Collegedunia NCERT Solutions

Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 4: Determinants

About this Chapter

The **determinant** of a square matrix is a single number that tells us whether the matrix is invertible and how it scales area or volume under a linear transformation. In this exercise we evaluate determinants of 2×2 and 3×3 matrices using the standard **expansion along a row or column**, and apply the property $|kA| = k^n\,|A|$ for an $n \times n$ matrix. The class-12 NCERT (2026-27) presents determinants up to order three with real entries only.

Topics covered: Determinant of order 2 • Determinant of order 3 • Expansion along a row/column • Scaling property $|kA| = k^n|A|$

Quick Formula Sheet

Order 2:
$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Order 3 (along R_1):
$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32})$
$-a_{12}(a_{21}a_{33} - a_{23}a_{31})$
$+a_{13}(a_{21}a_{32} - a_{22}a_{31})$

Scaling: $|kA| = k^n|A|$ for A of order n.

Exercise 4.1

Q 4.1 **Evaluate the determinant** $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$.

SOLUTION

Concept used. For a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the determinant is defined as

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$

This is the difference of the products of the two diagonals: the principal diagonal $a \cdot d$ minus the anti-diagonal $b \cdot c$.

Step 1. Identify the entries with $a = 2$, $b = 4$, $c = -5$, $d = -1$.

Step 2. Substitute in $ad - bc$:

$$\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = (2)(-1) - (4)(-5).$$

Step 3. Evaluate each product separately:

$$(2)(-1) = -2, \qquad (4)(-5) = -20.$$

Step 4. Combine, taking care of the sign:

$$-2 - (-20) = -2 + 20 = 18.$$

Final Answer: $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 18.$

☞ **Sign of the cross term**

The formula is $ad - bc$, not $ad + bc$. The negative sign in front of bc is what makes the determinant detect orientation: swap two columns and the sign flips.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Quick reading. A 2×2 determinant is a single arithmetic step. Read off the principal-diagonal product, the anti-diagonal product, and subtract.

Step 1. Principal diagonal: top-left $\times$ bottom-right $= 2 \times (-1) = -2$.

Step 2. Anti-diagonal: top-right $\times$ bottom-left $= 4 \times (-5) = -20$.

Step 3. Determinant = principal − anti $= -2 - (-20)$:

$$-2 - (-20) = -2 + 20 = 18.$$

Why this matters. The value $18 \neq 0$ tells us that the matrix $\begin{pmatrix} 2 & 4 \\ -5 & -1 \end{pmatrix}$ is non-singular, so it has an inverse, and the two column vectors $\binom{2}{-5}$ and $\binom{4}{-1}$ are linearly independent.

Final Answer: $|A| = 18.$

Q 4.2 **Evaluate the determinants:**

(i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ **(ii)** $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$.

SOLUTION

Concept used. For a 2×2 matrix the determinant is $ad-bc$. We also recall the Pythagorean identity $\sin^2\theta+\cos^2\theta=1$ for part (i) and the difference-of-squares $a^2-b^2=(a-b)(a+b)$ for part (ii).

Part (i).

Step 1. Identify $a=\cos\theta$, $b=-\sin\theta$, $c=\sin\theta$, $d=\cos\theta$.

Step 2. Apply $ad-bc$:

$$\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = (\cos\theta)(\cos\theta)-(-\sin\theta)(\sin\theta).$$

Step 3. Simplify the two products:

$$=\cos^2\theta+\sin^2\theta.$$

Step 4. Use $\sin^2\theta+\cos^2\theta=1$:

$$=1.$$

Final Answer: Value of (i) $=1$.

Part (ii).

Step 1. Here $a=x^2-x+1$, $b=x-1$, $c=x+1$, $d=x+1$.

Step 2. Apply $ad-bc$:

$$\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix} = (x^2-x+1)(x+1)-(x-1)(x+1).$$

Step 3. Expand the first product. Multiply (x^2-x+1) by $(x+1)$ term by term:

$$\begin{aligned}(x^2-x+1)(x+1) &= x^2\cdot x+x^2\cdot 1-x\cdot x-x\cdot 1+1\cdot x+1\cdot 1\\ &= x^3+x^2-x^2-x+x+1\\ &= x^3+1.\end{aligned}$$

(This is the sum-of-cubes factorisation $x^3+1=(x+1)(x^2-x+1)$ read backwards.)

