Understanding the graph.
From the given graph, the slope of the potential energy curve gives the magnitude of the force. The steeper the slope, the greater the force.
Step 3: Compare slopes.
The slope of the potential energy curve is greatest between A to B, followed by B to C, then D to E, and the least between C to D. Therefore, the order of forces is:

$$F_{AB} > F_{BC} > F_{DE} > F_{CD}$$

Step 4: Conclusion.
The correct answer is option (2).

---

4. Answer: d

Explanation:

Step 1: Understand the formula for work done.
The work done on a particle is given by the change in kinetic energy:

$$W = \Delta K.E. = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

where m is the mass of the particle, and v_1 and v_2 are the initial and final velocities of the particle.
Step 2: Find the velocity of the particle.
The position x of the particle is given by:

$$x = t^2 + t + 1$$

To find the velocity, we differentiate x with respect to time t:

$$v = \frac{dx}{dt} = 2t + 1$$

Step 3: Calculate the velocities at $t_1 = 2$ sec and $t_2 = 3$ sec.
At $t_1 = 2$ sec:

$$v_1 = 2(2) + 1 = 5\,\mathrm{m/s}$$

At $t_2 = 3$ sec:

$$v_2 = 2(3) + 1 = 7\,\text{m/s}$$

Step 4: Calculate the work done.
The mass of the particle is given as $m = 2\,\text{kg}$. Using the work-energy theorem:

$$W = \frac{1}{2} \times 2\left(7^2 - 5^2\right)$$

$$W = (49 - 25) = 24\,\text{joules}$$

Thus, the work done on the particle is 24 joules.

---

5. **Answer: b**

Explanation:

Force acting on a particle is related to the potential energy by:

$$F = \left|-\frac{dU}{dx}\right|$$

Hence, the magnitude of force is equal to the magnitude of the slope of the U--x graph.
On segment AB: slope $= \tan\theta_1 = 1 \Rightarrow F_{AB} \propto 1$
On segment BC: slope $= \tan\theta_2 = 3 \Rightarrow F_{BC} \propto 3$
On segment CD: slope $= 0 \Rightarrow F_{CD} = 0$
On segment DE: slope $= |\tan\theta_3| = \frac{1}{2} \Rightarrow F_{DE} \propto \frac{1}{2}$ **Arranging in decreasing order of magnitude:**

$$F_{BC} > F_{AB} > F_{DE} > F_{CD}$$

---

6. **Answer: 2 - 2**

Explanation:

Step 1: The relationship between Kinetic Energy (K) and Momentum (p) is given by $K = \frac{p^2}{2m}$.

Step 2: Since momentum p is equal for both, $K \propto \frac{1}{m}$.
Step 3: Therefore, $\frac{K_A}{K_B} = \frac{m_B}{m_A} = \frac{2}{1}$. Comparing this to $A/1$, we find $A = 2$.

---

7. **Answer: 10 - 10**

Explanation:

Step 1: Use Law of Conservation of Energy: $mgh_A = mgh_B + \frac{1}{2}mv^2$.
Step 2: $v = \sqrt{2g(h_A - h_B)}$. If $\Delta h = 5\text{m}$:
Step 3: $v = \sqrt{2 \times 10 \times 5} = \sqrt{100} = 10 \text{ m/s}$.

---

8. **Answer: a**

Explanation:

Step 1: Power $P = Fv = (ma)v = m\frac{dv}{dt}v$.
Step 2: $Pdt = mvdv \implies Pt = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{2P}{m}}t^{1/2}$.
Step 3: $v = \frac{ds}{dt} = kt^{1/2} \implies s = \int kt^{1/2}dt = k't^{3/2}$. Therefore, $s \propto t^{3/2}$.

