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Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 13: Probability

About this Chapter

The miscellaneous exercise revisits every theme of Chapter 13: conditional probability under set-inclusion, the independence test, binomial-style "at most" counts, structural arguments about leap-year weekdays, Bayes' theorem with three or four hypotheses, and the algebraic interplay between $P(A)$, $P(B)$, $P(A \cap B)$ and $P(A \cup B)$. The questions are deliberately mixed, so each one starts by identifying which Chapter-13 tool fits.

Topics covered: Subset/disjoint conditioning • Two-child families • Leap-year weekdays • Bayes' theorem • Logic of $P(A \mid B) > P(A)$

Quick Formula Sheet

Conditional probability:

$P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$, $P(A) \neq 0$

Special cases:

$A \subset B \Rightarrow P(A \cap B) = P(A)$, so $P(B \mid A) = 1$.

$A \cap B = \varnothing \Rightarrow P(B \mid A) = 0$.

Independence:

$P(A \cap B) = P(A)\,P(B)$

Bayes' theorem:

$P(E_i \mid A) = \dfrac{P(E_i)\,P(A \mid E_i)}{\sum_j P(E_j) P(A \mid E_j)}$

Miscellaneous Exercise

Q 13.1 A **and** B **are two events such that** $P(A) \neq 0$**. Find** $P(B \mid A)$**, if (i)** A **is a subset of** B **(ii)** $A \cap B = \varnothing$**.**

SOLUTION

Concept used. The defining relation

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)}, \quad P(A) \neq 0,$$

together with two set facts: $A \subset B \Rightarrow A \cap B = A$, and $A \cap B = \varnothing \Rightarrow P(A \cap B) = 0$.

Step 1. **(i)** $A \subset B$. Every outcome in A is also in B, so $A \cap B = A$ and therefore $P(A \cap B) = P(A)$. Substitute:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1.$$

Step 2. **(ii)** $A \cap B = \varnothing$. Then $P(A \cap B) = P(\varnothing) = 0$. Substitute:

$$P(B \mid A) = \frac{0}{P(A)} = 0.$$

Final Answer: (i) $P(B \mid A) = 1$ (ii) $P(B \mid A) = 0$.

♥ **Two extremes of conditioning**

"$A \subset B$" forces B once A happens; "$A \cap B = \varnothing$" rules B out once A happens. These two extremes generate $P(B \mid A) = 1$ and $P(B \mid A) = 0$ respectively.

EXPERT'S SOLUTION : *Aarav Reddy, M.Sc Mathematics, IIT Bombay*

Quick reading.

Step 1. (i) $A \cap B = A \Rightarrow$ ratio $= P(A)/P(A) = 1$.

Step 2. (ii) $A \cap B = \varnothing \Rightarrow$ ratio $= 0/P(A) = 0$.

Why this matters. Set relationships often reduce conditional-probability questions to a one-line answer. Always check whether one event sits inside the other or whether they are disjoint.

Final Answer: 1 and 0.

Q 13.2 **A couple has two children.**
(i) Find the probability that both children are males, if it is known that at least one of the children is male.
(ii) Find the probability that both children are females, if it is known that the elder child is a female.

SOLUTION

Concept used. Sample space (eldest listed first): $S = \{BB, BG, GB, GG\}$, equally likely. Use $P(E \mid F) = |E \cap F|/|F|$.

Step 1. **(i) Both male given at least one male.**

$E = \{BB\}$, $|E| = 1$.

F = "at least one male" $= \{BB, BG, GB\}$, $|F| = 3$.

$E \cap F = \{BB\}$, $|E \cap F| = 1$.

$$P(E \mid F) = \frac{1}{3}.$$

Step 2. **(ii) Both female given elder female.**

$E = \{GG\}$, $|E| = 1$.

F = "elder female" = outcomes with first letter $G = \{GB, GG\}$, $|F| = 2$.

$E \cap F = \{GG\}$, $|E \cap F| = 1$.

$$P(E \mid F) = \frac{1}{2}.$$

Final Answer: (i) $\frac{1}{3}$ (ii) $\frac{1}{2}$.

