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NCERT Exemplar Solutions

Solved NCERT Exemplar Problems for Class 12 Mathematics, Chapter 13 — Representative Set

Chapter 13: Probability

About this Chapter

Class 12 **Probability** extends the elementary classical probability of Class 11 into the world of **conditional probability**, the **multiplication theorem**, the **total probability theorem**, **Bayes' theorem**, **random variables**, their probability distributions, mean, variance, and the **Binomial distribution** obtained from independent Bernoulli trials. The Exemplar problems drill exactly the cases that the board paper (4–5-mark Bayes question) and JEE/CUET (mixed MCQs on independence, BMD, expectation) repeatedly target.

Topics covered: Conditional probability $P(A \mid B)$ • Multiplication theorem • Independent events • Total probability theorem • Bayes' theorem • Random variable • Probability distribution • Mean $E(X)$ • Variance σ^2 • Bernoulli trial • Binomial distribution $B(n, p)$

Quick Formula Sheet

Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, $P(B) > 0$

Multiplication theorem: $P(A \cap B) = P(A) \cdot P(B \mid A) = P(B) \cdot P(A \mid B)$

Independence: A, B indep. $\Leftrightarrow P(A \cap B) = P(A)P(B)$

Total probability: $P(A) = \sum_i P(E_i)\,P(A \mid E_i)$

Bayes' theorem: $P(E_i \mid A) = \dfrac{P(E_i)\,P(A \mid E_i)}{\sum_j P(E_j)\,P(A \mid E_j)}$

Binomial: $P(X = r) = \binom{n}{r} p^r q^{n-r}$; $\mu = np$, $\sigma^2 = npq$

I. Multiple Choice Questions (MCQ)

Q 13.1 **If** $P(A) = \frac{4}{5}$ **and** $P(A \cap B) = \frac{7}{10}$, **then** $P(B \mid A)$ **is equal to**

(A) $\frac{1}{10}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{17}{20}$

SOLUTION

Correct option: (C) $\frac{7}{8}$.

Concept used. The conditional probability of event B given that event A has occurred is defined as

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}, \qquad P(A) > 0.$$

This is the basic formula introduced in §13.2 of the NCERT chapter.

Step 1. Identify the given values: $P(A) = \frac{4}{5}$ and $P(A \cap B) = \frac{7}{10}$.

Step 2. Substitute into the conditional formula:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{7/10}{4/5}.$$

Step 3. Simplify the complex fraction:

$$\frac{7/10}{4/5} = \frac{7}{10} \times \frac{5}{4} = \frac{7 \times 5}{10 \times 4} = \frac{35}{40} = \frac{7}{8}.$$

Step 4. Sanity check: $0 \leq P(B \mid A) = 7/8 \leq 1$ ✓.

> **Final Answer:** $P(B \mid A) = \frac{7}{8}$; option **(C)**.

☞ **Order matters**

$P(B \mid A) = P(A \cap B)/P(A)$, but $P(A \mid B) = P(A \cap B)/P(B)$. The *numerator* is the same intersection, but the *denominator* is the probability of the *given* event.

EXPERT'S SOLUTION : *Aarav Sharma, M.Sc Mathematics, IIT Kanpur*

Direct substitution.

Concept used. $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$.

Step 1. Plug in: $\frac{7/10}{4/5}$.

Step 2. Re-write as multiplication by reciprocal: $\frac{7}{10} \cdot \frac{5}{4} = \frac{35}{40}$.

Step 3. Reduce: $\gcd(35, 40) = 5$, so $= \frac{7}{8}$.

> **Final Answer:** $\frac{7}{8}$; option **(C)**.

Q 13.2 **If** $P(A \cap B) = \frac{7}{10}$ **and** $P(B) = \frac{17}{20}$, **then** $P(A \mid B)$ **equals**

(A) $\frac{14}{17}$ **(B)** $\frac{17}{20}$ **(C)** $\frac{7}{8}$ **(D)** $\frac{1}{8}$

SOLUTION

Correct option: (A) $\frac{14}{17}$.

Concept used. Conditional probability formula: $P(A \mid B) = P(A \cap B)/P(B)$ provided $P(B) > 0$.

Step 1. Substitute: $P(A \mid B) = \frac{7/10}{17/20}$.

Step 2. Re-write as multiplication: $\frac{7}{10} \cdot \frac{20}{17} = \frac{7 \times 20}{10 \times 17} = \frac{140}{170}$.

Step 3. Reduce: $\gcd(140, 170) = 10$: $\frac{140}{170} = \frac{14}{17}$.

Step 4. Sanity: $14/17 \approx 0.824 \in [0, 1]$ ✓.

> **Final Answer:** $P(A \mid B) = \frac{14}{17}$; option **(A)**.

EXPERT'S SOLUTION *: Priya Iyer, Ph.D Mathematics, IISc Bangalore*

Cross-multiplication angle. $P(A \mid B) = P(A \cap B)/P(B)$:

$$\frac{7/10}{17/20} = \frac{7 \cdot 20}{10 \cdot 17} = \frac{140}{170} = \frac{14}{17}.$$

> **Final Answer:** $\frac{14}{17}$; option **(A)**.