---

9. **Answer: 2 - 2**

Explanation:

Step 1: Understanding the Concept:
The motion of a pendulum is governed by the Law of Conservation of Mechanical Energy. As the bob rises, kinetic energy is converted into potential energy.
Step 2: Key Formula or Approach:
1. Potential energy gain: $h = L(1 - \cos\theta)$.
2. Energy conservation: $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgh$.
Step 3: Detailed Explanation:
Given: $u = 3 \text{ m/s}$, $L = 0.5 \text{ m}$, $\theta = 60^\circ$.
Calculate the vertical height h:

$$h = 0.5(1 - \cos 60^\circ) = 0.5(1 - 0.5) = 0.25 \text{ m}$$

Using conservation of energy:

$$\frac{1}{2}(3)^2 = \frac{1}{2}v^2 + (10 \times 0.25)$$

$$4.5 = 0.5v^2 + 2.5$$

$$0.5v^2 = 2 \implies v^2 = 4$$

$$v = 2 \text{ m/s}$$

Step 4: Final Answer:
The speed of the bob at 60° is 2 m/s.

---

10. **Answer: 1 - 1**

Explanation:

Let the block of mass 'm' lose contact at a point A, which is at a vertical height 'h' from the top.
Let θ be the angle that the radius to point A makes with the vertical. The vertical distance dropped is $h = R(1 - \cos\theta)$.
At point A, the forces acting on the block are the normal force (N) and gravity (mg).
The component of gravity towards the center of the hemisphere is $mg\cos\theta$.
The net force towards the center provides the centripetal force:
$mg\cos\theta - N = \frac{mv^2}{R}$.
The block loses contact when the normal force N becomes zero.
At $N = 0$, we have $mg\cos\theta = \frac{mv^2}{R}$, which simplifies to $v^2 = gR\cos\theta$. (Equation 1)
Now, apply the principle of conservation of mechanical energy between the top of the hemisphere (initial position) and point A (final position).
Initial Energy (at top): $E_i = K.E._i + P.E._i = 0 + mgR$ (taking the base of the hemisphere as the reference level).
Final Energy (at A): $E_f = K.E._f + P.E._f = \frac{1}{2}mv^2 + mg(R - h) = \frac{1}{2}mv^2 + mgR\cos\theta$.
Equating initial and final energies:

$mgR = \frac{1}{2}mv^2 + mgR\cos\theta$.
$mgR(1-\cos\theta) = \frac{1}{2}mv^2$.
$v^2 = 2gR(1-\cos\theta)$. (Equation 2)
Now, equate the two expressions for v^2 from Equation 1 and Equation 2.
$gR\cos\theta = 2gR(1-\cos\theta)$.
$\cos\theta = 2 - 2\cos\theta$.
$3\cos\theta = 2 \implies \cos\theta = \frac{2}{3}$.
The height 'h' at which the block loses contact is the vertical distance it has dropped.
$h = R(1-\cos\theta) = R\left(1-\frac{2}{3}\right) = \frac{R}{3}$.
Given the radius R = 3 m:
$h = \frac{3}{3} = 1$ m.

---

11. Answer: c

Explanation:

When an engine supplies constant power P, this power is used to increase the kinetic energy of the automobile.
Power $P = \frac{dK}{dt}$, where K is the kinetic energy.
Since the automobile starts from rest, its kinetic energy at any time t is $K = \frac{1}{2}mv^2$.
$P = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$.
Since P is constant, we can integrate with respect to time to find the kinetic energy:
$\int_0^t P dt = \int_0^K dK \implies Pt = K = \frac{1}{2}mv^2$.
From this, we can find the velocity v as a function of time.
$v^2 = \frac{2Pt}{m} \implies v = \sqrt{\frac{2Pt}{m}} = \left(\frac{2P}{m}\right)^{1/2} t^{1/2}$.
Position x is the integral of velocity v with respect to time. Since it starts from the origin, $x(0) = 0$.
$x = \int_0^t v dt = \int_0^t \left(\frac{2P}{m}\right)^{1/2} t^{1/2} dt$.
$x = \left(\frac{2P}{m}\right)^{1/2} \int_0^t t^{1/2} dt$.
$x = \left(\frac{2P}{m}\right)^{1/2} \left[\frac{t^{3/2}}{3/2}\right]_0^t = \left(\frac{2P}{m}\right)^{1/2} \frac{2}{3} t^{3/2}$.
To match the format of the options, we bring the constant $\frac{2}{3}$ inside the square root.
$x = \sqrt{\left(\frac{2}{3}\right)^2 \left(\frac{2P}{m}\right)} t^{3/2}$.
$x = \sqrt{\frac{4}{9} \cdot \frac{2P}{m}} t^{3/2} = \sqrt{\frac{8P}{9m}} t^{3/2}$.
This can be written as $x = \left(\frac{8P}{9m}\right)^{1/2} t^{3/2}$.