Step 4. Expand the second product using $a^2-b^2=(a-b)(a+b)$:

$$(x-1)(x+1)=x^2-1.$$

Step 5. Subtract:

$$(x^3 + 1) - (x^2 - 1) = x^3 + 1 - x^2 + 1 = x^3 - x^2 + 2.$$

Final Answer: Value of (ii) $= x^3 - x^2 + 2$.

☞ Exam Tip

Part (i) is the determinant of a *rotation matrix* through angle θ. Rotation preserves area, so its determinant must be 1 — a useful sanity check that you have not slipped a sign.

EXPERT'S SOLUTION *: Pranav Iyer, M.Sc Mathematics, ISI Kolkata*

Structural observation. In (i) the matrix is the standard rotation matrix; in (ii) the rows have a hidden algebraic identity. Spotting the structure cuts the work in half.

Part (i).

Step 1. Write $ad - bc = \cos\theta \cdot \cos\theta - (-\sin\theta) \cdot \sin\theta$.

Step 2. Two negatives multiply to a positive: $-(-\sin^2\theta) = +\sin^2\theta$. So the expression is $\cos^2\theta + \sin^2\theta$.

Step 3. By the fundamental Pythagorean identity this equals 1.

Part (ii).

Step 1. Notice that $(x^2 - x + 1)(x + 1)$ is the standard expansion of $x^3 + 1$, since $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ with $a = x$, $b = 1$.

$$(x^2 - x + 1)(x + 1) = x^3 + 1^3 = x^3 + 1.$$

Step 2. Similarly $(x - 1)(x + 1)$ is the difference-of-squares pattern:

$$(x - 1)(x + 1) = x^2 - 1.$$

Step 3. Subtract the two:

$$(x^3 + 1) - (x^2 - 1) = x^3 - x^2 + 2.$$

Why this matters. Recognising algebraic identities ($a^3 + b^3$, $a^2 - b^2$) inside determinant entries is a recurring habit that will save many lines later in this chapter, especially when properties of determinants are applied to simplify 3×3 entries.

Final Answer: (i) 1; (ii) $x^3 - x^2 + 2$.

Q 4.3 **If $A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}$, then show that $|2A| = 4\,|A|$.**

SOLUTION

Concept used. Two facts.

(a) For a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $|A| = ad - bc$.

(b) If every entry of an $n \times n$ matrix is multiplied by a scalar k, the determinant is multiplied by k^n. That is,

$$|kA| \;=\; k^n\,|A|.$$

Here $n = 2$, so we expect $|2A| = 2^2\,|A| = 4\,|A|$.

Step 1. Compute $|A|$ directly:

$$|A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = (1)(2) - (2)(4) = 2 - 8 = -6.$$

Step 2. Form $2A$ by doubling every entry:

$$2A = \begin{pmatrix} 2\cdot 1 & 2\cdot 2 \\ 2\cdot 4 & 2\cdot 2 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 8 & 4 \end{pmatrix}.$$

Step 3. Compute $|2A|$ directly:

$$|2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = (2)(4) - (4)(8) = 8 - 32 = -24.$$

Step 4. Compute $4\,|A|$:

$$4\,|A| = 4 \times (-6) = -24.$$

Step 5. Compare: $|2A| = -24 = 4\,|A|$. The required equality holds.

> **Final Answer:** $|2A| = -24 = 4\,|A|$.

♥ Why k^n and not k

A scalar k scales every length by k, but a determinant measures *area* in 2D (volume in 3D). Area scales as the square of the length factor: doubling every entry of a 2×2 matrix scales the parallelogram spanned by its columns by $2^2 = 4$.

EXPERT'S SOLUTION : *Aanya Mehta, M.Tech CS, IIT Madras*

Strategic angle. The cleanest verification is to compute both sides and compare. We do it once via the entries, then re-derive $|kA| = k^n|A|$ symbolically so the result generalises.