Q 13.3 **If** $P(A) = \frac{3}{10}$, $P(B) = \frac{2}{5}$ **and** $P(A \cup B) = \frac{3}{5}$, **then** $P(B \mid A) + P(A \mid B)$ **equals**

(A) $\frac{1}{4}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{5}{12}$ **(D)** $\frac{7}{12}$

SOLUTION

Correct option: (D) $\frac{7}{12}$.

Concept used. We need $P(A \cap B)$ first (from the addition theorem), then use conditional probability twice.

Step 1. Find $P(A \cap B)$ using the **addition theorem** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$:

$$\frac{3}{5} = \frac{3}{10} + \frac{2}{5} - P(A \cap B).$$

Step 2. Combine right-hand side: $\frac{3}{10} + \frac{2}{5} = \frac{3}{10} + \frac{4}{10} = \frac{7}{10}$.

Step 3. So $\frac{3}{5} = \frac{7}{10} - P(A \cap B)$, giving $P(A \cap B) = \frac{7}{10} - \frac{3}{5} = \frac{7}{10} - \frac{6}{10} = \frac{1}{10}$.

Step 4. Now compute the two conditional probabilities:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{1/10}{3/10} = \frac{1}{3},$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{1/10}{2/5} = \frac{1}{10} \cdot \frac{5}{2} = \frac{1}{4}.$$

Step 5. Add them:

$$P(B \mid A) + P(A \mid B) = \frac{1}{3} + \frac{1}{4} = \frac{4+3}{12} = \frac{7}{12}.$$

> **Final Answer:** $P(B \mid A) + P(A \mid B) = \frac{7}{12}$; option **(D)**.

☞ Three given ⇒ extract $P(A \cap B)$

Whenever you are given $P(A),\ P(B),\ P(A \cup B)$, the very first step in 95% of questions is to compute $P(A \cap B)$ via the addition theorem. Almost everything else flows from there.

EXPERT'S SOLUTION *: Vikram Rao, M.Sc Mathematics, Delhi University*

Symmetry angle. Note that
$P(B \mid A) + P(A \mid B) = P(A \cap B)\left(\frac{1}{P(A)} + \frac{1}{P(B)}\right) = P(A \cap B) \cdot \frac{P(A)+P(B)}{P(A)P(B)}$.
Concept used. Pulling out the common factor $P(A \cap B)$.

Step 1. Compute $P(A \cap B) = 1/10$ as before.

Step 2. $\frac{P(A)+P(B)}{P(A)P(B)} = \frac{3/10 + 2/5}{(3/10)(2/5)} = \frac{7/10}{6/50} = \frac{7}{10} \cdot \frac{50}{6} = \frac{35}{6}$.

Step 3. Product: $\frac{1}{10} \cdot \frac{35}{6} = \frac{35}{60} = \frac{7}{12}$.

> **Final Answer:** $\frac{7}{12}$; option **(D)**.

Q 13.4 **If** $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ **and** $P(A \cap B) = \frac{1}{5}$, **then** $P(A' \mid B') \cdot P(B' \mid A')$ **is equal to**

(A) $\frac{5}{6}$ **(B)** $\frac{5}{7}$ **(C)** $\frac{25}{42}$ **(D)** 1

SOLUTION

Correct option: (C) $\frac{25}{42}$.

Concept used. For complements: $P(A') = 1 - P(A)$, and $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B)$.

Step 1. Compute
$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{5} + \frac{3}{10} - \frac{1}{5} = \frac{4}{10} + \frac{3}{10} - \frac{2}{10} = \frac{5}{10} = \frac{1}{2}$.

Step 2. $P(A') = 1 - \frac{2}{5} = \frac{3}{5}$. $P(B') = 1 - \frac{3}{10} = \frac{7}{10}$.
$P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{1}{2} = \frac{1}{2}$.

Step 3. Compute the two conditional probabilities:

$$P(A' \mid B') = \frac{P(A' \cap B')}{P(B')} = \frac{1/2}{7/10} = \frac{1}{2} \cdot \frac{10}{7} = \frac{10}{14} = \frac{5}{7}.$$

$$P(B' \mid A') = \frac{P(A' \cap B')}{P(A')} = \frac{1/2}{3/5} = \frac{1}{2} \cdot \frac{5}{3} = \frac{5}{6}.$$

Step 4. Product:

$$\frac{5}{7} \cdot \frac{5}{6} = \frac{25}{42}.$$

> **Final Answer:** $P(A' \mid B') \cdot P(B' \mid A') = \frac{25}{42}$; option **(C)**.

EXPERT'S SOLUTION : *Aarav Sharma, M.Sc Mathematics, IIT Kanpur*

Complement-trick angle. The denominator of both conditionals is a complement probability; the numerator $P(A' \cap B') = 1 - P(A \cup B)$ saves three steps.

Concept used. $P(A' \cap B') = P((A \cup B)')$ — De Morgan.

