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Step-by-step solutions for the 2026-27 NCERT (Latest Edition)

Chapter 11: Three Dimensional Geometry

About this Chapter

This exercise opens the chapter on Three Dimensional Geometry for Class 12. The five problems revolve around **direction cosines** and **direction ratios** of a line in space. You will compute direction cosines from given direction angles, from given direction ratios, prove collinearity of three points by checking proportionality of direction ratios, and find direction cosines of the sides of a triangle whose vertices are given in 3D. The toolkit is small but every step must be written out: the identity $l^2 + m^2 + n^2 = 1$, the normalising factor $\pm 1/\sqrt{a^2 + b^2 + c^2}$, and the two-point formula for direction cosines. We use 3D sketches throughout to anchor the algebra to a picture.

Topics covered: direction angles and direction cosines $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$; fundamental identity $l^2 + m^2 + n^2 = 1$; direction ratios and the proportionality $l : m : n = a : b : c$; two-point formula for direction cosines; collinearity of three points via proportional direction ratios.

Quick Formula Sheet

Direction cosines. $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$.

Identity. $l^2 + m^2 + n^2 = 1$.

From DRs a, b, c:

$$l = \pm\frac{a}{\sqrt{a^2 + b^2 + c^2}},$$

and similarly for m, n.

Segment PQ: DCs are $\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$.

Collinearity. A, B, C collinear iff $\overrightarrow{AB} \parallel \overrightarrow{BC}$.

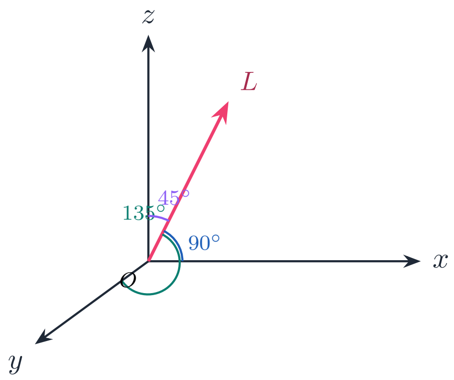
Q 11.1 **If a line makes angles 90°, 135°, 45° with the x, y and z-axes respectively, find its direction cosines.**

SOLUTION

Concept used. For a directed line in space that makes angles α, β, γ with the positive x, y and z-axes respectively, the **direction cosines** are

$$l = \cos\alpha, \quad m = \cos\beta, \quad n = \cos\gamma.$$

These satisfy the fundamental identity $l^2 + m^2 + n^2 = 1$, which we use here as a sanity check.



Step 1. Read off the direction angles from the problem statement:

$$\alpha = 90^\circ, \quad \beta = 135^\circ, \quad \gamma = 45^\circ.$$

Here α is the angle with the x-axis, β with the y-axis, γ with the z-axis.

Step 2. Apply $l = \cos\alpha$:

$$l = \cos 90^\circ = 0.$$

So the line is perpendicular to the x-axis.

Step 3. Apply $m = \cos\beta$:

$$m = \cos 135^\circ = -\cos 45^\circ = -\frac{1}{\sqrt{2}}.$$

The minus sign comes from 135° lying in the second quadrant of the cosine.

Step 4. Apply $n = \cos\gamma$:

$$n = \cos 45^\circ = \frac{1}{\sqrt{2}}.$$

Step 5. **Sanity check** using the identity $l^2 + m^2 + n^2 = 1$:

$$l^2 + m^2 + n^2 = 0^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 0 + \frac{1}{2} + \frac{1}{2} = 1. \checkmark$$

Final Answer: Direction cosines: $\left(0,\ -\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$

☞ **Quadrant of cosine**

cos is positive on $[0^\circ, 90^\circ)$ and negative on $(90^\circ, 180^\circ]$. So whenever a direction angle exceeds 90°, the corresponding direction cosine is negative.

EXPERT'S SOLUTION : *Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Picture-first. A direction in 3D is just a unit vector. The cosines of its angles with the three axes are precisely its components, so this question reduces to "write $\cos\alpha$, $\cos\beta$, $\cos\gamma$ in order, then verify the unit-length property."

Concept restated. A line in space has two opposite directions, hence two opposite triples $(\pm l, \pm m, \pm n)$. The values must satisfy $l^2 + m^2 + n^2 = 1$; this identity is equivalent to saying that the projections of a unit vector onto the three axes form a Pythagorean triple of squares summing to one.

