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Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 13: Probability

About this Chapter

Exercise 13.2 trains two closely related ideas. The **multiplication theorem** writes the intersection $P(A \cap B)$ in terms of conditional probability, and **independence** is the special case in which $P(A \cap B) = P(A)P(B)$. The exercise mixes verification of independence, computation of joint and union probabilities for independent events, and tree-style multistage drawing problems.

Topics covered: Multiplication theorem • Independence test $P(A\cap B) = P(A)P(B)$ • Drawing with/without replacement • Union of independent events

Quick Formula Sheet

Multiplication theorem:

$P(A \cap B) = P(A)\,P(B \mid A) = P(B)\,P(A \mid B)$

Independence (definition):

A, B independent $\Leftrightarrow P(A\cap B) = P(A)\,P(B)$

Consequences when A, B are independent:

$P(A \mid B) = P(A),\ P(B \mid A) = P(B)$

$P(A \cup B) = P(A) + P(B) - P(A)P(B)$

$P(A' \cap B') = 1 - P(A \cup B)$

$A', B';\ A, B';\ A', B$ also independent

Exercise 13.2

Q 13.1 **If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A \cap B)$ if A and B are independent events.**

SOLUTION

Concept used. Events A and B are **independent** if and only if

$$P(A \cap B) = P(A)\,P(B).$$

When independence is assumed, the intersection probability is simply the product.

Step 1. Apply the independence formula:

$$P(A \cap B) = P(A)\,P(B) = \frac{3}{5} \times \frac{1}{5}.$$

Step 2. Multiply the fractions: numerator $3 \times 1 = 3$, denominator $5 \times 5 = 25$.

$$P(A \cap B) = \frac{3}{25}.$$

Final Answer: $P(A \cap B) = \dfrac{3}{25}$.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc Mathematics, IIT Bombay*

Quick reading. Independence $\Rightarrow$ multiply.

Step 1. $P(A \cap B) = P(A)P(B) = \frac{3}{5} \cdot \frac{1}{5} = \frac{3}{25}$.

Step 2. Sanity: $\frac{3}{25} < \min(\frac{3}{5}, \frac{1}{5}) = \frac{1}{5} = \frac{5}{25}$ ✓.

Why this matters. For independent events, joint probability factorises; this single fact powers every later calculation in the exercise.

Final Answer: $\dfrac{3}{25}$.

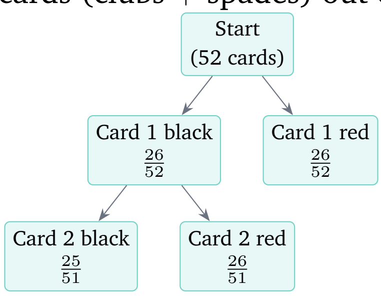
Q 13.2 **Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.**

SOLUTION

Concept used. **Multiplication theorem** (without independence, since the cards are drawn without replacement):

$$P(B_1 \cap B_2) = P(B_1)\,P(B_2 \mid B_1).$$

A standard pack has 26 black cards (clubs + spades) out of 52.



Step 1. Compute $P(B_1)$, the probability the first card is black:

$$P(B_1) = \frac{26}{52} = \frac{1}{2}.$$

Step 2. Given B_1, one black card has been removed, leaving 25 black cards in 51:

$$P(B_2 \mid B_1) = \frac{25}{51}.$$

Step 3. Apply the multiplication theorem:

$$P(B_1 \cap B_2) = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}.$$

Final Answer: $P(\text{both black}) = \dfrac{25}{102}.$

✗ Common Mistake

Drawing *without replacement* forces dependence. Writing $P(B_1) \cdot P(B_1) = (1/2)^2 = 1/4$ is wrong because the second draw's success rate depends on what happened in the first draw.

EXPERT'S SOLUTION : *Sneha Kapoor, Ph.D Mathematics, IIT Delhi*

Counting angle. Directly count favourable ordered pairs.

Step 1. Total ordered pairs: 52×51.

Step 2. Favourable (black, black) ordered pairs: 26×25.

Step 3. Probability:

$$\frac{26 \times 25}{52 \times 51} = \frac{26}{52} \cdot \frac{25}{51} = \frac{25}{102}.$$

Why this matters. The counting form and multiplication-theorem form are the same calculation re-arranged; either path lands at $25/102$.

