Collegedunia NCERT Solutions

Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 5: Continuity and Differentiability

About this Chapter

Exercise 5.3 covers **implicit differentiation** and **derivatives of inverse trigonometric functions**. When y is not explicit in x, differentiate both sides of the equation with respect to x, treat y as a function of x, then solve for dy/dx. Several questions disguise inverse-trig expressions like $\sin^{-1}(\frac{2x}{1+x^2})$ which reduce dramatically after a substitution $x = \tan\theta$.

Topics covered: Implicit differentiation • Derivatives of $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ • Trig substitutions • Domain restrictions

Quick Formula Sheet

Inverse trig derivatives:

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}.$$

Implicit step:

Differentiate both sides of $F(x,y) = 0$ w.r.t. x (treat y as $y(x)$) and solve for dy/dx.

Useful identity:

Putting $x = \tan\theta$ turns $\frac{2x}{1+x^2}$ into $\sin 2\theta$, and $\frac{1-x^2}{1+x^2}$ into $\cos 2\theta$.

Exercise 5.3

Q 5.1 **Find** $\frac{dy}{dx}$ **if** $2x + 3y = \sin x$.

SOLUTION

Concept used. **Implicit differentiation.** Differentiate both sides of the equation with respect to x, treating y as a function of x.

Step 1. Differentiate both sides:

$$\frac{d}{dx}(2x + 3y) = \frac{d}{dx}(\sin x).$$

Step 2. Compute term-by-term:

$$2 + 3\frac{dy}{dx} = \cos x.$$

Step 3. Solve for $\frac{dy}{dx}$:

$$3\frac{dy}{dx} = \cos x - 2 \implies \frac{dy}{dx} = \frac{\cos x - 2}{3}.$$

Final Answer: $\frac{dy}{dx} = \frac{\cos x - 2}{3}$.

EXPERT'S SOLUTION *: Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Quick reading. The equation is linear in y; explicit solve is also fast.

Step 1. Solve for y: $y = (\sin x - 2x)/3$.

Step 2. Differentiate: $dy/dx = (\cos x - 2)/3$.

Final Answer: $(\cos x - 2)/3$.

Q 5.2 **Find $\frac{dy}{dx}$ if $2x + 3y = \sin y$.**

SOLUTION

Concept used. Implicit differentiation; the right side involves y, so use chain rule $\frac{d}{dx}\sin y = \cos y \cdot \frac{dy}{dx}$.

Step 1. Differentiate both sides:

$$2 + 3\frac{dy}{dx} = \cos y \cdot \frac{dy}{dx}.$$

Step 2. Group dy/dx terms:

$$3\frac{dy}{dx} - \cos y \cdot \frac{dy}{dx} = -2 \implies \frac{dy}{dx}(3 - \cos y) = -2.$$

Step 3. Solve:

$$\frac{dy}{dx} = \frac{-2}{3 - \cos y} = \frac{2}{\cos y - 3}.$$

Final Answer: $\frac{dy}{dx} = \frac{2}{\cos y - 3}$.

EXPERT'S SOLUTION : *Sneha Iyer, Ph.D Mathematics, IIT Delhi*

Structural observation. Cannot solve for y explicitly; implicit differentiation is mandatory.

Step 1. LHS derivative: $2 + 3y'$.

Step 2. RHS derivative: $\cos y \cdot y'$.

Step 3. Solve: $y'(3 - \cos y) = -2 \Rightarrow y' = 2/(\cos y - 3)$.

Final Answer: $2/(\cos y - 3)$.

Q 5.3 **Find $\frac{dy}{dx}$ if $ax + by^2 = \cos y$.**

SOLUTION

Concept used. Implicit differentiation; $\frac{d}{dx}y^2 = 2y \cdot \frac{dy}{dx}$ by chain rule.

Step 1. Differentiate both sides:

$$a + 2by\frac{dy}{dx} = -\sin y \cdot \frac{dy}{dx}.$$

Step 2. Group dy/dx:

$$2by\frac{dy}{dx} + \sin y \cdot \frac{dy}{dx} = -a \implies \frac{dy}{dx}(2by + \sin y) = -a.$$

Step 3. Solve:

$$\frac{dy}{dx} = \frac{-a}{2by + \sin y}.$$

Final Answer: $\frac{dy}{dx} = -\frac{a}{2by + \sin y}$.

EXPERT'S SOLUTION : *Vivaan Gupta, M.Tech CS, IIT Madras*

Quick reading. Both sides involve y, so collect y' after differentiating.