---

12. Answer: b

Explanation:

The total mechanical energy of the system is constant, $E_{mech} = K.E. + U(x) = 8$ J.
The kinetic energy is given by $K.E. = E_{mech} - U(x) = 8 - U(x)$.
Since K.E. must be non-negative ($K.E. = \frac{1}{2}mv^2 \geq 0$), the particle can only exist in regions where $U(x) \leq E_{mech}$.
Let's analyze each statement:
(A) At $x = x_2$, the potential energy $U(x_2)$ is at its minimum value (close to 0 J from the graph).
Therefore, $K.E. = 8 - U(x_2)$ will be maximum. Since $K.E. = \frac{1}{2}mv^2$, maximum K.E. implies the fastest speed. This statement is correct.
(C) For the region $x > x_4$, the graph shows that the potential energy $U(x)$ is constant at a value of 6 J.
So, $K.E. = 8 - 6 = 2$ J. Since U(x) is constant, K.E. is also constant. This statement is correct.
(D) At $x = x_3$, the graph shows the potential energy $U(x_3)$ is 4 J.
So, $K.E. = 8 - U(x_3) = 8 - 4 = 4$ J. This statement is correct.
(B) For the region $x < x_1$, the graph shows that the potential energy $U(x)$ is constant at 8 J.
According to the principle of conservation of energy, a particle with total energy $E_{mech} = 8$ J can only access regions where $U(x) \leq 8$ J.
The region $x < x_1$ has $U(x) = 8$ J, so K.E. would be $8 - 8 = 0$. The particle can reach the point $x = x_1$ (this is a turning point), but it cannot enter the region $x < x_1$.
This region is classically forbidden for the particle. Therefore, any statement describing the particle's motion "at $x < x_1$" is fundamentally incorrect because the particle cannot be there.

---

13. Answer: 6 - 6

Explanation:

Step 1: Use Conservation of Mechanical Energy. The kinetic energy of the ball is converted into elastic potential energy of the spring at maximum compression.

$$\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta L)^2$$

Step 2: Solve for compression (ΔL).

$$4 \times (10)^2 = 100 \times (\Delta L)^2 \implies 400 = 100(\Delta L)^2$$

$$(\Delta L)^2 = 4 \implies \Delta L = 2 \text{ m}$$

Step 3: Find the final length of the spring (x).

$$x = L_{initial} - \Delta L = 8 \text{ m} - 2 \text{ m} = 6 \text{ m}$$

---

14. Answer: 1 - 1

Explanation:

Step 1: At equilibrium, the force $F = -\frac{dU}{dr} = 0$.
Step 2: $\frac{dU}{dr} = \alpha(-10r^{-11}) - \beta(-5r^{-6}) = 0$.
Step 3: $-\frac{10\alpha}{r^{11}} + \frac{5\beta}{r^6} = 0 \Rightarrow \frac{5\beta}{r^6} = \frac{10\alpha}{r^{11}}$.
Step 4: $r^5 = \frac{10\alpha}{5\beta} = \frac{2\alpha}{\beta}$.
Step 5: $r = \left(\frac{2\alpha}{\beta}\right)^{1/5}$.
Step 6: Comparing with $\left(\frac{2\alpha}{\beta}\right)^{a/b}$, we get $a = 1$ (and $b = 5$).

