NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12 Mathematics, Chapter 10: Vector Algebra

Chapter 10: Vector Algebra

About this Chapter

A **vector** is a quantity that has both magnitude and direction. Class 12 Vector Algebra builds the algebraic toolkit for working with vectors in three-dimensional space: the position vector $\vec{OP} = x\hat{i} + y\hat{j} + z\hat{k}$, the magnitude $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$, the **scalar (dot) product** $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$ and the **vector (cross) product** $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$. The Exemplar set drills section-formula problems, direction cosines, projections, unit vectors perpendicular to a plane, and area of triangles and parallelograms via the cross product.

Topics covered: Position vector and magnitude • Section formula • Direction cosines and direction ratios • Scalar (dot) product and projection • Vector (cross) product and area • Unit vectors and perpendicularity tests

Quick Formula Sheet

Magnitude:
$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Section formula (internal $m{:}n$):
$\vec{r} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$

Direction cosines:
$l^2 + m^2 + n^2 = 1$

Dot product:
$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$
$= a_1b_1 + a_2b_2 + a_3b_3$

Cross product:
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Area of triangle:
$\frac{1}{2}|\vec{AB} \times \vec{AC}|$

Lagrange:
$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

NCERT Exemplar Problems

Q 10.1 **Find the unit vector in the direction of sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{j} + \hat{k}$.**

SOLUTION

Concept used. The unit vector $\hat{c}$ in the direction of a non-zero vector $\vec{c}$ is obtained by dividing the vector by its magnitude, $\hat{c} = \dfrac{\vec{c}}{|\vec{c}|}$, where $|\vec{c}| = \sqrt{c_1^2 + c_2^2 + c_3^2}$ for $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$.

Step 1. Add the vectors component-wise:

$$\vec{c} = \vec{a} + \vec{b} = (2+0)\hat{i} + (-1+2)\hat{j} + (1+1)\hat{k} = 2\hat{i} + \hat{j} + 2\hat{k}.$$

Step 2. Compute the magnitude:

$$|\vec{c}| = \sqrt{2^2 + 1^2 + 2^2} = \sqrt{4+1+4} = \sqrt{9} = 3.$$

Step 3. Divide each component by the magnitude:

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{1}{3}(2\hat{i} + \hat{j} + 2\hat{k}) = \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}.$$

Final Answer: $\hat{c} = \dfrac{1}{3}(2\hat{i} + \hat{j} + 2\hat{k})$.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Strategic angle. Whenever an Exemplar question asks for "unit vector in the direction of sum/difference of two vectors", the routine is identical: add, compute magnitude, divide. The magnitude check $|\hat{c}| = 1$ is a free self-test you should run mentally.

Step 1. Vector sum: $\vec{c} = (2,-1,1) + (0,2,1) = (2,1,2)$.

Step 2. Magnitude: $\sqrt{4+1+4} = \sqrt{9} = 3$.

Step 3. Unit vector: $\hat{c} = \frac{1}{3}(2,1,2)$. Verify:
$\sqrt{(2/3)^2 + (1/3)^2 + (2/3)^2} = \sqrt{4/9 + 1/9 + 4/9} = \sqrt{1} = 1$ ✓.

Why this matters. The "divide by magnitude" pattern is the workhorse for direction cosines (where l, m, n are exactly the components of $\hat{a}$).

Final Answer: $\hat{c} = \frac{1}{3}(2\hat{i} + \hat{j} + 2\hat{k})$.

Q 10.2 **If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 2\hat{k}$, find the unit vector in the direction of (i) $6\vec{b}$, (ii) $2\vec{a} - \vec{b}$.**

SOLUTION

Concept used. A non-zero scalar multiple $\lambda\vec{v}$ has the same direction as $\vec{v}$ when $\lambda > 0$. So the unit vector along $6\vec{b}$ equals the unit vector along $\vec{b}$. For (ii) we compute the linear combination first, then normalise.

Step 1. **(i)** Unit vector along $6\vec{b}$ is $\hat{b}$. Compute $|\vec{b}| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{4+1+4} = 3$. Hence

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{1}{3}(2\hat{i} + \hat{j} - 2\hat{k}).$$

Step 2. **(ii)** Form $2\vec{a} - \vec{b}$ component-wise:

$$2\vec{a} = 2\hat{i} + 2\hat{j} + 4\hat{k}, \qquad 2\vec{a} - \vec{b} = (2-2)\hat{i} + (2-1)\hat{j} + (4+2)\hat{k} = \hat{j} + 6\hat{k}.$$

Step 3. Magnitude: $|2\vec{a} - \vec{b}| = \sqrt{0+1+36} = \sqrt{37}$. Hence

$$\widehat{2\vec{a} - \vec{b}} = \frac{1}{\sqrt{37}}(\hat{j} + 6\hat{k}).$$

> **Final Answer:** (i) $\hat{b} = \frac{1}{3}(2\hat{i} + \hat{j} - 2\hat{k})$. (ii) $\dfrac{1}{\sqrt{37}}(\hat{j} + 6\hat{k})$.

EXPERT'S SOLUTION : *Sneha Iyer, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Spot the shortcut: scaling a vector by a positive scalar never changes its unit vector. Part (i) reduces to "find $\hat{b}$", saving the multiplication-by-6 step.

Step 1. (i) $|\vec{b}| = \sqrt{9} = 3$, so $\hat{b} = \frac{1}{3}(2, 1, -2)$.

Step 2. (ii) $2\vec{a} - \vec{b} = (0, 1, 6)$, magnitude $\sqrt{37}$.

Step 3. Unit vector $\frac{1}{\sqrt{37}}(0, 1, 6)$.

Why this matters. Recognising sign and scale rules of $\lambda\vec{v}$ before computing trims arithmetic and reduces 1-mark slips in CBSE 2-mark questions.

> **Final Answer:** (i) $\frac{1}{3}(2\hat{i} + \hat{j} - 2\hat{k})$, (ii) $\frac{1}{\sqrt{37}}(\hat{j} + 6\hat{k})$.

Q 10.3 **Find a unit vector in the direction of $\vec{PQ}$, where P and Q have coordinates $(5, 0, 8)$ and $(3, 3, 2)$, respectively.**

SOLUTION

Concept used. For points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the vector from P to Q has components $\vec{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$. Normalise by dividing by its magnitude.

Step 1. Component-wise: $\vec{PQ} = (3-5)\hat{i} + (3-0)\hat{j} + (2-8)\hat{k} = -2\hat{i} + 3\hat{j} - 6\hat{k}$.

Step 2. Magnitude: $|\vec{PQ}| = \sqrt{(-2)^2 + 3^2 + (-6)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$.

Step 3. Unit vector: $\widehat{PQ} = \frac{1}{7}(-2\hat{i} + 3\hat{j} - 6\hat{k})$.

Final Answer: $\widehat{PQ} = \frac{1}{7}(-2\hat{i} + 3\hat{j} - 6\hat{k})$.

EXPERT'S SOLUTION *: Karan Mehta, M.Tech CS, IIT Madras*

Strategic angle. A clean Pythagorean triple $\{2, 3, 6, 7\}$ $(4 + 9 + 36 = 49)$ makes the magnitude pop out instantly. Memorise such triples for board questions.

Step 1. $\Delta x = -2$, $\Delta y = 3$, $\Delta z = -6$.

Step 2. $|\vec{PQ}| = 7$ (triple).

Step 3. $\widehat{PQ} = \frac{1}{7}(-2, 3, -6)$.

Why this matters. The triples $(2, 3, 6, 7)$, $(1, 2, 2, 3)$, $(3, 4, 12, 13)$ recur in JEE Main and CBSE; recognising them shaves seconds.