Step 1. Differentiate: $a + 2by\,y' = -\sin y \cdot y'$.

Step 2. Group: $y'(2by + \sin y) = -a$.

Step 3. $y' = -a/(2by + \sin y)$.

Final Answer: $-a/(2by + \sin y)$.

Q 5.4 **Find $\frac{dy}{dx}$ if $xy + y^2 = \tan x + y$.**

SOLUTION

Concept used. Implicit differentiation; use product rule on xy.

Step 1. Differentiate both sides w.r.t. x:

$$\frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = \frac{d}{dx}(\tan x) + \frac{d}{dx}(y).$$

Using product rule on xy:

$$y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}.$$

Step 2. Group dy/dx:

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} = \sec^2 x - y \implies \frac{dy}{dx}(x + 2y - 1) = \sec^2 x - y.$$

Step 3. Solve:

$$\frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}.$$

Final Answer: $\frac{dy}{dx} = \frac{\sec^2 x - y}{x + 2y - 1}$.

EXPERT'S SOLUTION *: Aanya Mehta, Ph.D Pure Mathematics, IISc Bangalore*

Structural observation. Product rule on xy produces both an $x\,y'$ term and a stand-alone y.

Step 1. $y + xy' + 2yy' = \sec^2 x + y'$.

Step 2. $y'(x + 2y - 1) = \sec^2 x - y$.

Step 3. $y' = (\sec^2 x - y)/(x + 2y - 1)$.

Final Answer: $(\sec^2 x - y)/(x + 2y - 1)$.

Q 5.5 **Find $\frac{dy}{dx}$ if $x^2 + xy + y^2 = 100$.**

SOLUTION

Concept used. Implicit differentiation with product rule on xy.

Step 1. Differentiate both sides:

$$2x + \frac{d}{dx}(xy) + 2y\frac{dy}{dx} = 0.$$

Step 2. Product rule on xy:

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0.$$

Step 3. Group dy/dx:

$$(x + 2y)\frac{dy}{dx} = -(2x + y).$$

Step 4. Solve:

$$\frac{dy}{dx} = -\frac{2x + y}{x + 2y}.$$

Final Answer: $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$.

EXPERT'S SOLUTION *: Priya Singh, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading. Symmetric expression; differentiated curve is the ellipse-like $x^2 + xy + y^2 = 100$.

Step 1. Differentiate: $2x + y + (x + 2y)y' = 0$.

Step 2. $y' = -(2x + y)/(x + 2y)$.

Final Answer: $-(2x + y)/(x + 2y)$.

Q 5.6 **Find $\frac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$.**

SOLUTION

Concept used. Implicit differentiation with product/chain rules on each mixed term.

Step 1. Differentiate term-by-term:

$$\begin{aligned} \frac{d}{dx}(x^3) &= 3x^2, \\ \frac{d}{dx}(x^2y) &= 2xy + x^2\frac{dy}{dx}, \\ \frac{d}{dx}(xy^2) &= y^2 + x \cdot 2y\frac{dy}{dx} = y^2 + 2xy\frac{dy}{dx}, \\ \frac{d}{dx}(y^3) &= 3y^2\frac{dy}{dx}. \end{aligned}$$

Sum equals 0:

$$3x^2 + 2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0.$$

Step 2. Group dy/dx:

$$(x^2 + 2xy + 3y^2)\frac{dy}{dx} = -(3x^2 + 2xy + y^2).$$

Step 3. Solve:

$$\frac{dy}{dx} = -\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}.$$

Final Answer: $\frac{dy}{dx} = -\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}$.

EXPERT'S SOLUTION *: Karan Joshi, M.Sc Mathematics, IIT Bombay*

Structural observation. Cubic in x and y; each mixed term needs product rule.

Step 1. Collect terms without y': $3x^2 + 2xy + y^2$.

Step 2. Collect coefficients of y': $x^2 + 2xy + 3y^2$.

Step 3. Ratio with a minus sign gives y'.

Final Answer: $-(3x^2 + 2xy + y^2)/(x^2 + 2xy + 3y^2)$.

Q 5.7 **Find** $\frac{dy}{dx}$ **if** $\sin^2 y + \cos xy = \kappa$ **(constant).**

SOLUTION

Concept used. Differentiate both sides; chain rule on $\sin^2 y$ and on $\cos(xy)$, treating y as a function of x.