✗ Common Mistake

The phrasing "at least one is male" picks out three outcomes, not two: it does NOT mean "exactly one is male". Triple-check the conditioning event before computing.

EXPERT'S SOLUTION : *Sneha Iyer, M.Sc Mathematics, ISI Kolkata*

Structural angle. The two parts use the same underlying sample space; only the conditioning event differs.

Step 1. (i) Restrict to "at least one boy": $\{BB, BG, GB\}$. BB is 1 of 3. Probability $1/3$.

Step 2. (ii) Restrict to "elder female": $\{GB, GG\}$. GG is 1 of 2. Probability $1/2$.

Why this matters. The boy-girl paradox uses exactly this asymmetry: specifying *which* child is female (eldest) gives more information than merely saying *some* child is female.

Final Answer: $\frac{1}{3}$ and $\frac{1}{2}$.

Q 13.3 **Suppose that 5% of men and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.**

SOLUTION

Concept used. Bayes' theorem. Hypotheses: male/female. Event: grey hair.

Step 1. Events: M = male, W = female (women), G = grey hair.

Step 2. Priors: $P(M) = P(W) = 1/2$ (equal numbers).

Step 3. Likelihoods: $P(G \mid M) = 5\% = 0.05$, $P(G \mid W) = 0.25\% = 0.0025$.

Step 4. Total probability:

$$P(G) = \frac{1}{2}(0.05) + \frac{1}{2}(0.0025) = 0.025 + 0.00125 = 0.02625.$$

Step 5. Bayes':

$$P(M \mid G) = \frac{(1/2)(0.05)}{0.02625} = \frac{0.025}{0.02625}.$$

Multiply numerator and denominator by 1000000 to clear decimals:

$$= \frac{25000}{26250} = \frac{20}{21}.$$

(Divide numerator and denominator by 1250: $25000 \div 1250 = 20$, $26250 \div 1250 = 21$.)

Final Answer: $P(\text{male} \mid \text{grey hair}) = \dfrac{20}{21}$.

EXPERT'S SOLUTION *: Vivaan Gupta, M.Tech CS, IIT Madras*

Frequency angle. Take an imaginary population of 40000 (20000 men, 20000 women).

- Grey-haired men: $20000 \times 0.05 = 1000$.
- Grey-haired women: $20000 \times 0.0025 = 50$.
- Total grey-haired: 1050.

Step 1. $P(M \mid G) = 1000/1050 = 20/21$.

Why this matters. Men are 20 times more likely to be grey than women here; the high posterior $20/21 \approx 0.952$ is just that likelihood ratio normalised.

Final Answer: $\dfrac{20}{21}$.

Q 13.4 **Suppose that 90% of people are right-handed. What is the probability that at most 6 of a random sample of 10 people are right-handed?**

SOLUTION

Concept used. Each person is independently right-handed with $p = 0.9$. The number of right-handed people in 10 trials, call it X, is **binomial**: $X \sim B(10, 0.9)$.

$$P(X = k) = \binom{10}{k} (0.9)^k (0.1)^{10-k}.$$

"At most 6" means $X \leq 6$. Using the complement:

$$P(X \leq 6) = 1 - P(X \geq 7) = 1 - \sum_{k=7}^{10} P(X = k).$$

Step 1. Compute $P(X = 10) = (0.9)^{10}$.

Step 2. Compute $P(X = 9) = \binom{10}{9}(0.9)^9(0.1)^1 = 10(0.9)^9(0.1)$.

Step 3. Compute $P(X = 8) = \binom{10}{8}(0.9)^8(0.1)^2 = 45(0.9)^8(0.01)$.

Step 4. Compute $P(X = 7) = \binom{10}{7}(0.9)^7(0.1)^3 = 120(0.9)^7(0.001)$.