Final Answer: $\dfrac{25}{102}.$

Q 13.3 **A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.**

SOLUTION

Concept used. For three sequential draws without replacement, the extended multiplication theorem gives

$$P(G_1 \cap G_2 \cap G_3) = P(G_1)\, P(G_2 \mid G_1)\, P(G_3 \mid G_1 \cap G_2),$$

where G_i is the event that the ith drawn orange is good.

Step 1. First draw: 12 good out of 15, so $P(G_1) = \frac{12}{15}$.

Step 2. Given G_1, 11 good remain out of 14: $P(G_2 \mid G_1) = \frac{11}{14}$.

Step 3. Given $G_1 \cap G_2$, 10 good remain out of 13: $P(G_3 \mid G_1 \cap G_2) = \frac{10}{13}$.

Step 4. Multiply:

$$P(\text{box approved}) = \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}.$$

Cancel a 5 between 15 and 10: $\frac{12}{3} \cdot \frac{11}{14} \cdot \frac{2}{13} = \frac{4}{1} \cdot \frac{11}{14} \cdot \frac{2}{13} = \frac{4 \times 11 \times 2}{14 \times 13} = \frac{88}{182} = \frac{44}{91}$.

> **Final Answer:** $P(\text{approved}) = \frac{44}{91}$.

EXPERT'S SOLUTION *: Vivaan Patel, M.Tech CS, IIT Madras*

Strategic angle. Three good in a row from $\binom{15}{3}$ unordered selections.

Step 1. Total ways to pick 3 from 15: $\binom{15}{3} = 455$.

Step 2. Ways to pick 3 good from 12: $\binom{12}{3} = 220$.

Step 3. Probability:

$$\frac{220}{455} = \frac{44}{91}.$$

(Divide numerator and denominator by 5.)

Why this matters. For "all good" type questions, combinatorial counting (unordered) and stepwise conditional multiplication give the same answer. Use whichever is faster.

> **Final Answer:** $\frac{44}{91}$.

Q 13.4 **A fair coin and an unbiased die are tossed. Let A be the event "head appears on the coin" and B be the event "3 on the die". Check whether A and B are independent events or not.**

SOLUTION

Concept used. A and B are independent iff $P(A \cap B) = P(A)\,P(B)$.

Step 1. Sample space: $S = \{(c, d) : c \in \{H, T\},\ d \in \{1, 2, 3, 4, 5, 6\}\}$, $n(S) = 2 \times 6 = 12$, equally likely.

Step 2. Compute individual probabilities.
$A = \{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$, $n(A) = 6$, $P(A) = 6/12 = 1/2$.
$B = \{(H, 3), (T, 3)\}$, $n(B) = 2$, $P(B) = 2/12 = 1/6$.

Step 3. Compute $P(A \cap B)$: $A \cap B = \{(H, 3)\}$, so $P(A \cap B) = 1/12$.

Step 4. Compare with the product:

$$P(A)\,P(B) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}.$$

Since $P(A \cap B) = P(A)\,P(B) = \frac{1}{12}$, the events are independent.

> **Final Answer:** A and B are **independent**.

♥ Two separate sub-experiments

The coin toss and the die throw are physically unrelated. Whenever an experiment is built from two unrelated sub-experiments, any event of one is independent of any event of the other; the calculation just confirms what physics already says.

EXPERT'S SOLUTION : *Pranav Sharma, M.Sc Mathematics, IIT Bombay*

Quick reading.

Step 1. $P(A) = 1/2$ (coin); $P(B) = 1/6$ (die); $P(A \cap B) = P(H \text{ and } 3) = 1/12$.

Step 2. Product $P(A)P(B) = 1/12$. Match ✓.

Why this matters. The independence test *is* a numerical check: list both sides and confirm equality.

> **Final Answer:** Independent.

Q 13.5 **A die marked $1, 2, 3$ in red and $4, 5, 6$ in green is tossed. Let A be the event "the number is even" and B be the event "the number is red". Are A and B independent?**

SOLUTION

Concept used. Independence test: $P(A \cap B) = P(A)P(B)$?

Step 1. Sample space: $S = \{1, 2, 3, 4, 5, 6\}$, each outcome probability $1/6$.