Step 1. Differentiate:

$$\frac{d}{dx}(\sin^2 y) + \frac{d}{dx}(\cos xy) = 0.$$

Step 2. Compute the two derivatives. First:

$$\frac{d}{dx}(\sin^2 y) = 2\sin y \cdot \cos y \cdot \frac{dy}{dx} = \sin(2y)\frac{dy}{dx}.$$

Second (product inside cosine via chain rule):

$$\frac{d}{dx}(\cos xy) = -\sin(xy) \cdot \frac{d}{dx}(xy) = -\sin(xy)\left(y + x\frac{dy}{dx}\right).$$

Step 3. Substitute:

$$\sin(2y)\frac{dy}{dx} - \sin(xy)\left(y + x\frac{dy}{dx}\right) = 0.$$

Expand:

$$\sin(2y)\frac{dy}{dx} - y\sin(xy) - x\sin(xy)\frac{dy}{dx} = 0.$$

Step 4. Group dy/dx:

$$\big(\sin(2y) - x\sin(xy)\big)\frac{dy}{dx} = y\sin(xy).$$

Step 5. Solve:

$$\frac{dy}{dx} = \frac{y\sin(xy)}{\sin(2y) - x\sin(xy)}.$$

Final Answer: $\dfrac{dy}{dx} = \dfrac{y\sin(xy)}{\sin(2y) - x\sin(xy)}.$

EXPERT'S SOLUTION : *Aditya Patel, M.Sc Mathematics, IIT Bombay*

Strategic angle. Two layers of chain rule (one for $\sin^2 y$, one for $\cos(xy)$); collect y' at the end.

Step 1. Term 1 derivative: $\sin 2y \cdot y'$.

Step 2. Term 2 derivative: $-\sin(xy)(y + xy')$.

Step 3. Set sum to zero, isolate y'.

Final Answer: $y\sin(xy)/(\sin 2y - x\sin(xy))$.

Q 5.8 **Find $\frac{dy}{dx}$ if $\sin^2 x + \cos^2 y = 1$.**

SOLUTION

Concept used. Differentiate implicitly.

Step 1. Differentiate:

$$2\sin x\cos x + 2\cos y(-\sin y)\frac{dy}{dx} = 0.$$

Use $2\sin\theta\cos\theta = \sin 2\theta$:

$$\sin 2x - \sin 2y \cdot \frac{dy}{dx} = 0.$$

Step 2. Solve:

$$\sin 2y \cdot \frac{dy}{dx} = \sin 2x \implies \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}.$$

Final Answer: $\frac{dy}{dx} = \frac{\sin 2x}{\sin 2y}$.

EXPERT'S SOLUTION *: Tara Pillai, Ph.D Mathematics, IIT Delhi*

Quick reading. Both sides differentiated produce $\sin 2x$ and $-\sin 2y \cdot y'$.

Step 1. $\sin 2x - \sin 2y \cdot y' = 0$.

Step 2. $y' = \sin 2x / \sin 2y$.

Final Answer: $\sin 2x / \sin 2y$.

Q 5.9 **Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$.**

SOLUTION

Concept used. Substitute $x = \tan\theta$, so that $\frac{2x}{1+x^2} = \frac{2\tan\theta}{1+\tan^2\theta} = \sin(2\theta)$. This simplifies y to a function of θ which is then differentiated.

Step 1. Let $x = \tan\theta$, with $\theta = \tan^{-1} x$. Domain $-1 < x < 1$ corresponds to $-\pi/4 < \theta < \pi/4$, so $-\pi/2 < 2\theta < \pi/2$ and $\sin^{-1}(\sin 2\theta) = 2\theta$.

Step 2. Therefore

$$y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1} x.$$

Step 3. Differentiate:

$$\frac{dy}{dx} = 2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}.$$

Final Answer: $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$.

☞ Exam Tip

Whenever you see $\dfrac{2x}{1+x^2}$ or $\dfrac{1-x^2}{1+x^2}$ inside an inverse-trig function, the substitution $x = \tan\theta$ collapses it to $\sin 2\theta$ or $\cos 2\theta$, often turning a hard derivative into a one-liner.

EXPERT'S SOLUTION *: Krishna Rao, M.Sc Mathematics, IIT Bombay*

Strategic angle. Trig substitution converts a complicated expression to $2\tan^{-1} x$.

Step 1. $x = \tan\theta \Rightarrow \frac{2x}{1+x^2} = \sin 2\theta$.

Step 2. $y = 2\theta = 2\tan^{-1} x$.

