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Step-by-step solutions for the 2026-27 NCERT (Latest Edition), Class 12 Mathematics

Chapter 10: Vector Algebra

About this Chapter

Exercise 10.2 puts vector arithmetic to work: computing **magnitudes** of $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, finding **unit vectors**, scalar/vector **components**, **direction cosines and ratios**, the **section formula** for internal and external division, and verifying collinearity and triangle-vertex configurations.

Topics covered: Magnitude • Unit vectors • Components • Direction cosines • Section formula

Quick Formula Sheet

Magnitude: If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Unit vector along $\vec{a}$: $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$.

Direction cosines: $l = a_1/|\vec{a}|$, $m = a_2/|\vec{a}|$, $n = a_3/|\vec{a}|$; with $l^2 + m^2 + n^2 = 1$.

Section formula (m:n):

Internal: $\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$.

External: $\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$.

Exercise 10.2

Q 10.1 **Compute the magnitude of the following vectors:**
$\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$; $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$.

SOLUTION

Concept used. The **magnitude** of a vector $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ is

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}.$$

Here v_1, v_2, v_3 are the scalar components along $\hat{i}, \hat{j}, \hat{k}$. We square each component, sum, and take the non-negative square root.

Step 1. **For** $\vec{a} = \hat{i} + \hat{j} + \hat{k}$**.** Components: $a_1 = 1,\ a_2 = 1,\ a_3 = 1.$

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}.$$

Step 2. **For** $\vec{b} = 2\hat{i} - 7\hat{j} - 3\hat{k}$**.** Components: $b_1 = 2,\ b_2 = -7,\ b_3 = -3.$

$$|\vec{b}| = \sqrt{2^2 + (-7)^2 + (-3)^2} = \sqrt{4 + 49 + 9} = \sqrt{62}.$$

Step 3. **For** $\vec{c} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} - \frac{1}{\sqrt{3}}\hat{k}$**.** Components: each $= \pm 1/\sqrt{3}$.

$$|\vec{c}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2} = \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = \sqrt{1} = 1.$$

The fact $|\vec{c}| = 1$ tells us $\vec{c}$ is a unit vector.

> **Final Answer:** $|\vec{a}| = \sqrt{3}, \quad |\vec{b}| = \sqrt{62}, \quad |\vec{c}| = 1.$

EXPERT'S SOLUTION *: Aanya Verma, Ph.D Mathematics, IIT Delhi*

Quick reading. The magnitude operation is just the Pythagorean length in three dimensions: square, sum, square-root.

Step 1. $\vec{a}$: $\sqrt{1 + 1 + 1} = \sqrt{3}$.

Step 2. $\vec{b}$: $\sqrt{4 + 49 + 9} = \sqrt{62}$.

Step 3. $\vec{c}$: each component squared is $1/3$; total 1, so $|\vec{c}| = 1$.

Why this matters. Vectors with magnitude 1 (like $\vec{c}$) are unit vectors. Recognising them at a glance saves the step of dividing by $|\vec{v}|$ later when you need a direction-only quantity.

> **Final Answer:** $\sqrt{3},\ \sqrt{62},\ 1.$

Q 10.2 Write two different vectors having same magnitude.

SOLUTION

Concept used. Two vectors have the same magnitude when $\sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{b_1^2 + b_2^2 + b_3^2}$. To be "different" they must differ in at least one component (i.e. have different directions or different sign-patterns), even if the sum of squares is the same.

Step 1. Choose $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. Then $|\vec{a}| = \sqrt{1+1+1} = \sqrt{3}$.

Step 2. Choose $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. Then $|\vec{b}| = \sqrt{1+1+1} = \sqrt{3}$.

Step 3. Both magnitudes equal $\sqrt{3}$, but $\vec{a} \neq \vec{b}$ since their $\hat{j}$-components are $+1$ and -1.

> **Final Answer:** $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ both have magnitude $\sqrt{3}$, yet are different vectors.

EXPERT'S SOLUTION *: Riya Kapoor, M.Sc Mathematics, IIT Bombay*

Structural observation. "Different but same magnitude" is satisfied by any pair related by a sign flip on one component.

Step 1. Pick any non-trivial vector, say $\vec{p} = 2\hat{i} + \hat{j}$. Then $|\vec{p}| = \sqrt{5}$.

Step 2. Flip the sign of one component: $\vec{q} = 2\hat{i} - \hat{j}$. Then $|\vec{q}| = \sqrt{4+1} = \sqrt{5}$.

Step 3. $\vec{p} \neq \vec{q}$ (the $\hat{j}$-component differs in sign) yet $|\vec{p}| = |\vec{q}|$.

Why this matters. An infinite family of vectors has the same magnitude r: geometrically, they are all the arrows from the origin to the surface of the sphere $x^2 + y^2 + z^2 = r^2$.

> **Final Answer:** $2\hat{i} + \hat{j}$ and $2\hat{i} - \hat{j}$, magnitudes both $\sqrt{5}$.

Q 10.3 Write two different vectors having same direction.