Step 1. Direct computation of $|A|$:

$$|A| = 1 \cdot 2 - 2 \cdot 4 = 2 - 8 = -6.$$

Step 2. Direct computation of $|2A|$ where $2A = \begin{pmatrix} 2 & 4 \\ 8 & 4 \end{pmatrix}$:

$$|2A| = 2 \cdot 4 - 4 \cdot 8 = 8 - 32 = -24.$$

Step 3. Symbolic check: for any 2×2 matrix,

$$|kA| = (ka)(kd) - (kb)(kc) = k^2(ad - bc) = k^2\,|A|.$$

Setting $k = 2$ gives $|2A| = 4\,|A|$ for every 2×2 matrix, in particular for ours.

Step 4. Conclude: $-24 = 4 \times (-6)$. The equality holds.

Why this matters. The same idea extends to order 3: $|2A| = 8\,|A|$, and in general $|kA| = k^n|A|$. Many JEE problems use this directly to avoid computing a fresh 3×3 determinant.

> **Final Answer:** $|2A| = -24 = 4\,|A|$, in agreement with $|kA| = k^2|A|$.

Q 4.4 **If** $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{pmatrix}$**, then show that** $|3A| = 27\,|A|$**.**

SOLUTION

Concept used. For an $n \times n$ matrix and any scalar k,

$$|kA| \;=\; k^n\,|A|.$$

Here $n = 3$, so $|3A| = 3^3\,|A| = 27\,|A|$. We also use the fact that the determinant of an **upper-triangular** matrix is the product of its diagonal entries (since cofactor expansion along the column with the zeros collapses to a single term repeatedly).

Step 1. Inspect A. The entries below the diagonal are all zero, so A is upper-triangular. Its determinant is the product of diagonal entries:

$$|A| = 1 \times 1 \times 4 = 4.$$

(One can verify by expanding along column 1, which has only one non-zero entry $a_{11} = 1$.)

Step 2. Form $3A$ by multiplying every entry by 3:

$$3A = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{pmatrix}.$$

This is also upper-triangular.

Step 3. Compute $|3A|$ as a product of diagonal entries:

$$|3A| = 3 \times 3 \times 12 = 108.$$

Step 4. Compute $27\,|A|$:

$$27\,|A| = 27 \times 4 = 108.$$

Step 5. Compare: $|3A| = 108 = 27\,|A|$. The required equality holds.

Final Answer: $|3A| = 108 = 27\,|A|$.

✗ Common Pitfall

Do not write $|3A| = 3\,|A|$. The scalar 3 multiplies *every* entry of A, so each of the three rows contributes a factor of 3 to the determinant, giving $3^3 = 27$.

EXPERT'S SOLUTION *: Vivaan Reddy, Ph.D Mathematics, IIT Delhi*

Structural observation. A is upper-triangular: every entry below the main diagonal is 0. For such matrices the determinant collapses to the diagonal product.

Step 1. Read off the diagonal of A: entries are $1, 1, 4$.

$$|A| = 1 \cdot 1 \cdot 4 = 4.$$

Step 2. Form $3A$. Its diagonal entries are $3, 3, 12$.

$$|3A| = 3 \cdot 3 \cdot 12 = 108.$$

Step 3. Symbolic check: pulling the factor 3 out of *each* of the three rows gives

$$|3A| = 3 \cdot 3 \cdot 3 \cdot |A| = 27\,|A| = 27 \times 4 = 108.$$

Why this matters. The triangular-matrix shortcut and the $|kA| = k^n|A|$ identity together let you handle order-3 determinants in seconds whenever the matrix has a sparse structure.

Final Answer: $|3A| = 108 = 27\,|A|$.

Q 4.5 **Evaluate the determinants:**

(i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ **(ii)** $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

(iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ **(iv)** $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$.

SOLUTION

Concept used. For a 3×3 matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, the determinant expanded along the first row is

$$|A| = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}.$$

The signs follow the chessboard pattern $\begin{smallmatrix} + & - & + \\ - & + & - \\ + & - & + \end{smallmatrix}$. Expansion along any row or column gives the same value; choosing the row/column with the most zeros saves work.

Part (i). Two entries of R_2 are zero, so expand along R_2.

Step 1. Along R_2, $|A| = -a_{21}\,M_{21} + a_{22}\,M_{22} - a_{23}\,M_{23}$ where M_{ij} is the 2×2 minor.

Step 2. With $a_{21} = 0$, $a_{22} = 0$, $a_{23} = -1$, only the last term survives:

$$|A| = -(-1)\,M_{23} = M_{23} = \begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix}.$$

Step 3. Compute the minor:

$$\begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = (3)(-5) - (-1)(3) = -15 + 3 = -12.$$

Final Answer: (i) $= -12$.