Step 3. $dy/dx = 2/(1+x^2)$.

Final Answer: $2/(1+x^2)$.

Q 5.10 **Find** $\dfrac{dy}{dx}$ **if** $y = \tan^{-1}\left(\dfrac{3x-x^3}{1-3x^2}\right)$, **where** $-\dfrac{1}{\sqrt{3}} < x < \dfrac{1}{\sqrt{3}}$.

SOLUTION

Concept used. Triple-angle formula for tangent: $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$. Substitution $x = \tan\theta$ collapses the bracket to $\tan 3\theta$.

Step 1. Let $x = \tan\theta$ with $\theta = \tan^{-1} x$. Given $|x| < 1/\sqrt{3}$, $|\theta| < \pi/6$, so $|3\theta| < \pi/2$ and $\tan^{-1}(\tan 3\theta) = 3\theta$.

Step 2. Thus

$$y = \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right) = \tan^{-1}(\tan 3\theta) = 3\theta = 3\tan^{-1} x.$$

Step 3. Differentiate:

$$\frac{dy}{dx} = 3 \cdot \frac{1}{1+x^2} = \frac{3}{1+x^2}.$$

Final Answer: $\dfrac{dy}{dx} = \dfrac{3}{1+x^2}$.

EXPERT'S SOLUTION : *Meera Chatterjee, M.Sc Mathematics, ISI Kolkata*

Structural observation. The bracket is exactly $\tan 3\theta$ in disguise.

Step 1. Recognise the triple-angle formula for $\tan$.

Step 2. $y = 3\tan^{-1} x$, derivative $3/(1+x^2)$.

Final Answer: $3/(1+x^2)$.

Q 5.11 **Find $\dfrac{dy}{dx}$ if $y = \cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$, $0 < x < 1$.**

SOLUTION

Concept used. Substitute $x = \tan\theta$, so $\dfrac{1-x^2}{1+x^2} = \cos 2\theta$.

Step 1. $x = \tan\theta$ with $\theta = \tan^{-1} x$. For $0 < x < 1$, $0 < \theta < \pi/4$, hence $0 < 2\theta < \pi/2$ and $\cos^{-1}(\cos 2\theta) = 2\theta$.

Step 2. Therefore

$$y = \cos^{-1}(\cos 2\theta) = 2\theta = 2\tan^{-1} x.$$

Step 3. Differentiate:

$$\frac{dy}{dx} = \frac{2}{1+x^2}.$$

Final Answer: $\dfrac{dy}{dx} = \dfrac{2}{1+x^2}$.

EXPERT'S SOLUTION : *Rohit Verma, M.Sc Mathematics, ISI Kolkata*

Quick reading. Double-angle cosine identity collapses the bracket to $\cos 2\theta$.

Step 1. $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$.

Step 2. $y = 2\tan^{-1} x$, $dy/dx = 2/(1+x^2)$.

Final Answer: $2/(1+x^2)$.

Q 5.12 **Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$.**

SOLUTION

Concept used. Substitute $x = \tan\theta$ and use $\sin\left(\frac{\pi}{2} - 2\theta\right) = \cos 2\theta$.

Step 1. Let $x = \tan\theta$. For $0 < x < 1$, $0 < \theta < \pi/4$, so $0 < 2\theta < \pi/2$.

Step 2. $\frac{1-x^2}{1+x^2} = \cos 2\theta = \sin\left(\frac{\pi}{2} - 2\theta\right)$. Since $0 < \pi/2 - 2\theta < \pi/2$, this lies in the principal range of $\sin^{-1}$:

$$y = \sin^{-1}\left(\sin\left(\tfrac{\pi}{2} - 2\theta\right)\right) = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1} x.$$

Step 3. Differentiate:

$$\frac{dy}{dx} = 0 - 2 \cdot \frac{1}{1+x^2} = -\frac{2}{1+x^2}.$$

Final Answer: $\frac{dy}{dx} = -\frac{2}{1+x^2}$.

EXPERT'S SOLUTION *: Yash Joshi, M.Tech CS, IIT Madras*

Strategic angle. Convert $\sin^{-1}(\cos 2\theta)$ to $\pi/2 - 2\theta$ via the co-function identity.

Step 1. Identity: $\sin^{-1}(\cos\alpha) = \pi/2 - \alpha$ for $\alpha \in [0, \pi]$.

Step 2. $y = \pi/2 - 2\tan^{-1} x$.