Step 5. Factor out $(0.9)^7$ from all four terms:

$$P(X \geq 7) = (0.9)^7 \Big[(0.9)^3 + 10(0.9)^2(0.1) + 45(0.9)(0.01) + 120(0.001)\Big].$$

Inside the bracket: $(0.9)^3 = 0.729$; $10(0.81)(0.1) = 0.81$; $45(0.9)(0.01) = 0.405$; $120(0.001) = 0.12$.
Sum: $0.729 + 0.81 + 0.405 + 0.12 = 2.064$.
Also $(0.9)^7$: $(0.9)^2 = 0.81$, $(0.9)^4 = 0.6561$, $(0.9)^6 = 0.6561 \cdot 0.81 = 0.531441$, $(0.9)^7 = 0.531441 \cdot 0.9 = 0.4782969$.
Hence

$$P(X \geq 7) = 0.4782969 \times 2.064 \approx 0.9872.$$

Therefore

$$P(X \leq 6) = 1 - 0.9872 \approx 0.0128.$$

In closed form,
$P(X \leq 6) = 1 - \Big[\binom{10}{7}(0.9)^7(0.1)^3 + \binom{10}{8}(0.9)^8(0.1)^2 + \binom{10}{9}(0.9)^9(0.1) + (0.9)^{10}\Big]$.

Final Answer: $P(X \leq 6) \approx 0.0128$.

☞ Exam Tip

For "at most k" / "at least k" binomial questions with k close to n, switch to the complement; the tail you need to sum is much shorter.

EXPERT'S SOLUTION : *Aanya Bhat, Ph.D Mathematics, IIT Delhi*

Structural angle. Factor $(0.9)^7$ out of the four-term complement to keep the arithmetic tractable.

Step 1. Compute the bracketed factor: $0.729 + 0.81 + 0.405 + 0.12 = 2.064$.

Step 2. Multiply by $(0.9)^7 \approx 0.4783$: $0.4783 \times 2.064 \approx 0.9872$.

Step 3. Subtract from 1: $1 - 0.9872 = 0.0128$.

Why this matters. A "small" tail probability ≈ 0.013 says it is unusual to see 6 or fewer right-handers in 10 people when the population rate is 90%; binomial intuition would predict a count near 9.

> **Final Answer:** ≈ 0.0128.

Q 13.5 **If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays?**

SOLUTION

Concept used. A leap year has 366 days = 52 complete weeks +2 extra days. The 52 weeks contribute exactly 52 of every weekday. The two extra days come as one of the following equally-likely consecutive pairs:

$$\{(\text{Sun,Mon}), (\text{Mon,Tue}), (\text{Tue,Wed}), (\text{Wed,Thu}), (\text{Thu,Fri}), (\text{Fri,Sat}), (\text{Sat,Sun})\}.$$

So the sample space has 7 equally-likely outcomes.

Step 1. Identify when "53 Tuesdays" occurs: when at least one of the two extra days is a Tuesday.

Step 2. Among the 7 pairs above, those containing Tuesday are (Mon,Tue) and (Tue,Wed): 2 outcomes.

Step 3. Apply the equally-likely formula:

$$P(53 \text{ Tuesdays}) = \frac{2}{7}.$$

> **Final Answer:** $P = \frac{2}{7}$.

♥ Structural counting beats brute force

We never had to look at the calendar; structural counting of the two "extra" days does the entire job.

EXPERT'S SOLUTION *: Karan Patel, M.Sc Mathematics, ISI Kolkata*

Quick reading.

Step 1. $366 = 52 \times 7 + 2$: two surplus days.

Step 2. Surplus days are some consecutive (day_i, day_{i+1}); there are 7 such pairs.

Step 3. Tuesday lies in 2 of these 7 pairs.

Step 4. Probability $= 2/7$.

Why this matters. The same logic gives $P(53 \text{ Sundays in a normal year}) = 1/7$ (only one surplus day) and various other variants seen in board papers.

Final Answer: $\frac{2}{7}$.

Q 13.6 **Suppose we have four boxes A, B, C, D containing coloured marbles as given below:**

Box	Red	White	Black
A	1	6	3
B	6	2	2
C	8	1	1
D	0	6	4

One of the boxes has been selected at random and a single marble is drawn from it. If the marble is red, what is the probability that it was drawn from box A? box B? box C?