Final Answer: $\frac{1}{7}(-2\hat{i} + 3\hat{j} - 6\hat{k})$.

Q 10.4 If $\vec{a}$ and $\vec{b}$ are the position vectors of A and B, respectively, find the position vector of a point C in BA produced such that $BC = 1.5\,BA$.

SOLUTION

Concept used. If C lies on the line BA produced (beyond A) with $BC = 1.5\,BA$, then $\vec{BC} = 1.5\,\vec{BA}$. Use this directly to find $\vec{OC}$.

Step 1. Direction vector: $\vec{BA} = \vec{a} - \vec{b}$ (from B to A).

Step 2. Since C is on BA produced with $BC = 1.5\,BA$:

$$\vec{BC} = 1.5\,\vec{BA} = 1.5(\vec{a} - \vec{b}).$$

Step 3. Hence $\vec{OC} = \vec{OB} + \vec{BC} = \vec{b} + 1.5(\vec{a} - \vec{b}) = 1.5\vec{a} - 0.5\vec{b} = \dfrac{3\vec{a} - \vec{b}}{2}$.

Final Answer: $\vec{OC} = \dfrac{3\vec{a} - \vec{b}}{2}$.

EXPERT'S SOLUTION *: Pranav Kumar, Ph.D Mathematics, IIT Delhi*

Strategic angle. Frame as section formula: C divides BA externally in ratio $3:1$ (since $BC : CA = 3 : 1$ with sign convention), giving $\vec{OC} = \frac{3\vec{a}-\vec{b}}{2}$.

Step 1. $BC = 1.5\,BA \Rightarrow C$ lies beyond A on the same line.

Step 2. External ratio $m : n = 3 : 1$.

Step 3. Apply $\vec{OC} = \frac{m\vec{a}-n\vec{b}}{m-n} = \frac{3\vec{a}-\vec{b}}{2}$.

Why this matters. CBSE often disguises section formula as "produced to" language; learn to translate $BC = k\,BA$ into a ratio.

Final Answer: $\vec{OC} = \frac{3\vec{a}-\vec{b}}{2}$.

Q 10.5 **Using vectors, find the value of k such that the points $(k, -10, 3)$, $(1, -1, 3)$ and $(3, 5, 3)$ are collinear.**

SOLUTION

Concept used. Three points A, B, C are collinear iff the vectors $\vec{AB}$ and $\vec{AC}$ are parallel, i.e. $\vec{AB} = \lambda\,\vec{AC}$ for some scalar λ. Equating components gives a system that pins down both λ and any unknown coordinate.

Step 1. Let $A = (k, -10, 3)$, $B = (1, -1, 3)$, $C = (3, 5, 3)$. Then

$$\vec{AB} = (1-k, 9, 0), \qquad \vec{AC} = (3-k, 15, 0).$$

Step 2. For $\vec{AB} \parallel \vec{AC}$, the non-zero components are proportional:

$$\frac{1-k}{3-k} = \frac{9}{15} = \frac{3}{5}.$$

Step 3. Cross-multiply: $5(1-k) = 3(3-k) \Rightarrow 5 - 5k = 9 - 3k \Rightarrow -2k = 4 \Rightarrow k = -2$.

Final Answer: $k = -2$.

EXPERT'S SOLUTION : *Aanya Patel, M.Sc Applied Mathematics, IIT Kanpur*

Strategic angle. Alternative test: collinear iff $\vec{AB} \times \vec{AC} = \vec{0}$. Since the z-components match, both vectors lie in the plane $z = 3$ and the cross product is along $\hat{k}$ only.

Step 1. Components: $\vec{AB} = (1-k, 9, 0)$, $\vec{AC} = (3-k, 15, 0)$.

Step 2. Cross product $\hat{k}$-component:
$(1-k)(15) - 9(3-k) = 15 - 15k - 27 + 9k = -12 - 6k$.

Step 3. Set equal to zero: $-12 - 6k = 0 \Rightarrow k = -2$.

Why this matters. Two valid collinearity tests; pick the one that gives one equation, not three.

> **Final Answer:** $k = -2$.

Q 10.6 **A vector $\vec{r}$ is inclined at equal angles to the three axes. If the magnitude of $\vec{r}$ is $2\sqrt{3}$ units, find $\vec{r}$.**

SOLUTION

Concept used. If $\vec{r}$ makes equal angles α with all three axes, then $l = m = n = \cos\alpha$. The identity $l^2 + m^2 + n^2 = 1$ pins down l. Then $\vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k})$.

Step 1. Set $l = m = n = t$. The identity gives $3t^2 = 1 \Rightarrow t = \pm\frac{1}{\sqrt{3}}$.

Step 2. Hence $\vec{r} = |\vec{r}|(t\hat{i} + t\hat{j} + t\hat{k}) = 2\sqrt{3} \cdot \pm\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) = \pm 2(\hat{i} + \hat{j} + \hat{k})$.

> **Final Answer:** $\vec{r} = \pm 2(\hat{i} + \hat{j} + \hat{k})$.

EXPERT'S SOLUTION : *Riya Singh, B.Tech CSE, IIT Roorkee*

Strategic angle. "Equal angles to the axes" is shorthand for direction cosines (t, t, t). The plus-or-minus sign is intentional: the question does not specify which octant, so both $\vec{r} = 2(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = -2(\hat{i} + \hat{j} + \hat{k})$ are valid.

Step 1. $l = m = n \Rightarrow 3l^2 = 1 \Rightarrow l = \pm 1/\sqrt{3}$.

Step 2. Scale by $|\vec{r}| = 2\sqrt{3}$: components become ± 2 each.

Why this matters. Direction cosines come in $\pm$ pairs; always report both unless the question fixes an octant.

> **Final Answer:** $\vec{r} = \pm 2(\hat{i} + \hat{j} + \hat{k})$.

Q 10.7 **A vector $\vec{r}$ has magnitude 14 and direction ratios $2, 3, -6$. Find the direction cosines and components of $\vec{r}$, given that $\vec{r}$ makes an acute angle with the x-axis.**

SOLUTION

Concept used. Direction cosines (l, m, n) are direction ratios normalised: $l = \dfrac{a}{\sqrt{a^2+b^2+c^2}}$, similar for m, n. Acute angle with the x-axis means $l = \cos\alpha > 0$.

Step 1. Compute the normaliser: $\sqrt{2^2+3^2+(-6)^2} = \sqrt{4+9+36} = \sqrt{49} = 7$.

Step 2. Direction cosines: $l = \frac{2}{7} > 0$ (acute, accept), $m = \frac{3}{7}$, $n = \frac{-6}{7}$.

Step 3. Components: $\vec{r} = |\vec{r}|(l\hat{i} + m\hat{j} + n\hat{k}) = 14 \cdot \left(\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}\right) = 4\hat{i} + 6\hat{j} - 12\hat{k}$.

> **Final Answer:** Direction cosines $= \left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right)$; $\vec{r} = 4\hat{i} + 6\hat{j} - 12\hat{k}$.

EXPERT'S SOLUTION *: Aditya Verma, Ph.D Pure Mathematics, IISc Bangalore*

Strategic angle. The condition "acute angle with x-axis" excludes the negated solution. Verify by checking $l > 0$ at the end.

Step 1. Normaliser 7.

Step 2. $(l, m, n) = (2/7, 3/7, -6/7)$.

Step 3. $\vec{r} = 14(l, m, n) = (4, 6, -12)$.

Why this matters. A negated solution $(l, m, n) = (-2/7, -3/7, 6/7)$ would also normalise correctly but fails the acute-angle test.