Step 1. List the three given angles paired with their axes:

$$\alpha = 90^\circ \text{ (with } x\text{)}, \ \beta = 135^\circ \text{ (with } y\text{)}, \ \gamma = 45^\circ \text{ (with } z\text{)}.$$

Step 2. Evaluate $l = \cos\alpha = \cos 90^\circ$. From the unit circle the cosine of a right angle is exactly zero:

$$l = 0.$$

Step 3. Evaluate $m = \cos\beta = \cos 135^\circ$. Using the supplement identity $\cos(180^\circ - \theta) = -\cos\theta$ with $\theta = 45^\circ$:

$$m = -\cos 45^\circ = -\frac{1}{\sqrt{2}}.$$

Step 4. Evaluate $n = \cos\gamma = \cos 45^\circ$. This is the standard exact value $1/\sqrt{2}$:

$$n = \frac{1}{\sqrt{2}}.$$

Step 5. Unit-length verification:

$$l^2 + m^2 + n^2 = 0 + \tfrac{1}{2} + \tfrac{1}{2} = 1,$$

confirming that (l, m, n) are valid direction cosines.

Step 6. Geometric reading: $l = 0$ means the line lies in the yz-plane; the equal magnitudes of m and n say the line bisects the angle between the negative y-axis and the positive z-axis.

Final Answer: $(l,\ m,\ n) = \left(0,\ -\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$

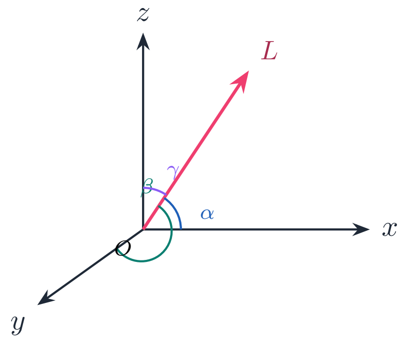
Q 11.2 **Find the direction cosines of a line which makes equal angles with the coordinate axes.**

SOLUTION

Concept used. Let α, β, γ be the angles the line makes with the x, y and z-axes. The direction cosines are $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$ and obey

$$l^2 + m^2 + n^2 = 1.$$

"Equal angles with the axes" means $\alpha = \beta = \gamma$, so $l = m = n$. Substituting $l = m = n$ in the identity pins down the common value.



Step 1. Write the equal-angle condition. Since $\alpha = \beta = \gamma$ we have

$$l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma \implies l = m = n.$$

Call this common value k.

Step 2. Substitute $l = m = n = k$ into $l^2 + m^2 + n^2 = 1$:

$$k^2 + k^2 + k^2 = 1 \implies 3k^2 = 1.$$

Step 3. Solve for k:

$$k^2 = \frac{1}{3} \implies k = \pm\frac{1}{\sqrt{3}}.$$

The two signs correspond to the two opposite directions of the same line.

Step 4. Therefore the direction cosines are

$$(l, m, n) = \pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right).$$

Step 5. Sanity check. $\left(\pm\frac{1}{\sqrt{3}}\right)^2 \times 3 = \frac{1}{3} \times 3 = 1.$ ✓

Final Answer: $(l,\ m,\ n) = \pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

♥ Geometric meaning

The vector $(1,1,1)/\sqrt{3}$ points along the principal diagonal of a unit cube whose edges lie along the coordinate axes. This is the unique line (up to sense) that is equally inclined to the three axes, and it is everywhere in solid geometry, crystallography and isotropic-material problems.

EXPERT'S SOLUTION *: Sneha Iyer, M.Sc Applied Mathematics, IIT Kanpur*

Strategic angle. Whenever a problem couples a symmetry condition ("equal angles", "equal distances") with the identity $l^2+m^2+n^2=1$, plug the symmetry into the identity and solve a single-variable equation. That is exactly what happens here.

Concept restated. The set of all unit vectors in $\mathbb{R}^3$ is the unit sphere $\{(l,m,n) : l^2+m^2+n^2=1\}$. Direction cosines are precisely the coordinates of a point on this sphere.

Step 1. Translate the wording. "Makes equal angles with the coordinate axes" $\equiv$ $\alpha=\beta=\gamma$, which by taking cosines gives

$$l=m=n.$$

We label this common scalar $k\in\mathbb{R}$.

