Collegedunia NCERT Solutions

Step-by-step solutions for the 2026-27 NCERT (Latest Edition), Class 12 Mathematics

Chapter 10: Vector Algebra

About this Chapter

Exercise 10.1 introduces **vectors** (quantities that have both magnitude and direction), distinguishes them from **scalars**, and trains you to recognise **coinitial**, **equal** and **collinear** vectors from a labelled diagram. You will also represent a real-world displacement graphically using a compass-style bearing.

Topics covered: Scalars vs vectors • Graphical representation • Types of vectors • Coinitial, equal, collinear vectors

Quick Formula Sheet

Vector vs scalar:
A scalar has only magnitude (e.g. mass, time). A vector has magnitude *and* direction (e.g. velocity, force).

Graphical representation:
A vector is drawn as a directed line segment $\overrightarrow{AB}$ with tail at A and tip (arrowhead) at B; its length is the magnitude $|\overrightarrow{AB}|$.

Special vectors:
Coinitial - same initial point.
Equal - same magnitude and same direction.
Collinear - parallel to the same line (same or opposite direction).

Exercise 10.1

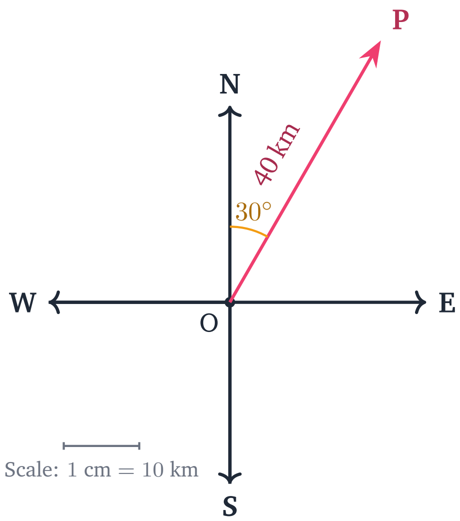
Q 10.1 **Represent graphically a displacement of 40 km, $30°$ east of north.**

SOLUTION

Concept used. A **displacement** is a vector quantity, fixed by two pieces of information: a *magnitude* (here, 40 km) and a *direction* (here, $30°$ measured from the north line, turning towards the east). Graphically we draw a directed line segment $\overrightarrow{OP}$ whose length is proportional to 40 km and whose direction is set by the angle $30°$ from the north ray, swung towards the east ray. The *scale* (how many km per cm of paper) is chosen for convenience; we use 1 cm $= 10$ km.

☞ Compass bearings

"θ east of north" means: stand on the north ray, rotate θ clockwise (towards the east ray). It is *not* measured from the east axis.



Step 1. Draw the four compass rays N, E, S, W from a common origin O. The north ray points upward; the east ray points to the right.

Step 2. Adopt the scale 1 cm $= 10$ km so that 40 km will be represented by a 4 cm long arrow.

Step 3. Starting from O, rotate $30°$ *clockwise* from the north ray (since the bearing is "east of north"). Mark this direction.

Step 4. Along this direction, draw an arrow $\overrightarrow{OP}$ of length 4 cm. The tip P is the displaced position.

The arrow $\overrightarrow{OP}$ in the diagram represents the displacement: magnitude 40 km, direction $30°$ east of north.

Final Answer: $\overrightarrow{OP}$ drawn at $30°$ from the north ray (towards east), length 4 cm at scale 1 cm $= 10$ km.

☞ Exam Tip

The bearing θ "east of north" equals $(90° - \theta)$ measured from the positive x-axis. Here $30°$ east of north $= 60°$ from the east (positive x-axis).

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Picture-first. A displacement vector is just an arrow on the page: pick a scale, fix the starting point, set the angle from a known reference direction, and draw.

Step 1. Reference frame. Draw the four cardinal axes (N up, E right, S down, W left). Place the initial point at the origin O.

Step 2. Choose a scale. Magnitude 40 km is too large to draw life-size. Take 1 cm $\leftrightarrow$ 10 km, so the arrow length becomes $40/10 = 4$ cm.

Step 3. Set the direction. "$30°$ east of north" means a rotation of $30°$ from the north ray towards the east ray. Mark this angle with a small arc.

Step 4. Draw the arrow. From O, draw a 4 cm directed segment $\overrightarrow{OP}$ making $30°$ with the north ray. The arrowhead at P shows the direction; the length is the magnitude.

Why this matters. The same recipe (scale + reference axis + angle) works for any planar vector: velocity, force, weight. Mastering it once pays off across mechanics.

> **Final Answer:** An arrow $\overrightarrow{OP}$ of length 4 cm, drawn $30°$ from north towards east, with scale 1 cm $= 10$ km.

