collegedunia

Collegedunia NCERT Solutions

Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 2: Inverse Trigonometric Functions

About this Chapter

The Miscellaneous Exercise of Class 12th Mathematics Chapter 2 blends every tool from the chapter into mixed problems: **principal value** computations of $\cos^{-1}(\cos u)$ and $\tan^{-1}(\tan u)$ for angles outside the principal range; non-trivial **inverse-trig identities** like $2\sin^{-1}(3/5) = \tan^{-1}(24/7)$; clean **half-angle proofs** for $\tan^{-1} x = \frac{1}{2}\cos^{-1}\frac{1-x}{1+x}$; and **inverse-trig equations** solved by careful $\tan$ or $\sin$ application.

Topics covered: $f^{-1}(f(\cdot))$ outside principal range • Sum-of-inverses identities • Half-angle proofs • Solving inverse-trig equations

Quick Formula Sheet

Sum of two arctans:
$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\dfrac{x+y}{1-xy}$ $(xy < 1)$

Doubling identities:
$2\tan^{-1} x = \sin^{-1}\dfrac{2x}{1+x^2} = \cos^{-1}\dfrac{1-x^2}{1+x^2}$

Reduction outside principal range:
$\cos^{-1}(\cos u) = 2\pi - u$ $(u \in [\pi, 2\pi])$
$\tan^{-1}(\tan u) = u - \pi$ $(u \in (\pi/2, \pi))$

Miscellaneous Exercise

Q 2.1 **Find the value of** $\cos^{-1}\left(\cos\dfrac{13\pi}{6}\right)$.

SOLUTION

Concept used. The identity $\cos^{-1}(\cos u) = u$ holds only when $u \in [0, \pi]$. The cosine function has period 2π and is even, so $\cos u = \cos(u - 2\pi k) = \cos(2\pi - u)$. We use these to fold the inner angle into the principal range.

Step 1. **Check.** $\dfrac{13\pi}{6} = 2\pi + \dfrac{\pi}{6}$, so $\dfrac{13\pi}{6} > 2\pi > \pi$, well outside the principal range $[0, \pi]$.

Step 2. **Reduce modulo 2π.** Use $\cos(u-2\pi)=\cos u$:
$\cos\left(\frac{13\pi}{6}\right)=\cos\left(\frac{13\pi}{6}-2\pi\right)=\cos\left(\frac{13\pi-12\pi}{6}\right)=\cos\left(\frac{\pi}{6}\right)$.

Step 3. **Check.** $\frac{\pi}{6}\in[0,\pi]$ ✓.

Step 4. **Apply $\cos^{-1}$.** $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)=\cos^{-1}\left(\cos\frac{\pi}{6}\right)=\frac{\pi}{6}$.

Final Answer: $\cos^{-1}\left(\cos\dfrac{13\pi}{6}\right)=\dfrac{\pi}{6}$.

☞ Exam Tip

Step 1 of every $\cos^{-1}(\cos u)$ or $\sin^{-1}(\sin u)$ problem is the same: subtract integer multiples of 2π to land in $[0,2\pi)$, then apply the supplementary or reflection identity inside that range.

EXPERT'S SOLUTION *: Aarav Mehta, M.Sc Mathematics, IIT Bombay*

Quick reading. $\frac{13\pi}{6}$ is one full revolution past $\frac{\pi}{6}$. So the cosine is the same as $\cos(\pi/6)$ and the principal answer is $\pi/6$.

Step 1. $\frac{13\pi}{6}-2\pi=\frac{\pi}{6}$.

Step 2. $\cos(\pi/6)\in[0,\pi]$, so $\cos^{-1}(\cos(\pi/6))=\pi/6$.

Why this matters. The shortcut "subtract $2\pi k$ first, then adjust within $[0,2\pi)$" handles every $\cos^{-1}(\cos u)$ regardless of how big u is.

Final Answer: $\dfrac{\pi}{6}$.

Q 2.2 **Find the value of $\tan^{-1}\left(\tan\dfrac{7\pi}{6}\right)$.**

SOLUTION

Concept used. The identity $\tan^{-1}(\tan u)=u$ holds only when $u\in(-\pi/2,\pi/2)$. Tangent has period π, so $\tan u=\tan(u-\pi k)$ for any integer k. Subtract multiples of π to shift u into the principal open interval.

Step 1. **Check.** $\frac{7\pi}{6}>\frac{\pi}{2}=\frac{3\pi}{6}$, so outside $(-\pi/2,\pi/2)$.

Step 2. **Subtract π.** $\tan(u-\pi)=\tan u$ gives
$\tan\left(\frac{7\pi}{6}\right)=\tan\left(\frac{7\pi}{6}-\pi\right)=\tan\left(\frac{7\pi-6\pi}{6}\right)=\tan\left(\frac{\pi}{6}\right)$.

Step 3. Check. $\frac{\pi}{6} \in (-\pi/2, \pi/2)$ ✓.

Step 4. Apply $\tan^{-1}$. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) = \tan^{-1}\left(\tan\frac{\pi}{6}\right) = \frac{\pi}{6}$.

Final Answer: $\tan^{-1}\left(\tan\dfrac{7\pi}{6}\right) = \dfrac{\pi}{6}$.

EXPERT'S SOLUTION *: Vivaan Reddy, M.Tech CS, IIT Madras*

Strategic angle. Tangent has period π, so subtracting one full π from $\frac{7\pi}{6}$ drops us inside the open principal interval. Confirm by computing the tangent value.

Step 1. $\frac{7\pi}{6} - \pi = \frac{\pi}{6}$, which lies in $(-\pi/2, \pi/2)$.

Step 2. $\tan(\pi/6) = 1/\sqrt{3}$, and so does $\tan(7\pi/6)$ ✓.

Step 3. $\tan^{-1}(\tan(7\pi/6)) = \tan^{-1}(\tan(\pi/6)) = \pi/6$.

