CBSE Class 12 Mathematics(Set 65/2/3) Question Paper

Time Allowed :3 Hour	**Maximum Marks :**70	**Total Questions :**38

General Instructions

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE Sections – Section A, B, C, D and E.

(iii) In Section A – Question Number 1 to 18 are Multiple Choice Questions (MCQs) and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B – Question Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.

(v) In Section C – Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D – Question Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.

(vii) In Section E – Question Number 36 to 38 are case study based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is NOT allowed.

1. Direction cosines of line $\frac{1-x}{0} = y = z$ **are**

(A) $1, 1, 1$
(B) $0, -\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$
(C) $1, 0, 0$
(D) $0, -1, -1$

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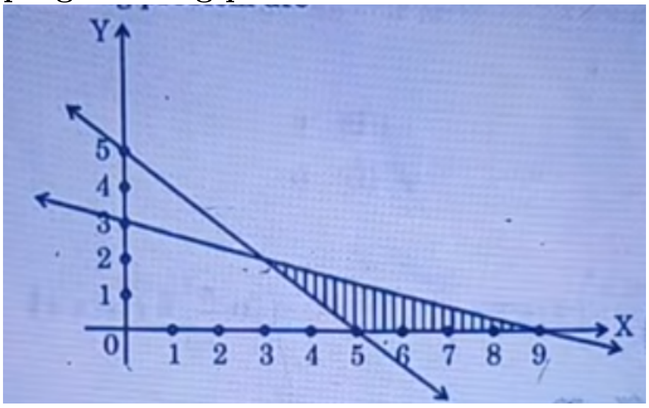
2. In a linear programming problem, the linear function which has to be maximized or minimized is called

(A) a feasible function
(B) an objective function
(C) an optimal function
(D) a constraint

3. **For the feasible region shown below, the non-trivial constraints of the linear programming problem are**



(A) $x + y \leq 5,\ x + 3y \leq 9$
(B) $x + y \leq 5,\ x + 3y \geq 9$
(C) $x + y \geq 5,\ x + 3y \leq 9$
(D) $x + y \geq 5,\ 3x + y \leq 9$

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4. **For two events A and B such that $P(A) \neq 0$ and $P(B) \neq 1$, $P(A'/B') =$**
(A) $1 - P(A/B)$
(B) $1 - P(A'/B)$
(C) $\dfrac{1 - P(A \cap B)}{P(B')}$
(D) $\dfrac{1 - P(A \cup B)}{P(B')}$

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5. **A relation R on set $A = \{1, 2, 3\}$ is defined as $R = \{(1,3), (3,3), (1,1), (2,2), (3,1)\}$ is**
(A) only reflexive and symmetric
(B) reflexive only
(C) only reflexive and transitive
(D) reflexive, symmetric and transitive

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6. **If A and B are square matrices of same order, then which of the following statements is/are always true?**
(i) $(A + B)(A - B) = A^2 - B^2$
(ii) $AB = BA$
(iii) $(A + B)^2 = A^2 + AB + BA + B^2$
(iv) $AB = 0 \Rightarrow A = 0$ or $B = 0$
(A) Only (i) and (iii)
(B) Only (ii) and (iii)
(C) Only (iii)
(D) Only (iii) and (iv)

7. If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ **is a symmetric matrix, then the value of** $3a + b + c$ **is**

(A) 2
(B) 6
(C) 4
(D) 0

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8. If $A = \begin{bmatrix} \tan x & \cot x \\ -\cot x & \tan x \end{bmatrix}$ **and** $A + A' = 2I$**, then value of** $x \in [0, \frac{\pi}{2}]$ **is**

(A) 0
(B) $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$
(D) $\frac{\pi}{2}$

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9. For a square matrix A**,** $(3A)^{-1} =$

(A) $3A^{-1}$
(B) $9A^{-1}$
(C) $\dfrac{1}{3}A^{-1}$
(D) $\dfrac{1}{9}A^{-1}$

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10. If

$$\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -4 \\ 0 & 4 & 2a \end{vmatrix} = -80,$$

then the sum of all possible values of a **is**

(A) 4
(B) 6
(C) -4
(D) 9

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11. If $x + y = xy$**, then** $\dfrac{dy}{dx}$ **is**