SOLUTION

Concept used. Two vectors $\vec{a}$ and $\vec{b}$ have the *same direction* iff one is a positive scalar multiple of the other, i.e. $\vec{b} = \lambda\vec{a}$ with $\lambda > 0$. They become different vectors when $\lambda \neq 1$, which forces $|\vec{a}| \neq |\vec{b}|$ even though the direction is the same.

Step 1. Take $\vec{a} = \hat{i} + \hat{j} + \hat{k}$.

Step 2. Take $\vec{b} = 2\hat{i} + 2\hat{j} + 2\hat{k} = 2\vec{a}$ (scalar $\lambda = 2 > 0$).

Step 3. Since $\lambda > 0$, $\vec{a}$ and $\vec{b}$ point in the same direction. But $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = 2\sqrt{3}$, so $\vec{a} \neq \vec{b}$.

> **Final Answer:** $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 2\hat{j} + 2\hat{k}$ have the same direction but different magnitudes.

EXPERT'S SOLUTION *: Karan Reddy, M.Tech CS, IIT Madras*

Structural observation. Positive scalar multiplication preserves direction; the magnitude scales by the factor.

Step 1. Start with any non-zero vector $\vec{u} = 3\hat{i} + 4\hat{j}$, $|\vec{u}| = 5$.

Step 2. Multiply by 3: $\vec{v} = 9\hat{i} + 12\hat{j}$, $|\vec{v}| = 15$. Same direction, magnitude tripled.

Why this matters. The unit vector $\hat{a} = \vec{a}/|\vec{a}|$ picks out the direction; every vector along the same direction is then $\lambda\hat{a}$ for some $\lambda > 0$.

Final Answer: $3\hat{i} + 4\hat{j}$ and $9\hat{i} + 12\hat{j}$ have the same direction.

Q 10.4 **Find the values of x and y so that the vectors $2\hat{i} + 3\hat{j}$ and $x\hat{i} + y\hat{j}$ are equal.**

SOLUTION

Concept used. Two vectors are **equal** iff their corresponding components are equal. So $a_1\hat{i} + a_2\hat{j} + a_3\hat{k} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ implies $a_1 = b_1$, $a_2 = b_2$, $a_3 = b_3$.

Step 1. Given $2\hat{i} + 3\hat{j} = x\hat{i} + y\hat{j}$.

Step 2. Equate the $\hat{i}$-components: $x = 2$.

Step 3. Equate the $\hat{j}$-components: $y = 3$.

Final Answer: $x = 2,\ y = 3$.

EXPERT'S SOLUTION *: Vivaan Joshi, M.Sc Mathematics, IIT Bombay*

Quick reading. Equating vectors $\iff$ equating components, one slot at a time.

Step 1. $\hat{i}$-slot: $2 = x \Rightarrow x = 2$.

Step 2. $\hat{j}$-slot: $3 = y \Rightarrow y = 3$.

Why this matters. The decomposition $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ is unique because $\hat{i}, \hat{j}, \hat{k}$ are linearly independent.

Final Answer: $x = 2,\ y = 3$.

Q 10.5 **Find the scalar and vector components of the vector with initial point $(2, 1)$ and terminal point $(-5, 7)$.**

SOLUTION

Concept used. Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, the vector $\overrightarrow{AB}$ from A to B is

$$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}.$$

The numbers $(x_2 - x_1)$ and $(y_2 - y_1)$ are the **scalar components** of $\overrightarrow{AB}$, while $(x_2 - x_1)\hat{i}$ and $(y_2 - y_1)\hat{j}$ are the corresponding **vector components** (each scalar component scaled by the appropriate unit vector).

Step 1. Identify the points: $A = (2, 1)$ as initial, $B = (-5, 7)$ as terminal.

Step 2. Subtract coordinates:

$$\overrightarrow{AB} = (-5 - 2)\hat{i} + (7 - 1)\hat{j} = -7\hat{i} + 6\hat{j}.$$

Step 3. Read off the scalar components: -7 (along $\hat{i}$) and 6 (along $\hat{j}$).

Step 4. Read off the vector components: $-7\hat{i}$ and $6\hat{j}$.

> **Final Answer:** Scalar components: -7 and 6. Vector components: $-7\hat{i}$ and $6\hat{j}$.

EXPERT'S SOLUTION *: Aditi Nair, M.Sc Mathematics, ISI Kolkata*

Picture-first. $\overrightarrow{AB}$ is the arrow from $A(2, 1)$ to $B(-5, 7)$: horizontal change -7, vertical change $+6$.

Step 1. Horizontal change: $x_2 - x_1 = -5 - 2 = -7$.

Step 2. Vertical change: $y_2 - y_1 = 7 - 1 = 6$.

Step 3. $\overrightarrow{AB} = -7\hat{i} + 6\hat{j}$.

Why this matters. Subtracting initial from terminal is the universal recipe: the *position-vector difference* formula extends directly to 3D as $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$.

> **Final Answer:** Scalar comps $-7, 6$; vector comps $-7\hat{i}, 6\hat{j}$.