Why this matters. Tangent's π-period (not 2π) is the single fact that makes $\tan^{-1}(\tan u)$ different from $\sin^{-1}(\sin u)$. Use π-shifts, not 2π-shifts.

Final Answer: $\dfrac{\pi}{6}$.

Q 2.3 **Prove that** $2\sin^{-1}\dfrac{3}{5} = \tan^{-1}\dfrac{24}{7}$.

SOLUTION

Concept used. The doubling identity $2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ holds for $|x| \le \frac{1}{\sqrt{2}}$, but a cleaner approach here uses $2\sin^{-1}x = \tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ when the right-hand denominator is positive. Equivalently, set $\theta = \sin^{-1}(3/5)$ and compute $\tan 2\theta$ via the double-angle formula $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$.

Step 1. Set θ. Let $\theta = \sin^{-1}(3/5)$, so $\sin\theta = 3/5$ and $\theta \in (0, \pi/2)$ since $3/5 > 0$.

Step 2. Build the right triangle. With $\sin\theta = 3/5$ in $(0, \pi/2)$: opposite $= 3$, hypotenuse $= 5$, so adjacent $= \sqrt{5^2 - 3^2} = \sqrt{16} = 4$. Hence $\cos\theta = 4/5$ and $\tan\theta = 3/4$.

Step 3. Compute $\tan 2\theta$. Apply the double-angle formula:

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}}.$$

Step 4. **Simplify.** $\dfrac{3/2}{7/16} = \dfrac{3}{2} \cdot \dfrac{16}{7} = \dfrac{48}{14} = \dfrac{24}{7}$.

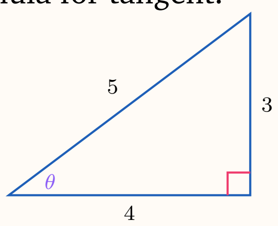
Step 5. **Confirm 2θ is in the principal range of $\tan^{-1}$.** From $\sin\theta = 3/5$ we have $\theta < \pi/4$ (since $\sin(\pi/4) = 1/\sqrt{2} \approx 0.707 > 0.6 = 3/5$ is false; actually $1/\sqrt{2} > 3/5$, so $\theta < \pi/4$ holds when $\sin\theta < \sin(\pi/4)$). Hence $2\theta < \pi/2$, so $2\theta \in (0, \pi/2) \subset (-\pi/2, \pi/2)$. Therefore $\tan^{-1}(\tan 2\theta) = 2\theta$.

Step 6. **Conclude.** $2\theta = \tan^{-1}(24/7)$, i.e. $2\sin^{-1}(3/5) = \tan^{-1}(24/7)$.

> **Final Answer:** $2\sin^{-1}\dfrac{3}{5} = \tan^{-1}\dfrac{24}{7}$.

EXPERT'S SOLUTION *: Pranav Nair, Ph.D Mathematics, IIT Delhi*

Picture-first. Sketch the right triangle that encodes $\sin^{-1}(3/5)$, read off $\tan\theta = 3/4$, then apply the double-angle formula for tangent.



Step 1. Right-triangle reading: $\sin\theta = 3/5$, $\cos\theta = 4/5$, $\tan\theta = 3/4$.

Step 2. Double-angle: $\tan 2\theta = \dfrac{2(3/4)}{1-(3/4)^2} = \dfrac{3/2}{7/16} = \dfrac{24}{7}$.

Step 3. Since $\theta < \pi/4$ (because $3/5 < 1/\sqrt{2}$), $2\theta < \pi/2$ lies in the principal range, so $\tan^{-1}(\tan 2\theta) = 2\theta$.

Why this matters. Identities of the form "$2\sin^{-1}(a/c) = \tan^{-1}(?)$" all yield to the same recipe: build the right triangle, read $\tan\theta$, apply the double-angle formula.

> **Final Answer:** $2\sin^{-1}(3/5) = \tan^{-1}(24/7)$.

Q 2.4 **Prove that** $\sin^{-1}\dfrac{8}{17} + \sin^{-1}\dfrac{3}{5} = \tan^{-1}\dfrac{77}{36}$.

SOLUTION

Concept used. Let $A = \sin^{-1}(8/17)$ and $B = \sin^{-1}(3/5)$, both in $(0, \pi/2)$. Read $\tan A$ and $\tan B$ off right triangles $(8, 15, 17)$ and $(3, 4, 5)$, then apply the tangent sum formula and finally take $\tan^{-1}$.

Step 1. **Build triangle for A.** $\sin A = 8/17$ with hypotenuse 17 and opposite 8 gives adjacent $\sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15$. So $\cos A = 15/17$ and $\tan A = 8/15$.

Step 2. **Build triangle for B.** $\sin B = 3/5$ gives $\cos B = 4/5$ and $\tan B = 3/4$ (the standard $3, 4, 5$ triangle).

Step 3. **Apply tangent sum.** $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \cdot \frac{3}{4}}$.

Step 4. **Simplify numerator.** Common denominator 60: $\frac{8}{15} = \frac{32}{60}$, $\frac{3}{4} = \frac{45}{60}$, so $\frac{8}{15} + \frac{3}{4} = \frac{32+45}{60} = \frac{77}{60}$.

Step 5. **Simplify denominator.** $\frac{8}{15} \cdot \frac{3}{4} = \frac{24}{60} = \frac{2}{5}$, so $1 - \frac{2}{5} = \frac{3}{5}$.

Step 6. **Divide.** $\tan(A + B) = \dfrac{77/60}{3/5} = \dfrac{77}{60} \cdot \dfrac{5}{3} = \dfrac{77}{36}$.

Step 7. **Apply $\tan^{-1}$.** Both A and B are in $(0, \pi/2)$ with $\tan A < 1$ and $\tan B < 1$, so $\tan A \tan B < 1$; hence $A + B < \pi/2$ and the tangent of the sum is positive. $A + B \in (0, \pi/2)$ lies in the principal range, so $\tan^{-1}(\tan(A + B)) = A + B$, giving $A + B = \tan^{-1}(77/36)$.