Step 2. Force unit length. The defining identity for direction cosines is

$$l^2+m^2+n^2=1.$$

With $l=m=n=k$, the left side becomes

$$k^2+k^2+k^2=3k^2.$$

Step 3. Equate and solve:

$$3k^2=1 \implies k^2=\frac{1}{3} \implies k=\pm\frac{1}{\sqrt{3}}.$$

Step 4. Interpret the two signs. The "+" sign labels the direction from the origin into the octant $x>0, y>0, z>0$ (the principal diagonal of the first octant); the "−" sign labels the opposite direction.

Step 5. Therefore the direction-cosine triple is

$$(l,m,n)=\pm\left(\tfrac{1}{\sqrt{3}},\tfrac{1}{\sqrt{3}},\tfrac{1}{\sqrt{3}}\right).$$

Final Answer: $(l,\,m,\,n)=\pm\left(\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}\right)$

Q 11.3 **If a line has the direction ratios $-18,\ 12,\ -4$, then what are its direction cosines?**

SOLUTION

Concept used. If a line has direction ratios a, b, c, then its direction cosines are obtained by normalising:

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm\frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \pm\frac{c}{\sqrt{a^2+b^2+c^2}}.$$

The denominator $\sqrt{a^2+b^2+c^2}$ is the length of the vector (a, b, c). The $\pm$ encodes that a line has two opposite directions.

Step 1. Identify the direction ratios:

$$a = -18, \quad b = 12, \quad c = -4.$$

Step 2. Compute the squares:

$$a^2 = (-18)^2 = 324, \quad b^2 = 12^2 = 144, \quad c^2 = (-4)^2 = 16.$$

Step 3. Add the squares:

$$a^2 + b^2 + c^2 = 324 + 144 + 16 = 484.$$

Step 4. Take the square root for the normalising factor:

$$\sqrt{a^2+b^2+c^2} = \sqrt{484} = 22.$$

(Since $22^2 = 484$.)

Step 5. Substitute into the formula for l, m, n:

$$l = \frac{-18}{22}, \quad m = \frac{12}{22}, \quad n = \frac{-4}{22}.$$

Step 6. Reduce each fraction by the common factor 2:

$$l = -\frac{9}{11}, \quad m = \frac{6}{11}, \quad n = -\frac{2}{11}.$$

Step 7. **Sanity check** using $l^2 + m^2 + n^2 = 1$:

$$l^2 + m^2 + n^2 = \frac{81 + 36 + 4}{121} = \frac{121}{121} = 1. \checkmark$$

Final Answer: $(l, m, n) = \left(-\frac{9}{11}, \frac{6}{11}, -\frac{2}{11}\right)$ (or its negative)

✗ Common Mistake

Direction ratios are not unique: $(-18, 12, -4)$, $(-9, 6, -2)$, $(18, -12, 4)$ all describe the same line. Direction *cosines*, however, are unique up to an overall sign because they must be unit-length. Do not skip the normalisation step or you will get $(-18, 12, -4)$ as your "direction cosines", which is wrong.

EXPERT'S SOLUTION *: Vivaan Patel, Ph.D Mathematics, IIT Delhi*

Quick reading. "Direction ratios" is just shorthand for "any vector along the line". To get direction cosines, divide by the vector's length. The arithmetic here is a clean perfect square, which is the only mildly interesting step.

Concept restated. For a line ℓ with parallel vector $\vec{v} = (a, b, c)$,

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|} = \left(\frac{a}{\|\vec{v}\|}, \frac{b}{\|\vec{v}\|}, \frac{c}{\|\vec{v}\|}\right),$$

where $\|\vec{v}\| = \sqrt{a^2 + b^2 + c^2}$. The components of $\hat{v}$ are the direction cosines.

Step 1. Read the vector along the line: $\vec{v} = (-18,\ 12,\ -4)$.

Step 2. Compute $\|\vec{v}\|^2$ term-by-term:

$$\|\vec{v}\|^2 = (-18)^2 + 12^2 + (-4)^2 = 324 + 144 + 16 = 484.$$

Step 3. Hence

$$\|\vec{v}\| = \sqrt{484} = 22.$$

Notice $484 = 4 \times 121 = (2 \times 11)^2 = 22^2$, a perfect square.