Q 10.6 **Find the sum of the vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.**

SOLUTION

Concept used. Vector addition is performed **component-wise**: if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$, $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$, then

$$\vec{a} + \vec{b} + \vec{c} = (a_1 + b_1 + c_1)\hat{i} + (a_2 + b_2 + c_2)\hat{j} + (a_3 + b_3 + c_3)\hat{k}.$$

Step 1. Sum of $\hat{i}$-components: $1 + (-2) + 1 = 0$.

Step 2. Sum of $\hat{j}$-components: $(-2) + 4 + (-6) = -4$.

Step 3. Sum of $\hat{k}$-components: $1 + 5 + (-7) = -1$.

Step 4. Combine: $\vec{a} + \vec{b} + \vec{c} = 0\,\hat{i} + (-4)\hat{j} + (-1)\hat{k} = -4\hat{j} - \hat{k}$.

> **Final Answer:** $\vec{a} + \vec{b} + \vec{c} = -4\hat{j} - \hat{k}$.

EXPERT'S SOLUTION *: Krishna Pillai, M.Sc Mathematics, IIT Bombay*

Strategic angle. Stack the three vectors as rows of a 3×3 "component table" and add column-by-column.

Step 1.

	$\hat{i}$	$\hat{j}$	$\hat{k}$
$\vec{a}$	1	-2	1
$\vec{b}$	-2	4	5
$\vec{c}$	1	-6	-7
Sum	0	-4	-1

Step 2. Read off: $\vec{a} + \vec{b} + \vec{c} = -4\hat{j} - \hat{k}$.

Why this matters. The same column-sum recipe works for any number of vectors; it is just the standard addition of triples (a, b, c).

> **Final Answer:** $-4\hat{j} - \hat{k}$.

Q 10.7 **Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.**

SOLUTION

Concept used. The **unit vector** in the direction of $\vec{a}$ is

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}, \quad \text{with } |\hat{a}| = 1.$$

We compute $|\vec{a}|$ by the magnitude formula, then divide each component of $\vec{a}$ by $|\vec{a}|$.

Step 1. Compute the magnitude:

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{1+1+4} = \sqrt{6}.$$

Step 2. Divide each component by $\sqrt{6}$:

$$\hat{a} = \frac{1}{\sqrt{6}}\,\hat{i} + \frac{1}{\sqrt{6}}\,\hat{j} + \frac{2}{\sqrt{6}}\,\hat{k}.$$

Step 3. **Sanity check:** $|\hat{a}|^2 = \frac{1}{6} + \frac{1}{6} + \frac{4}{6} = \frac{6}{6} = 1.$ ✓

> **Final Answer:** $\hat{a} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}.$

EXPERT'S SOLUTION *: Rohit Desai, M.Sc Mathematics, IIT Bombay*

Strategic angle. "Direction-only" = drop magnitude information by dividing the whole vector by its length.

Step 1. $|\vec{a}|^2 = 1 + 1 + 4 = 6$, so $|\vec{a}| = \sqrt{6}$.

Step 2. $\hat{a} = \dfrac{\vec{a}}{\sqrt{6}} = \dfrac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}.$

Why this matters. Unit vectors are the standard way to encode "which way"; combined with a magnitude, they reconstruct any vector via $\vec{a} = |\vec{a}|\,\hat{a}$.

> **Final Answer:** $\hat{a} = \dfrac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}.$

Q 10.8 **Find the unit vector in the direction of vector $\overrightarrow{PQ}$, where P and Q are the points $(1, 2, 3)$ and $(4, 5, 6)$ respectively.**

SOLUTION

Concept used. For points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the vector from P to Q is

$$\overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}.$$

The unit vector along $\overrightarrow{PQ}$ is $\widehat{PQ} = \overrightarrow{PQ}/|\overrightarrow{PQ}|$.

Step 1. Compute $\overrightarrow{PQ}$:

$$\overrightarrow{PQ} = (4-1)\hat{i} + (5-2)\hat{j} + (6-3)\hat{k} = 3\hat{i} + 3\hat{j} + 3\hat{k}.$$

Step 2. Compute its magnitude:

$$|\overrightarrow{PQ}| = \sqrt{3^2 + 3^2 + 3^2} = \sqrt{27} = 3\sqrt{3}.$$

Step 3. Divide by the magnitude:

$$\widehat{PQ} = \frac{3\hat{i} + 3\hat{j} + 3\hat{k}}{3\sqrt{3}} = \frac{1}{\sqrt{3}}\,\hat{i} + \frac{1}{\sqrt{3}}\,\hat{j} + \frac{1}{\sqrt{3}}\,\hat{k}.$$

Final Answer: $\widehat{PQ} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}.$

EXPERT'S SOLUTION *: Diya Bhat, M.Sc Mathematics, IIT Bombay*

Strategic angle. Same recipe as Q7, but with $\overrightarrow{PQ}$ derived from two given points first.

Step 1. $\overrightarrow{PQ} = (3, 3, 3)$.

Step 2. Length $= \sqrt{27} = 3\sqrt{3}$.

Step 3. Unit vector $= (1, 1, 1)/\sqrt{3}$.

Why this matters. A direction in 3D is one unit vector; here the direction is the body-diagonal direction of the cube along $(1, 1, 1)$.