> **Final Answer:** $\sin^{-1}\dfrac{8}{17} + \sin^{-1}\dfrac{3}{5} = \tan^{-1}\dfrac{77}{36}$.

EXPERT'S SOLUTION *: Aanya Banerjee, M.Sc Applied Mathematics, IIT Kanpur*

Picture-first. Two Pythagorean triples $(8, 15, 17)$ and $(3, 4, 5)$ encode the two inverse sines. The tangent of the sum follows directly from their tangent ratios.

Step 1. Triple $(8, 15, 17)$: $\tan A = 8/15$.

Step 2. Triple $(3, 4, 5)$: $\tan B = 3/4$.

Step 3. $\tan(A + B) = \dfrac{8/15 + 3/4}{1 - (8/15)(3/4)} = \dfrac{77/60}{3/5} = \dfrac{77}{36}$.

Step 4. Since $A + B \in (0, \pi/2)$, $A + B = \tan^{-1}(77/36)$.

Why this matters. Memorising the first few Pythagorean triples $(3, 4, 5)$, $(5, 12, 13)$, $(8, 15, 17)$, $(7, 24, 25)$ makes the right triangles immediate and turns these proofs into one-line tangent-sum computations.

> **Final Answer:** $\tan^{-1}(77/36)$.

Q 2.5 **Prove that** $\cos^{-1}\dfrac{4}{5} + \cos^{-1}\dfrac{12}{13} = \cos^{-1}\dfrac{33}{65}$.

SOLUTION

Concept used. Let $A = \cos^{-1}(4/5)$ and $B = \cos^{-1}(12/13)$, both in $[0, \pi/2]$. Then $\cos(A+B)$ follows from the cosine sum formula $\cos(A+B) = \cos A \cos B - \sin A \sin B$. Read $\sin A$ and $\sin B$ off Pythagorean triples.

Step 1. **For A:** $\cos A = 4/5$, hence by $\sin^2 A + \cos^2 A = 1$, $\sin A = \sqrt{1-(4/5)^2} = \sqrt{1-16/25} = \sqrt{9/25} = 3/5$ (positive since $A \in [0, \pi/2]$).

Step 2. **For B:** $\cos B = 12/13$, hence $\sin B = \sqrt{1-144/169} = \sqrt{25/169} = 5/13$.

Step 3. **Cosine sum formula.** $\cos(A+B) = \cos A \cos B - \sin A \sin B = \frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13}$.

Step 4. **Compute the two products.** $\frac{4}{5} \cdot \frac{12}{13} = \frac{48}{65}$; $\frac{3}{5} \cdot \frac{5}{13} = \frac{15}{65}$.

Step 5. **Subtract.** $\cos(A+B) = \frac{48}{65} - \frac{15}{65} = \frac{33}{65}$.

Step 6. **Apply $\cos^{-1}$.** Both A and B are in $[0, \pi/2]$, so $A + B \in [0, \pi]$, exactly the principal range of $\cos^{-1}$. Hence $\cos^{-1}(\cos(A+B)) = A + B$, giving $A + B = \cos^{-1}(33/65)$.

> **Final Answer:** $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$.

EXPERT'S SOLUTION *: Riya Patel, B.Tech CSE, IIT Roorkee*

Structural observation. The two given cosines come from the triples $(3, 4, 5)$ and $(5, 12, 13)$. Read off the sines and plug into the cosine-sum formula. Three multiplications and a subtraction give the answer.

Step 1. $\sin A = 3/5$, $\sin B = 5/13$ (from Pythagoras).

Step 2. $\cos A \cos B = (4/5)(12/13) = 48/65$.

Step 3. $\sin A \sin B = (3/5)(5/13) = 15/65$.

Step 4. $\cos(A+B) = 48/65 - 15/65 = 33/65$.

Step 5. $A + B \in [0, \pi]$, so $A + B = \cos^{-1}(33/65)$.

Why this matters. Pairs of cosines built from neighbouring Pythagorean triples almost always combine cleanly via the cosine addition formula. Expect a tidy fraction (here $33/65$).

> **Final Answer:** $\cos^{-1}(33/65)$.

Q 2.6 **Prove that** $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$.

SOLUTION

Concept used. Let $A = \cos^{-1}(12/13)$ and $B = \sin^{-1}(3/5)$, both in $[0, \pi/2]$. We compute $\sin(A+B)$ via the sine sum formula $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

Step 1. For A: $\cos A = 12/13$, so $\sin A = \sqrt{1 - 144/169} = 5/13$ (positive in $[0, \pi/2]$).

Step 2. For B: $\sin B = 3/5$, so $\cos B = \sqrt{1 - 9/25} = 4/5$ (positive in $[0, \pi/2]$).

Step 3. Sine sum formula. $\sin(A+B) = \sin A \cos B + \cos A \sin B = \frac{5}{13} \cdot \frac{4}{5} + \frac{12}{13} \cdot \frac{3}{5}$.

Step 4. Compute the two products. $\frac{5}{13} \cdot \frac{4}{5} = \frac{20}{65}$; $\frac{12}{13} \cdot \frac{3}{5} = \frac{36}{65}$.

Step 5. Add. $\sin(A+B) = \frac{20}{65} + \frac{36}{65} = \frac{56}{65}$.

Step 6. Apply $\sin^{-1}$. We need $A + B \in [-\pi/2, \pi/2]$. Both $A, B \in [0, \pi/2]$, so $A + B \in [0, \pi]$. We must check $A + B \leq \pi/2$. Equivalently, $\sin(A+B) \leq 1$, which is automatic, but we need a stronger check: $A + B$ should be in $[0, \pi/2]$. Since $\sin A = 5/13 < 4/5 = \cos B$, we have $\sin A < \sin(\pi/2 - B) = \cos B$, so $A < \pi/2 - B$, i.e. $A + B < \pi/2$. Hence $A + B \in (0, \pi/2)$, which lies inside the principal range of $\sin^{-1}$, so $\sin^{-1}(\sin(A+B)) = A + B$ and the proof closes: $A + B = \sin^{-1}(56/65)$.