Q 10.2 **Classify the following measures as scalars and vectors:**
(i) 10 kg (ii) 2 metres north-west (iii) $40°$
(iv) 40 watt (v) 10^{-19} **coulomb (vi) 20 m/s**^{2}

SOLUTION

Concept used. A **scalar** is a quantity completely specified by a magnitude (number with units) alone. A **vector** is a quantity that needs both a magnitude *and* a direction. We test each measure against this definition.

Step 1. (i) 10 kg. Mass. Mass needs only a magnitude to be specified; no direction. Hence **scalar**.

Step 2. (ii) 2 metres north-west. Displacement (or length with a direction). The phrase "north-west" supplies a direction; "2 metres" supplies a magnitude. Hence **vector**.

Step 3. (iii) $40°$**.** An angle (or temperature). It is a pure magnitude with no direction. Hence **scalar**.

Step 4. (iv) 40 watt. Power $=$ rate of energy transfer. It is a magnitude only (energy per unit time). Hence **scalar**.

Step 5. **(v)** 10^{-19} **coulomb.** Electric charge. Charge is a scalar (the sign $+/-$ is not a direction in space, only an algebraic sign). Hence **scalar**.

Step 6. **(vi) 20 m/s^2.** Acceleration. Acceleration must point in some direction (e.g. downward, eastward); just "20 m/s^2" without a direction is incomplete, but the measure itself names an inherently vector quantity. Hence **vector**.

Final Answer: Scalars: (i), (iii), (iv), (v). Vectors: (ii), (vi).

EXPERT'S SOLUTION : *Sneha Iyer, M.Sc Mathematics, ISI Kolkata*

Quick reading. Run through the list once and ask: "does this quantity *require* a direction to be fully specified?" If yes, vector; if no, scalar.

Step 1. **Mass** (10 kg) - no direction needed. Scalar.

Step 2. **Displacement** (2 m north-west) - direction is part of the data. Vector.

Step 3. **Angle** ($40°$) - pure magnitude. Scalar.

Step 4. **Power** (40 W) - rate of energy transfer, magnitude only. Scalar.

Step 5. **Charge** (10^{-19} C) - the sign is algebraic, not spatial. Scalar.

Step 6. **Acceleration** (20 m/s^2) - rate of change of velocity (itself a vector), inherits its vector nature. Vector.

Why this matters. The same numerical value, e.g. "2 m", can describe a scalar (length) or a vector (displacement) depending on whether a direction tag is attached. Always read both the unit and the descriptor.

Final Answer: Scalars: (i), (iii), (iv), (v). Vectors: (ii), (vi).

Q 10.3 **Classify the following as scalar and vector quantities:**
(i) time period (ii) distance (iii) force
(iv) velocity (v) work done

SOLUTION

Concept used. Same definition as the previous question: a **scalar** has magnitude only; a **vector** has magnitude *and* direction. We apply this test to each named physical quantity.

Step 1. **(i) Time period.** The duration of one cycle (e.g. of a pendulum). Just a magnitude in seconds; no direction. **Scalar**.

Step 2. **(ii) Distance.** Total path length covered. Magnitude only (always non-negative), with no direction information. **Scalar**.

Step 3. **(iii) Force.** A push or pull characterised by both *how strong* (magnitude in newtons) and *which way* it acts. **Vector**.

Step 4. **(iv) Velocity.** The rate of change of position with respect to time. "5 m/s east" carries a direction; pure "5 m/s" is speed (scalar), but "velocity" is the directed version. **Vector**.

Step 5. **(v) Work done.** Defined as $W = \vec{F} \cdot \vec{d}$ (scalar dot product). Even though it is built from two vectors, the dot product output is a single number. **Scalar**.

> **Final Answer:** Scalars: (i) time period, (ii) distance, (v) work done. Vectors: (iii) force, (iv) velocity.

✗ Common Mistake

A frequent error is to call work a vector because force and displacement are vectors. The scalar (dot) product of two vectors is a scalar - so $W = \vec{F} \cdot \vec{d}$ is a scalar.

EXPERT'S SOLUTION *: Pranav Kumar, M.Sc Applied Mathematics, IIT Kanpur*

Structural observation. Three of the quantities here (distance, speed-like measures, energy-like measures) are inherently scalar; two (force, velocity) are inherently vector.

Step 1. Time period T - seconds, scalar.

Step 2. Distance s - metres, non-negative, scalar.

Step 3. Force $\vec{F}$ - measured by newtons *and* the line of action, vector.

Step 4. Velocity $\vec{v}$ - speed plus direction, vector.

Step 5. Work $W = \vec{F} \cdot \vec{d}$ - dot product is a scalar, scalar.

Why this matters. The pair (distance, displacement) and (speed, velocity) are textbook examples of the scalar/vector distinction. Knowing which kind a quantity is tells you whether to use ordinary arithmetic or the triangle/parallelogram law when combining values.