Final Answer: $\widehat{PQ} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}.$

Q 10.9 **For given vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} - \hat{k}$, find the unit vector in the direction of the vector $\vec{a} + \vec{b}$.**

SOLUTION

Concept used. First find $\vec{a} + \vec{b}$ by component-wise addition, then compute its magnitude $|\vec{a} + \vec{b}|$, and divide.

Step 1. Component-wise addition:

$$\vec{a} + \vec{b} = (2 - 1)\hat{i} + (-1 + 1)\hat{j} + (2 - 1)\hat{k} = \hat{i} + 0\,\hat{j} + \hat{k} = \hat{i} + \hat{k}.$$

Step 2. Magnitude:

$$|\vec{a} + \vec{b}| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}.$$

Step 3. Unit vector:

$$\widehat{\vec{a}+\vec{b}} = \frac{\hat{i}+\hat{k}}{\sqrt{2}} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}.$$

Final Answer: $\widehat{\vec{a}+\vec{b}} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}.$

EXPERT'S SOLUTION *: Tara Singh, M.Sc Mathematics, IIT Bombay*

Quick reading. Add, find length, divide.

Step 1. $\vec{a}+\vec{b} = (1, 0, 1)$.

Step 2. $|\vec{a}+\vec{b}| = \sqrt{2}$.

Step 3. Unit vector $= (1, 0, 1)/\sqrt{2}$.

Why this matters. The direction of $\vec{a}+\vec{b}$ is the diagonal of the parallelogram with sides $\vec{a}, \vec{b}$; its unit vector is the standard handle on that diagonal.

Final Answer: $\frac{\hat{i}+\hat{k}}{\sqrt{2}}$.

Q 10.10 **Find a vector in the direction of vector $5\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 8 units.**

SOLUTION

Concept used. A vector of magnitude m in the direction of a given vector $\vec{v}$ is $m \cdot \hat{v}$, where $\hat{v} = \vec{v}/|\vec{v}|$.

Step 1. Call $\vec{v} = 5\hat{i} - \hat{j} + 2\hat{k}$.

Step 2. Magnitude:

$$|\vec{v}| = \sqrt{5^2 + (-1)^2 + 2^2} = \sqrt{25+1+4} = \sqrt{30}.$$

Step 3. Unit vector along $\vec{v}$:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{5\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{30}}.$$

Step 4. Scale by 8:

$$8\hat{v} = \frac{8(5\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{30}} = \frac{40\hat{i} - 8\hat{j} + 16\hat{k}}{\sqrt{30}}.$$

Final Answer: $\frac{40}{\sqrt{30}}\hat{i} - \frac{8}{\sqrt{30}}\hat{j} + \frac{16}{\sqrt{30}}\hat{k}$.

EXPERT'S SOLUTION : *Yash Chatterjee, M.Sc Mathematics, IIT Bombay*

Strategic angle. "Make a vector of length m along direction $\vec{v}$" = $m \cdot \hat{v}$.

Step 1. $|\vec{v}| = \sqrt{30}$.

Step 2. Multiply $\vec{v}$ by $8/\sqrt{30}$: result $= \frac{8}{\sqrt{30}}(5\hat{i} - \hat{j} + 2\hat{k})$.

Why this matters. This decoupling (direction $\hat{v}$, magnitude m) lets you scale forces, velocities, displacements without recomputing direction every time.

Final Answer: $\frac{8(5\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{30}}$.

Q 10.11 **Show that the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $-4\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear.**

SOLUTION

Concept used. Two vectors $\vec{a}$ and $\vec{b}$ are **collinear** iff $\vec{b} = \lambda\,\vec{a}$ for some scalar λ (positive $\lambda \Rightarrow$ same direction; negative $\lambda \Rightarrow$ opposite direction). Equivalently, the corresponding components are proportional.

Step 1. Write the two vectors. Let $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = -4\hat{i} + 6\hat{j} - 8\hat{k}$.

Step 2. Test whether each component of $\vec{b}$ is the same multiple of the corresponding component of $\vec{a}$:

$$\frac{-4}{2} = -2, \qquad \frac{6}{-3} = -2, \qquad \frac{-8}{4} = -2.$$

Step 3. All three ratios equal -2, so $\vec{b} = -2\,\vec{a}$.

Since $\vec{b} = -2\,\vec{a}$, the two vectors are scalar multiples of each other, hence collinear (in fact, anti-parallel because the scalar is negative).

Final Answer: $\vec{b} = -2\,\vec{a} \Rightarrow$ the vectors are collinear.

EXPERT'S SOLUTION *: Neha Gupta, M.Sc Applied Mathematics, IIT Kanpur*

Strategic angle. Compare component ratios; if all three are equal, the vectors are collinear.

Step 1. Ratios: $-4/2,\ 6/-3,\ -8/4$.

Step 2. Each equals -2, so $\vec{b} = -2\,\vec{a}$.

Why this matters. The common ratio $\lambda = -2$ also says $\vec{b}$ has twice the magnitude of $\vec{a}$ and is antiparallel.