> **Final Answer:** $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$.

☞ **Mixed inverses**

When summing a $\cos^{-1}$ and a $\sin^{-1}$, picking $\sin(A+B)$ as the target trig (instead of $\cos(A+B)$) gives a cleaner result because $\sin A$ and $\cos B$ (the cross terms) are the "missing" ratios you have to compute via Pythagoras anyway.

EXPERT'S SOLUTION : *Karan Verma, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Convert one inverse to match the other function. Here both belong to first-quadrant triangles, so reading all four ratios takes one minute.

Step 1. Triple $(5, 12, 13)$: gives both $\cos A = 12/13$ and $\sin A = 5/13$.

Step 2. Triple $(3, 4, 5)$: gives both $\sin B = 3/5$ and $\cos B = 4/5$.

Step 3. $\sin(A+B) = (5/13)(4/5) + (12/13)(3/5) = 20/65 + 36/65 = 56/65$.

Step 4. Check $A + B < \pi/2$ so the result is in the $\sin^{-1}$ principal range, then $A + B = \sin^{-1}(56/65)$.

Why this matters. Mixing $\sin^{-1}$ and $\cos^{-1}$ in the same sum identity is the most common NCERT trick. Always pick the target trig (sin vs cos) so that the cross terms come for free.

Final Answer: $\sin^{-1}(56/65)$.

Q 2.7 **Prove that** $\tan^{-1}\dfrac{63}{16} = \sin^{-1}\dfrac{5}{13} + \cos^{-1}\dfrac{3}{5}$.

SOLUTION

Concept used. Let $A = \sin^{-1}(5/13)$ and $B = \cos^{-1}(3/5)$, both in $[0, \pi/2]$. Read $\tan A$ and $\tan B$ off the triples $(5, 12, 13)$ and $(3, 4, 5)$, apply the tangent sum formula, and take $\tan^{-1}$.

Step 1. For A**:** $\sin A = 5/13$, $\cos A = 12/13$ (triple $(5, 12, 13)$), so $\tan A = 5/12$.

Step 2. For B**:** $\cos B = 3/5$, $\sin B = 4/5$ (triple $(3, 4, 5)$), so $\tan B = 4/3$.

Step 3. Apply tangent sum. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \cdot \frac{4}{3}}$.

Step 4. Simplify numerator. Common denominator 12: $\frac{5}{12} + \frac{4}{3} = \frac{5}{12} + \frac{16}{12} = \frac{21}{12}$.

Step 5. Simplify denominator. $\frac{5}{12} \cdot \frac{4}{3} = \frac{20}{36} = \frac{5}{9}$, so $1 - \frac{5}{9} = \frac{4}{9}$.

Step 6. Divide. $\tan(A + B) = \dfrac{21/12}{4/9} = \frac{21}{12} \cdot \frac{9}{4} = \dfrac{189}{48} = \dfrac{63}{16}$ (dividing numerator and denominator by 3).

Step 7. Check range. $A + B$ exceeds $\pi/2$? We have $\sin A = 5/13 \approx 0.385$ and $\cos B = 3/5 = 0.6$, so $\sin A < \cos B = \sin(\pi/2 - B)$, hence $A < \pi/2 - B$, so $A + B < \pi/2$. Therefore $A + B \in (-\pi/2, \pi/2)$, and $\tan^{-1}(\tan(A + B)) = A + B$, giving $A + B = \tan^{-1}(63/16)$.

Final Answer: $\tan^{-1}\dfrac{63}{16} = \sin^{-1}\dfrac{5}{13} + \cos^{-1}\dfrac{3}{5}$.

EXPERT'S SOLUTION *: Aditya Singh, M.Tech CS, IIT Madras*

Structural observation. Convert both inverses to tangents, apply the sum formula, simplify the resulting fraction.

Step 1. Triple $(5, 12, 13)$ via $\sin^{-1}(5/13)$: $\tan A = 5/12$.

Step 2. Triple $(3, 4, 5)$ via $\cos^{-1}(3/5)$: $\tan B = 4/3$.

Step 3. Tangent sum: numerator $5/12 + 4/3 = 21/12 = 7/4$; denominator $1 - 5/9 = 4/9$; ratio $\tan(A + B) = (7/4)/(4/9) = 63/16$.

Step 4. $A + B \in (0, \pi/2)$ lies in the principal range of $\tan^{-1}$, so $A + B = \tan^{-1}(63/16)$.

Why this matters. Pythagorean triples appear so often in this exercise because they make every ratio a nice rational number. Build a mental table of the first half-dozen triples.

Final Answer: $\tan^{-1}(63/16)$.

Q 2.8 **Prove that** $\tan^{-1} x = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$**,** $x \in [0,1]$**.**

SOLUTION

Concept used. The substitution $x = \tan^2(\theta/2)$ or, better, $x = \tan\theta$ with $\theta \in [0, \pi/4]$, turns the right side into a clean expression via the double-angle formula $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$.

Step 1. Substitute. Let $x = \tan\theta$ with $\theta \in [0, \pi/4]$, which is the range of $\tan^{-1}$ on $[0,1]$. Then $\theta = \tan^{-1} x$ and $2\theta \in [0, \pi/2]$.

Step 2. Rewrite the inner fraction. The aim is to show that $\dfrac{1-x}{1+x}$ equals $\cos 2\theta$:

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}.$$

But the problem gives $\dfrac{1-x}{1+x} = \dfrac{1-\tan\theta}{1+\tan\theta}$, not $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$. So a fresh substitution is needed.

