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Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 4: Determinants

About this Chapter

This exercise applies determinants to coordinate geometry. The **signed area** of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is $\frac{1}{2}|\Delta|$ where Δ is a specific 3×3 determinant. The same determinant set to zero gives the **collinearity condition** for three points, and the **equation of a line** through two given points.

Topics covered: Area of a triangle via determinant • Collinearity of three points • Equation of a line through two points

Quick Formula Sheet

Area of a triangle:

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|$$

Collinearity: the same determinant equals 0.

Line through (x_1, y_1) **and** (x_2, y_2):

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

Exercise 4.2

Q 4.1 Find area of the triangle with vertices at the points given in each of the following:

(i) $(1, 0), (6, 0), (4, 3)$ **(ii)** $(2, 7), (1, 1), (10, 8)$

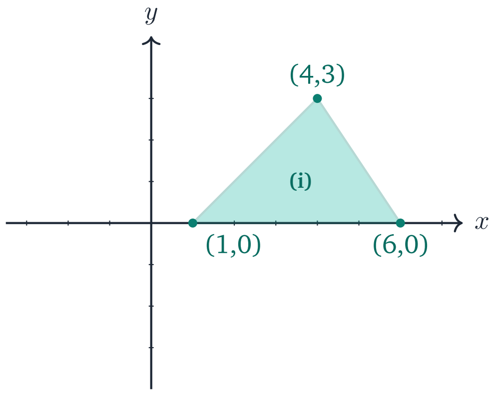
(iii) $(-2, -3), (3, 2), (-1, -8)$**.**

SOLUTION

Concept used. The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|.$$

The vertical bars around the determinant indicate *absolute value*, since area is always non-negative. The unsigned determinant equals $\pm 2 \times$ Area depending on the orientation (clockwise gives $-$, anticlockwise gives $+$).



Sketch of triangle (i). The base lies on the x-axis from $x = 1$ to $x = 6$, with height 3.

Part (i). Vertices $(1, 0), (6, 0), (4, 3)$.

Step 1. Set up the determinant:

$$\Delta = \begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}.$$

Step 2. Expand along C_2 (column 2 has two zeros, so most terms vanish):

$$\Delta = -0 \cdot M_{12} + 0 \cdot M_{22} - 3 \cdot M_{32},$$

where $M_{32} = \begin{vmatrix} 1 & 1 \\ 6 & 1 \end{vmatrix} = 1 - 6 = -5.$

Step 3. Hence $\Delta = -3 \times (-5) = 15$.

Step 4. Area $= \frac{1}{2} |\Delta| = \frac{1}{2} \times 15 = \frac{15}{2}$ square units.

Step 5. Sanity check from the picture: base $= 6 - 1 = 5$, height $= 3$, area $= \frac{1}{2} \cdot 5 \cdot 3 = \frac{15}{2}$. ✓

> **Final Answer:** Area of (i) $= \dfrac{15}{2}$ sq units.

Part (ii). Vertices $(2, 7), (1, 1), (10, 8)$.

Step 1. Set up:

$$\Delta = \begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}.$$

Step 2. Expand along R_1:

$$\Delta = 2 \begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} - 7 \begin{vmatrix} 1 & 1 \\ 10 & 1 \end{vmatrix} + 1 \begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix}.$$

Step 3. Minors:

$$\begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} = 1 - 8 = -7.$$

$$\begin{vmatrix} 1 & 1 \\ 10 & 1 \end{vmatrix} = 1 - 10 = -9.$$

$$\begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix} = 8 - 10 = -2.$$

Step 4. Combine:

$$\Delta = 2(-7) - 7(-9) + 1(-2) = -14 + 63 - 2 = 47.$$

Step 5. Area $= \frac{1}{2}|47| = \frac{47}{2}$ sq units.

> **Final Answer:** Area of (ii) $= \dfrac{47}{2}$ sq units.

Part (iii). Vertices $(-2,-3),(3,2),(-1,-8)$.

Step 1. Set up:

$$\Delta = \begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}.$$

Step 2. Expand along R_1:

$$\Delta = -2\begin{vmatrix} 2 & 1 \\ -8 & 1 \end{vmatrix} - (-3)\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} 3 & 2 \\ -1 & -8 \end{vmatrix}.$$

Step 3. Minors:

$$\begin{vmatrix} 2 & 1 \\ -8 & 1 \end{vmatrix} = 2 - (-8) = 10.$$

$$\begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = 3 - (-1) = 4.$$

$$\begin{vmatrix} 3 & 2 \\ -1 & -8 \end{vmatrix} = -24 - (-2) = -22.$$

Step 4. Combine:

$$\Delta = -2(10) + 3(4) + 1(-22) = -20 + 12 - 22 = -30.$$

Step 5. Area $= \frac{1}{2}|-30| = 15$ sq units.

> **Final Answer:** Area of (iii) $= 15$ sq units.

☞ Exam Tip

Always wrap the determinant in absolute-value bars before halving. Negative output of Δ only encodes the orientation of the vertex-listing, not a negative area.

EXPERT'S SOLUTION : *Aditya Kapoor, M.Sc Mathematics, IIT Bombay*

Strategic angle. The same determinant template handles all three triangles; only the entries change. Expand along whichever row or column has the most zeros or the smallest numbers.

Step 1. **(i)** $(1, 0), (6, 0), (4, 3)$. Column 2 has two zeros: $\Delta = -3\,(1 - 6) = -3(-5) = 15$, area $= 15/2$.

Step 2. **(ii)** $(2, 7), (1, 1), (10, 8)$. No zeros, but column 3 is all 1s, so expand along it:

$$\Delta = 1 \cdot \begin{vmatrix} 1 & 1 \\ 10 & 8 \end{vmatrix} - 1 \cdot \begin{vmatrix} 2 & 7 \\ 10 & 8 \end{vmatrix} + 1 \cdot \begin{vmatrix} 2 & 7 \\ 1 & 1 \end{vmatrix}.$$

$$= (8 - 10) - (16 - 70) + (2 - 7) = -2 - (-54) - 5 = -2 + 54 - 5 = 47.$$

Area $= 47/2$.

Step 3. **(iii)** $(-2, -3), (3, 2), (-1, -8)$. Expand along column 3:

$$\Delta = \begin{vmatrix} 3 & 2 \\ -1 & -8 \end{vmatrix} - \begin{vmatrix} -2 & -3 \\ -1 & -8 \end{vmatrix} + \begin{vmatrix} -2 & -3 \\ 3 & 2 \end{vmatrix}.$$

$$= (-24 + 2) - (16 - 3) + (-4 + 9) = -22 - 13 + 5 = -30.$$

Area $= 30/2 = 15$.

Why this matters. Expanding along the column of 1s in this determinant produces three direct 2×2 expressions, no further bookkeeping.

Final Answer: Areas: (i) $15/2$, (ii) $47/2$, (iii) 15 square units.

Q 4.2 **Show that points $A(a, b + c)$, $B(b, c + a)$, $C(c, a + b)$ are collinear.**

SOLUTION

Concept used. Three points are collinear if and only if the area of the triangle they form is zero. Equivalently,

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

We also use the **column operation** property: adding a multiple of one column to another does not change the value of a determinant.

Step 1. Set up the determinant for the three points:

$$\Delta = \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}.$$

Step 2. Apply the column operation $C_1 \to C_1 + C_2$. The new first column becomes

$$\big(a+(b+c),\ b+(c+a),\ c+(a+b)\big) = (a+b+c,\ a+b+c,\ a+b+c).$$

So

$$\Delta = \begin{vmatrix} a+b+c & b+c & 1 \\ a+b+c & c+a & 1 \\ a+b+c & a+b & 1 \end{vmatrix}.$$

Step 3. Pull the common factor $(a+b+c)$ out of C_1:

$$\Delta = (a+b+c)\begin{vmatrix} 1 & b+c & 1 \\ 1 & c+a & 1 \\ 1 & a+b & 1 \end{vmatrix}.$$

Step 4. Columns C_1 and C_3 are now identical (both are $(1,1,1)^T$). A determinant with two identical columns is zero:

$$\Delta = (a+b+c)\cdot 0 = 0.$$

Step 5. Since the determinant is zero, the area of triangle ABC is zero, so A, B, C are collinear.

Final Answer: $\Delta = 0$, hence A, B, C are collinear.

☞ **Identical-column rule**

If two columns (or rows) of a determinant are equal, the determinant is zero. This is because swapping the identical columns leaves the matrix unchanged yet must flip the sign of the determinant; the only number equal to its own negative is 0.

EXPERT'S SOLUTION *: Diya Nair, Ph.D Mathematics, IIT Delhi*

Structural observation. Each y-coordinate is the sum of the other two of a, b, c. Adding column 1 to column 2 (or to column 1 to column 2's data) produces $a+b+c$ everywhere. That symmetry forces the determinant to zero.

Step 1. Operation $C_2 \to C_1 + C_2$ in

$$\Delta = \begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

replaces C_2 by
$(a + (b + c),\ b + (c + a),\ c + (a + b)) = (a + b + c,\ a + b + c,\ a + b + c)$.

Step 2. Factor out $(a + b + c)$ from the new C_2:

$$\Delta = (a+b+c) \begin{vmatrix} a & 1 & 1 \\ b & 1 & 1 \\ c & 1 & 1 \end{vmatrix}.$$

Step 3. Now $C_2 = C_3$, both being the column of 1s, so $\Delta = 0$.

Why this matters. Whenever a determinant has rows or columns whose entries sum to a common quantity, the right move is to add the rest into one row or column and pull out the common factor. The next step (often) shows two equal rows/columns, killing the determinant.

Final Answer: $\Delta = 0$, so A, B, C are collinear.

Q 4.3 **Find values of k if area of triangle is 4 sq units and vertices are (i) $(k, 0), (4, 0), (0, 2)$ (ii) $(-2, 0), (0, 4), (0, k)$.**

SOLUTION

Concept used. If the area of a triangle is known, then

$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm \text{Area}.$$

Both signs are used because the determinant itself may be positive or negative depending on orientation; we want all values of the unknown.

Part (i).

Step 1. Form the determinant equation:

$$\frac{1}{2} \begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 4.$$

Step 2. Expand along C_2 (two zeros):

$$\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = -2\begin{vmatrix} k & 1 \\ 4 & 1 \end{vmatrix} = -2(k-4) = -2k+8.$$

(The sign is -2 because the cofactor sign at position $(3,2)$ is $(-1)^{3+2} = -1$, and the entry there is 2.)

Step 3. Substitute:

$$\frac{-2k+8}{2} = \pm 4 \implies -k+4 = \pm 4.$$

Step 4. Two cases. **Case $+$:** $-k+4 = 4 \Rightarrow k = 0$.
Case $-$: $-k+4 = -4 \Rightarrow k = 8$.

> **Final Answer:** (i) $k = 0$ or $k = 8$.

Part (ii).

Step 1. Form the equation:

$$\frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = \pm 4.$$

Step 2. Expand along C_1 (two zeros, top entry -2):

$$\Delta = -2\begin{vmatrix} 4 & 1 \\ k & 1 \end{vmatrix} = -2(4-k) = -8+2k.$$

Step 3. Substitute:

$$\frac{-8+2k}{2} = \pm 4 \implies -4+k = \pm 4.$$

Step 4. Two cases. **Case $+$:** $k-4 = 4 \Rightarrow k = 8$.
Case $-$: $k-4 = -4 \Rightarrow k = 0$.

> **Final Answer:** (ii) $k = 0$ or $k = 8$.

EXPERT'S SOLUTION *: Rohit Desai, M.Sc Mathematics, ISI Kolkata*

Quick reading. Both parts shrink to a linear equation in k after one column expansion, and the $\pm$ on the area gives two roots.

Step 1. **(i)** Determinant simplifies to $-2k+8$. Setting $(-2k+8)/2 = \pm 4$ gives $-k+4 = \pm 4$, hence $k = 0$ or $k = 8$.

Step 2. **(ii)** Determinant simplifies to $2k - 8$. Setting $(2k-8)/2 = \pm 4$ gives $k - 4 = \pm 4$, hence $k = 8$ or $k = 0$.

Step 3. Geometric reading of (i): two vertices lie on the x-axis, one on the y-axis at height 2. Base $= |k-4|$, height $= 2$, so area $= |k-4|$. Setting this $= 4$ gives $|k-4| = 4$, i.e. $k = 0$ or $k = 8$. ✓

Step 4. Geometric reading of (ii): two vertices on the y-axis $(0, 4)$ and $(0, k)$, one on the x-axis at $(-2, 0)$. Base $= |k-4|$, height $= 2$, area $= |k-4| = 4$, same result.

Why this matters. The $\pm$ in front of the area is critical. Forgetting it loses half the solutions every time.

Final Answer: (i) and (ii): $k \in \{0, 8\}$.

Q 4.4 **(i) Find equation of line joining $(1, 2)$ and $(3, 6)$ using determinants. (ii) Find equation of line joining $(3, 1)$ and $(9, 3)$ using determinants.**

SOLUTION

Concept used. The line through two distinct points (x_1, y_1) and (x_2, y_2) is the locus of points (x, y) such that the three points $(x, y), (x_1, y_1), (x_2, y_2)$ are collinear, i.e.

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0.$$

Expanding gives a linear equation in x and y.

Part (i). Through $(1, 2)$ and $(3, 6)$.

Step 1. Set up:

$$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0.$$

Step 2. Expand along R_1:

$$x \begin{vmatrix} 2 & 1 \\ 6 & 1 \end{vmatrix} - y \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 0.$$

Step 3. Minors:

$$\begin{vmatrix} 2 & 1 \\ 6 & 1 \end{vmatrix} = 2 - 6 = -4.$$

$$\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 1 - 3 = -2.$$

$$\begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6 - 6 = 0.$$

Step 4. Substitute:

$$x(-4) - y(-2) + 0 = 0 \implies -4x + 2y = 0.$$

Step 5. Divide by 2:

$$-2x + y = 0 \implies y = 2x.$$

Step 6. Verify: at $(1, 2)$, $y = 2 = 2(1)$. ✓ At $(3, 6)$, $y = 6 = 2(3)$. ✓

> **Final Answer:** (i) Line is $y = 2x$, i.e. $2x - y = 0$.

Part (ii). Through $(3, 1)$ and $(9, 3)$.

Step 1. Set up:

$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0.$$

Step 2. Expand along R_1:

$$x \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} - y \begin{vmatrix} 3 & 1 \\ 9 & 1 \end{vmatrix} + 1 \cdot \begin{vmatrix} 3 & 1 \\ 9 & 3 \end{vmatrix} = 0.$$

Step 3. Minors:

$$\begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 1 - 3 = -2.$$

$$\begin{vmatrix} 3 & 1 \\ 9 & 1 \end{vmatrix} = 3 - 9 = -6.$$

$$\begin{vmatrix} 3 & 1 \\ 9 & 3 \end{vmatrix} = 9 - 9 = 0.$$

Step 4. Substitute:

$$x(-2) - y(-6) + 0 = 0 \implies -2x + 6y = 0.$$

Step 5. Divide by -2:

$$x - 3y = 0 \implies x = 3y.$$

Step 6. Verify: at $(3, 1)$, $3 = 3(1)$. ✓ At $(9, 3)$, $9 = 3(3)$. ✓

> **Final Answer:** (ii) Line is $x = 3y$, i.e. $x - 3y = 0$.

♥ Determinant form vs slope form

The two-point determinant form $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ handles every case: horizontal lines, vertical lines, lines through the origin. The slope formula $y - y_1 = m(x - x_1)$ breaks down for vertical lines (infinite slope). That is why determinants are the preferred tool when a question says "using determinants".

EXPERT'S SOLUTION *: Ishita Singh, M.Tech CS, IIT Madras*

Picture-first. Both pairs of points happen to be collinear with the origin: $(1, 2)$ and $(3, 6)$ both satisfy $y = 2x$; $(3, 1)$ and $(9, 3)$ both satisfy $x = 3y$. The determinant must recover those relations.

Step 1. **(i)** Determinant expansion gives $-4x + 2y = 0$, which is $y = 2x$. The slope is $\frac{6-2}{3-1} = 2$, consistent.

Step 2. **(ii)** Determinant expansion gives $-2x + 6y = 0$, which is $x - 3y = 0$, or equivalently $y = x/3$. The slope is $\frac{3-1}{9-3} = \frac{2}{6} = \frac{1}{3}$, consistent.

Why this matters. The determinant approach is mechanical: write the matrix, expand, divide out the common factor. No case analysis on whether the line is vertical, horizontal, or through the origin.

Final Answer: (i) $2x - y = 0$; (ii) $x - 3y = 0$.

Q 4.5 **If area of triangle is 35 sq units with vertices $(2, -6), (5, 4)$ and $(k, 4)$, then k is (A) 12 (B) -2 (C) $-12, -2$ (D) $12, -2$.**

SOLUTION

Concept used. The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is

$$\text{Area} = \frac{1}{2}\left|\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|,$$

so $\Delta \equiv 2 \times$ Area in absolute value. Because we do not know the orientation, we set $\Delta = \pm 2\,$Area.

Step 1. Write the determinant:

$$\Delta = \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}.$$

Step 2. Expand along R_1:

$$\Delta = 2\begin{vmatrix}4 & 1\\4 & 1\end{vmatrix} - (-6)\begin{vmatrix}5 & 1\\k & 1\end{vmatrix} + 1\begin{vmatrix}5 & 4\\k & 4\end{vmatrix}.$$

Step 3. Minors:

$$\begin{vmatrix}4 & 1\\4 & 1\end{vmatrix} = 4 - 4 = 0.$$

$$\begin{vmatrix}5 & 1\\k & 1\end{vmatrix} = 5 - k.$$

$$\begin{vmatrix}5 & 4\\k & 4\end{vmatrix} = 20 - 4k.$$

Step 4. Combine:

$$\Delta = 2(0) + 6(5 - k) + (20 - 4k) = 30 - 6k + 20 - 4k = 50 - 10k.$$

Step 5. Apply the area condition:

$$\frac{1}{2}\,|50 - 10k| = 35 \implies |50 - 10k| = 70.$$

Drop the absolute value:

$$50 - 10k = \pm 70.$$

Step 6. Solve both cases. **Case** $+$**:** $50 - 10k = 70 \Rightarrow -10k = 20 \Rightarrow k = -2$.
Case $-$**:** $50 - 10k = -70 \Rightarrow -10k = -120 \Rightarrow k = 12$.

Step 7. So $k \in \{-2, 12\}$, matching option (D).

Final Answer: Option (D): $k = 12$ or $k = -2$.

✗ Common Pitfall

Forgetting the $\pm$ sign on $|\Delta| = 2 \times$ Area gives only one root and pushes you towards options (A) or (B). The MCQ stem (with two-valued options) is itself a hint that the equation is two-sided.

EXPERT'S SOLUTION *: Yash Chatterjee, B.Tech CSE, IIT Roorkee*

Structural observation. The second and third vertices share $y = 4$, so the side joining them is horizontal of length $|5 - k|$. The perpendicular distance from $(2, -6)$ to the line $y = 4$ is $|4 - (-6)| = 10$. Area $= \frac{1}{2} \cdot |5 - k| \cdot 10 = 5|5 - k|$.

Step 1. Set $5|5-k| = 35 \Rightarrow |5-k| = 7$.

Step 2. Hence $5-k=7$ or $5-k=-7$, giving $k=-2$ or $k=12$.

Why this matters. Geometric reasoning here is faster than the determinant. When two vertices share a coordinate, the side between them is axis-parallel and area = $\frac{1}{2} \cdot$ base $\cdot$ perpendicular distance.

Final Answer: $k = 12$ or $k = -2$; option (D).

Key Takeaways

- Area $= \frac{1}{2}\,|\Delta|$ where Δ is the vertices-as-rows determinant with a column of 1s.
- Three points are collinear iff that determinant equals 0.
- Equation of a line through two given points: set the $(x, y, 1)$ row above the two known-point rows and equate to 0.
- When the area is given, always solve $\Delta = \pm 2\,$Area to keep both orientations.
- If two columns (or rows) of a determinant are identical, the determinant is 0.

End of Exercise 4.2