> **Final Answer:** Collinear: $\vec{b} = -2\vec{a}$.

Q 10.12 **Find the direction cosines of the vector $\hat{i} + 2\hat{j} + 3\hat{k}$.**

SOLUTION

Concept used. If $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ has magnitude $|\vec{a}|$, then its **direction cosines** are

$$l = \frac{a_1}{|\vec{a}|}, \quad m = \frac{a_2}{|\vec{a}|}, \quad n = \frac{a_3}{|\vec{a}|},$$

and they satisfy $l^2 + m^2 + n^2 = 1$. They are the cosines of the angles $\vec{a}$ makes with the x, y, z axes.

Step 1. Magnitude:

$$|\vec{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}.$$

Step 2. Direction cosines:

$$l = \frac{1}{\sqrt{14}}, \quad m = \frac{2}{\sqrt{14}}, \quad n = \frac{3}{\sqrt{14}}.$$

Step 3. Check: $l^2 + m^2 + n^2 = \dfrac{1 + 4 + 9}{14} = \dfrac{14}{14} = 1.$ ✓

> **Final Answer:** $l = \dfrac{1}{\sqrt{14}},\ m = \dfrac{2}{\sqrt{14}},\ n = \dfrac{3}{\sqrt{14}}$.

EXPERT'S SOLUTION *: Aditya Rao, M.Sc Mathematics, IIT Bombay*

Quick reading. Direction cosines = components of the unit vector along $\vec{a}$.

Step 1. $|\vec{a}| = \sqrt{14}$.

Step 2. $\hat{a} = (1, 2, 3)/\sqrt{14}$, so $l = 1/\sqrt{14}$, $m = 2/\sqrt{14}$, $n = 3/\sqrt{14}$.

Why this matters. The constraint $l^2 + m^2 + n^2 = 1$ makes (l, m, n) a point on the unit sphere; conversely every direction in space is encoded by a single point on this sphere.

Final Answer: $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$.

Q 10.13 **Find the direction cosines of the vector joining the points $A(1, 2, -3)$ and $B(-1, -2, 1)$, directed from A to B.**

SOLUTION

Concept used. The vector from A to B is $\overrightarrow{AB} = (B - A)$ in component form. The direction cosines of $\overrightarrow{AB}$ are its components divided by $|\overrightarrow{AB}|$.

Step 1. Compute $\overrightarrow{AB}$:

$$\overrightarrow{AB} = (-1-1)\hat{i} + (-2-2)\hat{j} + (1-(-3))\hat{k} = -2\hat{i} - 4\hat{j} + 4\hat{k}.$$

Step 2. Magnitude:

$$|\overrightarrow{AB}| = \sqrt{(-2)^2 + (-4)^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6.$$

Step 3. Direction cosines (divide each component by 6):

$$l = \frac{-2}{6} = -\frac{1}{3}, \quad m = \frac{-4}{6} = -\frac{2}{3}, \quad n = \frac{4}{6} = \frac{2}{3}.$$

Step 4. Check: $l^2 + m^2 + n^2 = \frac{1}{9} + \frac{4}{9} + \frac{4}{9} = \frac{9}{9} = 1.$ ✓

Final Answer: Direction cosines of $\overrightarrow{AB}$: $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$.

EXPERT'S SOLUTION *: Sanya Patel, M.Sc Mathematics, IIT Bombay*

Quick reading. Subtract A from B, find length, divide.

Step 1. $\overrightarrow{AB} = (-2, -4, 4)$.

Step 2. $|\overrightarrow{AB}| = 6$.

Step 3. Direction cosines: $\left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$.

Why this matters. The sign of each cosine tells whether the vector points along $+$ or $-$ side of that axis: here it points away from $+x, +y$ but towards $+z$.

Final Answer: $\left(-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right)$.

Q 10.14 **Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axes OX, OY and OZ.**

SOLUTION

Concept used. A vector is *equally inclined* to the three coordinate axes iff its three direction cosines are equal. Equivalently, the angles α, β, γ that the vector makes with OX, OY, OZ are equal.

Step 1. Take $\vec{a}=\hat{i}+\hat{j}+\hat{k}$. All three components are equal to 1.

Step 2. Magnitude: $|\vec{a}|=\sqrt{1+1+1}=\sqrt{3}$.

Step 3. Direction cosines:

$$l=\frac{1}{\sqrt{3}}, \quad m=\frac{1}{\sqrt{3}}, \quad n=\frac{1}{\sqrt{3}}.$$

Step 4. Since $l=m=n$, the angles with the three axes satisfy $\cos\alpha=\cos\beta=\cos\gamma=1/\sqrt{3}$. Hence $\alpha=\beta=\gamma=\cos^{-1}(1/\sqrt{3})$.

The vector $\hat{i}+\hat{j}+\hat{k}$ therefore makes the same angle $\cos^{-1}(1/\sqrt{3})$ with each of the three coordinate axes.

Final Answer: All three direction cosines equal $\frac{1}{\sqrt{3}}$; hence $\vec{a}$ is equally inclined to OX, OY, OZ.