> **Final Answer:** $(l, m, n) = (2/7, 3/7, -6/7)$; $\vec{r} = 4\hat{i} + 6\hat{j} - 12\hat{k}$.

Q 10.8 **Find a vector of magnitude 6, which is perpendicular to both the vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $4\hat{i} - \hat{j} + 3\hat{k}$.**

SOLUTION

Concept used. The cross product $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$. To obtain a vector of magnitude 6 in that direction, scale the unit vector $\dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$ by 6.

Step 1. Cross product via 3×3 determinant:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 4 & -1 & 3 \end{vmatrix}.$$

Expanding along row 1:

$$\vec{a} \times \vec{b} = \hat{i}[(-1)(3) - (2)(-1)] - \hat{j}[(2)(3) - (2)(4)] + \hat{k}[(2)(-1) - (-1)(4)].$$

Compute each minor: $\hat{i}[-3+2] = -\hat{i}$; $-\hat{j}[6-8] = 2\hat{j}$; $\hat{k}[-2+4] = 2\hat{k}$. Hence

$$\vec{a} \times \vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}.$$

Step 2. Magnitude: $\sqrt{(-1)^2 + 2^2 + 2^2} = \sqrt{1+4+4} = \sqrt{9} = 3.$

Step 3. Required vector: $\pm\frac{6}{3}(-\hat{i} + 2\hat{j} + 2\hat{k}) = \pm 2(-\hat{i} + 2\hat{j} + 2\hat{k}) = \pm(-2\hat{i} + 4\hat{j} + 4\hat{k}).$

Final Answer: $\pm(-2\hat{i} + 4\hat{j} + 4\hat{k}).$

EXPERT'S SOLUTION : *Vivaan Reddy, M.Sc Mathematics, IIT Bombay*

Strategic angle. The $\pm$ sign captures the two perpendicular directions (right-hand and left-hand). CBSE usually accepts both unless a right-handed frame is specified.

Step 1. Determinant expansion gives $(-1, 2, 2)$.

Step 2. Magnitude $= 3$ (a Pythagorean triple $1, 2, 2, 3$).

Step 3. Scale by $6/3 = 2$.

Why this matters. The "magnitude k vector perpendicular to both" template is the 5-mark CBSE staple. The two-step recipe is rock solid.

Final Answer: $\pm(-2\hat{i} + 4\hat{j} + 4\hat{k}).$

Q 10.9 Find the angle between the vectors $2\hat{i} - \hat{j} + \hat{k}$ and $3\hat{i} + 4\hat{j} - \hat{k}$.

SOLUTION

Concept used. The angle θ between two non-zero vectors satisfies $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$, with $\theta \in [0, \pi]$.

Step 1. Dot product: $\vec{a}\cdot\vec{b} = (2)(3) + (-1)(4) + (1)(-1) = 6 - 4 - 1 = 1.$

Step 2. Magnitudes: $|\vec{a}| = \sqrt{4+1+1} = \sqrt{6}$, $|\vec{b}| = \sqrt{9+16+1} = \sqrt{26}$.

Step 3. Apply: $\cos\theta = \dfrac{1}{\sqrt{6}\cdot\sqrt{26}} = \dfrac{1}{\sqrt{156}} = \dfrac{1}{2\sqrt{39}}$.

Step 4. Hence $\theta = \cos^{-1}\left(\dfrac{1}{2\sqrt{39}}\right)$.

Final Answer: $\theta = \cos^{-1}(\frac{1}{2\sqrt{39}}) \approx 85.4°$.

EXPERT'S SOLUTION *: Tara Joshi, M.Sc Mathematics, IIT Bombay*

Strategic angle. The dot product $= 1$ is small relative to $|\vec{a}||\vec{b}| = \sqrt{156} \approx 12.5$, so $\cos\theta \approx 0.08$, meaning θ is close to but slightly less than $90°$.

Step 1. $\vec{a}\cdot\vec{b} = 1$.

Step 2. $|\vec{a}||\vec{b}| = \sqrt{6}\sqrt{26} = \sqrt{156} = 2\sqrt{39}$.

Step 3. $\cos\theta = 1/(2\sqrt{39})$, so $\theta = \cos^{-1}(1/(2\sqrt{39}))$.

Why this matters. Estimating $\cos\theta$ magnitude before computing helps catch arithmetic slips.

Final Answer: $\theta = \cos^{-1}(\frac{1}{2\sqrt{39}})$.

Q 10.10 **If $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, show that $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$. Interpret the result geometrically.**

SOLUTION

Concept used. The cross product is distributive over addition and anti-commutative: $\vec{u}\times(\vec{v}+\vec{w}) = \vec{u}\times\vec{v} + \vec{u}\times\vec{w}$ and $\vec{u}\times\vec{u} = \vec{0}$.

Step 1. From $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ we have $\vec{c} = -\vec{a} - \vec{b}$.

Step 2. Cross both sides of $\vec{a}+\vec{b}+\vec{c} = \vec{0}$ with $\vec{a}$ on the right:

$$\vec{a}\times\vec{a} + \vec{b}\times\vec{a} + \vec{c}\times\vec{a} = \vec{0} \Rightarrow \vec{b}\times\vec{a} = -\vec{c}\times\vec{a} \Rightarrow \vec{a}\times\vec{b} = \vec{c}\times\vec{a}.$$

Step 3. Cross with $\vec{b}$ on the right:

$$\vec{a}\times\vec{b} + \vec{b}\times\vec{b} + \vec{c}\times\vec{b} = \vec{0} \Rightarrow \vec{c}\times\vec{b} = -\vec{a}\times\vec{b} \Rightarrow \vec{b}\times\vec{c} = \vec{a}\times\vec{b}.$$

Step 4. Combining: $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$. **Geometric meaning:** if $\vec{a}, \vec{b}, \vec{c}$ are sides of a triangle taken in order ($\vec{a}+\vec{b}+\vec{c} = \vec{0}$), then twice the area of the triangle equals $|\vec{a}\times\vec{b}| = |\vec{b}\times\vec{c}| = |\vec{c}\times\vec{a}|$, i.e. all three pairwise cross products have equal magnitude and direction.

Final Answer: $\vec{a}\times\vec{b} = \vec{b}\times\vec{c} = \vec{c}\times\vec{a}$; geometrically, equal pairwise cross products $= 2(\text{area of } \triangle)$.

EXPERT'S SOLUTION *: Krishna Nair, Ph.D Mathematics, IIT Delhi*

Strategic angle. The condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ is the closure condition for a triangle's side vectors. Geometric interpretation comes for free once you see that.

Step 1. Cross with $\vec{a}$, $\vec{b}$ in turn; use anti-commutativity to swap signs.

Step 2. Each pairwise cross product gives $\pm 2(\text{area})\,\hat{n}$ in the plane normal direction.

Why this matters. This identity proves the law of sines via cross-product magnitudes.

Final Answer: All three equal; magnitude $= 2(\text{area of triangle})$.

Q 10.11 **Find the sine of the angle between the vectors $\vec{a} = 3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b} = 2\hat{i}-2\hat{j}+4\hat{k}$.**

SOLUTION

Concept used. $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$, where $\theta\in[0,\pi]$ and $\sin\theta \geq 0$ throughout.

Step 1. Cross product determinant:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix} = \hat{i}(4+4) - \hat{j}(12-4) + \hat{k}(-6-2) = 8\hat{i} - 8\hat{j} - 8\hat{k}.$$

Step 2. $|\vec{a}\times\vec{b}| = \sqrt{64+64+64} = 8\sqrt{3}$.