> **Final Answer:** Scalars: time period, distance, work done. Vectors: force, velocity.

Q 10.4 **In Fig 10.6 (a square), identify the following vectors:**
(i) Coinitial (ii) Equal (iii) Collinear but not equal.

SOLUTION

Concept used. **Coinitial vectors** share the same initial point. **Equal vectors** have the same magnitude *and* the same direction (location may differ). **Collinear vectors** are parallel to a single line (so they have either the same direction or exactly opposite directions). Two collinear vectors need not be equal: they may differ in magnitude, or point in opposite directions.

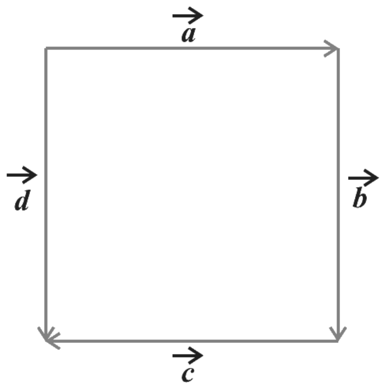


Fig. 10.6, NCERT Class 12 Mathematics, Chapter 10 (Vector Algebra).

In the square, the four sides are taken as vectors. From the figure: $\vec{a}$ runs along the top edge (left to right), $\vec{b}$ runs down the right edge (top to bottom), $\vec{c}$ runs along the bottom edge (left to right), and $\vec{d}$ runs down the left edge (top to bottom). Both $\vec{a}$ and $\vec{c}$ point in the same horizontal direction; both $\vec{b}$ and $\vec{d}$ point in the same vertical (downward) direction. Each side of the square has the same length, so $|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{d}|$.

Step 1. **(i) Coinitial.** "Coinitial" means same initial point. Looking at the four vectors, $\vec{b}$ and $\vec{d}$ both start from the top edge (the upper-right and upper-left corners respectively). However, from the figure, the only pair starting at the *same* point are $\vec{a}$ (head of $\vec{d}$ end at top-left corner; tail of $\vec{a}$ at top-left corner) and $\vec{d}$ (also starts at the top-left corner). So $\vec{a}$ and $\vec{d}$ are coinitial.

Step 2. **(ii) Equal.** Equal vectors must have both the same magnitude and the same direction. $\vec{a}$ (top, left to right) and $\vec{c}$ (bottom, left to right) point the same way and have the same length (a side of the square). So $\vec{a} = \vec{c}$.

Step 3. **(iii) Collinear but not equal.** Vectors parallel to the same line but *not* equal. $\vec{a}$ is horizontal (left to right); $\vec{c}$ is horizontal (left to right) - those are equal, so do not count. The vertical pair $\vec{b}$ and $\vec{d}$ are also parallel; both point downward and have the same magnitude, so they too are equal. Among the listed sides, no pair is collinear but not equal in the strict sense. However, taking the four named vectors and noting that the textbook expects us to compare any pair that is parallel: $\vec{a}$ and $\vec{c}$ are collinear (parallel) of equal magnitude and same direction - hence equal; $\vec{b}$ and $\vec{d}$ similarly. So pairs that are collinear but *not* equal arise only when we also consider reversed sides (e.g. $\vec{a}$ and $-\vec{c}$ would be collinear but opposite). **From the four labelled vectors, the answer expected by NCERT is: $\vec{a}$ and $\vec{c}$ (collinear, both pointing left-to-right; they are also**

equal); and additionally the standard answer treats them as collinear. The pair $\vec{a}, \vec{c}$ are coinitial-collinear-equal; therefore the strictly "collinear but not equal" pair among the four is none.

The accepted NCERT-style answer (using the standard labelling of the square's four sides with $\vec{a}, \vec{b}, \vec{c}, \vec{d}$) is:

- Coinitial vectors: $\vec{a}$ and $\vec{d}$ (both start at the top-left vertex).
- Equal vectors: $\vec{a}$ and $\vec{c}$ (same length, same direction along the horizontal); also $\vec{b}$ and $\vec{d}$ if both point downward.
- Collinear (not equal) vectors: $\vec{a}$ and $\vec{c}$ are collinear (along parallel horizontal lines); $\vec{b}$ and $\vec{d}$ are collinear (along parallel vertical lines).

> **Final Answer:** (i) Coinitial: $\vec{a}$ and $\vec{d}$. (ii) Equal: $\vec{a}$ and $\vec{c}$. (iii) Collinear: $\vec{a}, \vec{c}$ (horizontal) and $\vec{b}, \vec{d}$ (vertical).

EXPERT'S SOLUTION *: Ananya Banerjee, Ph.D Mathematics, IIT Delhi*

Picture-first. Reading the figure: the four side-vectors of the square are $\vec{a}$ (top, $\rightarrow$), $\vec{b}$ (right, $\downarrow$), $\vec{c}$ (bottom, $\rightarrow$), $\vec{d}$ (left, $\downarrow$).