Step 1. $P(A \cup B) = 0.4 + 0.3 - 0.2 = 0.5$, so $P(A' \cap B') = 0.5$.

Step 2. Conditionals: $P(A' \mid B') = 0.5/0.7 = 5/7$; $P(B' \mid A') = 0.5/0.6 = 5/6$.

Step 3. Product $= (5/7)(5/6) = 25/42$.

> **Final Answer:** $\frac{25}{42}$; option **(C)**.

Q 13.5 **Three persons A, B and C, fire at a target in turn, starting with A. Their probabilities of hitting the target are** $0.4,\ 0.3$ **and** 0.2 **respectively. The probability of two hits is**

(A) 0.024 **(B)** 0.188 **(C)** 0.336 **(D)** 0.452

SOLUTION

Correct option: (B) 0.188.

Concept used. The shots are *independent*, so we use $P(\text{specific outcome}) = \prod P_i^{\text{hit}} \cdot \prod (1 - P_j)^{\text{miss}}$. "Exactly two hits" means one of the three persons misses; we sum over all three "one-misser" cases.

Step 1. Let $a = P(A \text{ hits}) = 0.4,\ b = 0.3,\ c = 0.2$ and $a' = 0.6,\ b' = 0.7,\ c' = 0.8$.

Step 2. Three cases of "exactly two hits":

(1) A and B hit, C misses: $a \cdot b \cdot c' = 0.4(0.3)(0.8) = 0.096$.

(2) A and C hit, B misses: $a \cdot b' \cdot c = 0.4(0.7)(0.2) = 0.056$.

(3) B and C hit, A misses: $a' \cdot b \cdot c = 0.6(0.3)(0.2) = 0.036$.

Step 3. Add all three (mutually exclusive cases):

$$P(\text{exactly two hits}) = 0.096 + 0.056 + 0.036 = 0.188.$$

Final Answer: 0.188; option **(B)**.

☞ Sum over which one misses

"Exactly k hits out of n independent shots" has $\binom{n}{k}$ cases, each requiring a specific multiplication. Always enumerate them explicitly; don't try a shortcut formula unless all p_i are equal (then it's Binomial).

EXPERT'S SOLUTION *: Priya Iyer, Ph.D Mathematics, IISc Bangalore*

Generating-function angle. The probability of exactly k hits is the coefficient of z^k in $(0.6 + 0.4z)(0.7 + 0.3z)(0.8 + 0.2z)$.

Concept used. Multiply out and collect coefficients.

Step 1. Coefficient of z^2: pick z from two factors and the constant from one.

Step 2. Three ways:

$$(0.4)(0.3)(0.8) + (0.4)(0.7)(0.2) + (0.6)(0.3)(0.2) = 0.096 + 0.056 + 0.036 = 0.188.$$

Final Answer: 0.188; option **(B)**.

Q 13.6 **In a college, 30% of the students fail in Physics, 25% fail in Mathematics and 10% fail in both. One student is selected at random. The probability that the student fails in Physics given that he has failed in Mathematics is**

(A) $\frac{1}{10}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{9}{20}$ **(D)** $\frac{1}{3}$

SOLUTION

Correct option: (B) $\frac{2}{5}$.

Concept used. Translate percentages to probabilities, then apply conditional probability $P(F_P \mid F_M) = P(F_P \cap F_M)/P(F_M)$.

Step 1. Let F_P = fails Physics, F_M = fails Mathematics. Given

$P(F_P) = 0.30,\ P(F_M) = 0.25,\ P(F_P \cap F_M) = 0.10.$

Step 2. Apply conditional formula:

$$P(F_P \mid F_M) = \frac{P(F_P \cap F_M)}{P(F_M)} = \frac{0.10}{0.25}.$$

Step 3. Compute: $\frac{0.10}{0.25} = \frac{10}{25} = \frac{2}{5}$.

Final Answer: $P(F_P \mid F_M) = \frac{2}{5}$; option **(B)**.

EXPERT'S SOLUTION : *Vikram Rao, M.Sc Mathematics, Delhi University*

Reduced-sample-space angle. "Given failed in Math" restricts the universe to the 25% who failed Math. Of these, 10% also failed Physics. Fraction $= 10/25 = 2/5$.

Final Answer: $\frac{2}{5}$; option **(B)**.

Q 13.7 **If A and B are independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(A' \cap B')$ equals**

(A) $\frac{1}{12}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{2}{3}$

SOLUTION

Correct option: (C) $\frac{1}{2}$.

Concept used. Two events A, B are **independent** iff $P(A \cap B) = P(A)P(B)$. A standard *Theorem (NCERT §13.4)*: if A, B are independent, then so are A', B'. Therefore $P(A' \cap B') = P(A') \cdot P(B')$.

Step 1. Compute the complements: $P(A') = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$.
$P(B') = 1 - \frac{1}{4} = \frac{3}{4}$.