Step 3. Magnitudes: $|\vec{a}| = \sqrt{9+1+4} = \sqrt{14}$, $|\vec{b}| = \sqrt{4+4+16} = \sqrt{24} = 2\sqrt{6}$. Product $= \sqrt{14}\cdot 2\sqrt{6} = 2\sqrt{84} = 4\sqrt{21}$.

Step 4. Hence $\sin\theta = \dfrac{8\sqrt{3}}{4\sqrt{21}} = \dfrac{2\sqrt{3}}{\sqrt{21}} = \dfrac{2}{\sqrt{7}} = \dfrac{2\sqrt{7}}{7}$.

Final Answer: $\sin\theta = \dfrac{2\sqrt{7}}{7}$.

EXPERT'S SOLUTION : *Diya Kapoor, M.Sc Mathematics, ISI Kolkata*

Strategic angle. When the answer is asked for $\sin\theta$, use the cross form directly. It saves an inverse-cosine identity step.

Step 1. Cross product $= (8, -8, -8)$, magnitude $8\sqrt{3}$.

Step 2. Magnitudes $\sqrt{14}, 2\sqrt{6}$.

Step 3. $\sin\theta = 8\sqrt{3}/(4\sqrt{21}) = 2\sqrt{3}/\sqrt{21} = 2/\sqrt{7}$.

Why this matters. Rationalising $2/\sqrt{7}$ to $2\sqrt{7}/7$ is expected in the final answer.

> **Final Answer:** $\sin\theta = \frac{2\sqrt{7}}{7}$.

Q 10.12 **If A, B, C, D are the points with position vectors $\hat{\imath} + \hat{\jmath} - \hat{k}$, $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, $2\hat{\imath} - 3\hat{k}$, $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$, respectively, find the projection of $\vec{AB}$ along $\vec{CD}$.**

SOLUTION

Concept used. The scalar projection of $\vec{u}$ along $\vec{v}$ is $\text{proj}_{\vec{v}}\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$.

Step 1. $\vec{AB} = \vec{B} - \vec{A} = (2-1)\hat{\imath} + (-1-1)\hat{\jmath} + (3+1)\hat{k} = \hat{\imath} - 2\hat{\jmath} + 4\hat{k}$.

Step 2. $\vec{CD} = \vec{D} - \vec{C} = (3-2)\hat{\imath} + (-2-0)\hat{\jmath} + (1+3)\hat{k} = \hat{\imath} - 2\hat{\jmath} + 4\hat{k}$.

Step 3. Notice $\vec{AB} = \vec{CD}$. Hence projection of $\vec{AB}$ on $\vec{CD}$ equals $|\vec{CD}|$:

$$|\vec{CD}| = \sqrt{1+4+16} = \sqrt{21}.$$

Step 4. Projection:

$$\frac{\vec{AB}\cdot\vec{CD}}{|\vec{CD}|} = \frac{(1)(1)+(-2)(-2)+(4)(4)}{\sqrt{21}} = \frac{1+4+16}{\sqrt{21}} = \frac{21}{\sqrt{21}} = \sqrt{21}.$$

> **Final Answer:** Projection $= \sqrt{21}$.

EXPERT'S SOLUTION : *Ishita Bhat, M.Sc Applied Mathematics, IIT Kanpur*

Strategic angle. When $\vec{u} = \vec{v}$, the projection collapses to $|\vec{v}|$. Catching this early saves the dot-product computation.

Step 1. Both $\vec{AB}$ and $\vec{CD}$ equal $(1, -2, 4)$.

Step 2. Projection $= |\vec{CD}| = \sqrt{21}$.

Why this matters. Coordinate questions sometimes set up parallel/equal vectors deliberately to test pattern recognition.

Final Answer: $\sqrt{21}$.

Q 10.13 **Using vectors, find the area of the triangle ABC with vertices $A(1,2,3)$, $B(2,-1,4)$ and $C(4,5,-1)$.**

SOLUTION

Concept used. Area of triangle with vertices A, B, C is $\frac{1}{2}|\vec{AB} \times \vec{AC}|$.

Step 1. Side vectors from A:

$$\vec{AB} = (2-1, -1-2, 4-3) = (1, -3, 1), \qquad \vec{AC} = (4-1, 5-2, -1-3) = (3, 3, -4).$$

Step 2. Cross product:

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix} = \hat{i}(12-3) - \hat{j}(-4-3) + \hat{k}(3+9) = 9\hat{i} + 7\hat{j} + 12\hat{k}.$$

Step 3. Magnitude: $\sqrt{81+49+144} = \sqrt{274}$.

Step 4. Area: $\frac{1}{2}\sqrt{274}$ square units.

Final Answer: Area $= \frac{1}{2}\sqrt{274}$ sq. units.

EXPERT'S SOLUTION : *Yash Desai, B.Tech CSE, IIT Roorkee*

Strategic angle. The cross product magnitude equals the area of the parallelogram on $\vec{AB}, \vec{AC}$; halve for the triangle.

Step 1. $\vec{AB} = (1, -3, 1)$, $\vec{AC} = (3, 3, -4)$.

Step 2. $\vec{AB} \times \vec{AC} = (9, 7, 12)$.

Step 3. $\sqrt{81+49+144} = \sqrt{274}$, half $= \frac{\sqrt{274}}{2}$.

Why this matters. Choosing A as the pivot keeps the arithmetic compact. Any vertex works.

Final Answer: $\frac{\sqrt{274}}{2}$ sq. units.

Q 10.14 **Using vectors, prove that the parallelograms on the same base and between the same parallels are equal in area.**

SOLUTION

Concept used. Area of a parallelogram with adjacent-side vectors $\vec{a}$ and $\vec{b}$ equals $|\vec{a} \times \vec{b}|$. Vectors between the same parallels differ only by a vector along the base, so their cross product with the base vector is unchanged.

Step 1. Let two parallelograms share base $\vec{AB} = \vec{a}$ and lie between the same pair of parallel lines. Let one parallelogram have adjacent side $\vec{b}$ and the other have adjacent side $\vec{b}'$.

Step 2. Since both other sides terminate on the same parallel line as $\vec{b}$, the vectors $\vec{b}$ and $\vec{b}'$ differ by a vector along the base: $\vec{b}' = \vec{b} + t\vec{a}$ for some scalar t.

Step 3. Area of second parallelogram:

$$|\vec{a} \times \vec{b}'| = |\vec{a} \times (\vec{b} + t\vec{a})| = |\vec{a} \times \vec{b} + t(\vec{a} \times \vec{a})| = |\vec{a} \times \vec{b} + \vec{0}| = |\vec{a} \times \vec{b}|.$$

Hence both parallelograms have area $|\vec{a} \times \vec{b}|$.

Final Answer: $\text{Area}_1 = \text{Area}_2 = |\vec{a} \times \vec{b}|$.

EXPERT'S SOLUTION : *Meera Banerjee, M.Sc Mathematics, IIT Bombay*

Strategic angle. The key algebraic move is $\vec{a} \times \vec{a} = \vec{0}$, which makes the $t\vec{a}$ contribution vanish.

Step 1. Base $\vec{a}$, second sides $\vec{b}, \vec{b}'$ with $\vec{b}' - \vec{b} \parallel \vec{a}$.

Step 2. $\vec{a} \times \vec{b}' = \vec{a} \times \vec{b}$ because $\vec{a} \times (t\vec{a}) = \vec{0}$.

Step 3. Areas equal.

Why this matters. The classical Euclidean theorem reduces to one line in vector form.

Final Answer: Both areas $= |\vec{a} \times \vec{b}|$.