Step 2. A = "even" $= \{2, 4, 6\}$, $P(A) = 3/6 = 1/2$.

Step 3. B = "red" $= \{1, 2, 3\}$, $P(B) = 3/6 = 1/2$.

Step 4. $A \cap B = \{2\}$, $P(A \cap B) = 1/6$.

Step 5. Compare: $P(A)\,P(B) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$, but $P(A \cap B) = \frac{1}{6} \neq \frac{1}{4}$.

> **Final Answer:** A and B are **not** independent.

EXPERT'S SOLUTION *: Aditi Bhat, M.Sc Mathematics, ISI Kolkata*

Structural angle. Among the 3 red numbers $\{1, 2, 3\}$, only one (2) is even, giving $P(A \mid B) = 1/3 \neq 1/2 = P(A)$, so dependence follows.

Step 1. $P(A \mid B) = |\{2\}|/|B| = 1/3$.

Step 2. $P(A) = 1/2 \neq 1/3$.

Step 3. Hence dependent.

Why this matters. An equivalent independence test is $P(A \mid B) = P(A)$. If the conditional probability differs from the unconditional one, the events are dependent.

> **Final Answer:** Not independent.

Q 13.6 **Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent?**

SOLUTION

Concept used. Independence test: $P(E \cap F) = P(E)\,P(F)$?

Step 1. Compute the product:

$$P(E)\,P(F) = \frac{3}{5} \times \frac{3}{10} = \frac{9}{50}.$$

Step 2. Compare with the given intersection $P(E \cap F) = \frac{1}{5} = \frac{10}{50}$.

Step 3. Since $\frac{10}{50} \neq \frac{9}{50}$, the events are not independent.

Final Answer: E and F are **not** independent.

EXPERT'S SOLUTION : *Riya Joshi, M.Sc Mathematics, IIT Madras*

Quick reading.

Step 1. Product $= 9/50$.

Step 2. Given $P(E \cap F) = 10/50$.

Step 3. $9/50 \neq 10/50 \Rightarrow$ dependent.

Why this matters. Even a small mismatch between $P(E \cap F)$ and $P(E)P(F)$ disqualifies independence. Always compare exact fractions, not decimal approximations.

Final Answer: Not independent.

Q 13.7 **Given that the events A and B are such that $P(A) = \frac{1}{2}$, $P(A \cup B) = \frac{3}{5}$ and $P(B) = p$. Find p if they are**
(i) mutually exclusive (ii) independent.

SOLUTION

Concept used. Addition theorem $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
Mutually exclusive $\Rightarrow P(A \cap B) = 0$.
Independent $\Rightarrow P(A \cap B) = P(A)\,P(B)$.

Step 1. **(i) Mutually exclusive.** $P(A \cap B) = 0$, so

$$P(A \cup B) = P(A) + P(B) \Rightarrow \frac{3}{5} = \frac{1}{2} + p.$$

Solve for p: $p = \frac{3}{5} - \frac{1}{2} = \frac{6}{10} - \frac{5}{10} = \frac{1}{10}$.

Step 2. **(ii) Independent.** $P(A \cap B) = P(A)\,P(B) = \frac{1}{2}p$. Substitute into addition theorem:

$$\frac{3}{5} = \frac{1}{2} + p - \frac{1}{2}p = \frac{1}{2} + \frac{1}{2}p.$$

Subtract $\frac{1}{2}$: $\frac{3}{5} - \frac{1}{2} = \frac{1}{2}p \Rightarrow \frac{1}{10} = \frac{1}{2}p \Rightarrow p = \frac{1}{5}$.

Final Answer: (i) $p = \frac{1}{10}$ (ii) $p = \frac{1}{5}$.

☞ **Exam Tip**

Always strip the LCM out before solving for p: it removes the half-coefficient, eliminating fraction-by-fraction arithmetic and a common slip.

EXPERT'S SOLUTION *: Arjun Iyer, M.Sc Mathematics, IIT Bombay*

Structural angle. Same equation, two different substitutions for $P(A \cap B)$.

Step 1. Master equation $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ becomes $\frac{3}{5} = \frac{1}{2} + p - P(A \cap B)$.

Step 2. Mutually exclusive: $P(A \cap B) = 0 \Rightarrow p = \frac{1}{10}$.