♥ Body diagonal of a cube

The vector $\hat{i}+\hat{j}+\hat{k}$ runs along the body-diagonal of a unit cube whose edges lie on the axes. By symmetry this diagonal must lean at the same angle to each of the three axes, namely $\cos^{-1}(1/\sqrt{3}) \approx 54.74°$.

EXPERT'S SOLUTION *: Meera Joshi, M.Sc Mathematics, IIT Bombay*

Picture-first. The vector $(1,1,1)$ is the body diagonal of the unit cube; cubic symmetry forces equal inclination.

Step 1. Components are all 1; magnitude $\sqrt{3}$.

Step 2. Each direction cosine $=1/\sqrt{3}$.

Step 3. All angles equal $\cos^{-1}(1/\sqrt{3})$, so equal inclination.

Why this matters. Whenever the three components of a vector match, that vector is the body diagonal of a cube aligned with the axes, and the angle with each axis is the famous $\arccos(1/\sqrt{3}) \approx 54.7°$.

Final Answer: $l = m = n = 1/\sqrt{3}$, hence equally inclined.

Q 10.15 **Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively, in the ratio $2 : 1$**
(i) internally (ii) externally.

SOLUTION

Concept used. If $\vec{p}$ and $\vec{q}$ are the position vectors of P and Q, and R divides PQ in the ratio $m : n$, then

- **Internal division:** $\vec{r} = \dfrac{m\,\vec{q} + n\,\vec{p}}{m+n}$.
- **External division:** $\vec{r} = \dfrac{m\,\vec{q} - n\,\vec{p}}{m-n}$.

Here $m = 2$, $n = 1$, $\vec{p} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{q} = -\hat{i} + \hat{j} + \hat{k}$.

Step 1. **(i) Internal division.**

$$\vec{r}_{\text{int}} = \frac{2\vec{q} + 1\cdot\vec{p}}{2+1} = \frac{2(-\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 2\hat{j} - \hat{k})}{3}.$$

Expand numerator:

$$2\vec{q} = -2\hat{i} + 2\hat{j} + 2\hat{k}, \qquad 2\vec{q} + \vec{p} = (-2+1)\hat{i} + (2+2)\hat{j} + (2-1)\hat{k} = -\hat{i} + 4\hat{j} + \hat{k}.$$

Divide by 3:

$$\vec{r}_{\text{int}} = -\tfrac{1}{3}\hat{i} + \tfrac{4}{3}\hat{j} + \tfrac{1}{3}\hat{k}.$$

Step 2. **(ii) External division.**

$$\vec{r}_{\text{ext}} = \frac{2\vec{q} - 1\cdot\vec{p}}{2-1} = 2\vec{q} - \vec{p}.$$

Compute:

$$2\vec{q} - \vec{p} = (-2\hat{i} + 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - \hat{k}) = (-2-1)\hat{i} + (2-2)\hat{j} + (2+1)\hat{k} = -3\hat{i} + 0\,\hat{j} + 3\hat{k}.$$

So $\vec{r}_{\text{ext}} = -3\hat{i} + 3\hat{k}$.

Final Answer: (i) $\vec{r}_{\text{int}} = -\frac{1}{3}\hat{i} + \frac{4}{3}\hat{j} + \frac{1}{3}\hat{k}$. (ii) $\vec{r}_{\text{ext}} = -3\hat{i} + 3\hat{k}$.

✗ Common Mistake

The standard mistake is mis-assigning m, n. "R divides PQ in ratio $m : n$" means $PR : RQ = m : n$, so the *weight* of $\vec{q}$ in the formula is m (the part nearer Q), and of $\vec{p}$ is n. Cross-checking: for $m = n$, the formula should give the midpoint.

EXPERT'S SOLUTION : *Priya Sharma, M.Sc Mathematics, IIT Bombay*

Strategic angle. Apply the two section formulas mechanically; the only choices are the values of m, n and the sign.

Step 1. Internal. Weight of $\vec{q}$ is $m = 2$, weight of $\vec{p}$ is $n = 1$, denominator $m + n = 3$.

$$\vec{r}_{\text{int}} = \frac{2\vec{q} + \vec{p}}{3} = \frac{(-2, 2, 2) + (1, 2, -1)}{3} = \frac{(-1, 4, 1)}{3}.$$

Step 2. External. Same numerator structure but a minus:

$$\vec{r}_{\text{ext}} = \frac{2\vec{q} - \vec{p}}{2 - 1} = 2\vec{q} - \vec{p} = (-3, 0, 3).$$

Why this matters. The external division formula is the internal one with $n \to -n$; that single sign-flip captures the difference between "inside" and "outside" the segment PQ.

Final Answer: Internal: $\left(-\frac{1}{3}, \frac{4}{3}, \frac{1}{3}\right)$. External: $(-3, 0, 3)$.

Q 10.16 **Find the position vector of the midpoint of the vector joining the points $P(2, 3, 4)$ and $Q(4, 1, -2)$.**

SOLUTION

Concept used. The midpoint M of PQ divides PQ in the ratio $1 : 1$, so by the section formula

$$\vec{m} = \frac{\vec{p} + \vec{q}}{2}.$$

Step 1. Position vectors: $\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{q} = 4\hat{i} + \hat{j} - 2\hat{k}$.