SOLUTION

Concept used. Bayes' theorem with four hypotheses (box selected) and event R = red marble drawn.

Step 1. Priors. Boxes chosen uniformly: $P(A) = P(B) = P(C) = P(D) = 1/4$.

Step 2. Likelihoods. Each box has 10 marbles.

$$P(R \mid A) = \frac{1}{10},\ P(R \mid B) = \frac{6}{10},\ P(R \mid C) = \frac{8}{10},\ P(R \mid D) = \frac{0}{10} = 0.$$

Step 3. Total probability:

$$P(R) = \frac{1}{4}\left(\frac{1}{10} + \frac{6}{10} + \frac{8}{10} + 0\right) = \frac{1}{4} \cdot \frac{15}{10} = \frac{15}{40} = \frac{3}{8}.$$

Step 4. Bayes' for each box (with $P(R) = 3/8$):

$$P(A \mid R) = \frac{(1/4)(1/10)}{3/8} = \frac{1/40}{3/8} = \frac{1}{40} \times \frac{8}{3} = \frac{8}{120} = \frac{1}{15}.$$

$$P(B \mid R) = \frac{(1/4)(6/10)}{3/8} = \frac{6/40}{3/8} = \frac{6}{40} \times \frac{8}{3} = \frac{48}{120} = \frac{2}{5}.$$

$$P(C \mid R) = \frac{(1/4)(8/10)}{3/8} = \frac{8/40}{3/8} = \frac{8}{40} \times \frac{8}{3} = \frac{64}{120} = \frac{8}{15}.$$

Check: $\frac{1}{15} + \frac{2}{5} + \frac{8}{15} + 0 = \frac{1+6+8}{15} = 1$ ✓.

Final Answer: $P(A \mid R) = \frac{1}{15}$, $P(B \mid R) = \frac{2}{5}$, $P(C \mid R) = \frac{8}{15}$.

EXPERT'S SOLUTION : *Tara Joshi, M.Sc Mathematics, IIT Bombay*

Frequency angle. Imagine choosing 40 boxes (10 of each kind) and drawing one marble per box. The expected counts of red are $10 \cdot 1/10 = 1$ from A-boxes, 6 from B-boxes, 8 from C-boxes, 0 from D-boxes. Total reds $= 15$.

Step 1. Posteriors are ratios within the 15 expected reds:

$$P(A \mid R) = \frac{1}{15}, \quad P(B \mid R) = \frac{6}{15} = \frac{2}{5}, \quad P(C \mid R) = \frac{8}{15}.$$

Why this matters. Box D never produces a red marble, so its posterior is forced to zero. The remaining mass redistributes in proportion to the boxes' red counts.

Final Answer: $\frac{1}{15}; \frac{2}{5}; \frac{8}{15}$.

Q 13.7 **Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduces the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga.**

SOLUTION

Concept used. Bayes' theorem with two hypotheses (yoga vs drug).

Step 1. Events: Y = patient chose yoga, G = patient chose drug, H = patient suffers heart attack.

Step 2. Priors: $P(Y) = P(G) = 1/2$.

Step 3. Likelihoods. Baseline heart-attack rate is $40\% = 0.4$.
Yoga *reduces* risk by 30%: $P(H \mid Y) = 0.4 \times (1 - 0.30) = 0.4 \times 0.70 = 0.28$.
Drug reduces risk by 25%: $P(H \mid G) = 0.4 \times (1 - 0.25) = 0.4 \times 0.75 = 0.30$.

Step 4. Total probability:

$$P(H) = \frac{1}{2}(0.28) + \frac{1}{2}(0.30) = 0.14 + 0.15 = 0.29.$$

Step 5. Bayes':

$$P(Y \mid H) = \frac{(1/2)(0.28)}{0.29} = \frac{0.14}{0.29} = \frac{14}{29}.$$

Final Answer: $P(\text{yoga} \mid \text{heart attack}) = \frac{14}{29}$.

✗ Common Mistake

"Reduces the risk by 30%" means the new risk is 70% of the original, not $40\% - 30\% = 10\%$. Read percentage-reduction language carefully: it is a multiplicative reduction.