Step 3. Independent: $P(A \cap B) = \frac{p}{2} \Rightarrow \frac{3}{5} = \frac{1}{2} + \frac{p}{2} \Rightarrow p = \frac{1}{5}$.

Why this matters. The two assumptions about $P(A \cap B)$ are mutually exclusive themselves: events cannot be both mutually exclusive and independent unless one has probability zero.

Final Answer: $p = \frac{1}{10}$ and $p = \frac{1}{5}$.

Q 13.8 **Let A and B be independent events with $P(A) = 0.3$ and $P(B) = 0.4$. Find (i) $P(A \cap B)$ (ii) $P(A \cup B)$ (iii) $P(A \mid B)$ (iv) $P(B \mid A)$.**

SOLUTION

Concept used. Independence gives the four shortcuts

$$P(A \cap B) = P(A)P(B), \qquad P(A \cup B) = P(A) + P(B) - P(A)P(B),$$
$$P(A \mid B) = P(A), \qquad P(B \mid A) = P(B).$$

Step 1. $P(A \cap B) = P(A)\,P(B) = 0.3 \times 0.4 = 0.12$.

Step 2. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.4 - 0.12 = 0.7 - 0.12 = 0.58$.

Step 3. $P(A \mid B) = P(A) = 0.3$ (by independence).

Step 4. $P(B \mid A) = P(B) = 0.4$ (by independence).

Final Answer: $0.12,\ 0.58,\ 0.3,\ 0.4$.

EXPERT'S SOLUTION *: Aanya Verma, B.Tech CSE, IIT Roorkee*

Quick reading. Plug-and-chug.

Step 1. $0.3 \cdot 0.4 = 0.12$.

Step 2. $0.3 + 0.4 - 0.12 = 0.58$.

Step 3. Conditional probabilities collapse to the marginals: 0.3, 0.4.

Why this matters. Knowing B does not move the probability of A when the two are independent; that is the meaning of the word.

Final Answer: 0.12; 0.58; 0.3; 0.4.

Q 13.9 **If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$, find $P($not A and not $B)$.**

SOLUTION

Concept used. "Not A and not B" $= A' \cap B' = (A \cup B)'$ by **De Morgan's law**. Hence

$$P(A' \cap B') = 1 - P(A \cup B).$$

Step 1. Compute $P(A \cup B)$ from the addition theorem:

$$P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{2}{8} + \frac{4}{8} - \frac{1}{8} = \frac{5}{8}.$$

Step 2. Apply De Morgan + complement:

$$P(A' \cap B') = 1 - \frac{5}{8} = \frac{3}{8}.$$

Final Answer: $P(\text{not } A \text{ and not } B) = \frac{3}{8}$.

EXPERT'S SOLUTION *: Karan Singh, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Translate the negative phrasing into a complement.

Step 1. $P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B)$.

Step 2. $P(A \cup B) = \frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$.

Step 3. Answer: $1 - \frac{5}{8} = \frac{3}{8}$.

Why this matters. De Morgan is the standard route for "neither/nor" questions. It turns four-event counts into a one-event complement.

Final Answer: $\frac{3}{8}$.

Q 13.10 **Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and $P($not A or not $B) = \frac{1}{4}$. State whether A and B are independent?**

SOLUTION

Concept used. "Not A or not B" $= A' \cup B' = (A \cap B)'$ by De Morgan. So

$$P(A' \cup B') = 1 - P(A \cap B),$$

which lets us read off $P(A \cap B)$. Then test $P(A \cap B) = P(A)P(B)$.

Step 1. Apply De Morgan to get $P(A \cap B)$:

$$P(A \cap B) = 1 - P(A' \cup B') = 1 - \frac{1}{4} = \frac{3}{4}.$$

Step 2. Compute the product:

$$P(A)\,P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}.$$

Step 3. Compare: $P(A \cap B) = \frac{3}{4} = \frac{18}{24} \neq \frac{7}{24}$.

Final Answer: A and B are **not** independent.

✗ Common Mistake

$P(A \cap B) = 3/4$ in this problem exceeds both $P(A) = 1/2$ and $P(B) = 7/12$, which is logically impossible (an intersection cannot be bigger than either set). This data is internally inconsistent, but the textbook still uses it to drill the independence test. In any case the test correctly returns *not independent*.