Step 2. Sum:

$$\vec{p} + \vec{q} = (2 + 4)\hat{i} + (3 + 1)\hat{j} + (4 - 2)\hat{k} = 6\hat{i} + 4\hat{j} + 2\hat{k}.$$

Step 3. Divide by 2:

$$\vec{m} = \tfrac{1}{2}(6\hat{i} + 4\hat{j} + 2\hat{k}) = 3\hat{i} + 2\hat{j} + \hat{k}.$$

Final Answer: Midpoint $M = (3, 2, 1)$, i.e. $\vec{m} = 3\hat{i} + 2\hat{j} + \hat{k}$.

EXPERT'S SOLUTION : *Siddharth Bhat, M.Sc Mathematics, IIT Bombay*

Quick reading. Average the two endpoints, coordinate by coordinate.

Step 1. $(x, y, z)_M = \left(\frac{2+4}{2}, \frac{3+1}{2}, \frac{4+(-2)}{2}\right) = (3, 2, 1)$.

Why this matters. Midpoint formula is the section formula with $m = n = 1$; the same averaging idea extends to centroids $(1 : 1 : 1)$ and other weighted means.

Final Answer: $3\hat{i} + 2\hat{j} + \hat{k}$.

Q 10.17 **Show that the points A, B and C with position vectors $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$, respectively form the vertices of a right-angled triangle.**

SOLUTION

Concept used. The vertices form a right-angled triangle iff one of the three side vectors is perpendicular to another, equivalently (by the converse of Pythagoras' theorem) iff the squared length of one side equals the sum of the squared lengths of the other two.

Step 1. Compute the three side vectors:

$$\overrightarrow{AB} = \vec{b} - \vec{a} = (2-3)\hat{i} + (-1+4)\hat{j} + (1+4)\hat{k} = -\hat{i} + 3\hat{j} + 5\hat{k}.$$

$$\overrightarrow{BC} = \vec{c} - \vec{b} = (1-2)\hat{i} + (-3+1)\hat{j} + (-5-1)\hat{k} = -\hat{i} - 2\hat{j} - 6\hat{k}.$$

$$\overrightarrow{CA} = \vec{a} - \vec{c} = (3-1)\hat{i} + (-4+3)\hat{j} + (-4+5)\hat{k} = 2\hat{i} - \hat{j} + \hat{k}.$$

Step 2. Squared lengths:

$$|\overrightarrow{AB}|^2 = (-1)^2 + 3^2 + 5^2 = 1 + 9 + 25 = 35.$$

$$|\overrightarrow{BC}|^2 = (-1)^2 + (-2)^2 + (-6)^2 = 1 + 4 + 36 = 41.$$

$$|\overrightarrow{CA}|^2 = 2^2 + (-1)^2 + 1^2 = 4 + 1 + 1 = 6.$$

Step 3. Test Pythagoras: $|\overrightarrow{AB}|^2 + |\overrightarrow{CA}|^2 = 35 + 6 = 41 = |\overrightarrow{BC}|^2$.

Step 4. Hence the triangle is right-angled with the right angle at A (because $\overrightarrow{AB} \perp \overrightarrow{AC}$, i.e. the legs meeting at A).

Final Answer: $|AB|^2 + |AC|^2 = |BC|^2$, so $\triangle ABC$ is right-angled at A.

EXPERT'S SOLUTION : *Ankit Kapoor, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Compute the three squared side-lengths; if one is the sum of the other two, the triangle is right-angled at the opposite vertex.

Step 1. $|\overrightarrow{AB}|^2 = 35,\ |\overrightarrow{BC}|^2 = 41,\ |\overrightarrow{CA}|^2 = 6.$

Step 2. $35 + 6 = 41$, so $|AB|^2 + |CA|^2 = |BC|^2$.

Step 3. Right angle is at A (the vertex opposite to the longest side BC).

Why this matters. Squared lengths avoid square roots entirely; this is the cleanest computational test for right triangles in 3D.

Final Answer: Pythagoras' converse satisfied at A - right-angled triangle.

Q 10.18 **In triangle ABC (Fig 10.18), which of the following is not true:**

(A) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$
(B) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}$
(C) $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$
(D) $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA} = \vec{0}$

SOLUTION

Concept used. The triangle law of addition: in any triangle, the sum of the three side-vectors taken in order is the zero vector. Specifically,

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}.$$

Also $\overrightarrow{XY} = -\overrightarrow{YX}$ for any two points.

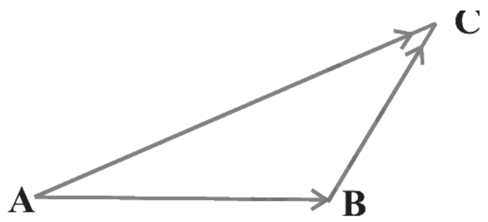

Fig 10.18

Fig. 10.18, NCERT Class 12 Mathematics, Chapter 10 (Vector Algebra).