EXPERT'S SOLUTION *: Diya Verma, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading.

Step 1. Yoga risk $= 0.4 \times 0.7 = 0.28$; Drug risk $= 0.4 \times 0.75 = 0.30$.

Step 2. Numerator: $(1/2)(0.28) = 0.14$.

Step 3. Denominator: $0.14 + 0.15 = 0.29$.

Step 4. Ratio: $14/29$.

Why this matters. Yoga reduces risk more effectively, so a heart-attack victim is slightly more likely to have used the drug option. The posterior $14/29 \approx 0.483 < 0.5$ reflects this.

Final Answer: $\frac{14}{29}$.

Q 13.8 **If each element of a second order determinant is either zero or one, what is the probability that the value of the determinant is positive? (Assume that the individual entries of the determinant are chosen independently, each value being assumed with probability $\frac{1}{2}$.)**

SOLUTION

Concept used. A 2×2 determinant

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc,$$

with each of $a, b, c, d \in \{0, 1\}$ independent and equally likely. There are $2^4 = 16$ equally-likely matrices.
The value $ad - bc$ is positive iff $ad - bc > 0$, i.e. $ad = 1$ and $bc = 0$ (the only way to get a positive integer with each term in $\{0, 1\}$).

Step 1. Count matrices with $ad = 1$: both $a = 1$ and $d = 1$. The remaining entries $b, c \in \{0, 1\}$ are free: 4 matrices.

Step 2. Of these 4, count those with $bc = 0$: exclude $b = c = 1$. So $4 - 1 = 3$ matrices have $ad = 1, bc = 0$.

Step 3. Probability:

$$P(\det > 0) = \frac{3}{16}.$$

Final Answer: $P(\det > 0) = \frac{3}{16}$.

EXPERT'S SOLUTION : *Krishna Banerjee, M.Sc Mathematics, IIT Bombay*

Enumeration angle. List the 3 matrices with positive determinant:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

Each has $\det = 1 > 0$.

Step 1. Total possibilities: $2^4 = 16$.

Step 2. Favourable: 3 (listed above).

Step 3. Probability: $3/16$.

Why this matters. The determinant takes only three values here: $-1, 0, 1$. Knowing the value set lets you enumerate matrices by determinant value directly.

Final Answer: $\frac{3}{16}$.

Q 13.9 **An electronic assembly consists of two subsystems, say, A and B. From previous testing procedures, the following probabilities are assumed to be known:**
$P(A$ **fails**$) = 0.2$, $P(B$ **fails alone**$) = 0.15$, $P(A$ **and** B **fail**$) = 0.15$.
Evaluate the following probabilities
(i) $P(A$ **fails** $\mid B$ **has failed**$)$ **(ii)** $P(A$ **fails alone**$)$.

SOLUTION

Concept used. Let A and B denote the failure events of the two subsystems. Given: $P(A) = 0.2$, $P(B \cap A') = P(\text{B fails alone}) = 0.15$, $P(A \cap B) = 0.15$.
We can now read off $P(B)$ from the decomposition $B = (B \cap A) \cup (B \cap A')$ (disjoint):

$$P(B) = P(A \cap B) + P(B \cap A') = 0.15 + 0.15 = 0.30.$$

Step 1. **(i)** $P(A \mid B)$:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.30} = \frac{1}{2}.$$

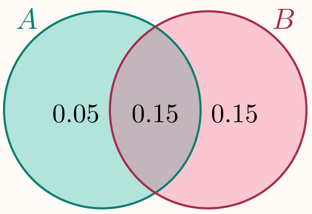
Step 2. **(ii)** $P(A$ **fails alone**$)$ **=** $P(A \cap B')$**.** Decompose $A = (A \cap B) \cup (A \cap B')$ (disjoint):

$$P(A \cap B') = P(A) - P(A \cap B) = 0.20 - 0.15 = 0.05.$$

Final Answer: (i) $\frac{1}{2}$ (ii) 0.05.

EXPERT'S SOLUTION : *Yash Chatterjee, M.Tech CS, IIT Madras*

Structural observation.