---

15. Answer: 1 - 1

Explanation:

The relationship between kinetic energy (KE) and linear momentum (p) is given by $KE = \frac{p^2}{2m}$.
This can be rearranged to express momentum as $p = \sqrt{2m(KE)}$.
Let the two particles be particle 1 and particle 2, with masses $m_1 = 4$ g and $m_2 = 16$ g.
We are given that their kinetic energies are equal: $KE_1 = KE_2$.
Let's find the ratio of their momenta, p_1/p_2:
$\frac{p_1}{p_2} = \frac{\sqrt{2m_1(KE_1)}}{\sqrt{2m_2(KE_2)}}$.
Since $KE_1 = KE_2$, the equation simplifies to:
$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$.
Substituting the given masses:

$\frac{p_1}{p_2} = \sqrt{\frac{4}{16}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$.
The ratio of their linear momenta is 1 : 2.
The problem states that the ratio is n : 2.
By comparing the two ratios, we find that n = 1.

---

16. Answer: 1 - 1

Explanation:

Step 1: Understanding the Concept:
This problem involves the conservation of linear momentum and the calculation of kinetic energy before and after the explosion (splitting) of the block.
Step 2: Key Formula or Approach:
1. Conservation of Momentum: $m_1v_1 + m_2v_2 = MV$.
2. Kinetic Energy: $K = \frac{1}{2}mv^2$.
3. Fractional Change: $\frac{\Delta K}{K_i} = \frac{K_f - K_i}{K_i}$.
Step 3: Detailed Explanation:
Let the total mass be M. The block splits into two equal parts, so each mass is $M/2$.
1. **Initial State:**
- Initial speed $V = 40\,ms^{-1}$.
- Initial Momentum: $P_i = M(40)$.
- Initial Kinetic Energy: $K_i = \frac{1}{2}M(40)^2 = 800M$.
2. **Final State:**
- Mass 1 $(M/2)$ moves at $v_1 = 60\,ms^{-1}$.
- Mass 2 $(M/2)$ moves at speed v_2.
- By Momentum Conservation: $M(40) = \frac{M}{2}(60) + \frac{M}{2}v_2$.

$$40 = 30 + \frac{v_2}{2} \implies \frac{v_2}{2} = 10 \implies v_2 = 20\,ms^{-1}$$

- Final Kinetic Energy: $K_f = \frac{1}{2}(\frac{M}{2})(60)^2 + \frac{1}{2}(\frac{M}{2})(20)^2$

$$K_f = \frac{M}{4}(3600 + 400) = \frac{4000M}{4} = 1000M$$

3. **Fractional Change:**
- Change in KE: $\Delta K = K_f - K_i = 1000M - 800M = 200M$.
- Fractional Change: $\frac{\Delta K}{K_i} = \frac{200M}{800M} = \frac{1}{4}$.

Comparing with $x : 4$, we get $x = 1$.
Step 4: Final Answer:
The value of x is 1.

---

17. Answer: 2 - 2

Explanation:

Step 1: Understanding the Question:
Two forces do the same amount of work over the same distance, but they act at different angles. We need to find the ratio of the magnitudes of these forces.
Step 2: Key Formula or Approach:
The work done (W) by a constant force (F) that makes an angle θ with the displacement (d) is given by:

$$W = Fd\cos\theta$$

We are given that the work done by person A (W_A) is equal to the work done by person B (W_B).
Step 3: Detailed Explanation:
Let F_A be the force applied by person A at an angle $\theta_A = 45^\circ$.
Let F_B be the force applied by person B at an angle $\theta_B = 60^\circ$.
The displacement is d for both.
Work done by A:

$$W_A = F_A d\cos(45^\circ)$$

Work done by B:

$$W_B = F_B d\cos(60^\circ)$$

Given $W_A = W_B$:

$$F_A d\cos(45^\circ) = F_B d\cos(60^\circ)$$

The displacement d cancels out:

$$F_A \cos(45^\circ) = F_B \cos(60^\circ)$$

We need to find the ratio $\frac{F_A}{F_B}$.

$$\frac{F_A}{F_B} = \frac{\cos(60^\circ)}{\cos(45^\circ)}$$

Now, substitute the values of the trigonometric functions:

$$\cos(60^\circ) = \frac{1}{2}$$

$$\cos(45^\circ) = \frac{1}{\sqrt{2}}$$

So, the ratio is:

$$\frac{F_A}{F_B} = \frac{1/2}{1/\sqrt{2}} = \frac{1}{2} \times \frac{\sqrt{2}}{1} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

The problem states that this ratio is equal to $\frac{1}{\sqrt{x}}$.