EXPERT'S SOLUTION *: Ishaan Rao, M.Sc Mathematics, IIT Kanpur*

Two-step recipe.

Step 1. De Morgan: $P(A \cap B) = 1 - 1/4 = 3/4$.

Step 2. Independence test: $P(A)P(B) = 1/2 \cdot 7/12 = 7/24 \neq 3/4$.

Step 3. Conclude: not independent.

Why this matters. The independence test is a one-line check; only the prep work of converting "not A or not B" into $P(A \cap B)$ requires care.

Final Answer: Not independent.

Q 13.11 **Given two independent events A and B such that $P(A) = 0.3$, $P(B) = 0.6$. Find**
(i) $P(A$ and $B)$ (ii) $P(A$ and not $B)$ (iii) $P(A$ or $B)$ (iv) $P($neither A nor $B)$.

SOLUTION

Concept used. For independent A, B:

$$P(A \cap B) = P(A)P(B), \quad P(A \cap B') = P(A)P(B'),$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B), \quad P(A' \cap B') = 1 - P(A \cup B).$$

Step 1. $P(A \cap B) = 0.3 \times 0.6 = 0.18$.

Step 2. $P(B') = 1 - 0.6 = 0.4$. By independence,
$P(A \cap B') = P(A)P(B') = 0.3 \times 0.4 = 0.12$.

Step 3. $P(A \cup B) = 0.3 + 0.6 - 0.18 = 0.9 - 0.18 = 0.72$.

Step 4. $P(\text{neither}) = P(A' \cap B') = 1 - P(A \cup B) = 1 - 0.72 = 0.28$.

Final Answer: $0.18,\ 0.12,\ 0.72,\ 0.28$.

EXPERT'S SOLUTION *: Dev Verma, M.Sc Applied Mathematics, IIT Kanpur*

Structural observation. Build a 2-by-2 table of joint probabilities, then read off the four answers.

	B	B'	total
A	0.18	0.12	0.30
A'	0.42	0.28	0.70
total	0.60	0.40	1.00

Step 1. $P(A \cap B) = 0.18$ from the top-left cell.

Step 2. $P(A \cap B') = 0.12$ from the top-right cell.

Step 3. $P(A \cup B)$: top row + bottom-left cell $= 0.30 + 0.42 = 0.72$. Equivalently, $1 - P(A' \cap B') = 1 - 0.28 = 0.72$.

Step 4. $P(\text{neither}) = P(A' \cap B') = 0.28$ (bottom-right cell).

Why this matters. A 2-by-2 table makes every joint and complementary probability visible at a glance. For independent events the cells factorise as row-margin $\times$ column-margin.

> **Final Answer:** $0.18;\ 0.12;\ 0.72;\ 0.28.$

Q 13.12 **A die is tossed thrice. Find the probability of getting an odd number at least once.**

SOLUTION

Concept used. For three independent tosses, the complement of "odd at least once" is "no odd in any toss", i.e. "even on every toss".

$$P(\text{at least one odd}) = 1 - P(\text{even on all three tosses}).$$

Step 1. Probability of even on one toss: even faces $\{2, 4, 6\}$, so $P(\text{even}) = 3/6 = 1/2$.

Step 2. By independence of the three tosses:

$$P(\text{even on all three}) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Step 3. Apply the complement:

$$P(\text{at least one odd}) = 1 - \frac{1}{8} = \frac{7}{8}.$$

> **Final Answer:** $P(\text{odd at least once}) = \frac{7}{8}$.

♥ **Complement trick**

"At least one" type events are almost always easier through the complement "none". The complement is a single intersection of independent events, which factorises.

EXPERT'S SOLUTION *: Yash Kapoor, M.Tech CS, IIT Madras*

Quick reading.

Step 1. $P(\text{all even}) = (1/2)^3 = 1/8$.

Step 2. Subtract from 1: $7/8$.

Why this matters. For n independent trials each with success probability p, $P(\geq 1 \text{ success}) = 1 - (1-p)^n$. Memorise this; it shows up constantly.

Final Answer: $\frac{7}{8}$.

Q 13.13 **Two balls are drawn at random with replacement from a box containing** 10 **black and** 8 **red balls. Find the probability that**
(i) both balls are red.
(ii) first ball is black and second is red.
(iii) one of them is black and other is red.