Step 2. Use independence of A', B':

$$P(A' \cap B') = P(A') \cdot P(B') = \frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12} = \frac{1}{2}.$$

Final Answer: $P(A' \cap B') = \frac{1}{2}$; option **(C)**.

☞ **Independence is preserved under complement**

If A and B are independent, so are: A, B'; A', B; A', B'. So you can simply multiply complement probabilities directly — no need to compute $P(A \cup B)$ first.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Kanpur*

Direct-multiplication angle. For independent events, complements are independent. So

$$P(A' \cap B') = (1 - P(A))(1 - P(B)) = \tfrac{2}{3} \cdot \tfrac{3}{4} = \tfrac{1}{2}.$$

Final Answer: $\frac{1}{2}$; option **(C)**.

Q 13.8 **A die is thrown. Let A be the event "odd number turns up" and B the event "a number ≤ 4 turns up". Then $P(A \mid B)$ is**

(A) $\frac{1}{2}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$

SOLUTION

Correct option: (A) $\frac{1}{2}$.

Concept used. For equiprobable finite sample spaces, conditional probability can be computed by counting: $P(A \mid B) = |A \cap B|/|B|$.

Step 1. Sample space $S = \{1, 2, 3, 4, 5, 6\}$.

Step 2. Event A (odd): $A = \{1, 3, 5\}$, $|A| = 3$.

Step 3. Event B (≤ 4): $B = \{1, 2, 3, 4\}$, $|B| = 4$.

Step 4. $A \cap B = \{1, 3\}$, $|A \cap B| = 2$.

Step 5. $P(A \mid B) = \frac{|A \cap B|}{|B|} = \frac{2}{4} = \frac{1}{2}$.

Final Answer: $P(A \mid B) = \frac{1}{2}$; option **(A)**.

EXPERT'S SOLUTION *: Priya Iyer, Ph.D Mathematics, IISc Bangalore*

Counting angle.
Concept used. In an equiprobable sample space, $P(A \mid B) =$ (favorable outcomes in B) / (total outcomes in B).

Step 1. Restrict universe to $B = \{1, 2, 3, 4\}$: 4 outcomes.

Step 2. Within B, the odd ones are $\{1, 3\}$: 2 outcomes.

Step 3. Probability $= 2/4 = 1/2$.

Final Answer: $\frac{1}{2}$; option **(A)**.

Q 13.9 **If A and B are two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, and $P(A \mid B) = \frac{1}{4}$, then $P(A' \cap B')$ equals**

(A) $\frac{1}{12}$ **(B)** $\frac{3}{16}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{1}{4}$

SOLUTION

Correct option: (D) $\frac{1}{4}$.
Concept used. Use $P(A \cap B) = P(B)P(A \mid B)$, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, and finally $P(A' \cap B') = 1 - P(A \cup B)$.

Step 1. Compute $P(A \cap B)$:

$$P(A \cap B) = P(B) \cdot P(A \mid B) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}.$$

Step 2. Compute $P(A \cup B)$:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{2} + \frac{1}{3} - \frac{1}{12}.$$

Step 3. Common denominator 12: $\frac{6}{12}+\frac{4}{12}-\frac{1}{12}=\frac{9}{12}=\frac{3}{4}$.

Step 4. Apply De Morgan: $P(A'\cap B')=P((A\cup B)')=1-P(A\cup B)=1-\frac{3}{4}=\frac{1}{4}$.

> **Final Answer:** $P(A'\cap B')=\frac{1}{4}$; option **(D)**.

☞ **The chain of formulas**

$P(A\mid B)\xrightarrow{\times P(B)}P(A\cap B)\xrightarrow{\text{add thm}}P(A\cup B)\xrightarrow{\text{De Morgan}}P(A'\cap B')$. Three steps, purely mechanical.

EXPERT'S SOLUTION : *Vikram Rao, M.Sc Mathematics, Delhi University*

Walk-the-chain.

Step 1. $P(A\cap B)=1/12$.

Step 2. $P(A\cup B)=6/12+4/12-1/12=9/12=3/4$.

Step 3. $P(A'\cap B')=1-3/4=1/4$.

> **Final Answer:** $\frac{1}{4}$.

Q 13.10 **If X is a random variable with the probability distribution**

X	0	1	2	3
$P(X)$	k	$2k$	$3k$	$4k$

then $E(X)$ equals

(A) 1 **(B)** $\frac{3}{2}$ **(C)** 2 **(D)** $\frac{5}{2}$

SOLUTION

Correct option: (C) 2.

Concept used. A discrete probability distribution requires $\sum p_i=1$. Use this to find k. The **mean** (or **expectation**) is $E(X)=\sum x_i\,p_i$.

Step 1. Find k from the normalisation $\sum p_i=1$:

$$k+2k+3k+4k=10k=1\ \Rightarrow\ k=\frac{1}{10}.$$

Step 2. Verify each $p_i \in [0,1]$: $p_0 = 0.1,\ p_1 = 0.2,\ p_2 = 0.3,\ p_3 = 0.4$ ✓.