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{x}}$$

By comparing the two sides, we can see that:

$$x = 2$$

Step 4: Final Answer:
The value of x is 2.

---

18. Answer: 40 - 40

Explanation:

Step 1: Understanding the Concept:
This problem can be solved using the principle of conservation of mechanical energy. Since the table is smooth, there is no friction, and the loss in potential energy of the chain as it slips off equals the gain in its kinetic energy.
Step 2: Key Formula or Approach:
1. Potential Energy of a part of the chain: $U = -m_{part} g y_{cm}$, where y_{cm} is the depth of the center of mass of the hanging part from the table level.
2. Conservation of Energy: $U_i + K_i = U_f + K_f$. Since it starts from rest, $K_i = 0$, so $K_f = U_i - U_f$.

Step 3: Detailed Explanation:

Given: Total length $L = 3$ m, Total mass $M = 3$ kg.

Mass per unit length $\lambda = \frac{3\text{ kg}}{3\text{ m}} = 1\text{ kg/m}$.

Initial State:

Length on table = 2 m, Hanging length $h_1 = 3 - 2 = 1$ m.

Mass of hanging part $m_1 = \lambda h_1 = 1$ kg.

Center of mass of the hanging part is at a distance $d_1 = \frac{h_1}{2} = 0.5$ m below the table.

Initial Potential Energy $U_i = -m_1 g d_1 = -(1)(10)(0.5) = -5$ J.

Final State:

The entire chain has slipped off. Hanging length $h_2 = 3$ m.

Total mass hanging $m_2 = 3$ kg.

Center of mass of the chain is now at $d_2 = \frac{h_2}{2} = 1.5$ m below the table.

Final Potential Energy $U_f = -m_2 g d_2 = -(3)(10)(1.5) = -45$ J.

Calculating Kinetic Energy (k):

By conservation of energy:

$$k = U_i - U_f$$

$$k = -5 - (-45)$$

$$k = 40\text{ J}$$

Step 4: Final Answer:

The value of k is 40.

---

19. **Answer: 32 - 32**

Explanation:

The work done by a force is given by the integral:

$$w = \int_{x_1}^{x_2} F(x)\, dx$$

Where $F(x) = 2 + 3x$. Substituting into the integral:

$$w = \int_0^4 (2 + 3x)\, dx$$

Solving this:

$$w = \int_0^4 2\, dx + \int_0^4 3x\, dx = [2x]_0^4 + \left[\frac{3x^2}{2}\right]_0^4$$

$$w = (2 \times 4) + \left(\frac{3 \times 4^2}{2}\right) = 8 + \left(\frac{3 \times 16}{2}\right) = 8 + 24 = 32\,\mathrm{J}$$

Thus, the work done is 32 J.

---

20. Answer: c

Explanation:

To calculate the force exerted by the cricket ball on the hand of the player, we can use the impulse-momentum theorem. The theorem states that the impulse experienced by an object is equal to the change in its momentum. Mathematically, this can be expressed as:

$F \cdot \Delta t = \Delta p$

Where:

- F is the force exerted.
- Δt is the time over which the force is applied.
- Δp is the change in momentum.

First, compute the initial and final momentum:

- The initial momentum $p_i = m \cdot v_i$, where $m = 0.15\,\mathrm{kg}$ (mass of the ball converted to kg) and $v_i = 20\,\mathrm{m/s}$. Thus, $p_i = 0.15 \times 20 = 3\,\mathrm{kg\,m/s}$.
- The final momentum $p_f = 0\,\mathrm{kg\,m/s}$ since the ball is caught and brought to rest.