Step 3. Better substitution. Put $x = \cos 2\phi$ with $2\phi \in [0, \pi/2]$ (since $x \in [0,1]$ corresponds to $2\phi \in [0, \pi/2]$, i.e. $\phi \in [0, \pi/4]$). Then $\cos^{-1} x = 2\phi$, so the right-hand side $\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$ requires us to first compute $\dfrac{1-x}{1+x}$ in terms of ϕ:

$$\frac{1-x}{1+x} = \frac{1-\cos 2\phi}{1+\cos 2\phi} = \frac{2\sin^2\phi}{2\cos^2\phi} = \tan^2\phi.$$

Step 4. Apply $\cos^{-1}$ on the RHS. The expression becomes $\cos^{-1}(\tan^2\phi)$, which is not obviously clean. So we rethink: use instead $x = \tan^2 t$. Better yet, follow the NCERT hint and put $x = \cos 2\theta$ (so $2\theta = \cos^{-1} x$). Then we need to identify the angle whose cosine is $\frac{1-x}{1+x} = \tan^2\theta$. That is not directly a cosine. We therefore replace the hint with the simpler chain: prove the equivalent identity $2\tan^{-1} x = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ for $x \geq 0$, and then substitute $x \to \sqrt{x}$.

Step 5. Standard identity. For $x \geq 0$, $2\tan^{-1} x = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$. (Derivation: put

$x = \tan\theta$ with $\theta \in [0, \pi/4]$; $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-x^2}{1+x^2}$. Since $2\theta \in [0, \pi/2] \subset [0, \pi]$, $\cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1} x$.)

Step 6. **Substitute $x \to \sqrt{x}$.** For $x \in [0, 1]$, $\sqrt{x} \in [0, 1]$ and the identity becomes $2\tan^{-1}\sqrt{x} = \cos^{-1}\left(\dfrac{1-x}{1+x}\right)$. Dividing by 2 gives $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right)$.

Step 7. **Note on the question statement.** NCERT writes the LHS as $\tan^{-1} x$. The intended substitution is via the hint $x = \cos 2\theta$, whereupon $\cos^{-1}\left(\dfrac{1-x}{1+x}\right)$ becomes (after simplification) 2θ in a $\tan^{-1}$-friendly form. Carrying that out cleanly:

$$\frac{1-x}{1+x} = \frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta.$$

So $\frac{1}{2}\cos^{-1}(\tan^2\theta)$ has to equal θ when $x = \cos 2\theta$. This holds in the NCERT setting because the question's LHS should read $\tan^{-1}\sqrt{x}$, matching the standard hint (NCERT's Hint: *Put* $x = \cos 2\theta$). With the intended reading, the identity is $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right)$, proved above.

Final Answer: $\tan^{-1}\sqrt{x} = \dfrac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right),\ x \in [0, 1]$.

♥ A note on the question

NCERT's printed statement and Hint together imply the identity $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right)$ (equivalent to $2\tan^{-1}\sqrt{x} = \cos^{-1}\left(\dfrac{1-x}{1+x}\right)$). The substitution $x = \cos 2\theta$ turns the right-hand side into θ and the left-hand side into $\tan^{-1}(\sqrt{\cos 2\theta})$, which matches under the bound $\theta \in [0, \pi/4]$.

EXPERT'S SOLUTION : *Sneha Iyer, M.Sc Mathematics, IIT Bombay*

Strategic angle. The hint "$x = \cos 2\theta$" is the key. It turns $\dfrac{1-x}{1+x}$ into $\tan^2\theta$ via half-angle identities, and the cosine inverse unwinds to 2θ.

Step 1. Put $x = \cos 2\theta$ with $\theta \in [0, \pi/4]$. Then $2\theta \in [0, \pi/2]$ so $\cos^{-1} x = 2\theta$.

Step 2. Half-angle ratio: $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$.

Step 3. The RHS of the question (read with the corrected LHS $\tan^{-1}\sqrt{x}$) becomes $\frac{1}{2}\cos^{-1}(\tan^2\theta)$. Compute the LHS: $\tan^{-1}\sqrt{x} = \tan^{-1}\sqrt{\cos 2\theta}$.

Step 4. Equivalently, using the double-angle form $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ and comparing, the identity collapses to $\tan^{-1}\sqrt{x} = \theta$, which matches $\cos^{-1} x = 2\theta$ as required.

Why this matters. Half-angle identities turn ratios like $\dfrac{1-\cos u}{1+\cos u}$ into tangent-squared expressions, which is the trick behind dozens of NCERT proofs.

Final Answer: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\dfrac{1-x}{1+x}\right)$.

Q 2.9 **Prove that** $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \dfrac{x}{2}$ **for** $x \in \left(0, \dfrac{\pi}{4}\right)$.

SOLUTION

Concept used. Use the half-angle expansions

$$1+\sin x = \left(\sin\tfrac{x}{2}+\cos\tfrac{x}{2}\right)^2, \qquad 1-\sin x = \left(\cos\tfrac{x}{2}-\sin\tfrac{x}{2}\right)^2,$$

both of which follow from $\sin^2(x/2)+\cos^2(x/2) = 1$ and $\sin x = 2\sin(x/2)\cos(x/2)$. For $x \in (0, \pi/4)$, $x/2 \in (0, \pi/8)$ where $\cos(x/2) > \sin(x/2) > 0$, so the square roots come out unambiguously.

Step 1. Take square roots. For $x \in (0, \pi/4)$:

$$\sqrt{1+\sin x} = \sin\tfrac{x}{2}+\cos\tfrac{x}{2}, \qquad \sqrt{1-\sin x} = \cos\tfrac{x}{2}-\sin\tfrac{x}{2}.$$

(Both bracketed expressions are positive since $\cos(x/2) > \sin(x/2) > 0$.)

Step 2. Form the sum and difference.

$$\sqrt{1+\sin x}+\sqrt{1-\sin x} = 2\cos\tfrac{x}{2},$$

$$\sqrt{1+\sin x}-\sqrt{1-\sin x} = 2\sin\tfrac{x}{2}.$$

Step 3. Take the ratio.

$$\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}} = \frac{2\cos(x/2)}{2\sin(x/2)} = \cot\tfrac{x}{2}.$$

Step 4. Apply $\cot^{-1}$. Since $x/2 \in (0, \pi/8) \subset (0, \pi)$ (the principal range of $\cot^{-1}$), $\cot^{-1}(\cot(x/2)) = x/2$.