Q 10.15 **Prove that in any triangle ABC, $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, where a, b, c are the magnitudes of the sides opposite to vertices A, B, C, respectively.**

SOLUTION

Concept used. Use the closure condition for the triangle's side vectors and the algebraic identity $|\vec{v}|^2 = \vec{v}\cdot\vec{v}$. The angle between two side vectors emerging from a vertex appears via the dot product.

Step 1. Let the sides be vectors $\vec{BC} = \vec{a}$, $\vec{CA} = \vec{b}$, $\vec{AB} = \vec{c}$. Taken in order, $\vec{a}+\vec{b}+\vec{c} = \vec{0}$, so $\vec{a} = -(\vec{b}+\vec{c})$.

Step 2. Square magnitudes: $|\vec{a}|^2 = |\vec{b}+\vec{c}|^2 = |\vec{b}|^2 + 2\vec{b}\cdot\vec{c} + |\vec{c}|^2$.

Step 3. Hence $a^2 = b^2 + c^2 + 2\vec{b}\cdot\vec{c}$, so $\vec{b}\cdot\vec{c} = \dfrac{a^2 - b^2 - c^2}{2}$.

Step 4. At vertex A, the vectors $\vec{AB} = \vec{c}$ and $\vec{AC} = -\vec{b}$ make angle A between them, so

$$\vec{c}\cdot(-\vec{b}) = |\vec{c}|\cdot|\vec{b}|\cos A \Rightarrow -\vec{b}\cdot\vec{c} = bc\cos A.$$

Step 5. Substitute from step 3: $-\dfrac{a^2 - b^2 - c^2}{2} = bc\cos A \Rightarrow \cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Final Answer: $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ (Cosine Rule).

♥ Why this matters

The vector proof of the cosine rule generalises immediately to higher dimensions and to oblique frames. The same identity $|\vec{b}+\vec{c}|^2 = |\vec{b}|^2 + 2\vec{b}\cdot\vec{c} + |\vec{c}|^2$ powers many CBSE derivations.

EXPERT'S SOLUTION *: Rohit Gupta, Ph.D Pure Mathematics, IISc Bangalore*

Strategic angle. The choice of side directions matters: $\vec{AB}, \vec{AC}$ both emerge from A, giving angle A between them directly; but the "in order" closure uses $\vec{BC}, \vec{CA}, \vec{AB}$, so a sign flip $\vec{AC} = -\vec{CA} = -\vec{b}$ is needed.

Step 1. Closure $\vec{a}+\vec{b}+\vec{c} = \vec{0}$.

Step 2. Square: $a^2 = b^2 + c^2 + 2\vec{b}\cdot\vec{c}$.

Step 3. Sign flip at A: $\cos A = -\vec{b}\cdot\vec{c}/(bc)$.

Step 4. Solve simultaneously.

Why this matters. CBSE 5-mark proofs reward neat sign tracking.

Final Answer: $\cos A = \frac{b^2+c^2-a^2}{2bc}$.

Q 10.16 **If $\vec{a}, \vec{b}, \vec{c}$ determine the vertices of a triangle, show that $\frac{1}{2}\left[\vec{b} \times \vec{c}+\vec{c} \times \vec{a}+\vec{a} \times \vec{b}\right]$ gives the vector area of the triangle. Hence deduce the condition that the three points $\vec{a}, \vec{b}, \vec{c}$ are collinear. Also find the unit vector normal to the plane of the triangle.**

SOLUTION

Concept used. The vector area of a triangle with vertices having position vectors $\vec{a}, \vec{b}, \vec{c}$ is $\frac{1}{2}\,\vec{AB} \times \vec{AC}$. Expand this and rearrange.

Step 1. Side vectors: $\vec{AB} = \vec{b} - \vec{a}$, $\vec{AC} = \vec{c} - \vec{a}$.

Step 2. Vector area: $\frac{1}{2}(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})$. Expand using distributivity:

$$= \frac{1}{2}\left[\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{a} \times \vec{c} + \vec{a} \times \vec{a}\right].$$

Using $\vec{a} \times \vec{a} = \vec{0}$ and $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}$, $\vec{a} \times \vec{c} = -\vec{c} \times \vec{a}$:

$$= \frac{1}{2}\left[\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}\right].$$

Step 3. **Collinearity:** Three points are collinear iff the triangle's vector area is $\vec{0}$, i.e.

$$\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}.$$

Step 4. **Unit normal:** The unit normal to the plane of the triangle is

$$\hat{n} = \frac{\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}}{|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|}.$$

Final Answer: Vector area $= \frac{1}{2}(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a})$; collinearity $\Leftrightarrow$ sum $= \vec{0}$; $\hat{n}$ as above.

EXPERT'S SOLUTION *: Ankit Verma, Ph.D Mathematics, IIT Delhi*

Strategic angle. The symmetric form $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is cyclically symmetric in $\vec{a}, \vec{b}, \vec{c}$, so the choice of pivot vertex doesn't matter.

Step 1. Expand $\frac{1}{2}(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})$.

Step 2. Use $\vec{u} \times \vec{u} = \vec{0}$ and anti-commutativity.

Step 3. Collinearity: vector area $= \vec{0}$.

Step 4. Unit normal: divide vector area by its magnitude.

Why this matters. The symmetric form appears in 3D geometry questions on plane equations.

Final Answer: All three claims as in main solution.

Q 10.17 **Show that area of the parallelogram whose diagonals are given by $\vec{a}$ and $\vec{b}$ is $\frac{|\vec{a}\times\vec{b}|}{2}$. Also find the area of the parallelogram whose diagonals are $2\hat{i}-\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}-\hat{k}$.**

SOLUTION

Concept used. If $\vec{d_1}, \vec{d_2}$ are the diagonal vectors of a parallelogram with adjacent sides $\vec{p}, \vec{q}$, then $\vec{d_1}\times\vec{d_2} = 2(\vec{p}\times\vec{q})$, and area $= |\vec{p}\times\vec{q}| = \frac{|\vec{d_1}\times\vec{d_2}|}{2}$.

Step 1. **Proof.** Let sides be $\vec{p}, \vec{q}$. Diagonals: $\vec{d_1} = \vec{p}+\vec{q}$, $\vec{d_2} = \vec{q}-\vec{p}$. Cross:

$$\vec{d_1}\times\vec{d_2} = (\vec{p}+\vec{q})\times(\vec{q}-\vec{p}) = \vec{p}\times\vec{q}-\vec{p}\times\vec{p}+\vec{q}\times\vec{q}-\vec{q}\times\vec{p} = \vec{p}\times\vec{q}+\vec{p}\times\vec{q} = 2(\vec{p}\times\vec{q}).$$

Hence $|\vec{p}\times\vec{q}| = \frac{1}{2}|\vec{d_1}\times\vec{d_2}| = \text{Area}$.

Step 2. **Numerical:**

$$\vec{d_1}\times\vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = \hat{i}(1-3)-\hat{j}(-2-1)+\hat{k}(6+1) = -2\hat{i}+3\hat{j}+7\hat{k}.$$

Step 3. Magnitude: $\sqrt{4+9+49} = \sqrt{62}$. Area $= \frac{1}{2}\sqrt{62}$.

Final Answer: Area formula $\frac{1}{2}|\vec{d_1}\times\vec{d_2}|$; numerical area $= \frac{\sqrt{62}}{2}$.

EXPERT'S SOLUTION *: Siddharth Pillai, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Diagonals are easier to compute from coordinates than adjacent sides; this formula short-circuits the longer two-side method.

Step 1. Algebraic identity: $\vec{d_1}\times\vec{d_2} = 2(\vec{p}\times\vec{q})$.

Step 2. Numerical: $\vec{d_1}\times\vec{d_2} = (-2, 3, 7)$, $|\cdot| = \sqrt{62}$.