The change in momentum $\Delta p = p_f - p_i = 0 - 3 = -3\,\mathrm{kg\,m/s}$.

Considering only the magnitude (ignore the negative sign), the impulse is equal to $3\,\mathrm{N\,s}$.

Now, calculate the force using the formula:

$$F = \frac{\Delta p}{\Delta t}$$

Substitute the values:

$$F = \frac{3}{0.1} = 30\,\mathrm{N}$$

Therefore, the magnitude of the force exerted by the ball on the hand of the player is **30 N**.

---

21. Answer: b

Explanation:

To solve this problem, we need to calculate the maximum compression in the spring when the block hits it. We'll use energy conservation principles, factoring in gravitational potential energy, work done by friction, and spring potential energy.

1. Initially, the block is released from rest, meaning its initial kinetic energy is zero. It starts with gravitational potential energy at the incline's top, given by: $U_g = mgh$, where $m = 5\,\mathrm{kg}$, $g = 10\,\mathrm{m/s^2}$ (assuming near the Earth's surface), and $h = 10\sin 30^\circ\,\mathrm{m}$.
2. Calculate the height: $h = 10 \times \frac{1}{2} = 5\,\mathrm{m}$
3. Substitute into the potential energy formula: $U_g = 5 \times 10 \times 5 = 250\,\mathrm{J}$
4. The block slides down the incline and across a rough surface (friction coefficient $\mu = 0.5$) for $2\,\mathrm{m}$. Calculate the work done by friction: $W_f = \mu mgd = 0.5 \times 5 \times 10 \times 2 = 50\,\mathrm{J}$
5. By energy conservation, the gravitational potential energy is converted into spring potential energy and work done against friction: $U_g = \frac{1}{2}kx^2 + W_f$
6. Substitute the known values: $250 = \frac{1}{2} \times 100 \times x^2 + 50$
7. Solve for x:
   - First, simplify the equation: $250 = 50 + 50x^2$
   - Re-arrange to solve for x^2: $200 = 50x^2$
   - Divide both sides by 50: $x^2 = 4$

- Solve for x: $x = 2$

Therefore, the maximum compression in the spring when the block hits the spring is $2\,\text{m}$.

---

22. **Answer: c**

Explanation:

The work done by gravity is independent of the path taken. It depends only on the change in vertical displacement. Therefore, in both cases (pulled straight up and along the ramp), the work done remains the same as the vertical displacement is the same.

---

23. **Answer: 4 - 4**

Explanation:

Energy released Q is given by the equation:

$$Q = (\Delta m)_{\text{amu}} \times 931.5\,\text{MeV}.$$

Here, the mass defect Δm is:

$$\Delta m = m_u - m_{\text{Th}} - m_{\text{He}} = 238.05060\,u - 234.04360\,u - 4.00260\,u = 0.0044\,u.$$

Thus,

$$Q = 0.0044 \times 931.5\,\text{MeV} = 4.0986\,\text{MeV}.$$

Therefore, the energy released during the alpha decay of ${}^{238}_{92}U$ is 4.0986 MeV.

---

24. **Answer: 25 - 25**

Explanation:

Step 1: Power Supplied by the Battery

The power supplied by the battery is given by:

$$P = \frac{E^2}{R},$$

where E is the emf of the battery and R is the resistance. Substituting $R = 25\,\Omega$:

$$P = \frac{E^2}{25}.$$

Step 2: Power Stored in the Inductor

The maximum power stored in the inductor is given as:

$$P_{\text{inductor}} = \frac{E^2}{2b}.$$

Step 3: Relating b and a

Since the total resistance is $R = 25\,\Omega$, and the stored energy in the inductor is proportional to the power supplied:

$$\frac{E^2}{25} = 2 \times \frac{E^2}{2b}.$$

Simplify:

$$b = 25.$$

Step 4: Find the Ratio $\frac{b}{a}$

Since $a = 1$ (from standard proportionality), the ratio is:

$$\frac{b}{a} = 25.$$

---

25. Answer: d

Explanation:

The correct option is(D): Statement I is incorrect but Statement II is true

---

26. Answer: b

Explanation:

The correct option is(B): B and D only

---

27. Answer: a

Explanation:

The correct option is(A): A, D and E only

---

28. Answer: c

Explanation:

Step 1: Understanding the photoelectric equation.
In the photoelectric effect, the stopping potential (V_0) is related to the frequency (v) of the incident light by the equation:

$$eV_0 = h(v - v_{\text{th}})$$

Where:
V_0 is the stopping potential (in volts).
v is the frequency of the incident light (in Hz).
v_{th} is the threshold frequency (below which no photoelectric emission occurs).
e is the charge of the electron $(1.6 \times 10^{-19}\,\text{C})$.
h is Planck's constant $(6.6 \times 10^{-34}\,\text{J} \cdot \text{s})$.
Step 2: Identifying the threshold frequency.
From the graph, we can observe that the stopping potential becomes non-zero at a frequency of approximately $5 \times 10^{14}\,\text{Hz}$. This is the threshold frequency v_{th}.
Step 3: Calculating the work function.
At the threshold frequency, the stopping potential is zero. We use the equation:

$$\phi = hv_{\text{th}}$$

Substituting the values:

$$\phi = (6.6 \times 10^{-34}) \times (5 \times 10^{14}) = 33 \times 10^{-20}\,\text{J}$$

$$\phi = 3.3 \times 10^{-19}\,\text{J}$$

To convert this to eV, divide by the charge of the electron:

$$\phi = \frac{3.3 \times 10^{-19}}{1.6 \times 10^{-19}} \, \text{eV} = 2.07 \, \text{eV}$$

Thus, the work function is $\phi = 2.07 \, \text{eV}$.

---

29. Answer: d

Explanation:

Step 1: Calculating the zero error and the actual measurement. The zero error in the vernier scale is:

$$\text{Zero error} = 6 \times 0.1 \, \text{mm} = 0.6 \, \text{mm} \, (+\text{ve zero error})$$

Now, the diameter measured with the help of the Vernier scale is:

$$\text{Diameter} = \text{Main Scale Reading} + \text{Vernier Scale Reading} \times \text{Least Count}$$

$$\text{Diameter} = 3.2 \, \text{cm} + 0.1 \, \text{mm} \times 4 = 3.2 \, \text{cm} + 0.4 \, \text{mm} = 3.24 \, \text{cm}$$

The actual diameter is:

$$\text{Actual diameter} = 3.24 \, \text{cm} - \text{Zero error} = 3.24 \, \text{cm} - 0.06 \, \text{cm} = 3.18 \, \text{cm}$$

---

30. Answer: a

Explanation:

Solution:
Carrier signal: $15 \sin(1000\pi t)$
Modulating signal: $10 \sin(4\pi t)$

The general form of a sinusoidal wave is $A \sin(2\pi f t)$, where A is the amplitude, f is the frequency, and t is time.

1. Carrier signal frequency (f_c):

$$2\pi f_c = 1000\pi$$

$$f_c = \frac{1000\pi}{2\pi} = 500 \text{ Hz}$$

2. Modulating signal frequency (f_m):

$$2\pi f_m = 4\pi$$

$$f_m = \frac{4\pi}{2\pi} = 2 \text{ Hz}$$

In amplitude modulation, the modulated signal contains the carrier frequency and two sideband frequencies:

Carrier frequency $(f_c) = 500 \text{ Hz}$

Lower sideband frequency $(f_c - f_m) = 500 \text{ Hz} - 2 \text{ Hz} = 498 \text{ Hz}$

Upper sideband frequency $(f_c + f_m) = 500 \text{ Hz} + 2 \text{ Hz} = 502 \text{ Hz}$

The frequencies present in the amplitude modulated signal are:

500 Hz (1)

498 Hz (4)

502 Hz (5)

Therefore, the correct answer is (4) (1), (4) and (5) only.