Part (ii). Expand along R_1.

Step 1. Write the expansion:

$$|A| = 3\begin{vmatrix}1 & -2\\3 & 1\end{vmatrix} - (-4)\begin{vmatrix}1 & -2\\2 & 1\end{vmatrix} + 5\begin{vmatrix}1 & 1\\2 & 3\end{vmatrix}.$$

Step 2. Each minor:

$$\begin{vmatrix}1 & -2\\3 & 1\end{vmatrix} = (1)(1) - (-2)(3) = 1 + 6 = 7.$$

$$\begin{vmatrix}1 & -2\\2 & 1\end{vmatrix} = (1)(1) - (-2)(2) = 1 + 4 = 5.$$

$$\begin{vmatrix}1 & 1\\2 & 3\end{vmatrix} = (1)(3) - (1)(2) = 3 - 2 = 1.$$

Step 3. Combine:

$$|A| = 3(7) + 4(5) + 5(1) = 21 + 20 + 5 = 46.$$

Final Answer: (ii) $= 46$.

Part (iii). The corner entry $a_{11} = 0$. Expand along R_1.

Step 1. Expand:

$$|A| = 0 \cdot M_{11} - 1\begin{vmatrix}-1 & -3\\-2 & 0\end{vmatrix} + 2\begin{vmatrix}-1 & 0\\-2 & 3\end{vmatrix}.$$

Step 2. First minor:

$$\begin{vmatrix}-1 & -3\\-2 & 0\end{vmatrix} = (-1)(0) - (-3)(-2) = 0 - 6 = -6.$$

Step 3. Second minor:

$$\begin{vmatrix}-1 & 0\\-2 & 3\end{vmatrix} = (-1)(3) - (0)(-2) = -3 - 0 = -3.$$

Step 4. Combine:

$$|A| = 0 - 1 \times (-6) + 2 \times (-3) = 6 - 6 = 0.$$

Final Answer: (iii) $= 0$.

Part (iv). Expand along R_1.

Step 1. Expand:

$$|A| = 2\begin{vmatrix}2 & -1\\-5 & 0\end{vmatrix} - (-1)\begin{vmatrix}0 & -1\\3 & 0\end{vmatrix} + (-2)\begin{vmatrix}0 & 2\\3 & -5\end{vmatrix}.$$

Step 2. First minor:

$$\begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} = (2)(0) - (-1)(-5) = 0 - 5 = -5.$$

Step 3. Second minor:

$$\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} = (0)(0) - (-1)(3) = 0 + 3 = 3.$$

Step 4. Third minor:

$$\begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} = (0)(-5) - (2)(3) = 0 - 6 = -6.$$

Step 5. Combine:

$$|A| = 2(-5) + 1(3) + (-2)(-6) = -10 + 3 + 12 = 5.$$

Final Answer: (iv) $= 5$.

EXPERT'S SOLUTION *: Riya Banerjee, Ph.D Pure Mathematics, IISc Bangalore*

Strategic angle. For each part, scan for the row or column with the most zeros and expand along it. Zeros kill terms before they cost any work.

Step 1. (i) $R_2 = (0, 0, -1)$ has two zeros. Expand along R_2:

$$|A| = -(0) \cdot \ldots + 0 \cdot \ldots - (-1) \begin{vmatrix} 3 & -1 \\ 3 & -5 \end{vmatrix} = 1 \cdot (-15 + 3) = -12.$$

Step 2. (ii) No zero entries. Expand along R_1 directly:

$$\begin{aligned} |A| &= 3(1 \cdot 1 - (-2) \cdot 3) - (-4)(1 \cdot 1 - (-2) \cdot 2) + 5(1 \cdot 3 - 1 \cdot 2) \\ &= 3(7) + 4(5) + 5(1) = 21 + 20 + 5 = 46. \end{aligned}$$

Step 3. (iii) $a_{11} = 0$. Expand along R_1:

$$|A| = -1 \cdot (0 - 6) + 2 \cdot (-3 - 0) = 6 - 6 = 0.$$

Sanity check: $|A| = 0$ means the rows are linearly dependent. Indeed, $R_1 + R_2 + R_3 = (0, 1, 2) + (-1, 0, -3) + (-2, 3, 0) = (-3, 4, -1)$ is not zero, so the dependence is more subtle, but the determinant has the final say.

