Collegedunia NCERT Solutions

Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 3: Matrices

About this Chapter

Exercise 3.4 is the gateway to **invertible matrices**. A square matrix A has an inverse A^{-1} only when both products AB and BA collapse to the identity I. This single-question exercise tests whether you have internalised the definition. Both directions $AB = I$ *and* $BA = I$ are required; failure of either rules out inverses.

Topics covered: Inverse of a matrix • Identity matrix • Two-sided condition $AB = BA = I$ • Square-matrix requirement • Uniqueness of inverse

Quick Formula Sheet

Definition of inverse:
B is the inverse of A iff $AB = BA = I$, in which case we write $B = A^{-1}$.

Two-sided condition:
Both $AB = I$ and $BA = I$ are required; one alone is not enough.

Order:
Only square matrices can have inverses.

Uniqueness:
If A^{-1} exists, it is unique.

Exercise 3.4

Q 3.1 **Matrices A and B will be inverse of each other only if:**

(A) $AB = BA$ **(B)** $AB = BA = 0$ **(C)** $AB = 0,\ BA = I$ **(D)** $AB = BA = I$.

SOLUTION

Concept used. **Definition of inverse matrix.** If A is a square matrix of order n, then a square matrix B of the same order is called the *inverse* of A if

$$AB = BA = I_n,$$

where I_n is the $n \times n$ identity matrix (ones on the diagonal, zeros elsewhere). In this case we write $B = A^{-1}$ and equivalently $A = B^{-1}$. The condition is *two-sided*: both products must equal I.

> ☞ **Two-sided condition**
>
> For square matrices of the same order, requiring $AB = I$ alone is actually enough to force $BA = I$ as well (a non-trivial theorem). But the definition demands both, and beginners must check both — it is the cleaner, safer rule.

Step 1. **Option (A)** $AB = BA$**.** This says A and B commute, but does not say their product equals I. For example, $A = B = 2I$ gives $AB = BA = 4I \neq I$, yet (A) holds. So (A) is *not sufficient*.

Step 2. **Option (B)** $AB = BA = 0$**.** Then $AB = O$, which means at least one of the matrices may behave as a zero-divisor; in particular $B = O$ satisfies this trivially. Inverses cannot give zero, so (B) is wrong.

Step 3. **Option (C)** $AB = 0,\ BA = I$**.** This forces AB to be the zero matrix and BA to be the identity, a contradiction in general. In fact for square matrices AB and BA have the same trace, so $AB = O$ and $BA = I$ simultaneously is impossible unless $I = O$ (which is false). So (C) is wrong.

Step 4. **Option (D)** $AB = BA = I$**.** This is exactly the definition of A and B being inverses of each other.

Step 5. Therefore the correct choice is (D).

> **Final Answer:** Correct answer: (D) $AB = BA = I$.

✗ Common Mistake

A frequent error is to settle for $AB = I$ alone and *declare* B to be the inverse. For square matrices of the same order this turns out to be enough, but the definition in the NCERT chapter is the two-sided $AB = BA = I$. Stick with the definition during exams.

♥ Why This Matters

The condition $AB = BA = I$ ties together three deep facts: (a) only square matrices can have inverses, (b) the inverse, when it exists, is unique, and (c) the set of invertible matrices forms a group under multiplication. All of these flow from the two-sided condition.

EXPERT'S SOLUTION : *Aarav Mehta, M.Sc Mathematics, IIT Bombay*

Quick reading. The definition says: B is the inverse of A iff their products in both orders give the identity. Read each option against that definition.

Step 1. Option (A): only commutativity, no identity. Fails.

Step 2. Option (B): both products are zero; $B = O$ satisfies this with any A, so inverse cannot be zero. Fails.

Step 3. Option (C): $AB = O$ and $BA = I$ for square matrices is impossible (trace argument: $\text{tr}(AB) = \text{tr}(BA)$, so $0 = n \neq 0$). Fails.

Step 4. Option (D): exactly the definition.

Why this matters. Pinning the definition down rules out near-misses (one-sided products, commuting non-identity pairs) and sets you up for the determinant-based inverse formula in Ch 4.

Final Answer: (D).

Key Takeaways

- A square matrix A has an inverse A^{-1} iff there exists a square matrix B of the same order with $AB = BA = I$.
- Only square matrices can have inverses (rectangular matrices cannot satisfy both AB and BA being defined and equal).
- Two-sided condition: $AB = I$ together with $BA = I$. Both halves are required.
- The inverse, when it exists, is unique: if $AB = BA = I$ and $AC = CA = I$, then $B = C$.
- Reversal law (used heavily in later chapters): $(AB)^{-1} = B^{-1}A^{-1}$ whenever both inverses exist.

End of Exercise 3.4