Step 4. Normalise:

$$\hat{v} = \frac{1}{22}(-18,\ 12,\ -4) = \left(-\frac{18}{22}, \frac{12}{22}, -\frac{4}{22}\right).$$

Step 5. Simplify in lowest terms (divide numerator and denominator by $\gcd(18, 22) = 2$, $\gcd(12, 22) = 2$, $\gcd(4, 22) = 2$):

$$\hat{v} = \left(-\frac{9}{11}, \frac{6}{11}, -\frac{2}{11}\right).$$

Step 6. Verify unit length:

$$\left(-\tfrac{9}{11}\right)^2 + \left(\tfrac{6}{11}\right)^2 + \left(-\tfrac{2}{11}\right)^2 = \frac{81 + 36 + 4}{121} = 1.$$

Final Answer: $(l, m, n) = \left(-\frac{9}{11}, \frac{6}{11}, -\frac{2}{11}\right)$ (or the negation)

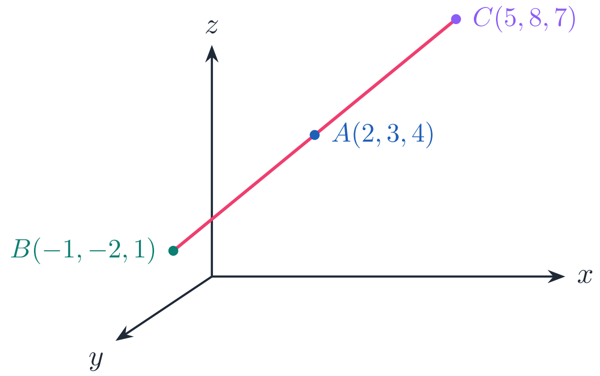
Q 11.4 **Show that the points $(2,3,4)$, $(-1,-2,1)$, $(5,8,7)$ are collinear.**

SOLUTION

Concept used. Three points A, B, C in space are **collinear** when they lie on a single line. A convenient algebraic test: compute the direction ratios of $\overrightarrow{AB}$ and of $\overrightarrow{BC}$. If

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2},$$

then $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$, and since both share the common point B, the three points lie on the same line. The direction ratios of the segment from $P(x_1, y_1, z_1)$ to $Q(x_2, y_2, z_2)$ are $(x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$.



Step 1. Name the points: let $A = (2,3,4)$, $B = (-1,-2,1)$, $C = (5,8,7)$.

Step 2. Compute the direction ratios of $\overrightarrow{AB}$:

$$\overrightarrow{AB} = (-1-2,\ -2-3,\ 1-4) = (-3,\ -5,\ -3).$$

Step 3. Compute the direction ratios of $\overrightarrow{BC}$:

$$\overrightarrow{BC} = (5-(-1),\ 8-(-2),\ 7-1) = (6,\ 10,\ 6).$$

Step 4. Test for proportionality. Form the three ratios:

$$\frac{-3}{6} = -\frac{1}{2}, \quad \frac{-5}{10} = -\frac{1}{2}, \quad \frac{-3}{6} = -\frac{1}{2}.$$

All three ratios are equal to $-\frac{1}{2}$.

Step 5. Conclude. Since the corresponding components are in the same ratio, $\overrightarrow{BC} = -2 \cdot \overrightarrow{AB}$, so the two vectors are parallel. Since B lies on both directed segments, the three points A, B, C share a common line. Hence they are collinear.

Final Answer: The three points are collinear, with $\overrightarrow{BC} = -2\,\overrightarrow{AB}$.

EXPERT'S SOLUTION : *Aanya Gupta, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Three points are collinear iff the vector from any one to a second is a scalar multiple of the vector from that point to the third. We can also use the distance test ($AB + BC = AC$ for some ordering), but the vector-proportionality test involves no square roots and is preferred in 3D.

Concept restated. Vectors $\vec{u}, \vec{v} \in \mathbb{R}^3$ are parallel iff there exists $\lambda \in \mathbb{R}$ with $\vec{v} = \lambda\vec{u}$. Component-wise this is $v_x/u_x = v_y/u_y = v_z/u_z = \lambda$ (with the usual care when any $u_i = 0$).

Step 1. Label: $A(2, 3, 4)$, $B(-1, -2, 1)$, $C(5, 8, 7)$.