Step 3. Compute $E(X) = \sum x_i\, p_i$:

$$E(X) = 0(0.1) + 1(0.2) + 2(0.3) + 3(0.4) = 0 + 0.2 + 0.6 + 1.2 = 2.0.$$

Final Answer: $E(X) = 2$; option **(C)**.

☞ **Normalise first, expect second**

Two-line trick: first force $\sum p_i = 1$ to extract k; then compute $E(X) = \sum x_i p_i$ with the now-numerical p_i values.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Kanpur*

Normalise + sum.

Step 1. $10k = 1 \Rightarrow k = 0.1$.

Step 2. Weighted sum: $0.0 + 0.2 + 0.6 + 1.2 = 2.0$.

Final Answer: $E(X) = 2$; option **(C)**.

II. Short Answer Questions (SA)

Q 13.1 **An unbiased coin is tossed** 5 **times. Find the probability of getting exactly** 3 **heads.**

SOLUTION

Concept used. Binomial distribution. The number of heads X in n independent tosses of a fair coin follows $X \sim B\big(n, p = \frac{1}{2}\big)$ with PMF

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}.$$

Step 1. Identify parameters: $n = 5$, $p = \dfrac{1}{2}$, $q = 1 - p = \dfrac{1}{2}$, $r = 3$.

Step 2. Substitute:

$$P(X = 3) = \binom{5}{3} \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2.$$

Step 3. Compute $\binom{5}{3}$:

$$\binom{5}{3} = \frac{5!}{3!\,2!} = \frac{5 \cdot 4}{2} = 10.$$

Step 4. Combine: $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$.

Step 5. Multiply:

$$P(X = 3) = 10 \cdot \frac{1}{32} = \frac{10}{32} = \frac{5}{16}.$$

> **Final Answer:** $P(X = 3) = \dfrac{5}{16} = 0.3125.$

☞ Binomial PMF mnemonic

"Choose × powers": $\binom{n}{r}$ counts the ways to place the successes; $p^r q^{n-r}$ weights each specific arrangement. They never appear separately.

EXPERT'S SOLUTION *: Priya Iyer, Ph.D Mathematics, IISc Bangalore*

Sample-space counting angle. For a fair coin, all $2^5 = 32$ sequences are equally likely. Number of sequences with exactly 3 heads: $\binom{5}{3} = 10$. So $P = 10/32 = 5/16$.
Concept used. Equiprobable sample-space approach (works because $p = 1/2$).

> **Final Answer:** $\dfrac{5}{16}$.

Q 13.2 **Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability that both cards are kings.**

SOLUTION

Concept used. Multiplication theorem for dependent events:
$P(A \cap B) = P(A) \cdot P(B \mid A)$. The two draws are dependent because the first king is not replaced.

Step 1. Let A = first card is a king, B = second card is a king. $P(A) = \dfrac{4}{52}$ (four kings in 52).

Step 2. Given A, the deck has 51 cards, 3 kings remaining. So

$$P(B \mid A) = \frac{3}{51}.$$

Step 3. Multiplication theorem:

$$P(A \cap B) = P(A) \cdot P(B \mid A) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652}.$$

Step 4. Simplify: $\frac{12}{2652} = \frac{1}{221}$ (divide num and denom by 12).

Step 5. Verification by alternative count: $\binom{4}{2}/\binom{52}{2} = 6/1326 = 1/221$ ✓.

Final Answer: $P(\text{both kings}) = \frac{1}{221}$.

✗ Don't multiply $\frac{4}{52} \cdot \frac{4}{52}$

That would correspond to drawing *with replacement*; the second king-probability becomes $4/52$ again only if the first card is put back. Without replacement, use $3/51$ for the second draw.

EXPERT'S SOLUTION *: Vikram Rao, M.Sc Mathematics, Delhi University*

Combinations angle. $P(\text{both kings}) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} = \frac{1}{221}$.

Concept used. Symmetric counting — "ways to choose 2 kings out of 4" over "ways to choose any 2 cards out of 52".

Final Answer: $\frac{1}{221}$.

Q 13.3 The probability distribution of a random variable X is given by

X	0	1	2	3	4
$P(X)$	0.1	k	0.3	$2k$	0.1

Find k and the variance of X.

SOLUTION

Concept used. Use $\sum p_i = 1$ to find k; then compute $E(X)$ and $E(X^2)$; finally use $\text{Var}(X) = E(X^2) - [E(X)]^2$.

Step 1. Normalisation:

$$0.1 + k + 0.3 + 2k + 0.1 = 1 \;\Rightarrow\; 0.5 + 3k = 1 \;\Rightarrow\; k = \frac{0.5}{3} = \frac{1}{6} \approx 0.1667.$$

So $k = \frac{1}{6}$ and $2k = \frac{1}{3}$.

Step 2. Re-write the distribution with numerical probabilities:

X	0	1	2	3	4
P	0.1	$\frac{1}{6}$	0.3	$\frac{1}{3}$	0.1

Or with common denominator 30: $\frac{3}{30}, \frac{5}{30}, \frac{9}{30}, \frac{10}{30}, \frac{3}{30}$. Sum $= 30/30 = 1$ ✓.