Step 3. Area $= \sqrt{62}/2$.

Why this matters. Recognise the "diagonal-based area" formula as a CBSE shortcut.

Final Answer: $\frac{\sqrt{62}}{2}$ sq. units.

Q 10.18 **If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.**

SOLUTION

Concept used. The system $\vec{a} \times \vec{c} = \vec{b}$, $\vec{a} \cdot \vec{c} = 3$ has a unique solution when $\vec{a} \cdot \vec{b} = 0$ (a consistency condition). Use the identity $\vec{a} \times (\vec{a} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{a})\vec{c}$.

Step 1. Check consistency: $\vec{a} \cdot \vec{b} = (1)(0) + (1)(1) + (1)(-1) = 0$. ✓

Step 2. Cross $\vec{a}$ with $\vec{a} \times \vec{c} = \vec{b}$ from the left:

$$\vec{a} \times (\vec{a} \times \vec{c}) = \vec{a} \times \vec{b}.$$

Using the triple-cross identity: $(\vec{a} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{a})\vec{c} = \vec{a} \times \vec{b}$.

Step 3. $\vec{a} \cdot \vec{a} = 3$, $\vec{a} \cdot \vec{c} = 3$. So $3\vec{a} - 3\vec{c} = \vec{a} \times \vec{b} \Rightarrow \vec{c} = \vec{a} - \frac{1}{3}(\vec{a} \times \vec{b})$.

Step 4. Compute $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \hat{i}(-1-1) - \hat{j}(-1-0) + \hat{k}(1-0) = -2\hat{i} + \hat{j} + \hat{k}$.

Step 5. Hence $\vec{c} = (\hat{i} + \hat{j} + \hat{k}) - \frac{1}{3}(-2\hat{i} + \hat{j} + \hat{k}) = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$.

Step 6. Verify: $\vec{a} \cdot \vec{c} = \frac{5}{3} + \frac{2}{3} + \frac{2}{3} = \frac{9}{3} = 3$. ✓

> **Final Answer:** $\vec{c} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$.

✗ Common Mistake

A common slip is to assume $\vec{c}$ is a scalar multiple of $\vec{b}$. It is not. The dot-product constraint adds a component along $\vec{a}$.

EXPERT'S SOLUTION : *Neha Rao, M.Sc Mathematics, IIT Bombay*

Strategic angle. The triple cross identity $\vec{a} \times (\vec{a} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{a} - |\vec{a}|^2\vec{c}$ converts a vector equation into a scalar-multiple equation for $\vec{c}$.

Step 1. Verify $\vec{a} \cdot \vec{b} = 0$.

Step 2. $\vec{c} = \vec{a} - \frac{1}{3}\vec{a} \times \vec{b}$.

Step 3. $\vec{a} \times \vec{b} = (-2, 1, 1)$, hence $\vec{c} = (5/3, 2/3, 2/3)$.

Why this matters. JEE Main 2024 carried a near-identical "vector equation" question; the triple-cross identity is the standard tool.

> **Final Answer:** $\vec{c} = \frac{1}{3}(5\hat{i} + 2\hat{j} + 2\hat{k})$.

Q 10.19 **The vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 is:**

(A) $\hat{i} - 2\hat{j} + 2\hat{k}$

(B) $\dfrac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$

(C) $3(\hat{i} - 2\hat{j} + 2\hat{k})$

(D) $9(\hat{i} - 2\hat{j} + 2\hat{k})$.

SOLUTION

Correct option: (C) $3(\hat{i} - 2\hat{j} + 2\hat{k})$.

Concept used. A vector of magnitude M along $\vec{v}$ is $M\hat{v} = M\dfrac{\vec{v}}{|\vec{v}|}$.

Step 1. $|\vec{v}| = \sqrt{1+4+4} = 3$.

Step 2. Required vector $= 9 \cdot \dfrac{1}{3}(\hat{i} - 2\hat{j} + 2\hat{k}) = 3(\hat{i} - 2\hat{j} + 2\hat{k})$.

Final Answer: Option **(C)**: $3(\hat{i} - 2\hat{j} + 2\hat{k})$.

Q 10.20 **The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio $3:1$ is:**

(A) $\dfrac{3\vec{a} - 2\vec{b}}{2}$

(B) $\dfrac{7\vec{a} - 8\vec{b}}{4}$

(C) $\dfrac{3\vec{a}}{4}$

(D) $\dfrac{5\vec{a}}{4}$.

SOLUTION

Correct option: (D) $\dfrac{5\vec{a}}{4}$.

Concept used. Internal section formula: $\vec{r} = \dfrac{m\vec{Q} + n\vec{P}}{m+n}$ when R divides PQ in ratio $m:n$ internally.

Step 1. Here $P = 2\vec{a} - 3\vec{b}$, $Q = \vec{a} + \vec{b}$, $m:n = 3:1$.

Step 2. $\vec{r} = \dfrac{3(\vec{a}+\vec{b}) + 1(2\vec{a} - 3\vec{b})}{3+1} = \dfrac{3\vec{a} + 3\vec{b} + 2\vec{a} - 3\vec{b}}{4} = \dfrac{5\vec{a}}{4}$.

Final Answer: Option **(D)**: $\dfrac{5\vec{a}}{4}$.

Q 10.21 **The vector having initial and terminal points as $(2, 5, 0)$ and $(-3, 7, 4)$, respectively, is:**

(A) $-\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$
(B) $5\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$
(C) $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$
(D) $\hat{\imath} + \hat{\jmath} + \hat{k}$.

SOLUTION

Correct option: (C) $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$.
Concept used. For initial point $P(x_1, y_1, z_1)$ and terminal point $Q(x_2, y_2, z_2)$:
$\vec{PQ} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$.
Step 1. $\vec{PQ} = (-3 - 2, 7 - 5, 4 - 0) = (-5, 2, 4)$.

> **Final Answer:** Option **(C)**: $-5\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$.

Q 10.22 **The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 4, respectively, and $\vec{a} \cdot \vec{b} = 2\sqrt{3}$ is:**

(A) $\pi/6$ **(B)** $\pi/3$ **(C)** $\pi/2$ **(D)** $5\pi/2$.

SOLUTION

Correct option: (B) $\pi/3$.

Concept used. $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$.

Step 1. $\cos\theta = \dfrac{2\sqrt{3}}{\sqrt{3} \cdot 4} = \dfrac{2\sqrt{3}}{4\sqrt{3}} = \frac{1}{2}$.

Step 2. $\theta = \cos^{-1}(1/2) = \pi/3$.

> **Final Answer:** Option **(B)**: $\pi/3$.

Q 10.23 **Find the value of λ such that the vectors $\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ are orthogonal:**

(A) 0 **(B)** 1 **(C)** $3/2$ **(D)** $-5/2$.

SOLUTION

Correct option: **(D)** $-5/2$.
Concept used. Two non-zero vectors are orthogonal iff $\vec{a} \cdot \vec{b} = 0$.

Step 1. $\vec{a} \cdot \vec{b} = (2)(1) + (\lambda)(2) + (1)(3) = 2 + 2\lambda + 3 = 5 + 2\lambda$.

Step 2. Set $= 0$: $2\lambda = -5 \Rightarrow \lambda = -5/2$.

Final Answer: Option **(D)**: $\lambda = -5/2$.

Q 10.24 **The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is:**
(A) $2/3$ **(B)** $3/2$ **(C)** $5/2$ **(D)** $2/5$.

SOLUTION

Correct option: **(A)** $2/3$.
Concept used. Two vectors are parallel iff corresponding components are proportional: $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$.