Step 2. Form $\overrightarrow{AB}$ by subtracting coordinates:

$$\overrightarrow{AB} = B - A = (-1 - 2,\ -2 - 3,\ 1 - 4) = (-3,\ -5,\ -3).$$

Step 3. Form $\overrightarrow{AC}$ similarly (we use $A \to C$ this time, instead of $B \to C$, to vary the check):

$$\overrightarrow{AC} = C - A = (5 - 2,\ 8 - 3,\ 7 - 4) = (3,\ 5,\ 3).$$

Step 4. Compare components:

$$\frac{3}{-3} = -1, \quad \frac{5}{-5} = -1, \quad \frac{3}{-3} = -1.$$

All three ratios are equal to -1, so $\overrightarrow{AC} = -1 \cdot \overrightarrow{AB}$.

Step 5. Conclude. $\overrightarrow{AC}$ is a (negative) scalar multiple of $\overrightarrow{AB}$, both share the point A, so A, B, C lie on one line.

Step 6. Locate A on segment BC. From $\overrightarrow{AC} = -\overrightarrow{AB}$ we get $AB = AC$, hence A is the midpoint of BC. Indeed $\frac{1}{2}(B + C) = \frac{1}{2}(-1 + 5,\ -2 + 8,\ 1 + 7) = (2, 3, 4) = A$. ✓

Final Answer: A, B, C are collinear, with A the midpoint of BC.

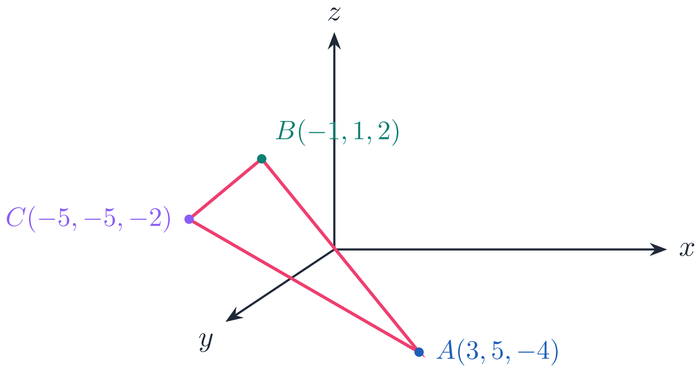
Q 11.5 **Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, -4)$, $(-1, 1, 2)$ and $(-5, -5, -2)$.**

SOLUTION

Concept used. The direction cosines of the segment from $P(x_1, y_1, z_1)$ to $Q(x_2, y_2, z_2)$ are

$$\left(\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}\right), \quad PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

A triangle has three sides AB, BC and CA; we apply this formula to each side.



Let $A = (3, 5, -4)$, $B = (-1, 1, 2)$, $C = (-5, -5, -2)$.

Side AB (from A to B).

Step 1. Differences:

$$x_2 - x_1 = -1 - 3 = -4, \quad y_2 - y_1 = 1 - 5 = -4, \quad z_2 - z_1 = 2 - (-4) = 6.$$

Step 2. Length:

$$AB = \sqrt{(-4)^2 + (-4)^2 + 6^2} = \sqrt{16 + 16 + 36} = \sqrt{68} = 2\sqrt{17}.$$

Step 3. Direction cosines of AB:

$$\left(\frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}\right) = \left(\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}\right).$$

Side BC (from B to C).

Step 1. Differences:

$$-5 - (-1) = -4, \quad -5 - 1 = -6, \quad -2 - 2 = -4.$$

Step 2. Length:

$$BC = \sqrt{(-4)^2 + (-6)^2 + (-4)^2} = \sqrt{16 + 36 + 16} = \sqrt{68} = 2\sqrt{17}.$$

Step 3. Direction cosines of BC:

$$\left(\frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}\right) = \left(\frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}\right).$$

Side CA (from C to A).