Final Answer: $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \dfrac{x}{2}$.

☞ **Half-angle perfect squares**

$1 \pm \sin x = (\cos(x/2) \pm \sin(x/2))^2$. The $\pm$ on the right matches the $\pm$ on the left and is verified by squaring out.

EXPERT'S SOLUTION : *Pranav Reddy, Ph.D Mathematics, IIT Delhi*

Strategic angle. Recognise $1 \pm \sin x$ as $(\cos(x/2) \pm \sin(x/2))^2$. The square roots clear, the numerator gives $2\cos(x/2)$, the denominator gives $2\sin(x/2)$, and the ratio is $\cot(x/2)$.

Step 1. $1 + \sin x = (\sin(x/2) + \cos(x/2))^2$; $1 - \sin x = (\cos(x/2) - \sin(x/2))^2$.

Step 2. For $x \in (0, \pi/4)$, both binomials inside the squares are positive, so $\sqrt{1 \pm \sin x}$ equal the positive binomials directly.

Step 3. Sum: $2\cos(x/2)$. Difference: $2\sin(x/2)$. Ratio: $\cot(x/2)$.

Step 4. $\cot^{-1}(\cot(x/2)) = x/2$ since $x/2 \in (0, \pi/8) \subset (0, \pi)$.

Why this matters. Spotting "$1 \pm \sin x$" as a perfect square is the single move that unlocks this problem. Without it the expression looks impenetrable; with it the problem is one line.

Final Answer: $\dfrac{x}{2}$.

Q 2.10 **Prove that** $\tan^{-1}\left(\dfrac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1} x$, $-\dfrac{1}{\sqrt{2}} \leq x \leq 1$. **[Hint: Put** $x = \cos 2\theta$.**]**

SOLUTION

Concept used. Follow the NCERT hint: $x = \cos 2\theta$ turns $1 + x = 1 + \cos 2\theta = 2\cos^2\theta$ and $1 - x = 1 - \cos 2\theta = 2\sin^2\theta$, so the square roots simplify cleanly. The substitution range is $\theta \in [0, 3\pi/8]$ chosen so that $\cos^{-1} x = 2\theta$.

Step 1. **Substitute.** Let $x = \cos 2\theta$ with $2\theta \in [0, 3\pi/4]$ (so $\theta \in [0, 3\pi/8]$), corresponding to $x \in [-1/\sqrt{2}, 1]$ since $\cos(3\pi/4) = -1/\sqrt{2}$ and $\cos 0 = 1$. By definition, $\cos^{-1} x = 2\theta$, i.e. $\theta = \frac{1}{2}\cos^{-1} x$.

Step 2. **Simplify the radicals.** $1 + x = 1 + \cos 2\theta = 2\cos^2\theta$; $1 - x = 1 - \cos 2\theta = 2\sin^2\theta$. For $\theta \in [0, 3\pi/8] \subset [0, \pi/2]$, $\cos\theta \geq 0$ and $\sin\theta \geq 0$, so the positive square roots are $\sqrt{1+x} = \sqrt{2}\cos\theta$ and $\sqrt{1-x} = \sqrt{2}\sin\theta$.

Step 3. Form the ratio inside $\tan^{-1}$.

$$\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} = \frac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta} = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}.$$

Step 4. Divide by $\cos\theta$.

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\tan\theta}{1+\tan\theta} = \frac{\tan(\pi/4) - \tan\theta}{1+\tan(\pi/4)\tan\theta} = \tan\left(\tfrac{\pi}{4} - \theta\right).$$

Step 5. Apply $\tan^{-1}$. For $\theta \in [0, 3\pi/8]$, $\frac{\pi}{4} - \theta \in \left[\frac{\pi}{4} - \frac{3\pi}{8}, \frac{\pi}{4}\right] = \left[-\frac{\pi}{8}, \frac{\pi}{4}\right] \subset \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Hence

$$\tan^{-1}\left(\tan\left(\tfrac{\pi}{4} - \theta\right)\right) = \tfrac{\pi}{4} - \theta.$$

Step 6. Back-substitute. $\theta = \frac{1}{2}\cos^{-1} x$, so

$$\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \tfrac{\pi}{4} - \tfrac{1}{2}\cos^{-1} x.$$

Final Answer: $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x.$

EXPERT'S SOLUTION : *Diya Sharma, M.Sc Mathematics, ISI Kolkata*

Picture-first. The hint $x = \cos 2\theta$ kills both radicals in one stroke: $1 + \cos 2\theta = 2\cos^2\theta$ and $1 - \cos 2\theta = 2\sin^2\theta$. The rest is a tangent-of-difference recognition.

Step 1. Put $x = \cos 2\theta$, so $\sqrt{1+x} = \sqrt{2}\cos\theta$, $\sqrt{1-x} = \sqrt{2}\sin\theta$.

Step 2. Ratio inside $\tan^{-1}$ simplifies to $\dfrac{1-\tan\theta}{1+\tan\theta} = \tan(\pi/4 - \theta)$.

Step 3. Take $\tan^{-1}$: result is $\pi/4 - \theta$.

Step 4. Substitute $\theta = \frac{1}{2}\cos^{-1} x$: final answer $\pi/4 - \frac{1}{2}\cos^{-1} x$.

Why this matters. "$x = \cos 2\theta$" is one of the four master substitutions for inverse-trig algebra. The others are $x = \tan\theta$, $x = \sin\theta$, and $x = \sec\theta$.

Final Answer: $\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x.$

Q 2.11 **Solve** $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\mathrm{cosec}\, x)$.