Step 3. Compute $E(X) = \sum x_i p_i$:

$$E(X) = 0 \cdot \tfrac{3}{30} + 1 \cdot \tfrac{5}{30} + 2 \cdot \tfrac{9}{30} + 3 \cdot \tfrac{10}{30} + 4 \cdot \tfrac{3}{30}.$$

Compute each: $0 + \tfrac{5}{30} + \tfrac{18}{30} + \tfrac{30}{30} + \tfrac{12}{30} = \tfrac{65}{30} = \tfrac{13}{6}$.

Step 4. Compute $E(X^2) = \sum x_i^2 p_i$:

$$E(X^2) = 0 + 1 \cdot \tfrac{5}{30} + 4 \cdot \tfrac{9}{30} + 9 \cdot \tfrac{10}{30} + 16 \cdot \tfrac{3}{30}.$$

Compute each: $0 + \tfrac{5}{30} + \tfrac{36}{30} + \tfrac{90}{30} + \tfrac{48}{30} = \tfrac{179}{30}$.

Step 5. Variance:

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = \frac{179}{30} - \left(\frac{13}{6}\right)^2.$$

Step 6. Compute $(13/6)^2 = 169/36$. Common denominator 180: $\frac{179}{30} = \frac{1074}{180}$, $\frac{169}{36} = \frac{845}{180}$. Subtract: $\frac{1074 - 845}{180} = \frac{229}{180} \approx 1.2722$.

Final Answer: $k = \frac{1}{6}$; $\mathrm{Var}(X) = \frac{229}{180} \approx 1.27$.

☞ Use common denominator

Distributions with mixed decimals and fractions are error-prone. Convert everything to a single common denominator (here 30) before summing.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Kanpur*

Tabular angle. Build a single table with columns x, p, xp, x^2p and total at the bottom.
Concept used. Direct tabular evaluation of $E(X)$ and $E(X^2)$ side by side.

Step 1. $\sum p = 1 \Rightarrow k = 1/6$.

Step 2. $\sum xp = 13/6 \approx 2.1667$.

Step 3. $\sum x^2p = 179/30 \approx 5.9667$.

Step 4. $\text{Var} = 5.9667 - (2.1667)^2 = 5.9667 - 4.6944 = 1.2722$.

Final Answer: $k = 1/6$; $\text{Var}(X) \approx 1.272$.

Q 13.4 **In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper, and 20% read both. A student is selected at random. Find the probability that he reads neither Hindi nor English newspapers.**

SOLUTION

Concept used. Use complement of the union:
$P(\text{neither}) = 1 - P(\text{at least one}) = 1 - P(H \cup E)$, with
$P(H \cup E) = P(H) + P(E) - P(H \cap E)$.

Step 1. Let H = reads Hindi, E = reads English. Given
$P(H) = 0.6,\ P(E) = 0.4,\ P(H \cap E) = 0.2$.

Step 2. Compute $P(H \cup E)$ by the addition theorem:

$$P(H \cup E) = 0.6 + 0.4 - 0.2 = 0.8.$$

Step 3. "Neither" is the complement: $P(H' \cap E') = 1 - P(H \cup E) = 1 - 0.8 = 0.2$.

Final Answer: $P(\text{neither}) = 0.2$.

EXPERT'S SOLUTION *: Priya Iyer, Ph.D Mathematics, IISc Bangalore*

Inclusion–exclusion + De Morgan. One line: $P(\text{neither}) = 1 - (0.6 + 0.4 - 0.2) = 0.2$.

Final Answer: 0.2.

Q 13.5 **Two dice are thrown together. What is the probability that the sum of the numbers on the two dice is 8, if it is known that the second die always exhibits an odd number?**

SOLUTION

Concept used. Conditional probability on the reduced sample space defined by "second die odd".

Step 1. Sample space restricted by "second die odd": second die can be $1, 3, 5$, so the restricted sample space is $\{(i, j) : i \in \{1, \dots, 6\},\ j \in \{1, 3, 5\}\}$ with $6 \times 3 = 18$ equally likely outcomes.

Step 2. Sum equals 8 on this restricted space: need $i + j = 8$ with $j \in \{1, 3, 5\}$.

(1) $j = 1 \Rightarrow i = 7$: impossible ($i \leq 6$).

(2) $j = 3 \Rightarrow i = 5$: $(5, 3)$ ✓.

(3) $j = 5 \Rightarrow i = 3$: $(3, 5)$ ✓.

Two outcomes.

Step 3. Conditional probability:

$$P(\text{sum} = 8 \mid \text{second die odd}) = \frac{2}{18} = \frac{1}{9}.$$

Final Answer: $P = \frac{1}{9}$.

☞ **Restrict, then count**

For dice conditional questions, list out the restricted sample space explicitly. Sample-space counting is faster and less error-prone than algebraic conditional-probability manipulation.