(A) $\dfrac{y}{x-1}$
(B) $\dfrac{1}{x-1}$

(C) $\dfrac{y-1}{x-1}$
(D) $\dfrac{1-y}{x-1}$

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12. $\displaystyle\int \frac{dx}{\sec x + \tan x}$ **is equal to**
(A) $\log|\sec x + \tan x| + C$
(B) $\log|\sec x - \tan x| + C$
(C) $\log|1 + \cos x| + C$
(D) $\log|1 + \sin x| + C$

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13. For $f(x) = x + \dfrac{1}{x}$ $(x \neq 0)$**,**
(A) local maximum value is 2
(B) local minimum value is -2
(C) local maximum value is -2
(D) local minimum value $<$ local maximum value

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14. Which of the following expressions will give the area of region bounded by the curve $y = x^2$ **and line** $y = 16$**?**
(A) $\displaystyle\int_0^4 x^2\,dx$
(B) $2\displaystyle\int_0^4 x^2\,dx$
(C) $\displaystyle\int_0^{16} \sqrt{y}\,dy$
(D) $2\displaystyle\int_0^{16} \sqrt{y}\,dy$

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15. The general solution of the differential equation : $x^2dy + y^2dx = 0$ **is**
(A) $x^3 + y^3 = k$
(B) $\dfrac{1}{y} - \dfrac{1}{x} = k$
(C) $\dfrac{1}{y} + \dfrac{1}{x} = k$
(D) $\log y^2 + \log x^2 = k$

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16. The integrating factor of the differential equation $2x\dfrac{dy}{dx} - y = 3$ **is**

(A) $\sqrt{x}$
(B) $\dfrac{1}{\sqrt{x}}$
(C) e^x
(D) e^{-x}

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17. If $|\vec{a}| = 5$ and $-2 \le \lambda \le 1$, then the sum of greatest and the smallest value of $|\lambda\vec{a}|$ is
(A) -5
(B) 5
(C) 10
(D) 15

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18. Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is
(A) $\hat{i} + 2\sqrt{2}\hat{j}$
(B) $3\hat{k}$
(C) $\dfrac{3\sqrt{2}}{2}\hat{i} + \dfrac{3\sqrt{2}}{2}\hat{j}$
(D) $\sqrt{3}\hat{i} + \sqrt{3}\hat{j} + \sqrt{3}\hat{k}$

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19. For two vectors $\vec{a}$ and $\vec{b}$
Assertion (A) : $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$
Reason (R) : $|\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b})\tan\theta,\ (\theta \ne \frac{\pi}{2})$
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

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20. Assertion (A) : A line can have direction cosines $\langle 1, 1, 1\rangle$
Reason (R) : $\cos\theta = 1$ is possible for $\theta = 0$.
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

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21. Find the co-ordinates of the point on line $x = \frac{y-1}{2} = \frac{z-2}{3}$ whose y-coordinate is 3 times the x-coordinate.

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22. (a) Check whether $f : \mathbb{R} - \{3\} \to \mathbb{R}$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not.
OR
(b) Check whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ (where $\mathbb{Z}$ is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.

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23. If $x = e^{\sin^{-1} t},\ y = e^{\cos^{-1} t}$, find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$.

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24. (a) Find the absolute maximum value of $f(x) = \cos x + \sin^2 x,\ x \in [0, \pi]$.
OR
(b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.

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25. If $\overrightarrow{AB} = \hat{j} + \hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$ represent the two vectors along the sides AB and AC of $\triangle ABC$, prove that the median $\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$, where D is midpoint of BC. Hence, find the length of median AD.

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26. (a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed.

The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

(i) target is hit
(ii) at least one shot misses the target.
OR
(b) Mother, Father and Son line up at random for a family picture. Let events E: Son on one end and F: Father in the middle. Find $P(E/F)$.