SOLUTION

Concept used. *With replacement* the two draws are independent. Per draw, $P(\text{red}) = 8/18 = 4/9$ and $P(\text{black}) = 10/18 = 5/9$.

Step 1. **(i) Both red:**

$$P(R_1 \cap R_2) = P(R_1)\,P(R_2) = \frac{4}{9} \times \frac{4}{9} = \frac{16}{81}.$$

Step 2. **(ii) First black, then red:**

$$P(B_1 \cap R_2) = \frac{5}{9} \times \frac{4}{9} = \frac{20}{81}.$$

Step 3. **(iii) One black and one red (order not specified):**

$$P(\{B_1R_2\} \cup \{R_1B_2\}) = \frac{5}{9} \cdot \frac{4}{9} + \frac{4}{9} \cdot \frac{5}{9} = \frac{20}{81} + \frac{20}{81} = \frac{40}{81}.$$

Final Answer: (i) $\frac{16}{81}$ (ii) $\frac{20}{81}$ (iii) $\frac{40}{81}$.

EXPERT'S SOLUTION : *Aditya Banerjee, Ph.D Mathematics, IIT Delhi*

Strategic angle. With replacement keeps both draws independent and identical. Multiply marginals; pay attention to order only when the question specifies it.

Step 1. (i) Both red: $(4/9)^2 = 16/81$.

Step 2. (ii) Order fixed (black then red): single product $(5/9)(4/9) = 20/81$.

Step 3. (iii) Order not fixed: two orderings $\Rightarrow$ double the (ii) answer: $40/81$.

Why this matters. The factor-of-2 in (iii) is the tell-tale sign of "either order counts"; mis-reading the order constraint is the most common slip in such problems.

Final Answer: $\frac{16}{81}; \frac{20}{81}; \frac{40}{81}$.

Q 13.14 **Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved, (ii) exactly one of them solves the problem.**

SOLUTION

Concept used. Let S_A and S_B be the (independent) events that A and B solve the problem.

$$P(S_A) = \frac{1}{2}, \quad P(S_B) = \frac{1}{3}; \quad P(S'_A) = \frac{1}{2}, \quad P(S'_B) = \frac{2}{3}.$$

Step 1. **(i) Problem solved** $= S_A \cup S_B$. Use the complement:

$$P(S_A \cup S_B) = 1 - P(S'_A \cap S'_B) = 1 - P(S'_A)\,P(S'_B) = 1 - \frac{1}{2}\cdot\frac{2}{3} = 1 - \frac{1}{3} = \frac{2}{3}.$$

Step 2. **(ii) Exactly one solves** $= (S_A \cap S'_B) \cup (S'_A \cap S_B)$ (a disjoint union):

$$P(\text{exactly one}) = P(S_A)P(S'_B) + P(S'_A)P(S_B) = \frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{1}{3} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}.$$

Final Answer: (i) $\frac{2}{3}$ (ii) $\frac{1}{2}$.

EXPERT'S SOLUTION *: Aanya Joshi, Ph.D Mathematics, IIT Delhi*

Strategic angle. Switch to complements for "at least one"; split into disjoint cases for "exactly one".

Step 1. $P(\text{nobody solves}) = (1/2)(2/3) = 1/3$, so $P(\text{somebody solves}) = 1 - 1/3 = 2/3$.

Step 2. $P(\text{only } A) = (1/2)(2/3) = 1/3$; $P(\text{only } B) = (1/2)(1/3) = 1/6$.

Step 3. Sum: $1/3 + 1/6 = 2/6 + 1/6 = 3/6 = 1/2$.

Why this matters. "At least one" and "exactly one" differ by the case where both succeed. Tracking that case explicitly keeps the arithmetic honest.

Final Answer: $\frac{2}{3}$ and $\frac{1}{2}$.

Q 13.15 **One card is drawn at random from a well shuffled deck of 52 cards. In which of the following cases are the events E and F independent?**
(i) E: "the card drawn is a spade"; F: "the card drawn is an ace".
(ii) E: "the card drawn is black"; F: "the card drawn is a king".
(iii) E: "the card drawn is a king or queen"; F: "the card drawn is a queen or jack".