Step 1. Differences:

$$3 - (-5) = 8, \quad 5 - (-5) = 10, \quad -4 - (-2) = -2.$$

Step 2. Length:

$$CA = \sqrt{8^2 + 10^2 + (-2)^2} = \sqrt{64 + 100 + 4} = \sqrt{168} = 2\sqrt{42}.$$

Step 3. Direction cosines of CA:

$$\left(\frac{8}{2\sqrt{42}}, \frac{10}{2\sqrt{42}}, \frac{-2}{2\sqrt{42}}\right) = \left(\frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}\right).$$

Sanity check. For AB:

$$\left(\tfrac{-2}{\sqrt{17}}\right)^2 + \left(\tfrac{-2}{\sqrt{17}}\right)^2 + \left(\tfrac{3}{\sqrt{17}}\right)^2 = \frac{4+4+9}{17} = 1. \checkmark$$

Final Answer: $AB : \left(\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}\right)$; $BC : \left(\frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}\right)$; $CA : \left(\frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}\right)$.

☞ Exam Tip

Boards often ask for direction cosines of a triangle's sides in a follow-up to "show the triangle is isosceles / right-angled". Note $AB = BC = 2\sqrt{17}$ here, so this is an isosceles triangle, a fact worth flagging if the next part of the question goes there.

EXPERT'S SOLUTION *: Pranav Mehta, B.Tech CSE, IIT Roorkee*

Structural observation. Three sides ⇒ three identical sub-problems. Each subproblem is "given two points, write the unit vector from one to the other". Tabulating the arithmetic side-by-side avoids errors.

Concept restated. For points $P, Q \in \mathbb{R}^3$ the unit vector $\hat{u}_{PQ}$ from P to Q is $\vec{PQ}/\|\vec{PQ}\|$. Its components are the direction cosines of the directed segment from P to Q. Reversing direction flips all three signs.

Step 1. Set vertex labels: $A(3,5,-4)$, $B(-1,1,2)$, $C(-5,-5,-2)$.

Step 2. For each ordered pair, compute the displacement vector and its length:

$$\vec{AB} = (-4,-4,6), \quad \|\vec{AB}\| = \sqrt{16+16+36} = \sqrt{68} = 2\sqrt{17}.$$

$$\vec{BC} = (-4,-6,-4), \quad \|\vec{BC}\| = \sqrt{16+36+16} = \sqrt{68} = 2\sqrt{17}.$$

$$\vec{CA} = (8,10,-2), \quad \|\vec{CA}\| = \sqrt{64+100+4} = \sqrt{168} = 2\sqrt{42}.$$

Step 3. Divide each component by the corresponding length to obtain unit vectors:

$$\hat{u}_{AB} = \tfrac{1}{2\sqrt{17}}(-4,-4,6) = \tfrac{1}{\sqrt{17}}(-2,-2,3).$$

$$\hat{u}_{BC} = \tfrac{1}{2\sqrt{17}}(-4,-6,-4) = \tfrac{1}{\sqrt{17}}(-2,-3,-2).$$

$$\hat{u}_{CA} = \tfrac{1}{2\sqrt{42}}(8,10,-2) = \tfrac{1}{\sqrt{42}}(4,5,-1).$$

Step 4. Verify unit length for $\hat{u}_{CA}$:

$$\left(\tfrac{4}{\sqrt{42}}\right)^2 + \left(\tfrac{5}{\sqrt{42}}\right)^2 + \left(\tfrac{-1}{\sqrt{42}}\right)^2 = \frac{16+25+1}{42} = 1. \checkmark$$

Step 5. Bonus geometric reading. $AB = BC = 2\sqrt{17}$ while $CA = 2\sqrt{42}$, so the triangle is isosceles with apex at B.

Final Answer: $AB:\ \frac{1}{\sqrt{17}}(-2,-2,3)$; $BC:\ \frac{1}{\sqrt{17}}(-2,-3,-2)$; $CA:\ \frac{1}{\sqrt{42}}(4,5,-1)$.

Key Takeaways

- Direction cosines $l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$ are the cosines of the angles a directed line makes with the positive x, y, z-axes. They satisfy $l^2 + m^2 + n^2 = 1$.
- Direction ratios a, b, c are any numbers proportional to l, m, n. To convert, divide by $\sqrt{a^2+b^2+c^2}$:

$$l = \pm\frac{a}{\sqrt{a^2+b^2+c^2}},\ \text{etc.}$$

- For a segment PQ, the direction ratios are $(x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1)$ and the direction cosines are these divided by PQ.
- Three points A, B, C are collinear iff $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are proportional (have equal ratios component-by-component).
- The principal-diagonal direction $\pm\frac{1}{\sqrt{3}}(1,1,1)$ is the unique line equally inclined to the three axes.