Step 1. Ratios: $\frac{3}{2} = \frac{-6}{-4} = \frac{3}{2}$. The z-component must satisfy $\frac{1}{\lambda} = \frac{3}{2} \Rightarrow \lambda = 2/3$.

Final Answer: Option **(A)**: $\lambda = 2/3$.

Q 10.25 **The vectors from origin to points A and B are $\vec{a} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$, respectively, then the area of triangle OAB is:**
(A) $\sqrt{340}$ **(B)** $\sqrt{25}$ **(C)** $\sqrt{229}$ **(D)** $\frac{1}{2}\sqrt{229}$.

SOLUTION

Correct option: **(D)** $\frac{1}{2}\sqrt{229}$.
Concept used. Area of triangle with one vertex at origin is $\frac{1}{2}|\vec{a} \times \vec{b}|$.

Step 1. Cross: $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = \hat{i}(-3-6) - \hat{j}(2-4) + \hat{k}(6+6) = -9\hat{i} + 2\hat{j} + 12\hat{k}$.

Step 2. Magnitude: $\sqrt{81 + 4 + 144} = \sqrt{229}$. Area $= \frac{1}{2}\sqrt{229}$.

Final Answer: Option **(D)**: $\frac{1}{2}\sqrt{229}$.

Q 10.26 **For any vector $\vec{a}$, the value of $(\vec{a}\times\hat{i})^2+(\vec{a}\times\hat{j})^2+(\vec{a}\times\hat{k})^2$ is equal to:**
(A) $\vec{a}^2$ **(B)** $3\vec{a}^2$ **(C)** $4\vec{a}^2$ **(D)** $2\vec{a}^2$.

SOLUTION

Correct option: **(D)** $2\vec{a}^2$.
Concept used. If $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$, compute each cross with a basis vector explicitly.

Step 1. $\vec{a}\times\hat{i}=(0,a_3,-a_2)$, so $|\vec{a}\times\hat{i}|^2=a_2^2+a_3^2$.

Step 2. Similarly $|\vec{a}\times\hat{j}|^2=a_1^2+a_3^2$, $|\vec{a}\times\hat{k}|^2=a_1^2+a_2^2$.

Step 3. Sum: $2(a_1^2+a_2^2+a_3^2)=2|\vec{a}|^2$.

Final Answer: Option **(D)**: $2|\vec{a}|^2$.

Q 10.27 **If $|\vec{a}|=10$, $|\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=12$, then value of $|\vec{a}\times\vec{b}|$ is:**
(A) 5 (B) 10 (C) 14 (D) 16.

SOLUTION

Correct option: **(D)** 16.
Concept used. Lagrange's identity: $|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$.

Step 1. $|\vec{a}|^2|\vec{b}|^2=100\cdot 4=400$.

Step 2. $|\vec{a}\times\vec{b}|^2=400-144=256$, so $|\vec{a}\times\vec{b}|=16$.

Final Answer: Option **(D)**: 16.

Q 10.28 **The vectors $\lambda\hat{i}+\hat{j}+2\hat{k}$, $\hat{i}+\lambda\hat{j}-\hat{k}$ and $2\hat{i}-\hat{j}+\lambda\hat{k}$ are coplanar if:**
(A) $\lambda=-2$ **(B)** $\lambda=0$ **(C)** $\lambda=1$ **(D)** $\lambda=-1$.

SOLUTION

Correct option: (A) $\lambda = -2$.

Concept used. Three vectors are coplanar iff their scalar triple product is zero:

$$\begin{vmatrix} \lambda & 1 & 2 \\ 1 & \lambda & -1 \\ 2 & -1 & \lambda \end{vmatrix} = 0.$$

Step 1. Expand:

$$\lambda(\lambda^2 - 1) - 1(\lambda + 2) + 2(-1 - 2\lambda) = \lambda^3 - \lambda - \lambda - 2 - 2 - 4\lambda = \lambda^3 - 6\lambda - 4.$$

Step 2. Set $= 0$: $\lambda^3 - 6\lambda - 4 = 0$. Test $\lambda = -2$: $-8 + 12 - 4 = 0$. ✓

> **Final Answer:** Option **(A)**: $\lambda = -2$.

Q 10.29 **If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is:**

(A) 1 (B) 3 (C) $-3/2$ (D) None of these.

SOLUTION

Correct option: (C) $-3/2$.

Concept used. Square the closure $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ via the dot product.

Step 1. $|\vec{a} + \vec{b} + \vec{c}|^2 = 0$. Expand:

$$|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0.$$

Step 2. Unit vectors: $3 + 2S = 0 \Rightarrow S = -3/2$.

> **Final Answer:** Option **(C)**: $-3/2$.

Q 10.30 **Projection vector of $\vec{a}$ on $\vec{b}$ is:**

(A) $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\vec{b}$

(B) $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$

(C) $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

(D) $\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}\right)\hat{b}$.

SOLUTION

Correct option: (A) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

Concept used. The vector projection of $\vec{a}$ on $\vec{b}$ is the scalar projection times the unit vector $\hat{b}$, i.e. $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\hat{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$.

Step 1. Definition: $\text{proj}_{\vec{b}}\vec{a} = (\vec{a}\cdot\hat{b})\hat{b} = \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\cdot\frac{\vec{b}}{|\vec{b}|} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$.

Final Answer: Option **(A)**.

Q 10.31 **If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=2$, $|\vec{b}|=3$, $|\vec{c}|=5$, then value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is:**

(A) 0 **(B)** 1 **(C)** -19 **(D)** 38.

SOLUTION

Correct option: (C) -19.

Concept used. Square the closure and read off the pairwise-dot sum.

Step 1. $|\vec{a}+\vec{b}+\vec{c}|^2 = 0 \Rightarrow 4+9+25+2S=0 \Rightarrow 38+2S=0 \Rightarrow S=-19$.

Final Answer: Option **(C)**: -19.

Q 10.32 **If $|\vec{a}|=4$ and $-3\le\lambda\le 2$, then the range of $|\lambda\vec{a}|$ is:**

(A) $[0,8]$ **(B)** $[-12,8]$ **(C)** $[0,12]$ **(D)** $[8,12]$.

SOLUTION

Correct option: (C) $[0,12]$.

Concept used. $|\lambda\vec{a}| = |\lambda|\,|\vec{a}|$.

Step 1. $|\lambda|$ ranges over $[0,3]$ for $\lambda\in[-3,2]$.

Step 2. Hence $|\lambda\vec{a}| = 4|\lambda| \in [0,12]$.

Final Answer: Option **(C)**: $[0,12]$.

Q 10.33 **The number of vectors of unit length perpendicular to the vectors $\vec{a}=$**

$2\hat{i} + \hat{j} + 2\hat{k}$ **and** $\vec{b} = \hat{j} + \hat{k}$ **is:**

(A) one (B) two (C) three (D) infinite.

SOLUTION

Correct option: (B) two.

Concept used. For any pair of non-parallel non-zero vectors, exactly two unit vectors are perpendicular to both, namely $\pm\dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$.

Step 1. $\vec{a} \nparallel \vec{b}$ (components not proportional). Two unit normals exist.

> **Final Answer:** Option **(B)**: two.

Q 10.34 **The vector $\vec{a} + \vec{b}$ bisects the angle between the non-collinear vectors $\vec{a}$ and $\vec{b}$ if ________.**

SOLUTION

Concept used. The diagonal of the parallelogram with sides $\vec{a}, \vec{b}$ bisects the angle between them iff the parallelogram is a rhombus, i.e. $|\vec{a}| = |\vec{b}|$.

Step 1. For the sum $\vec{a} + \vec{b}$ to bisect the angle, the sides must have equal magnitudes: $|\vec{a}| = |\vec{b}|$.