Step 1. A-only $= P(A) - P(A \cap B) = 0.20 - 0.15 = 0.05$. Tick.

Step 2. B-only $= P(B \cap A') = 0.15$ (given).

Step 3. Both $= 0.15$ (given).

Step 4. Therefore $P(B) = 0.15 + 0.15 = 0.30$ and $P(A \mid B) = 0.15/0.30 = 1/2$.

Why this matters. A Venn-diagram bookkeeping for joint failures keeps the four pieces (A-only, B-only, both, neither) straight; every probability in the problem then reads off as one or a sum of these pieces.

> **Final Answer:** $\frac{1}{2}$ and 0.05.

Q 13.10 **Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.**

SOLUTION

Concept used. Bayes' theorem on the colour of the *transferred* ball, given that the second draw was red.

Step 1. Events: T_R = transferred ball is red, T_B = transferred ball is black. R = ball drawn from Bag II is red.

Step 2. Priors (from Bag I): $P(T_R) = 3/7,\ P(T_B) = 4/7$.

Step 3. Likelihoods. After transfer, Bag II has 10 balls.
If T_R: Bag II contains $4 + 1 = 5$ red and 5 black. $P(R \mid T_R) = 5/10 = 1/2$.
If T_B: Bag II contains 4 red and $5 + 1 = 6$ black. $P(R \mid T_B) = 4/10 = 2/5$.

Step 4. Total probability:

$$P(R) = \frac{3}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{2}{5} = \frac{3}{14} + \frac{8}{35}.$$

Common denominator 70: $\frac{15}{70} + \frac{16}{70} = \frac{31}{70}$.

Step 5. Bayes':

$$P(T_B \mid R) = \frac{(4/7)(2/5)}{31/70} = \frac{8/35}{31/70} = \frac{8}{35} \times \frac{70}{31} = \frac{8 \cdot 2}{31} = \frac{16}{31}.$$

> **Final Answer:** $P(\text{transferred black} \mid \text{drew red}) = \frac{16}{31}$.

♥ **Two-stage experiment**
The transfer step changes Bag II's composition before the draw. Track both bags' compositions on each branch; Bayes' theorem then runs as usual.

EXPERT'S SOLUTION *: Rohit Nair, M.Sc Mathematics, IIT Bombay*

Strategic angle.

Step 1. Branch $T_R \to R$: $(3/7)(1/2) = 3/14 = 15/70$.

Step 2. Branch $T_B \to R$: $(4/7)(2/5) = 8/35 = 16/70$.

Step 3. Posterior for T_B: $16/(15+16) = 16/31$.

Why this matters. Notice $P(T_B \mid R) > P(T_B)$ would be a paradox here? Actually $P(T_B) = 4/7 \approx 0.571$ and $P(T_B \mid R) = 16/31 \approx 0.516$. Observing red slightly *reduces* the chance the transferred ball was black, exactly as intuition suggests.

Final Answer: $\dfrac{16}{31}$.

Q 13.11 **If A and B are two events such that $P(A) \neq 0$ and $P(B \mid A) = 1$, then**
(A) $A \subset B$ **(B)** $B \subset A$ **(C)** $B = \varnothing$ **(D)** $A = \varnothing$.

SOLUTION

Concept used. $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$. Setting this equal to 1 gives $P(A \cap B) = P(A)$.

Step 1. From $P(A \cap B) = P(A)$ and $A \cap B \subset A$, we conclude $A \cap B = A$ (a strict subset would have strictly smaller probability when $P(A) > 0$).

Step 2. $A \cap B = A$ is equivalent to $A \subset B$.

Step 3. So option (A).

Final Answer: Option (A): $A \subset B$.

EXPERT'S SOLUTION *: Aditya Pillai, M.Sc Mathematics, ISI Kolkata*

Quick reading. $P(B \mid A) = 1$ means B is certain to occur whenever A does, which is the set-theoretic statement $A \subset B$.

Step 1. $P(A \cap B) = P(A) \Rightarrow A \cap B = A \Rightarrow A \subset B$.