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27. Find : $\displaystyle\int \frac{3x-1}{\sqrt{x^2-4x}}\,dx$

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28. (a) Evaluate : $\displaystyle\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$
OR
(b) Evaluate : $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)\,dx$

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29. If $\dfrac{d}{dx}(F(x)) = \dfrac{1}{e^x-1}$**, then find** $F(x)$ **given that** $F(0) = \log\dfrac{1}{2}$**.**

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30. (a) Solve the following differential equation :

$$x\frac{dy}{dx} = y - x\sin^2\left(\frac{y}{x}\right), \quad \text{given that } y(1) = \frac{\pi}{6}.$$

OR
(b) Find the general solution of the differential equation :

$$y\log y\frac{dx}{dy} + x = \frac{2}{y}.$$

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31. Solve the following linear programming problem graphically :
Maximize $Z = 4500x + 5000y$
Subject to constraints

$$x + y \leq 250$$

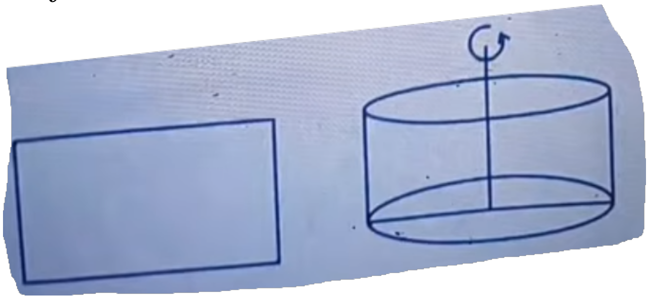
$$25x + 40y \leq 7000$$

$$x \geq 0,\ y \geq 0$$

32. (a) Find the sub-interval of $(0, \frac{\pi}{2})$ in which $f(x) = \log(\sin x + \cos x)$ is increasing and decreasing.

OR

(b) A rectangle of perimeter 30 cm is revolved along one of its sides to sweep out a cylinder of maximum volume. Find the dimensions of the rectangle.



33. Find the domain of $g(x) = \cos^{-1}(4x^2 - 3)$. Hence, find the value of x for which $g(x) = 0$. Also, write the range of $3g(x) - \pi$.

34. A line passing through the points $A(1, 2, 3)$ and $B(5, 8, 11)$ intersects the line

$$\vec{r} = 4\hat{i} + \hat{j} + \lambda(5\hat{i} + 2\hat{j} + \hat{k}).$$

Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.

35. (a)If $P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 6 \end{bmatrix}$, find (QP) and hence solve the following system of equations using matrices :

$$x - y = 3, \quad 2x + 3y + 4z = 17, \quad y + 2z = 7$$

OR

(b) Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y, z.

Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that

$$x^{-1} + y^{-1} + z^{-1} = -1.$$

36. Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.

The equation of one such track is given as follows :

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \leq x < 3 \\ x^2 + 40, & x \geq 3 \end{cases}$$

Based on given information, answer the following questions :

(i) Find $f'(x)$ for $0 < x \leq 3$.

(ii) Find $f'(4)$.

(iii) (a) Test for continuity of $f(x)$ at $x = 3$.

OR

(iii) (b) Test for differentiability of $f(x)$ at $x = 3$.

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37. A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems.

In a colony, 50 people were found to be smokers, of which 30 are males. A person is selected at random from these 50 people and tested for lung related problems.
Based on the given information answer the following questions:

(i) What is the probability that the selected person is a female?

(ii) If a male person is selected, what is the probability that he will not be suffering from lung problems?

(iii) (a) A person selected at random is detected with lung complications. Find the probability that the selected person is a female.
OR
(b) A person selected at random is not having lung problems. Find the probability that the person is a male.

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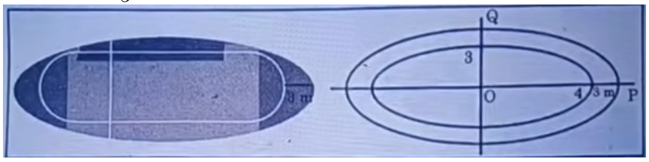
38. A racing track is built around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below.



Based on given information, answer the following questions :

(i) Express y as a function of x from the given equation of ellipse.

(ii) Integrate the function obtained in (i) with respect to x.

(iii) (a) Find the area of the region enclosed within the elliptical ground excluding the track using integration.

OR

(iii) (b) Write the co-ordinates of the points P and Q where the outer edge of the track cuts x-axis and y-axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration.