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Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 3: Matrices

About this Chapter

A **matrix** is a rectangular array of numbers written in rows and columns. Exercise 3.1 builds the language of matrices: the **order** $m \times n$, the meaning of the entry a_{ij}, how many entries a matrix of a given order contains, how to construct a matrix from a rule for a_{ij}, and what it means for two matrices to be **equal**. These ideas underpin every later operation on matrices.

Topics covered: Order of a matrix • Number of elements • Entry a_{ij} • Construction of a matrix • Equality of matrices • Types of matrices

Quick Formula Sheet

Order of a matrix:
$A = [a_{ij}]_{m\times n}$ has m rows and n columns.

Number of entries:
$A_{m\times n}$ has exactly $m \times n$ entries.

Equality of matrices:
$A = B \iff$ same order and $a_{ij} = b_{ij}$ for all i, j.

Square matrix:
$A_{m\times n}$ is square iff $m = n$.

Exercise 3.1

Q 3.1 **In the matrix** $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$**, write: (i) the order of the matrix, (ii) the number of elements, (iii) the elements** $a_{13},\ a_{21},\ a_{33},\ a_{24},\ a_{23}$**.**

SOLUTION

Concept used. A matrix is a rectangular arrangement of numbers in rows and columns. If a matrix A has m rows and n columns, its **order** is written as $m \times n$. The total number of **elements** (or entries) equals $m \times n$. The symbol a_{ij} denotes the entry sitting in the i-th row and j-th column; i is the row index and j is the column index.

☞ **Reading a_{ij}**

a_{ij} means: "go to row i, then move to column j." Always read the row first, the column second.

Step 1. **Find the order.** Count the horizontal lines (rows) and the vertical lines (columns). The given matrix A has

$$\text{rows} = 3, \qquad \text{columns} = 4.$$

Hence the order is 3×4.

Step 2. **Number of elements.** The total number of entries is

$$(\text{rows}) \times (\text{columns}) = 3 \times 4 = 12.$$

Step 3. **Locate a_{13}.** Row 1, column 3. Reading across row 1: $2, 5, 19, -7$. The third entry is 19. So $a_{13} = 19$.

Step 4. **Locate a_{21}.** Row 2, column 1. Row 2 is $35, -2, \frac{5}{2}, 12$. First entry: $a_{21} = 35$.

Step 5. **Locate a_{33}.** Row 3, column 3. Row 3 is $\sqrt{3}, 1, -5, 17$. Third entry: $a_{33} = -5$.

Step 6. **Locate a_{24}.** Row 2, column 4. Row 2 is $35, -2, \frac{5}{2}, 12$. Fourth entry: $a_{24} = 12$.

Step 7. **Locate a_{23}.** Row 2, column 3. Row 2 is $35, -2, \frac{5}{2}, 12$. Third entry: $a_{23} = \dfrac{5}{2}$.

Final Answer: Order $= 3 \times 4$; number of elements $= 12$; $a_{13} = 19,\ a_{21} = 35,\ a_{33} = -5,\ a_{24} = 12,\ a_{23} = \frac{5}{2}$.

✗ Common Mistake

A common slip is to swap the indices and look up column i, row j instead. Always go *row first, column second*: $a_{ij} = \text{row } i, \text{ column } j$.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Picture-first. Think of the matrix as a grid. Highlight rows top-to-bottom, columns left-to-right. Then each requested entry is just "which cell?" I will treat the matrix as a 3×4 table and read off each cell systematically.

Step 1. Lay out the entries by row.

$$\text{Row 1: } (2,\ 5,\ 19,\ -7)$$

$$\text{Row 2: } (35,\ -2,\ \tfrac{5}{2},\ 12)$$

$$\text{Row 3: } (\sqrt{3},\ 1,\ -5,\ 17)$$

Three rows and four columns, so the order is 3×4.

Step 2. The count of cells in a 3×4 grid is $3 \cdot 4 = 12$, so the matrix has 12 elements.

Step 3. Build a quick table of $(i, j) \to a_{ij}$:

(i, j)	Cell location	Value a_{ij}
$(1, 3)$	R1, C3	19
$(2, 1)$	R2, C1	35
$(3, 3)$	R3, C3	-5
$(2, 4)$	R2, C4	12
$(2, 3)$	R2, C3	$\frac{5}{2}$

Step 4. Confirm count: 5 entries asked for, 5 values delivered.

Why this matters. A clean mental "row-then-column" habit prevents 90% of indexing errors later in matrix multiplication, where the entry $(AB)_{ij}$ is row i of A paired with column j of B.

Final Answer: $3 \times 4,\ 12,\ a_{13} = 19,\ a_{21} = 35,\ a_{33} = -5,\ a_{24} = 12,\ a_{23} = \frac{5}{2}.$

Q 3.2 If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

SOLUTION

Concept used. A matrix of order $m \times n$ has exactly $m \cdot n$ entries. Therefore the possible orders for a matrix with N elements are exactly the ordered pairs (m, n) of positive integers whose product is N. Equivalently, m is any positive divisor of N, and $n = N/m$.

Step 1. **Case $N = 24$.** List all positive divisors of 24:

$$1,\ 2,\ 3,\ 4,\ 6,\ 8,\ 12,\ 24.$$

For each divisor m, set $n = 24/m$:

$$\begin{aligned} m = 1 &\Rightarrow n = 24, & m = 2 &\Rightarrow n = 12, \\ m = 3 &\Rightarrow n = 8, & m = 4 &\Rightarrow n = 6, \\ m = 6 &\Rightarrow n = 4, & m = 8 &\Rightarrow n = 3, \\ m = 12 &\Rightarrow n = 2, & m = 24 &\Rightarrow n = 1. \end{aligned}$$

Count: 8 ordered pairs, so 8 possible orders.

Step 2. Write them out as orders:

$$1 \times 24,\ 2 \times 12,\ 3 \times 8,\ 4 \times 6,\ 6 \times 4,\ 8 \times 3,\ 12 \times 2,\ 24 \times 1.$$

Step 3. **Case $N = 13$.** Since 13 is prime, its only positive divisors are 1 and 13. So the only factorisations are 1×13 and 13×1. That gives 2 possible orders.

Final Answer: 24 elements → 8 orders: 1×24, 2×12, 3×8, 4×6, 6×4, 8×3, 12×2, 24×1. 13 elements → 2 orders: $1\times13,\ 13\times1$.

♥ Why This Matters

"Possible orders" is a divisor-counting problem in disguise. For any N, the number of possible matrix orders equals the number of positive divisors $d(N)$.

EXPERT'S SOLUTION : *Pranav Iyer, M.Sc Applied Mathematics, IIT Kanpur*

Structural observation. "How many orders?" is just "how many ways can I factor $N = m \cdot n$ with $m, n \in \mathbb{Z}^+$?" Equivalently, count the ordered divisor pairs of N. For $N = p_1^{a_1} \cdots p_k^{a_k}$, the number of positive divisors is $d(N) = (a_1 + 1) \cdots (a_k + 1)$.

Step 1. Factor $24 = 2^3 \cdot 3^1$. Then

$$d(24) = (3+1)(1+1) = 4 \cdot 2 = 8.$$

So there are 8 ordered factorisations $m \cdot n = 24$, hence 8 matrix orders. Enumerate:

$$1\times24,\ 2\times12,\ 3\times8,\ 4\times6,\ 6\times4,\ 8\times3,\ 12\times2,\ 24\times1.$$

Step 2. For 13, since $13 = 13^1$ is prime,

$$d(13) = 1 + 1 = 2.$$

Only 1×13 and 13×1.

Step 3. Pattern check: a prime number of entries always forces a row matrix or a column matrix. A composite number admits more layouts.

Why this matters. The same idea tells you when a square arrangement is possible: $N = k^2$ is needed for an $k\times k$ square matrix, e.g. $36 = 6^2$ admits a square layout but 24 does not.

Final Answer: $24 \to 8$ orders; $13 \to 2$ orders $(1\times13,\ 13\times1)$.

Q 3.3 **If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?**

SOLUTION

Concept used. Same as Q2: an $m \times n$ matrix has $m \cdot n$ entries, so the possible orders are the ordered pairs of positive integers whose product equals the given number of entries.

Step 1. **Case $N = 18$.** Find all positive divisors of 18:

$$1,\ 2,\ 3,\ 6,\ 9,\ 18.$$

For each m, set $n = 18/m$:

$$m = 1 \Rightarrow n = 18;\ m = 2 \Rightarrow n = 9;\ m = 3 \Rightarrow n = 6;$$

$$m = 6 \Rightarrow n = 3;\ m = 9 \Rightarrow n = 2;\ m = 18 \Rightarrow n = 1.$$

That gives 6 ordered pairs.

Step 2. Possible orders:

$$1\times18,\ 2\times9,\ 3\times6,\ 6\times3,\ 9\times2,\ 18\times1.$$

Step 3. **Case $N = 5$.** Since 5 is prime, the divisors are 1 and 5. Only orders: 1×5 and 5×1.

> **Final Answer:** 18 elements $\to$ 6 orders: $1\times18,\ 2\times9,\ 3\times6,\ 6\times3,\ 9\times2,\ 18\times1.$
> 5 elements $\to$ 2 orders: $1\times5,\ 5\times1.$

EXPERT'S SOLUTION *: Sneha Patel, M.Sc Mathematics, ISI Kolkata*

Structural observation. Use the divisor formula directly. $18 = 2^1 \cdot 3^2$, so $d(18) = (1+1)(2+1) = 2\cdot3 = 6$. Six divisors, six ordered factorisations, six matrix orders.

Step 1. Divisor count for 18: $d(18) = 2 \cdot 3 = 6$. List the divisors in increasing order, pair each with $18/d$:

$$(1,18),\ (2,9),\ (3,6),\ (6,3),\ (9,2),\ (18,1).$$

Step 2. Divisor count for 5 (prime): $d(5) = 2$. Pair: $(1,5),\ (5,1)$.

Step 3. Notice the list for 18 is a palindrome around the centre $(3,6)\,(6,3)$ since pairs (m,n) and (n,m) both count.

Why this matters. Comparing 24 ($d = 8$) with 18 ($d = 6$) shows that more prime factors with higher exponents produce more possible orders. A number with very few divisors (like 5 or 13, both prime) can only be stored as a row or a column.

> **Final Answer:** $18 \to 6$ orders; $5 \to 2$ orders ($1\times5,\ 5\times1$).

Q 3.4 **Construct a 2×2 matrix $A = [a_{ij}]$ whose elements are given by:**
(i) $a_{ij} = \dfrac{(i+j)^2}{2}$, **(ii)** $a_{ij} = \dfrac{i}{j}$, **(iii)** $a_{ij} = \dfrac{(i+2j)^2}{2}$.

SOLUTION

Concept used. To construct a matrix from a rule $a_{ij} = f(i,j)$, plug every valid pair (i,j) into the rule and place the result in row i, column j. For a 2×2 matrix, (i,j) ranges over $\{(1,1),(1,2),(2,1),(2,2)\}$.

Step 1. **(i) Rule** $a_{ij} = \dfrac{(i+j)^2}{2}$. Compute each entry:

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2,$$
$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},$$
$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2},$$
$$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8.$$

Assemble:

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}.$$

Step 2. **(ii) Rule** $a_{ij} = \dfrac{i}{j}$. Compute:

$$a_{11} = \tfrac{1}{1} = 1, \quad a_{12} = \tfrac{1}{2},$$
$$a_{21} = \tfrac{2}{1} = 2, \quad a_{22} = \tfrac{2}{2} = 1.$$
$$A = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}.$$

Step 3. **(iii) Rule** $a_{ij} = \dfrac{(i+2j)^2}{2}$. Compute:

$$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2},$$
$$a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2},$$
$$a_{21} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8,$$
$$a_{22} = \frac{(2+4)^2}{2} = \frac{36}{2} = 18.$$
$$A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}.$$

Final Answer: (i) $\begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$, (ii) $\begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$, (iii) $\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$.

EXPERT'S SOLUTION : *Vivaan Kapoor, M.Tech CS, IIT Madras*

Quick reading. Each rule is a function of i and j. Evaluate it at the four lattice points $(1,1), (1,2), (2,1), (2,2)$ and drop the values into the right cells. The structure of the answer often reflects the rule: $\dfrac{(i+j)^2}{2}$ is symmetric in i, j, so the resulting matrix is symmetric.

Step 1. Rule (i) $a_{ij} = \frac{(i+j)^2}{2}$ is symmetric: swapping $i \leftrightarrow j$ leaves the formula unchanged. So $a_{12} = a_{21}$. Computed values: $a_{11} = \frac{4}{2} = 2$, $a_{12} = a_{21} = \frac{9}{2}$, $a_{22} = \frac{16}{2} = 8$.

$$\begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}.$$

Step 2. Rule (ii) $a_{ij} = \frac{i}{j}$ is *not* symmetric. $a_{11} = 1,\ a_{12} = \frac{1}{2},\ a_{21} = 2,\ a_{22} = 1$.

$$\begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}.$$

The diagonal is 1 because $\frac{i}{i} = 1$.

Step 3. Rule (iii) $a_{ij} = \frac{(i+2j)^2}{2}$ is not symmetric (coefficients of i and j differ). Compute the four squares: $3^2 = 9$, $5^2 = 25$, $4^2 = 16$, $6^2 = 36$, then halve: $\frac{9}{2}, \frac{25}{2}, 8, 18$.

$$\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}.$$

Why this matters. Spotting symmetry in the formula ($a_{ij} = a_{ji}$) instantly halves the work and tells you the matrix will be symmetric, a property used heavily later in this chapter.

Final Answer: (i), (ii), (iii) as boxed above.

Q 3.5 **Construct a 3×4 matrix whose elements are given by:**

(i) $a_{ij} = \dfrac{1}{2}|-3i + j|$, **(ii)** $a_{ij} = 2i - j$.

SOLUTION

Concept used. For a 3×4 matrix, $i \in \{1, 2, 3\}$ and $j \in \{1, 2, 3, 4\}$, giving 12 entries. Plug each pair into the rule; collect the results into a 3×4 array.

Step 1. **(i) Rule** $a_{ij} = \frac{1}{2}|-3i + j|$. Compute $-3i + j$ for each (i, j), take absolute value, halve.

Row $i = 1$: $-3 + j$. So $j = 1 \Rightarrow -2$, $j = 2 \Rightarrow -1$, $j = 3 \Rightarrow 0$, $j = 4 \Rightarrow 1$. Absolute values: $2, 1, 0, 1$. Halved: $1, \frac{1}{2}, 0, \frac{1}{2}$.

Row $i = 2$: $-6 + j$. So $-5, -4, -3, -2$. Absolute: $5, 4, 3, 2$. Halved: $\frac{5}{2}, 2, \frac{3}{2}, 1$.

Row $i = 3$: $-9 + j$. So $-8, -7, -6, -5$. Absolute: $8, 7, 6, 5$. Halved: $4, \frac{7}{2}, 3, \frac{5}{2}$.

Assemble:

$$A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}.$$

Step 2. **(ii) Rule** $a_{ij} = 2i - j$. Compute by row.

Row $i = 1$: $2 - j$. So $1, 0, -1, -2$.

Row $i = 2$: $4 - j$. So $3, 2, 1, 0$.

Row $i = 3$: $6 - j$. So $5, 4, 3, 2$.

$$A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}.$$

Final Answer: (i) and (ii) as displayed above.

✗ Common Mistake

Forgetting the absolute-value bars in (i) gives negative entries on the top-left, which is wrong. The bars force every entry to be non-negative.

EXPERT'S SOLUTION *: Aditi Banerjee, Ph.D Mathematics, IIT Delhi*

Quick reading. Both rules are affine in i, j. The smart move is to compute a single row's pattern once, then read off the others by shifting.

Step 1. (i) Notice $-3i + j$ increases by 1 as j steps up, and decreases by 3 as i steps up. After absolute-value and halving, the row pattern is $\left(\frac{|3i-1|}{2}, \frac{|3i-2|}{2}, \frac{|3i-3|}{2}, \frac{|3i-4|}{2}\right)$. Substitute $i = 1, 2, 3$ and you get the three rows shown.

Step 2. (ii) The rule $a_{ij} = 2i - j$ is just "constant per row, minus j." Row i starts at

$2i - 1$ and steps by -1 across the row.

$$i = 1 : 1, 0, -1, -2$$
$$i = 2 : 3, 2, 1, 0$$
$$i = 3 : 5, 4, 3, 2.$$

Step 3. Sanity check on entry $(3, 2)$: rule gives $2(3) - 2 = 4$. Matches.

Why this matters. Recognising the pattern in i, j saves time on larger matrices and reduces arithmetic mistakes.

Final Answer: Same matrices as the main solution.

Q 3.6 Find the values of x, y, z from the following equations:

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$, **(ii)** $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, **(iii)** $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$.

SOLUTION

Concept used. Two matrices are **equal** iff (a) they have the same order, and (b) every pair of corresponding entries is equal: $a_{ij} = b_{ij}$ for all i, j. So an equation between matrices splits into a system of scalar equations.

Step 1. **(i)** Compare entry-by-entry:

$$a_{11} : 4 = y; \quad a_{12} : 3 = z; \quad a_{21} : x = 1; \quad a_{22} : 5 = 5 \checkmark .$$

So $x = 1,\ y = 4,\ z = 3$.

Step 2. **(ii)** Compare entries:

$$x + y = 6, \qquad 5 + z = 5, \qquad xy = 8.$$

From $5 + z = 5$: $z = 0$. From $x + y = 6$ and $xy = 8$, x and y are the roots of $t^2 - 6t + 8 = 0$, so

$$t = \frac{6 \pm \sqrt{36 - 32}}{2} = \frac{6 \pm 2}{2} = 4 \text{ or } 2.$$

Thus $\{x, y\} = \{2, 4\}$, giving two solutions: $(x, y) = (2, 4)$ or $(x, y) = (4, 2)$, with $z = 0$ in both.

Step 3. **(iii)** The three scalar equations are

$$x + y + z = 9 \quad (1), \quad x + z = 5 \quad (2), \quad y + z = 7 \quad (3).$$

Subtract (2) from (1): $(x+y+z)-(x+z)=9-5$, so $y=4$. Subtract (3) from (1): $(x+y+z)-(y+z)=9-7$, so $x=2$. Use (2): $z=5-x=5-2=3$. Check (3): $y+z=4+3=7$ ✓.

Final Answer: (i) $x=1,\ y=4,\ z=3$. (ii) $x=2, y=4, z=0$ or $x=4, y=2, z=0$. (iii) $x=2,\ y=4,\ z=3$.

EXPERT'S SOLUTION *: Karan Mehta, Ph.D Pure Mathematics, IISc Bangalore*

Strategic angle. Matrix equality = a system of scalar equations. Solve each scalar equation; whenever one variable can be isolated, eliminate.

Step 1. (i) Direct read-off. The (1,1) entry forces $y=4$; the (1,2) forces $z=3$; the (2,1) forces $x=1$. Done.

Step 2. (ii) The clean entry is the off-diagonal $5+z=5 \Rightarrow z=0$. Then $x+y=6$ and $xy=8$ are sum-and-product, so x, y are roots of $t^2-6t+8=(t-2)(t-4)=0$.

$$\{x,y\}=\{2,4\}.$$

Both orderings are valid because the equations are symmetric in x, y.

Step 3. (iii) Subtract pairs:

$$(1)-(2):\ y=4; \qquad (1)-(3):\ x=2;$$

$$\text{then (2): } z=5-x=3.$$

Final check in (3): $4+3=7$ ✓.

Why this matters. Recognising a sum-and-product pair (like $x+y=6,\ xy=8$) lets you build a quadratic in t rather than slogging through substitution.

Final Answer: (i) $(1,4,3)$. (ii) $z=0$ and $\{x,y\}=\{2,4\}$. (iii) $(2,4,3)$.

Q 3.7 **Find the values of a, b, c, d from the equation** $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$.

SOLUTION

Concept used. Same as Q6: matrix equality is entry-wise equality. The four scalar equations from the four entries are

$$a - b = -1 \ (1), \quad 2a + c = 5 \ (2), \quad 2a - b = 0 \ (3), \quad 3c + d = 13 \ (4).$$

Step 1. Subtract (1) from (3) to eliminate b:

$$(2a - b) - (a - b) = 0 - (-1) \implies a = 1.$$

Step 2. Put $a = 1$ into (1):

$$1 - b = -1 \implies b = 1 - (-1) = 2.$$

Step 3. Put $a = 1$ into (2):

$$2(1) + c = 5 \implies c = 5 - 2 = 3.$$

Step 4. Put $c = 3$ into (4):

$$3(3) + d = 13 \implies 9 + d = 13 \implies d = 4.$$

Step 5. Verify in (3): $2a - b = 2(1) - 2 = 0$ ✓.

Final Answer: $a = 1, \ b = 2, \ c = 3, \ d = 4.$

☞ Exam Tip

When a system has more unknowns than "clean" equations, look for a pair you can subtract to instantly eliminate a variable, as we did with (3) - (1) to isolate a.

EXPERT'S SOLUTION *: Rohit Verma, M.Sc Mathematics, IIT Bombay*

Strategic angle. Write the four scalar equations, then look for the cheapest elimination. Here equations (1) and (3) share the $-b$, so subtracting them isolates a.

Step 1. Equations (1) $a - b = -1$ and (3) $2a - b = 0$. Subtract: $(2a - b) - (a - b) = 0 - (-1)$, giving $a = 1$.

Step 2. Back-substitute $a = 1$ in (1): $1 - b = -1 \Rightarrow b = 2$.

Step 3. Put $a = 1$ in (2): $c = 5 - 2 = 3$.

Step 4. Put $c = 3$ in (4): $d = 13 - 9 = 4$.

Step 5. Cross-check (3): $2(1) - 2 = 0$ ✓.

Why this matters. Always scan the system for a "free" elimination (two equations differing in one variable). This skill appears repeatedly when solving systems via matrices in Ch 4.

Final Answer: $(a, b, c, d) = (1, 2, 3, 4)$.

Q 3.8 $A = [a_{ij}]_{m\times n}$ **is a square matrix, if**
(A) $m < n$ **(B)** $m > n$ **(C)** $m = n$ **(D) None of these.**

SOLUTION

Concept used. By definition, a matrix is a **square matrix** when its number of rows equals its number of columns, i.e. the order $m \times n$ satisfies $m = n$.

Step 1. Option (A) $m < n$: this gives more columns than rows, e.g. 2×3. That is a *rectangular* (non-square) matrix.

Step 2. Option (B) $m > n$: this gives more rows than columns, e.g. 3×2. Again rectangular, not square.

Step 3. Option (C) $m = n$: equal rows and columns, e.g. 2×2 or 3×3. This is exactly the definition of a square matrix.

Step 4. Option (D) is therefore wrong.

Final Answer: Correct answer: (C).

EXPERT'S SOLUTION *: Ishita Joshi, B.Tech CSE, IIT Roorkee*

Quick reading. "Square" literally means equal sides; a 3×3 matrix can be drawn as a square block of entries.

Step 1. For a matrix to fit into a square outline, its row count and column count must be equal, $m = n$.

Step 2. This rules out options (A) $m < n$ and (B) $m > n$, which both produce rectangles.

Step 3. Option (C) is the precise statement of squareness.

Why this matters. Squareness is required for many later ideas: determinant, inverse, trace, eigenvalue, A^n, all of which demand $m = n$.

Final Answer: (C) $m = n$.

Q 3.9 **Which of the given values of x and y make the following pair of matrices**

equal: $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$**?**

(A) $x = -\frac{1}{3},\ y = 7$**,** **(B) Not possible to find,** **(C)** $y = 7,\ x = -\frac{2}{3}$**,** **(D)** $x = -\frac{1}{3},\ y = -\frac{2}{3}$**.**

SOLUTION

Concept used. Matrix equality forces all four corresponding entries to match simultaneously. So we must solve the system

$$3x+7=0, \quad 5=y-2, \quad y+1=8, \quad 2-3x=4$$

at the same time.

Step 1. From $3x+7=0$: $x=-\frac{7}{3}$.

Step 2. From $2-3x=4$: $-3x=2$, so $x=-\frac{2}{3}$.

Step 3. The two equations for x contradict each other: $-\frac{7}{3} \neq -\frac{2}{3}$.

Step 4. Therefore no single x satisfies both entries; the matrices cannot be equal for any pair (x, y).

> **Final Answer:** Correct answer: (B) Not possible to find.

✗ Common Mistake

A frequent error is to use only one of the equations involving x (say only $3x+7=0$) and announce $x=-\frac{7}{3}$. Matrix equality demands *every* entry to match: failure of any one entry kills the equality.

EXPERT'S SOLUTION *: Diya Nair, M.Sc Mathematics, IIT Bombay*

Strategic angle. Set up all four entry equations. The four options offered are all single (x, y) pairs; if the system is inconsistent, then no pair works and (B) is forced.

Step 1. Top-left entry: $3x+7=0 \Rightarrow x=-\frac{7}{3}$.

Step 2. Bottom-right entry: $2-3x=4 \Rightarrow x=-\frac{2}{3}$.

Step 3. These two values are different, so the system is inconsistent in x alone.

Step 4. No consistent value of x exists, regardless of y. So no (x, y) pair from the options will make the matrices equal.

Why this matters. "Matrix equation" is a compact way of writing several scalar equations at once. They all have to hold. This is precisely why systems of linear equations later get encoded as $AX = B$.

Final Answer: (B) Not possible to find.

Q 3.10 **The number of all possible matrices of order 3×3 with each entry 0 or 1 is:**
(A) 27 **(B)** 18 **(C)** 81 **(D)** 512.

SOLUTION

Concept used. Multiplication principle of counting: if a process involves k independent slots and slot i has n_i choices, the total number of outcomes is $n_1 \cdot n_2 \cdots n_k$. A 3×3 matrix has 9 entries; each entry independently can be 0 or 1, giving 2 choices per slot.

Step 1. Count the entries (slots) of a 3×3 matrix:

$$3 \times 3 = 9.$$

Step 2. Each slot can hold 0 or 1, so it has 2 choices.

Step 3. By the multiplication principle, the total number of 3×3 matrices with $\{0, 1\}$ entries is

$$\underbrace{2 \cdot 2 \cdots 2}_{9 \text{ factors}} = 2^9.$$

Step 4. Evaluate: $2^9 = 2^{10}/2 = 1024/2 = 512$.

Final Answer: Correct answer: (D) 512.

♥ **Why This Matters**

The count 2^{mn} generalises: a $0/1$ matrix of order $m \times n$ has 2^{mn} choices. This is the same combinatorial principle that counts subsets of an mn-element set.

EXPERT'S SOLUTION *: Ananya Reddy, M.Sc Mathematics, ISI Kolkata*

Counting-first. Treat each of the 9 cells as an independent binary switch. Total configurations $= 2^9$.

Step 1. Cells in a 3×3 grid: $3 \cdot 3 = 9$.

Step 2. Each cell: 2 choices (0 or 1).

Step 3. Configurations: 2^9. Compute by powers of two:
$2^1 = 2,\ 2^2 = 4,\ 2^3 = 8,\ \ldots,\ 2^9 = 512$.

Why this matters. This is the binary-matrix count used in combinatorics, graph adjacency matrices, and information theory.

Final Answer: (D) 512.

Key Takeaways

- A matrix of order $m \times n$ has m rows, n columns, and exactly mn entries. a_{ij} lives in row i, column j.
- The number of possible orders for a matrix with N entries equals the number of positive divisors of N.
- To build a matrix from a rule $a_{ij} = f(i, j)$, evaluate f at every valid (i, j) and place the result in the right cell.
- Two matrices are equal iff they have the same order and every pair of corresponding entries is equal. This converts a matrix equation into a system of scalar equations.
- A square matrix has $m = n$. The number of $0/1$ matrices of order 3×3 is $2^9 = 512$.

End of Exercise 3.1