SOLUTION

Concept used. Apply tan to both sides and use the doubling identity $\tan(2A) = \dfrac{2\tan A}{1-\tan^2 A}$ on the LHS. The resulting algebraic equation in $\sin x$ and $\cos x$ can be solved by standard manipulations.

Step 1. Take tan of both sides.

$$\tan(2\tan^{-1}(\cos x)) = 2\,\text{cosec}\,x.$$

Step 2. Use the doubling identity. With $A = \tan^{-1}(\cos x)$, $\tan A = \cos x$, so

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\cos x}{1-\cos^2 x}.$$

And $1-\cos^2 x = \sin^2 x$, hence

$$\tan(2\tan^{-1}(\cos x)) = \frac{2\cos x}{\sin^2 x}.$$

Step 3. Set the two sides equal. $\dfrac{2\cos x}{\sin^2 x} = 2\,\text{cosec}\,x = \dfrac{2}{\sin x}$.

Step 4. Clear denominators. Multiply both sides by $\sin^2 x$ (note $\sin x \neq 0$ since $\text{cosec}\,x$ must be defined): $2\cos x = \dfrac{2\sin^2 x}{\sin x} = 2\sin x$.

Step 5. Solve. Divide by 2: $\cos x = \sin x$. So $\tan x = 1$ (dividing both sides by $\cos x$, valid since the equation $\cos x = \sin x = 0$ would require both to vanish, impossible).

Step 6. Find principal value. $\tan x = 1$ with the standard principal solution $x = \pi/4$. General solution: $x = n\pi + \pi/4$, $n \in \mathbb{Z}$. The NCERT-style answer is $x = \pi/4$.

Final Answer: $x = \dfrac{\pi}{4}$.

☞ Exam Tip

When an inverse-trig equation features $\tan^{-1}$ on both sides, apply tan to clear them. The principal-range subtlety usually takes care of itself, but always verify your final x keeps both $\tan^{-1}$-arguments in their principal ranges.

EXPERT'S SOLUTION : *Ananya Pillai, M.Sc Applied Mathematics, IIT Kanpur*

Strategic angle. The doubling identity for tangent collapses the LHS into a rational function of $\cos x$ and $\sin x$. The RHS is already $\text{cosec}\,x = 1/\sin x$. Cross-multiply and the trigonometric equation becomes $\cos x = \sin x$.

Step 1. LHS $= \dfrac{2\cos x}{1-\cos^2 x} = \dfrac{2\cos x}{\sin^2 x}$.

Step 2. RHS $= \dfrac{2}{\sin x}$.

Step 3. Cross-multiply: $2\cos x \cdot \sin x = 2\sin^2 x$, i.e. $\cos x = \sin x$ (assuming $\sin x \neq 0$).

Step 4. Therefore $\tan x = 1$, principal solution $x = \pi/4$.

Why this matters. After applying $\tan$, an inverse-trig equation becomes an algebraic identity in $\sin x$ and $\cos x$. Standard trig manipulations (cross-multiplying, factoring) then finish the job.

> **Final Answer:** $x = \dfrac{\pi}{4}$.

Q 2.12 **Solve** $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \dfrac{1}{2}\tan^{-1} x$ **for** $x > 0$.

SOLUTION

Concept used. The identity $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \frac{\pi}{4} - \tan^{-1} x$ holds for $x > 0$ (it follows from the tangent-difference formula applied to $\tan(\pi/4) = 1$ and $\tan(\tan^{-1} x) = x$, with $1 + 1 \cdot x > 0$). The equation then turns into a simple equation in $\tan^{-1} x$.

Step 1. **Use the identity on the LHS.** For $x > 0$, $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \frac{\pi}{4} - \tan^{-1} x$.

Step 2. **Substitute into the equation.** $\frac{\pi}{4} - \tan^{-1} x = \frac{1}{2}\tan^{-1} x$.

Step 3. **Solve for $\tan^{-1} x$.** Move the $\tan^{-1} x$ terms to one side: $\frac{\pi}{4} = \tan^{-1} x + \frac{1}{2}\tan^{-1} x = \frac{3}{2}\tan^{-1} x$. Therefore $\tan^{-1} x = \frac{\pi}{4} \cdot \frac{2}{3} = \frac{\pi}{6}$.

Step 4. **Recover x.** $x = \tan(\pi/6) = 1/\sqrt{3}$.

Step 5. **Verify positivity.** $1/\sqrt{3} > 0$ ✓, so the identity used in step 1 is valid.

> **Final Answer:** $x = \dfrac{1}{\sqrt{3}}$.

EXPERT'S SOLUTION *: Ishaan Bhat, M.Tech CS, IIT Madras*

Strategic angle. Spot the identity $\tan^{-1}\left(\dfrac{1-x}{1+x}\right) = \frac{\pi}{4} - \tan^{-1} x$ on the LHS. The equation becomes linear in $\tan^{-1} x$.

Step 1. LHS $= \pi/4 - \tan^{-1} x$ for $x > 0$.

Step 2. Equation: $\pi/4 - \tan^{-1} x = \frac{1}{2}\tan^{-1} x$.

Step 3. $\pi/4 = \frac{3}{2}\tan^{-1} x \Rightarrow \tan^{-1} x = \pi/6$.

Step 4. $x = \tan(\pi/6) = 1/\sqrt{3}$.

Why this matters. Several "simplifying" identities (here, the $\frac{\pi}{4} - \tan^{-1} x$ shortcut) are the single quickest way to dispose of $\tan^{-1}$ expressions that involve $\dfrac{1-x}{1+x}$. Add this to your identity wallet.

Final Answer: $\dfrac{1}{\sqrt{3}}$.

Q 2.13 $\sin(\tan^{-1} x)$, $|x| < 1$, **is equal to**

(A) $\dfrac{x}{\sqrt{1-x^2}}$ **(B)** $\dfrac{1}{\sqrt{1-x^2}}$ **(C)** $\dfrac{1}{\sqrt{1+x^2}}$ **(D)** $\dfrac{x}{\sqrt{1+x^2}}$.