Step 4. (iv) Expand along R_1:

$$|A| = 2(0 - 5) + 1(0 + 3) - 2(0 - 6) = -10 + 3 + 12 = 5.$$

Why this matters. Picking the right row/column for expansion converts a potentially 9-multiplication slog into 1–2 line arithmetic. Make zero-spotting reflexive.

Final Answer: (i) -12, (ii) 46, (iii) 0, (iv) 5.

Q 4.6 **If** $A = \begin{pmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{pmatrix}$**, find** $|A|$**.**

SOLUTION

Concept used. For a 3×3 matrix, the determinant by expansion along the first row is

$$|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31}).$$

Step 1. Read off the entries:

$$a_{11} = 1,\ a_{12} = 1,\ a_{13} = -2,$$
$$a_{21} = 2,\ a_{22} = 1,\ a_{23} = -3,$$
$$a_{31} = 5,\ a_{32} = 4,\ a_{33} = -9.$$

Step 2. First minor (cofactor of a_{11}):

$$M_{11} = \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} = (1)(-9) - (-3)(4) = -9 + 12 = 3.$$

Step 3. Second minor:

$$M_{12} = \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} = (2)(-9) - (-3)(5) = -18 + 15 = -3.$$

Step 4. Third minor:

$$M_{13} = \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix} = (2)(4) - (1)(5) = 8 - 5 = 3.$$

Step 5. Combine with the alternating signs:

$$|A| = 1 \cdot M_{11} - 1 \cdot M_{12} + (-2) \cdot M_{13}.$$

$$|A| = (1)(3) - (1)(-3) + (-2)(3).$$
$$|A| = 3 + 3 - 6 = 0.$$

Final Answer: $|A| = 0$.

♥ A zero determinant carries information

$|A| = 0$ tells us A is **singular**, so A^{-1} does not exist. Equivalently, the three columns of A are linearly dependent. Check: $C_3 = -3\,C_1 + C_2$? Yes: $-3(1) + 1 = -2$, $-3(2) + 1 = -5$ (no), so this particular dependence fails. The right combination is $R_3 = 2R_2 + R_1$: $2(2, 1, -3) + (1, 1, -2) = (5, 3, -8) \neq R_3$, so the dependence is not a simple two-row sum either, but the determinant is still zero.

EXPERT'S SOLUTION : *Karan Joshi, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading. Expand along R_1, evaluate three 2×2 minors, plug in.

Step 1. Three minors:

$$M_{11} = 1 \cdot (-9) - (-3) \cdot 4 = -9 + 12 = 3,$$
$$M_{12} = 2 \cdot (-9) - (-3) \cdot 5 = -18 + 15 = -3,$$
$$M_{13} = 2 \cdot 4 - 1 \cdot 5 = 8 - 5 = 3.$$

Step 2. Combine using the chessboard signs $(+, -, +)$ on row 1:

$$|A| = 1(3) - 1(-3) + (-2)(3) = 3 + 3 - 6 = 0.$$

Step 3. Cross-check by expansion along C_1 (entries $1, 2, 5$):

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 2\begin{vmatrix} 1 & -2 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} 1 & -2 \\ 1 & -3 \end{vmatrix}.$$

$$= 1(-9 + 12) - 2(-9 + 8) + 5(-3 + 2) = 3 - 2(-1) + 5(-1) = 3 + 2 - 5 = 0. \checkmark$$

Why this matters. Cross-checking by expanding along a second row or column is the safest way to catch a sign slip in 3×3 determinants.

Final Answer: $|A| = 0$.

Q 4.7 **Find values of x, if**

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ **(ii)** $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$.

SOLUTION

Concept used. A 2×2 determinant equals $ad - bc$. Setting the two determinants equal gives a polynomial equation in x that we then solve.

Part (i).