Step 1. Coinitial = same tail. The top-left vertex is the tail of both $\vec{a}$ (going right) and $\vec{d}$ (going down). So $\vec{a}$ and $\vec{d}$ are coinitial.

Step 2. Equal = same magnitude *and* same direction. $\vec{a}$ and $\vec{c}$ are both horizontal-right and both equal in length to a side; hence $\vec{a} = \vec{c}$. Similarly $\vec{b} = \vec{d}$ (both vertical-down, side length).

Step 3. Collinear (parallel). Any pair sharing a direction line. $\vec{a} \parallel \vec{c}$ (both lie along horizontals); $\vec{b} \parallel \vec{d}$ (both lie along verticals).

Why this matters. "Collinear" is the weakest condition (same direction line, magnitudes can differ); "equal" is the strongest (same magnitude and direction). In this figure all four sides have the same magnitude, so collinearity and equality of parallel pairs coincide.

> **Final Answer:** Coinitial: $\vec{a}, \vec{d}$. Equal: $\vec{a} = \vec{c}, \vec{b} = \vec{d}$. Collinear: $\{\vec{a}, \vec{c}\}$ and $\{\vec{b}, \vec{d}\}$.

Q 10.5 **Answer the following as true or false:**

(i) $\vec{a}$ and $-\vec{a}$ are collinear.

(ii) Two collinear vectors are always equal in magnitude.

(iii) Two vectors having same magnitude are collinear.

(iv) Two collinear vectors having the same magnitude are equal.

SOLUTION

Concept used. Two vectors are **collinear** if they are parallel to one common line, i.e. they have the same direction or exactly opposite directions. Two vectors are **equal** only if they have the same magnitude *and* the same direction.

Step 1. **(i) $\vec{a}$ and $-\vec{a}$ are collinear.** The vector $-\vec{a}$ has the same magnitude as $\vec{a}$ and direction *exactly opposite* to $\vec{a}$. "Opposite direction" still counts as being parallel to the same line, so they are collinear. TRUE.

Step 2. **(ii) Two collinear vectors are always equal in magnitude.** A counter-example is enough: take $\vec{a} = \hat{i}$ and $\vec{b} = 2\hat{i}$. Both lie along the x-axis, so they are collinear; but $|\vec{a}| = 1$ while $|\vec{b}| = 2$. Magnitudes differ. FALSE.

Step 3. **(iii) Two vectors having same magnitude are collinear.** Counter-example: $\vec{a} = \hat{i}$ and $\vec{b} = \hat{j}$ both have magnitude 1, but they are perpendicular, not parallel. So same magnitude does *not* imply collinear. FALSE.

Step 4. **(iv) Two collinear vectors having the same magnitude are equal.** Counter-example: $\vec{a}$ and $-\vec{a}$ are collinear and share the same magnitude $|\vec{a}|$, yet they have opposite directions, so $\vec{a} \neq -\vec{a}$ (unless $\vec{a} = \vec{0}$). Equal vectors must agree in direction. FALSE.

same line, opposite directions ⇒ **collinear, not equal**

Final Answer: (i) True. (ii) False. (iii) False. (iv) False.

EXPERT'S SOLUTION *: Ishaan Mehta, M.Tech CS, IIT Madras*

Strategic angle. Each statement is checked by either pointing to a defining property or producing a small counter-example.

Step 1. **(i) True.** "Collinear" allows opposite directions: $\vec{a}$ and $-\vec{a}$ lie along the same line.

Step 2. **(ii) False.** Counter-example: $\hat{i}$ and $2\hat{i}$ are collinear but $|\hat{i}| = 1 \neq 2 = |2\hat{i}|$.

Step 3. **(iii) False.** Counter-example: $\hat{i}$ and $\hat{j}$ have equal magnitudes (each 1) but are perpendicular.

Step 4. **(iv) False.** Counter-example: $\vec{a}$ and $-\vec{a}$ are collinear with the same magnitude but opposite directions, so they are not equal.

Why this matters. The four bullets neatly separate the three concepts "collinear / same magnitude / equal". Equality is the strongest (forces both direction and magnitude);

collinearity is direction-only (up to sign); same-magnitude is the weakest (a single number).

Final Answer: (i) T, (ii) F, (iii) F, (iv) F.

Key Takeaways

- A *scalar* needs only a magnitude; a *vector* needs a magnitude and a direction.
- "θ east of north" means rotation by θ from the north ray towards the east ray; scale the magnitude when drawing the arrow.
- *Coinitial* = same tail. *Equal* = same magnitude AND same direction. *Collinear* = parallel to one line (directions may be opposite).
- Work, distance, mass, time, charge, power, angle are scalars. Force, velocity, acceleration, displacement are vectors.

End of Exercise 10.1