Step 1. **Check (A).** $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ is the triangle law itself, so it is **true**.

Step 2. **Check (B).** Replace $\overrightarrow{CA}$ in (A) by $-\overrightarrow{AC}$:

$$\overrightarrow{AB} + \overrightarrow{BC} + (-\overrightarrow{AC}) = \vec{0} \iff \overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{AC} = \vec{0}.$$

So (B) is **true**.

Step 3. **Check (C).** Statement says $\overrightarrow{AB} + \overrightarrow{BC} - \overrightarrow{CA} = \vec{0}$. From (A), $\overrightarrow{AB} + \overrightarrow{BC} = -\overrightarrow{CA}$, so the left side becomes $-\overrightarrow{CA} - \overrightarrow{CA} = -2\overrightarrow{CA}$. This equals $\vec{0}$ only when $\overrightarrow{CA} = \vec{0}$, which is false for a genuine triangle. Hence (C) is **not true**.

Step 4. **Check (D).** $\overrightarrow{AB} - \overrightarrow{CB} + \overrightarrow{CA}$. Note $-\overrightarrow{CB} = \overrightarrow{BC}$. So

$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0},$$

which is the triangle law. So (D) is **true**.

Final Answer: The option that is not true is **(C)**.

EXPERT'S SOLUTION *: Pooja Mehta, M.Sc Mathematics, IIT Bombay*

Quick reading. Verify each option by reducing to the triangle law $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.

Step 1. (A) is the triangle law itself - true.

Step 2. (B): swap $\overrightarrow{CA} = -\overrightarrow{AC}$ - true.

Step 3. (C): would force $-2\overrightarrow{CA} = \vec{0}$, impossible for a triangle - false.

Step 4. (D): $-\overrightarrow{CB} = \overrightarrow{BC}$, reduces to triangle law - true.

Why this matters. The sign-conversion $\overrightarrow{XY} = -\overrightarrow{YX}$ is the single trick that turns any apparent variant into the canonical triangle law - or exposes it as wrong.

Final Answer: Option **(C)** is not true.

Q 10.19 **If $\vec{a}$ and $\vec{b}$ are two collinear vectors, then which of the following are incorrect:**

(A) $\vec{b} = \lambda\vec{a}$, for some scalar λ

(B) $\vec{a} = \pm\vec{b}$

(C) the respective components of $\vec{a}$ and $\vec{b}$ are not proportional

(D) both the vectors $\vec{a}$ and $\vec{b}$ have same direction, but different magnitudes.

SOLUTION

Concept used. Two vectors $\vec{a}, \vec{b}$ are collinear $\iff$ $\vec{b} = \lambda\vec{a}$ for some scalar λ (positive or negative, possibly zero if $\vec{b} = \vec{0}$). Equivalently, their components are proportional. Collinearity does *not* force equal magnitudes nor identical signs.

Step 1. **(A) $\vec{b} = \lambda\vec{a}$.** This is the definition of collinearity - **correct**.

Step 2. **(B)** $\vec{a} = \pm\vec{b}$. This forces $\lambda = \pm 1$, i.e. equal magnitudes. But collinear vectors can have any magnitudes (e.g. $\vec{a} = \hat{i}$, $\vec{b} = 3\hat{i}$). **Incorrect**.

Step 3. **(C) Components are not proportional.** For collinear vectors, components *are* proportional (ratios equal λ). So this statement is **incorrect**.

Step 4. **(D) Same direction but different magnitudes.** Collinearity also allows *opposite* directions (e.g. $\vec{a}$ and $-\vec{a}$), and even equal magnitudes (when $\lambda = -1$ or $+1$). So (D) is too narrow - **incorrect**.

> **Final Answer:** The incorrect options are **(B), (C) and (D)**.

EXPERT'S SOLUTION *: Aanya Chatterjee, M.Sc Mathematics, ISI Kolkata*

Strategic angle. (A) is the exact definition; (B), (C), (D) each impose extra unnecessary conditions.

Step 1. (A) - correct definition.

Step 2. (B) - fails when magnitudes differ.

Step 3. (C) - reverses the truth; components *are* proportional.

Step 4. (D) - misses opposite-direction case.

Why this matters. The single equation $\vec{b} = \lambda\vec{a}$ captures all the freedom (sign of λ, magnitude $|\lambda|$, even $\lambda = 0$ degeneracy) in one place.

> **Final Answer:** Incorrect: **(B), (C), (D)**.

Key Takeaways

- Magnitude: $|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$. Unit vector: $\hat{v} = \vec{v}/|\vec{v}|$.
- Direction cosines (l, m, n) are components of $\hat{v}$; they satisfy $l^2 + m^2 + n^2 = 1$.
- Section formula in ratio $m : n$: internal $\dfrac{m\vec{q} + n\vec{p}}{m + n}$, external $\dfrac{m\vec{q} - n\vec{p}}{m - n}$.
- Collinearity test: $\vec{b} = \lambda\vec{a} \Leftrightarrow$ components proportional.
- Right-angled triangle from position vectors: apply the converse of Pythagoras on squared side-lengths.

End of Exercise 10.2