EXPERT'S SOLUTION *: Vikram Rao, M.Sc Mathematics, Delhi University*

Pair-spotter angle. Need ordered pair (i, j) with $i + j = 8$ and j odd. Two ordered pairs: $(5, 3),\ (3, 5)$. Restricted total $= 18$.
Concept used. Count favourable, divide by restricted total.

Final Answer: $\frac{1}{9}$.

III. Long Answer Questions (LA)

Q 13.1 **In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, $5,\ 4,\ 2$ per cent are defective bolts. A bolt is drawn at random from the product and is found to be**

defective. What is the probability that it is manufactured by machine B?

SOLUTION

Concept used. Bayes' theorem. Let E_1 = bolt is from A, E_2 = from B, E_3 = from C (a partition of the sample space). Let D = bolt is defective. We want $P(E_2 \mid D)$:

$$P(E_2 \mid D) = \frac{P(E_2)\,P(D \mid E_2)}{P(E_1)P(D \mid E_1) + P(E_2)P(D \mid E_2) + P(E_3)P(D \mid E_3)}.$$

Step 1. *Tabulate priors and likelihoods:*

Machine	$P(E_i)$	$P(D \mid E_i)$
A	0.25	0.05
B	0.35	0.04
C	0.40	0.02

Step 2. *Compute joint probabilities* $P(E_i)P(D \mid E_i)$*:*

$$P(E_1)P(D \mid E_1) = 0.25 \times 0.05 = 0.0125,$$
$$P(E_2)P(D \mid E_2) = 0.35 \times 0.04 = 0.0140,$$
$$P(E_3)P(D \mid E_3) = 0.40 \times 0.02 = 0.0080.$$

Step 3. *Total probability of defect:*

$$P(D) = 0.0125 + 0.0140 + 0.0080 = 0.0345.$$

Step 4. *Apply Bayes':*

$$P(E_2 \mid D) = \frac{0.0140}{0.0345}.$$

Step 5. Multiply numerator and denominator by $10\,000$: $\frac{140}{345}$. Reduce by $\gcd(140, 345) = 5$: $\frac{28}{69}$.

Step 6. *Numerical value:* $28/69 \approx 0.4058$.

Step 7. *Sanity check.* Verify $P(E_1 \mid D) + P(E_2 \mid D) + P(E_3 \mid D) = 1$: $\frac{125}{345} + \frac{140}{345} + \frac{80}{345} = \frac{345}{345} = 1$ ✓.

Final Answer: $P(E_2 \mid D) = \frac{28}{69} \approx 0.406.$

☞ **Bayes table is the time-saver**

For every Bayes question, draw a 3-column table: machine, prior, likelihood. Then add a fourth column for the product, and a fifth for the posterior (divide by the column total).

This makes the arithmetic mistake-proof.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Kanpur*

Probability-tree angle. Branch out: machine choice ($A/B/C$) at the first level, defect / no-defect at the second. The posterior $P(B \mid \text{defect})$ is the conditional weight of the B branch among the defect leaves.
Concept used. Bayes' theorem as a normalised reweighting of priors by likelihoods.

Step 1. Defect leaf weights: $A \to 25 \cdot 5 = 125$, $B \to 35 \cdot 4 = 140$, $C \to 40 \cdot 2 = 80$. Sum $= 345$.

Step 2. Posterior for B: $140/345 = 28/69$.

Final Answer: $\dfrac{28}{69}$.

Q 13.2 **Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the mean and standard deviation of the number of aces.**

SOLUTION

Concept used. Hypergeometric-style discrete distribution. X = number of aces in two cards drawn from a 52-card deck, with 4 *aces and* 48 *non-aces*. So $X \in \{0, 1, 2\}$.

Step 1. *Total ways to draw* 2 *cards:* $\binom{52}{2} = 1326$.

Step 2. *Compute* $P(X = 0)$*:* 2 non-aces from 48.

$$P(X = 0) = \frac{\binom{48}{2}}{\binom{52}{2}} = \frac{1128}{1326} = \frac{188}{221}.$$

Step 3. *Compute* $P(X = 1)$*:* 1 ace from 4 and 1 non-ace from 48.

$$P(X = 1) = \frac{\binom{4}{1}\binom{48}{1}}{\binom{52}{2}} = \frac{4 \cdot 48}{1326} = \frac{192}{1326} = \frac{32}{221}.$$

Step 4. *Compute* $P(X = 2)$*:* 2 aces from 4.

$$P(X = 2) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{6}{1326} = \frac{1}{221}.$$

Step 5. *Verification:* $\dfrac{188 + 32 + 1}{221} = \dfrac{221}{221} = 1$ ✓.

Step 6. *Distribution table:*

X	0	1	2
$P(X)$	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

Step 7. *Compute mean $E(X)$:*

$$E(X) = 0 \cdot \tfrac{188}{221} + 1 \cdot \tfrac{32}{221} + 2 \cdot \tfrac{1}{221} = \frac{0+32+2}{221} = \frac{34}{221} = \frac{2}{13}.$$

(Divide: $34 = 2 \cdot 17,\ 221 = 13 \cdot 17$, cancel 17.)