Step 1. Compute the LHS:

$$\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = (2)(1) - (4)(5) = 2 - 20 = -18.$$

Step 2. Compute the RHS in terms of x:

$$\begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix} = (2x)(x) - (4)(6) = 2x^2 - 24.$$

Step 3. Equate and solve:

$$-18 = 2x^2 - 24.$$

Add 24 to both sides:

$$6 = 2x^2.$$

Divide by 2:

$$x^2 = 3.$$

Take the square root (both signs):

$$x = \pm\sqrt{3}.$$

> **Final Answer:** (i) $x = \pm\sqrt{3}$.

Part (ii).

Step 1. LHS:

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = (2)(5) - (3)(4) = 10 - 12 = -2.$$

Step 2. RHS:

$$\begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix} = (x)(5) - (3)(2x) = 5x - 6x = -x.$$

Step 3. Equate:

$$-2 = -x \implies x = 2.$$

> **Final Answer:** (ii) $x = 2$.

☞ **Why both signs in (i)**

$x^2 = 3$ gives two real roots, $+\sqrt{3}$ and $-\sqrt{3}$, because squaring loses sign information. Always keep both unless the question rules one out.

EXPERT'S SOLUTION : *Sneha Verma, B.Tech CSE, IIT Roorkee*

Strategic angle. Reduce each side to a number or a polynomial in x first; only then set them equal.

Step 1. **(i)** LHS $= 2 - 20 = -18$; RHS $= 2x^2 - 24$. Solve

$$2x^2 - 24 = -18 \implies 2x^2 = 6 \implies x^2 = 3 \implies x = \pm\sqrt{3}.$$

Step 2. **(ii)** LHS $= 10 - 12 = -2$; RHS $= 5x - 6x = -x$. So $-x = -2$, giving $x = 2$.

Why this matters. When a determinant is set equal to another determinant, the underlying problem is just an algebraic equation. The determinant machinery is only there to define the equation; the solving step is plain algebra.

Final Answer: (i) $x = \pm\sqrt{3}$; (ii) $x = 2$.

Q 4.8 **If** $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$**, then** x **is equal to**

(A) 6 **(B)** ± 6 **(C)** -6 **(D)** 0.

SOLUTION

Concept used. A 2×2 determinant equals $ad - bc$. Once both sides are reduced to numbers (or to a polynomial in x), the equation is straightforward.

Step 1. Evaluate the RHS:

$$\begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix} = (6)(6) - (2)(18) = 36 - 36 = 0.$$

Step 2. Evaluate the LHS as a polynomial in x:

$$\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = (x)(x) - (2)(18) = x^2 - 36.$$

Step 3. Set LHS = RHS:

$$x^2 - 36 = 0.$$

Factorise as a difference of squares:

$$(x - 6)(x + 6) = 0.$$

Hence $x = 6$ or $x = -6$, i.e. $x = \pm 6$.

Step 4. Match with options: option (B) ± 6 is correct.

Final Answer: Option (B): $x = \pm 6$.

✗ Common Pitfall

Picking only $x = 6$ because $x = 6$ *also* makes the LHS look like the RHS is wrong. The equation is quadratic, and $x = -6$ is an equally valid solution: $(-6)^2 - 36 = 0$ as well.

EXPERT'S SOLUTION *: Aditi Pillai, M.Sc Mathematics, IIT Bombay*

Quick reading. The RHS is zero (its two diagonal products are both 36). The LHS as a function of x is $x^2 - 36$, which vanishes at $x = \pm 6$.

Step 1. RHS: $6 \cdot 6 - 2 \cdot 18 = 36 - 36 = 0$.

Step 2. LHS: $x^2 - 36$.

Step 3. Solve $x^2 - 36 = 0$. Both roots ± 6 are real.

Step 4. Therefore option (B) is correct.

Why this matters. MCQ options with $\pm$ entries usually mean the underlying equation is quadratic. Spotting that early saves you from prematurely choosing the single-value option.

Final Answer: $x = \pm 6$, option (B).

Key Takeaways

- For a 2×2 matrix, $|A| = ad - bc$. The minus sign on bc is what makes the determinant detect orientation.
- For a 3×3 matrix, expand along the row or column with the most zeros and use the chessboard sign pattern.
- If every entry of an $n \times n$ matrix is scaled by k, the determinant is scaled by k^n, not k.
- The determinant of an upper- (or lower-) triangular matrix is the product of its diagonal entries.
- $|A| = 0$ means A is singular and its rows/columns are linearly dependent.

End of Exercise 4.1