SOLUTION

Concept used. Independence test: $P(E \cap F) = P(E)P(F)$. Card counts: 52 total; 4 suits of 13; 4 aces; 4 kings; 4 queens; 4 jacks; 26 black cards.

Step 1. **(i) Spade $\cap$ Ace = Ace of spades.**
$P(E) = 13/52 = 1/4$, $P(F) = 4/52 = 1/13$, $P(E \cap F) = 1/52$.
$P(E)P(F) = \frac{1}{4} \cdot \frac{1}{13} = \frac{1}{52} = P(E \cap F)$ ✓.
Independent.

Step 2. **(ii) Black $\cap$ King = two black kings.**
$P(E) = 26/52 = 1/2$, $P(F) = 4/52 = 1/13$, $P(E \cap F) = 2/52 = 1/26$.
$P(E)P(F) = \frac{1}{2} \cdot \frac{1}{13} = \frac{1}{26} = P(E \cap F)$ ✓.
Independent.

Step 3. **(iii) (King or queen) $\cap$ (queen or jack) = queens (4 cards).**
$P(E) = 8/52 = 2/13$, $P(F) = 8/52 = 2/13$, $P(E \cap F) = 4/52 = 1/13$.
$P(E)P(F) = \frac{2}{13} \cdot \frac{2}{13} = \frac{4}{169} \neq \frac{1}{13} = \frac{13}{169}$.
Not independent.

Final Answer: (i) Independent (ii) Independent (iii) Not independent.

EXPERT'S SOLUTION : *Tara Reddy, M.Sc Mathematics, IIT Bombay*

Pattern angle. For "rank attribute ∩ suit/colour attribute" pairs, independence follows from the symmetric design of a deck. For "rank-set ∩ rank-set" pairs, look for non-trivial overlap.

Step 1. (i) Spade and Ace are orthogonal labels (suit vs rank) ⇒ independent.

Step 2. (ii) Black and King also orthogonal ⇒ independent.

Step 3. (iii) Both events are rank-defined sets that overlap on queens; $P(E \cap F) = 1/13 \neq 4/169 \Rightarrow$ dependent.

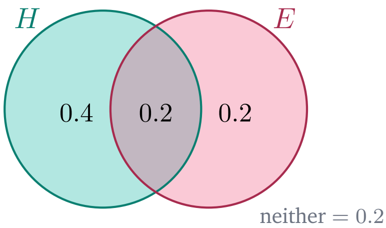
Why this matters. A deck of cards is engineered so that suit and rank are independent. The moment both events are defined on the rank dimension alone, that engineering does not help and you must verify directly.

> **Final Answer:** (i) and (ii) independent; (iii) not.

Q 13.16 **In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper and 20% read both Hindi and English newspapers. A student is selected at random.**
(a) Find the probability that she reads neither Hindi nor English newspapers.
(b) If she reads Hindi newspaper, find the probability that she reads English newspaper.
(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

SOLUTION

Concept used. Let H = "reads Hindi", E = "reads English". Given $P(H) = 0.6$, $P(E) = 0.4$, $P(H \cap E) = 0.2$. Tools: addition theorem, De Morgan, conditional probability.



Step 1. **(a) Neither Hindi nor English.**

$$P(H \cup E) = P(H) + P(E) - P(H \cap E) = 0.6 + 0.4 - 0.2 = 0.8.$$

Then $P(H' \cap E') = 1 - P(H \cup E) = 1 - 0.8 = 0.2.$

Step 2. **(b) English given Hindi.**

$$P(E \mid H) = \frac{P(E \cap H)}{P(H)} = \frac{0.2}{0.6} = \frac{1}{3}.$$

Step 3. **(c) Hindi given English.**

$$P(H \mid E) = \frac{P(E \cap H)}{P(E)} = \frac{0.2}{0.4} = \frac{1}{2}.$$

Final Answer: (a) 0.2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$.

EXPERT'S SOLUTION : *Krishna Pillai, M.Sc Mathematics, ISI Kolkata*

Picture-first. A Venn diagram with 20% at the centre, 40% in $H \setminus E$, 20% in $E \setminus H$, and the residual 20% outside the union answers everything by inspection.