> **Final Answer:** Blank: $|\vec{a}| = |\vec{b}|$.

Q 10.35 **If $\vec{r} \cdot \vec{a} = 0$, $\vec{r} \cdot \vec{b} = 0$ and $\vec{r} \cdot \vec{c} = 0$ for some non-zero vector $\vec{r}$, then the value of $\vec{a} \cdot (\vec{b} \times \vec{c})$ is ________.**

SOLUTION

Concept used. If a non-zero $\vec{r}$ is perpendicular to all of $\vec{a}, \vec{b}, \vec{c}$, then $\vec{a}, \vec{b}, \vec{c}$ are coplanar (they all lie in the plane normal to $\vec{r}$). Coplanar vectors have zero scalar triple product.

Step 1. Coplanarity $\Leftrightarrow [\vec{a}\,\vec{b}\,\vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = 0$.

> **Final Answer:** Blank: 0.

Q 10.36 **The vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} - 2\hat{k}$ are the adjacent sides of a parallelogram. The acute angle between its diagonals is ________.**

SOLUTION

Concept used. Diagonals of the parallelogram: $\vec{d_1} = \vec{a} + \vec{b}$, $\vec{d_2} = \vec{a} - \vec{b}$. Use $\cos\theta = \dfrac{\vec{d_1} \cdot \vec{d_2}}{|\vec{d_1}||\vec{d_2}|}$ and take the acute branch.

Step 1. $\vec{d_1} = (2, -2, 0)$, $\vec{d_2} = (4, -2, 4)$.

Step 2. Dot: $\vec{d_1} \cdot \vec{d_2} = 8 + 4 + 0 = 12$.

Step 3. Magnitudes: $|\vec{d_1}| = \sqrt{4+4} = 2\sqrt{2}$, $|\vec{d_2}| = \sqrt{16+4+16} = 6$.

Step 4. $\cos\theta = \dfrac{12}{2\sqrt{2} \cdot 6} = \dfrac{12}{12\sqrt{2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = \pi/4$.

Final Answer: Blank: $\pi/4$ (i.e. $45°$).

Q 10.37 **The values of k for which $|k\vec{a}| < |\vec{a}|$ and $k\vec{a} + \frac{1}{2}\vec{a}$ is parallel to $\vec{a}$ holds true are ________.**

SOLUTION

Concept used. $|k\vec{a}| = |k||\vec{a}| < |\vec{a}|$ gives $|k| < 1$. Any non-zero scalar combination of $\vec{a}$ is parallel to $\vec{a}$, so the second condition is automatic, except we must exclude $k = -1/2$ (which makes the sum the zero vector).

Step 1. $|k| < 1$ gives $k \in (-1, 1)$.

Step 2. Combined with $k \neq -1/2$: $k \in (-1, 1) \setminus \{-1/2\}$.

Final Answer: Blank: $k \in (-1, 1)$, $k \neq -1/2$.

Q 10.38 **The value of the expression $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$ is ________.**

SOLUTION

Concept used. Lagrange's identity: $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$.

Step 1. Direct application.

Final Answer: Blank: $|\vec{a}|^2|\vec{b}|^2$.

Q 10.39 **If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to ________.**

SOLUTION

Concept used. Lagrange's identity.

Step 1. $|\vec{a}|^2|\vec{b}|^2 = 144 \Rightarrow 16|\vec{b}|^2 = 144 \Rightarrow |\vec{b}|^2 = 9 \Rightarrow |\vec{b}| = 3.$

Final Answer: Blank: $|\vec{b}| = 3$.

Q 10.40 **If $\vec{a}$ is any non-zero vector, then $(\vec{a} \cdot \hat{i})\hat{i} + (\vec{a} \cdot \hat{j})\hat{j} + (\vec{a} \cdot \hat{k})\hat{k}$ equals ________.**

SOLUTION

Concept used. The scalar projections of $\vec{a}$ on the basis vectors are exactly the components a_1, a_2, a_3 of $\vec{a}$.

Step 1. $\vec{a} \cdot \hat{i} = a_1$, $\vec{a} \cdot \hat{j} = a_2$, $\vec{a} \cdot \hat{k} = a_3$.

Step 2. Sum: $a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = \vec{a}$.

Final Answer: Blank: $\vec{a}$.

Q 10.41 **If $|\vec{a}| = |\vec{b}|$, then necessarily it implies $\vec{a} = \pm\vec{b}$. True or False?**

SOLUTION

Answer: False.

Concept used. Magnitude tells us the length only, not the direction. Two vectors can have equal magnitudes yet point in entirely different directions.

Step 1. Counter-example: $\vec{a} = \hat{i}$, $\vec{b} = \hat{j}$. Both have magnitude 1 but $\vec{a} \neq \pm\vec{b}$.

Final Answer: False (equal magnitude $\neq$ equal direction).

Q 10.42 **Position vector of a point P is a vector whose initial point is origin. True or False?**

SOLUTION

Answer: True.
Concept used. By definition, the position vector of a point P is $\vec{OP}$, where O is the origin.

Step 1. Standard definition. True.

Final Answer: True.

Q 10.43 **If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then the vectors $\vec{a}$ and $\vec{b}$ are orthogonal. True or False?**

SOLUTION

Answer: True.
Concept used. Square both sides and expand:
$|\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = |\vec{a}|^2 - 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 \Rightarrow 4\vec{a}\cdot\vec{b} = 0 \Rightarrow \vec{a}\cdot\vec{b} = 0$, i.e. orthogonal.

Step 1. Square; cancel; conclude orthogonality.

Final Answer: True.

Q 10.44 **The formula $(\vec{a}+\vec{b})^2 = \vec{a}^2 + \vec{b}^2 + 2\vec{a}\times\vec{b}$ is valid for non-zero vectors $\vec{a}$ and $\vec{b}$. True or False?**

SOLUTION

Answer: False.
Concept used. The correct expansion is $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b}$, with the *dot* product, not the *cross* product. The cross product returns a vector and cannot be added to scalars $|\vec{a}|^2$ and $|\vec{b}|^2$.

Step 1. Correct identity uses $\vec{a}\cdot\vec{b}$, not $\vec{a}\times\vec{b}$.

Final Answer: False.

Q 10.45 **If $\vec{a}$ and $\vec{b}$ are adjacent sides of a rhombus, then $\vec{a}\cdot\vec{b} = 0$. True or False?**

SOLUTION

Answer: False.

Concept used. A rhombus has all four sides of equal length, but adjacent sides are not generally perpendicular (that would make it a square). The diagonals of a rhombus are perpendicular, not the sides.

Step 1. In general $\vec{a} \cdot \vec{b} \neq 0$ for a rhombus. The condition $\vec{a} \cdot \vec{b} = 0$ identifies a square.

Final Answer: False.

Key Takeaways

- A unit vector is obtained by dividing a non-zero vector by its magnitude: $\hat{a} = \vec{a}/|\vec{a}|$. Direction cosines always satisfy $l^2 + m^2 + n^2 = 1$.
- Section formula (internal $m{:}n$): $\vec{r} = (m\vec{b} + n\vec{a})/(m+n)$; external: $(m\vec{b} - n\vec{a})/(m-n)$. Translating "produced to" into a ratio is the standard CBSE trick.
- Dot product handles angles and projections; cross product handles areas, perpendiculars, and parallelism. Use Lagrange's identity $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$ to convert between them.
- Area of triangle $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$ and area of parallelogram by diagonals $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$ are the two highest-frequency 5-mark templates.
- Three vectors are coplanar iff their scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$, equivalently iff their 3×3 determinant vanishes.

End of NCERT Exemplar Problems