Why this matters. Conditional probability $= 1$ corresponds to set inclusion; conditional probability $= 0$ corresponds to disjointness. Memorise both directions.

Final Answer: (A) $A \subset B$.

Q 13.12 **If $P(A \mid B) > P(A)$, then which of the following is correct?**

(A) $P(B \mid A) < P(B)$ **(B)** $P(A \cap B) < P(A) \cdot P(B)$

(C) $P(B \mid A) > P(B)$ **(D)** $P(B \mid A) = P(B)$**.**

SOLUTION

Concept used. Conditional-probability definitions:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}, \quad P(B \mid A) = \frac{P(A \cap B)}{P(A)}.$$

Step 1. Multiply both sides of $P(A \mid B) > P(A)$ by $P(B)$ (positive): $P(A \cap B) > P(A)P(B)$.

Step 2. Divide both sides by $P(A)$ (positive): $\dfrac{P(A \cap B)}{P(A)} > P(B)$, i.e. $P(B \mid A) > P(B)$.

Step 3. This is option (C). Also, option (B) is the opposite inequality ($P(A \cap B) < P(A)P(B)$), so it is wrong. Options (A) and (D) are direct contradictions of the derived inequality.

Final Answer: Option (C): $P(B \mid A) > P(B)$.

♥ Symmetric "positive correlation"

The relation "A raises the probability of B" is symmetric: it is equivalent to "B raises the probability of A". Bayes' theorem is the bridge.

EXPERT'S SOLUTION *: Sneha Verma, Ph.D Mathematics, IIT Delhi*

Quick reading.

Step 1. $P(A \mid B) > P(A) \Leftrightarrow P(A \cap B) > P(A)P(B) \Leftrightarrow P(B \mid A) > P(B)$.

Step 2. Hence (C).

Why this matters. The three inequalities are mathematically equivalent; recognising this saves you from going down algebraic rabbit holes.

Final Answer: (C).

Q 13.13 **If A and B are any two events such that $P(A) + P(B) - P(A$ and $B) = P(A)$, then**

(A) $P(B \mid A) = 1$ **(B)** $P(A \mid B) = 1$

(C) $P(B \mid A) = 0$ **(D)** $P(A \mid B) = 0$.

SOLUTION

Concept used. The given equation is the addition theorem written as $P(A \cup B) = P(A)$.

Step 1. Recognise that $P(A) + P(B) - P(A \cap B) = P(A \cup B)$. The given equation therefore reduces to

$$P(A \cup B) = P(A).$$

Step 2. Combined with $A \subset A \cup B$, this forces $A \cup B = A$ (up to a null set), which means $B \subset A$.

Step 3. From $B \subset A$ we get $A \cap B = B$, so $P(A \cap B) = P(B)$.

Step 4. Compute $P(A \mid B)$:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1,$$

which is option (B).

Final Answer: Option (B): $P(A \mid B) = 1$.

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Quick reading.

Step 1. Rearrange $P(A) + P(B) - P(A \cap B) = P(A)$: subtract $P(A)$ to get $P(B) = P(A \cap B)$.

Step 2. Then $P(A \mid B) = P(A \cap B)/P(B) = P(B)/P(B) = 1$.

Step 3. Option (B).

Why this matters. The given algebraic identity is shorthand for $B \subset A$. Whenever $P(B) = P(A \cap B)$, B is entirely contained in A (up to probability zero) and conditioning on B makes A certain.

Final Answer: (B).

Key Takeaways

- Conditional probability of 1 means set inclusion; conditional probability of 0 means disjointness.
- $P(A \mid B) > P(A)$, $P(A \cap B) > P(A)P(B)$ and $P(B \mid A) > P(B)$ are three equivalent statements of positive association.
- For "at least one" / "at most k" counts in independent trials, switch to a binomial complement when the tail is short.
- Leap-year / weekday questions reduce to counting the surplus days after 52 complete weeks: 1 surplus day for a normal year, 2 for a leap year.
- Bayes' theorem with multiple hypotheses scales linearly: list priors, list likelihoods, multiply along each branch, then normalise.

End of Miscellaneous Exercise