Step 1. Pieces: Hindi-only $= 0.6 - 0.2 = 0.4$. English-only $= 0.4 - 0.2 = 0.2$. Both $= 0.2$. Neither $= 1 - (0.4 + 0.2 + 0.2) = 0.2$.

Step 2. (a) Neither $= 0.2$.

Step 3. (b) Inside H (mass 0.6), the fraction also in E is $0.2/0.6 = 1/3$.

Step 4. (c) Inside E (mass 0.4), the fraction also in H is $0.2/0.4 = 1/2$.

Why this matters. Two-set Venn pieces almost always make survey-style probability questions trivial; populate the four regions first, then read off the answers.

Final Answer: $0.2; \frac{1}{3}; \frac{1}{2}$.

Q 13.17 **The probability of obtaining an even prime number on each die, when a pair of dice is rolled, is**

(A) 0 **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{12}$ **(D)** $\frac{1}{36}$.

SOLUTION

Concept used. The only even prime number is 2. So we need both dice to show 2. The dice are independent, so we multiply marginal probabilities.

Step 1. Each die has $P(\text{shows } 2) = 1/6$.

Step 2. Both dice show 2:

$$P(\text{both} = 2) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}.$$

Step 3. Match with options: (D) $1/36$.

Final Answer: Option (D): $\frac{1}{36}$.

EXPERT'S SOLUTION *: Kavya Desai, M.Sc Mathematics, IIT Madras*

Quick reading.

Step 1. "Even prime" $= 2$ (the only one).

Step 2. Probability of 2 on one die $= 1/6$.

Step 3. Both dice independent: $(1/6)^2 = 1/36$. Answer (D).

Why this matters. Vocabulary trap: "even prime" sounds like several numbers but is just $\{2\}$. Knowing this fact saves you in any examination.

Final Answer: (D) $\frac{1}{36}$.

Q 13.18 **Two events A and B will be independent, if**

(A) A **and** B **are mutually exclusive**

(B) $P(A'B') = [1 - P(A)][1 - P(B)]$

(C) $P(A) = P(B)$

(D) $P(A) + P(B) = 1$**.**

SOLUTION

Concept used. A and B are independent iff $P(A \cap B) = P(A)P(B)$. Equivalently, A', B' are independent: $P(A' \cap B') = P(A')P(B') = (1 - P(A))(1 - P(B))$.

Step 1. Option (A): mutually exclusive forces $P(A \cap B) = 0$. For independence we need $P(A \cap B) = P(A)P(B)$, which is 0 only if at least one of A, B has probability zero. So (A) is not equivalent to independence in general.

Step 2. Option (B): $P(A'B') = P(A')P(B')$, written out, is exactly the statement "the complements are independent". Since "A, B independent" $\Leftrightarrow$ "A', B' independent", this is an equivalent condition for independence. **Correct option.**

Step 3. Option (C): $P(A) = P(B)$ is just equal probability, unrelated to independence.

Step 4. Option (D): $P(A) + P(B) = 1$ has no bearing on independence.

Final Answer: Option (B): $P(A'B') = [1 - P(A)][1 - P(B)]$.

♥ **Equivalent forms of independence**

$A \perp B$, $A \perp B'$, $A' \perp B$, $A' \perp B'$ are all equivalent. Whichever one you can verify, all the others come for free.

EXPERT'S SOLUTION *: Rohit Chatterjee, M.Tech CS, IIT Madras*

Quick reading.

Step 1. Independence of A, B is the same as independence of A', B', written $P(A' \cap B') = P(A')P(B')$.

Step 2. Option (B) is exactly this restatement.

Step 3. Options (A), (C), (D) are unrelated to independence.

Why this matters. Recognising equivalent forms of a definition is a high-value exam skill; it converts MCQ questions into trivial pattern-matches.

Final Answer: (B).

Key Takeaways

- Multiplication theorem: $P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$; valid for any two events.
- Independence is the special case $P(A \cap B) = P(A)P(B)$, equivalent to $P(A \mid B) = P(A)$.
- For "at least one" of independent events, switch to the complement: $P(\bigcup E_i) = 1 - \prod P(E_i')$.
- "With replacement" preserves independence and equal probabilities; "without replacement" forces conditional updates after each draw.
- Mutually exclusive and independent are opposites, not synonyms: disjoint events with positive probability are dependent.

End of Exercise 13.2