Step 8. *Compute $E(X^2)$:*

$$E(X^2) = 0 + 1 \cdot \tfrac{32}{221} + 4 \cdot \tfrac{1}{221} = \frac{36}{221}.$$

Step 9. *Variance:*

$$\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = \frac{36}{221} - \left(\frac{2}{13}\right)^2.$$

Compute $(2/13)^2 = 4/169$. Common denominator: $221 = 13 \cdot 17$, $169 = 13 \cdot 13$. LCM $= 13 \cdot 13 \cdot 17 = 2873$. So $\dfrac{36}{221} = \dfrac{36 \cdot 13}{2873} = \dfrac{468}{2873}$. $\dfrac{4}{169} = \dfrac{4 \cdot 17}{2873} = \dfrac{68}{2873}$. Subtract: $\dfrac{468 - 68}{2873} = \dfrac{400}{2873}$. Reduce: $\gcd(400, 2873) = 1$, so $\dfrac{400}{2873} \approx 0.1393$.

Step 10. *Standard deviation:* $\sigma = \sqrt{\mathrm{Var}(X)} = \sqrt{\dfrac{400}{2873}} \approx 0.373.$

Final Answer: $E(X) = \dfrac{2}{13}$; $\sigma = \sqrt{\dfrac{400}{2873}} \approx 0.373.$

☞ Hypergeometric vs Binomial

"Drawing without replacement" makes this *not* a Binomial distribution; use combinations $\binom{K}{k}\binom{N-K}{n-k}/\binom{N}{n}$ instead. Distinguishing between these two settings is a frequent CBSE trick.

EXPERT'S SOLUTION *: Priya Iyer, Ph.D Mathematics, IISc Bangalore*

Trial-by-trial angle. Compute mean via linearity of expectation: $X = I_1 + I_2$ where $I_k = 1$ if the kth card is an ace.

Concept used. $E(I_k) = P(k\text{th card is an ace}) = 4/52 = 1/13$ by symmetry (every position is equally likely to be an ace).

Step 1. $E(X) = E(I_1) + E(I_2) = 2 \cdot \frac{1}{13} = \frac{2}{13}$. Match!

Step 2. For variance, use $E(X^2)$ from the distribution (the indicators are dependent, so Var is not just a sum): $\mathrm{Var}(X) = 36/221 - 4/169 = 400/2873$.

Step 3. Standard deviation $\sigma = \sqrt{\mathrm{Var}(X)} = \sqrt{400/2873} \approx 0.373$.

Final Answer: $E(X) = \dfrac{2}{13}$; $\sigma = \sqrt{\dfrac{400}{2873}} \approx 0.373.$

Q 13.3 **A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.**

SOLUTION

Concept used. Binomial distribution with $n = 4$ independent trials, where "success" is a doublet (both dice show the same number) on a single throw of two dice.

Step 1. *Probability of a doublet on one throw.* Sample space of throwing two dice has 36 outcomes. Doublets are $(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)$: 6 outcomes. So $p = \dfrac{6}{36} = \dfrac{1}{6}$, and $q = 1 - p = \dfrac{5}{6}$.

Step 2. *Binomial setup.* X = number of doublets in $n = 4$ independent throws. $X \sim B(n = 4,\ p = 1/6)$ with PMF

$$P(X = r) = \binom{4}{r}\left(\tfrac{1}{6}\right)^r\left(\tfrac{5}{6}\right)^{4-r}.$$

Step 3. *Compute $P(X = 2)$.* Plug $r = 2$:

$$P(X = 2) = \binom{4}{2}\left(\tfrac{1}{6}\right)^2\left(\tfrac{5}{6}\right)^2.$$

Compute $\binom{4}{2} = 6$. Compute $(1/6)^2 = 1/36$ and $(5/6)^2 = 25/36$. So

$$P(X = 2) = 6 \cdot \frac{1}{36} \cdot \frac{25}{36} = \frac{6 \cdot 25}{36 \cdot 36} = \frac{150}{1296}.$$

Step 4. Reduce: $\gcd(150, 1296) = 6$: $\dfrac{150}{1296} = \dfrac{25}{216}$.

Step 5. Decimal check: $25/216 \approx 0.1157$.

Final Answer: $P(X = 2) = \dfrac{25}{216} \approx 0.116.$

☞ Spot the Binomial setup

Binomial requires four conditions (BINS): **B**inary outcomes, **I**ndependent trials, fixed **N** trials, and a constant **S**uccess probability. All four hold here: doublet vs. not-doublet, 4 independent throws, $p = 1/6$ same each time.

EXPERT'S SOLUTION *: Vikram Rao, M.Sc Mathematics, Delhi University*

Step-by-step substitution.

Step 1. Doublet probability $p = 6/36 = 1/6$.

Step 2. Apply Binomial: $P(X = 2) = \binom{4}{2}(1/6)^2(5/6)^2$.

Step 3. $= 6 \cdot (1/36) \cdot (25/36) = 150/1296 = 25/216$.

Final Answer: $\frac{25}{216}$.