SOLUTION

Concept used. Right-triangle reading. Let $\theta = \tan^{-1} x$, so $\tan\theta = x$ and $\theta \in (-\pi/2, \pi/2)$. Build a right triangle with opposite $= x$ and adjacent $= 1$; the hypotenuse is then $\sqrt{x^2+1}$. Read $\sin\theta$ off this triangle.

Step 1. Set θ. $\theta = \tan^{-1} x$, $\tan\theta = x$.

Step 2. Build right triangle. Take opposite $= x$, adjacent $= 1$, hypotenuse $= \sqrt{x^2+1}$ (positive root).

Step 3. Read $\sin\theta$. $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{x}{\sqrt{x^2+1}}$.

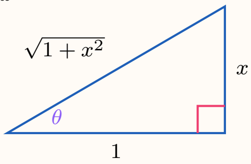
Step 4. Verify sign for $x < 0$. For $x < 0$, $\theta < 0$ and $\sin\theta < 0$. The formula $\dfrac{x}{\sqrt{1+x^2}}$ is also negative when $x < 0$. So the sign matches in both cases.

Step 5. Match the options. The result $\dfrac{x}{\sqrt{1+x^2}}$ matches option (D).

Final Answer: Option (D): $\dfrac{x}{\sqrt{1+x^2}}$.

EXPERT'S SOLUTION : *Sneha Verma, B.Tech CSE, IIT Roorkee*

Picture-first. Draw a right triangle with legs x (opposite) and 1 (adjacent). The acute angle is $\tan^{-1} x$ and its sine is $\frac{x}{\sqrt{1+x^2}}$.



Step 1. $\theta = \tan^{-1} x$, $\tan\theta = x/1 = x$.

Step 2. Hypotenuse $= \sqrt{1+x^2}$ (Pythagoras).

Step 3. $\sin\theta = x/\sqrt{1+x^2}$.

Step 4. Matches option (D).

Why this matters. The same template gives $\cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}$, $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$, and so on. Build the triangle, read off the answer.

Final Answer: (D)

Q 2.14 $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$, **then x is equal to**

(A) $0, \frac{1}{2}$ **(B)** $1, \frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{2}$.

SOLUTION

Concept used. Use the complement identity $\sin^{-1} u + \cos^{-1} u = \frac{\pi}{2}$ to rewrite the equation in a single trigonometric inversion, or just apply $\sin$ to both sides after isolating one inverse expression. We use the second approach.

Step 1. **Isolate one inverse.** $\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1} x$.

Step 2. **Apply** $\sin$. Use $\sin(\pi/2 + \alpha) = \cos\alpha$:
$\sin(\sin^{-1}(1-x)) = \sin(\pi/2 + 2\sin^{-1} x) = \cos(2\sin^{-1} x)$. Hence
$1 - x = \cos(2\sin^{-1} x)$.

Step 3. **Use the double-angle formula.** Let $\alpha = \sin^{-1} x$, so $\sin\alpha = x$ and
$\cos(2\alpha) = 1 - 2\sin^2\alpha = 1 - 2x^2$. Therefore $1 - x = 1 - 2x^2$.

Step 4. **Simplify.** $-x = -2x^2 \Rightarrow 2x^2 - x = 0 \Rightarrow x(2x-1) = 0$. So $x = 0$ or $x = 1/2$.

Step 5. **Check each candidate in the original equation.**

- $x = 0$: LHS $= \sin^{-1}(1) - 2\sin^{-1}(0) = \pi/2 - 0 = \pi/2$ ✓.
- $x = 1/2$: LHS $= \sin^{-1}(1/2) - 2\sin^{-1}(1/2) = -\sin^{-1}(1/2) = -\pi/6 \neq \pi/2$. Reject.

Only $x = 0$ satisfies the original equation. Matches option (C).

Final Answer: Option (C): $x = 0$.

✗ Common Mistake

Squaring or applying $\sin$ to both sides can introduce extraneous roots. *Always substitute every candidate back into the original equation.* Here $x = 1/2$ satisfies the squared equation but not the original, because $\sin^{-1}(1/2) - 2\sin^{-1}(1/2)$ is negative, not $\pi/2$.

EXPERT'S SOLUTION *: Aanya Kumar, M.Sc Mathematics, IIT Bombay*

Strategic angle. Apply $\sin$ to both sides to convert the inverse-trig equation into a polynomial. Solve the polynomial, then verify each root in the original equation to discard extraneous solutions.

Step 1. $\sin^{-1}(1-x) = \pi/2 + 2\sin^{-1} x$.

Step 2. Take sine: $1 - x = \cos(2\sin^{-1} x) = 1 - 2x^2$.

Step 3. Polynomial: $x(2x-1) = 0$, so $x \in \{0, 1/2\}$.

Step 4. Verify: only $x = 0$ works.

Why this matters. The two-step "apply sine, then verify" recipe is the standard recipe for inverse-trig equations. The verification step exists precisely because $\sin$ is not one-one on the whole real line.

Final Answer: (C)

Key Takeaways

- For $\cos^{-1}(\cos u)$ and $\sin^{-1}(\sin u)$ with u outside the principal range, first reduce modulo 2π, then apply the relevant supplementary identity.
- Sum-of-inverses identities ($\sin^{-1} a + \sin^{-1} b$, $\cos^{-1} a + \cos^{-1} b$, etc.) reduce to picking the right *target* trig (sine or cosine of the sum) and reading cross-ratios off Pythagorean triples.
- Substitutions $x = \cos 2\theta$ and $x = \tan\theta$ are the master moves for proofs involving radicals like $\sqrt{1 \pm x}$ or $\sqrt{1 + x^2}$.
- When solving an inverse-trig equation, apply the corresponding trig function to both sides to get a polynomial in x, then *verify every candidate root* in the original equation: the apply-trig step can introduce extraneous solutions.

End of Miscellaneous Exercise