Step 3. Derivative $-2/(1+x^2)$.

Final Answer: $-2/(1+x^2)$.

Q 5.13 **Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right)$, $-1 < x < 1$.**

SOLUTION

Concept used. Substitute $x = \tan\theta$; $\dfrac{2x}{1+x^2} = \sin 2\theta = \cos\left(\frac{\pi}{2} - 2\theta\right)$.

Step 1. $x = \tan\theta$, $-1 < x < 1 \Rightarrow -\pi/4 < \theta < \pi/4 \Rightarrow -\pi/2 < 2\theta < \pi/2$, hence $0 < \pi/2 - 2\theta < \pi$. This lies in the principal range of $\cos^{-1}$.

Step 2. Therefore

$$y = \cos^{-1}\left(\cos\left(\tfrac{\pi}{2} - 2\theta\right)\right) = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1}x.$$

Step 3. Differentiate:

$$\frac{dy}{dx} = -\frac{2}{1+x^2}.$$

Final Answer: $\dfrac{dy}{dx} = -\dfrac{2}{1+x^2}$.

EXPERT'S SOLUTION *: Dev Iyer, Ph.D Mathematics, IIT Delhi*

Quick reading. Sister of Q9 with $\cos^{-1}$ instead of $\sin^{-1}$. Differ by a sign.

Step 1. $\cos^{-1}(\sin 2\theta) = \pi/2 - 2\theta$.

Step 2. $y = \pi/2 - 2\tan^{-1}x$.

Step 3. $dy/dx = -2/(1+x^2)$.

Final Answer: $-2/(1+x^2)$.

Q 5.14 **Find** $\dfrac{dy}{dx}$ **if** $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$, $-\dfrac{1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}$.

SOLUTION

Concept used. Substitute $x = \sin\theta$, so that $\sqrt{1-x^2} = \cos\theta$ and $2x\sqrt{1-x^2} = 2\sin\theta\cos\theta = \sin 2\theta$.

Step 1. $x = \sin\theta$ with $\theta = \sin^{-1}x$. For $|x| < 1/\sqrt{2}$, $|\theta| < \pi/4$, so $|2\theta| < \pi/2$ and $\sin^{-1}(\sin 2\theta) = 2\theta$.

Step 2. Therefore

$$y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x.$$

Step 3. Differentiate:

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}.$$

Final Answer: $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-x^2}}$.

EXPERT'S SOLUTION *: Ishita Reddy, Ph.D Mathematics, IIT Delhi*

Structural observation. The argument $2x\sqrt{1-x^2}$ is $\sin 2\theta$ after $x = \sin\theta$.

Step 1. $\sin^{-1}$ undoes $\sin$ inside the principal range.

Step 2. $y = 2\sin^{-1} x$, $dy/dx = 2/\sqrt{1-x^2}$.

Final Answer: $2/\sqrt{1-x^2}$.

Q 5.15 **Find** $\dfrac{dy}{dx}$ **if** $y = \sec^{-1}\left(\dfrac{1}{2x^2-1}\right)$, $0 < x < \dfrac{1}{\sqrt{2}}$.

SOLUTION

Concept used. Recall $\sec^{-1}(1/u) = \cos^{-1}(u)$ when $|u| \le 1$ (the reciprocal-with-secant identity). Then substitute $x = \cos\theta$ to use the double-angle identity $2\cos^2\theta - 1 = \cos 2\theta$.

Step 1. Use $\sec^{-1}\left(\frac{1}{u}\right) = \cos^{-1} u$. Here $u = 2x^2 - 1$, so

$$y = \cos^{-1}(2x^2 - 1).$$

Step 2. Let $x = \cos\theta$ with $\theta = \cos^{-1} x$. Domain $0 < x < 1/\sqrt{2}$ gives $\pi/4 < \theta < \pi/2$, so $\pi/2 < 2\theta < \pi$, which lies in the principal range of $\cos^{-1}$.

Step 3. Then $2x^2 - 1 = 2\cos^2\theta - 1 = \cos 2\theta$, so

$$y = \cos^{-1}(\cos 2\theta) = 2\theta = 2\cos^{-1} x.$$

Step 4. Differentiate:

$$\frac{dy}{dx} = 2 \cdot \left(-\frac{1}{\sqrt{1-x^2}}\right) = -\frac{2}{\sqrt{1-x^2}}.$$

Final Answer: $\dfrac{dy}{dx} = -\dfrac{2}{\sqrt{1-x^2}}$.

EXPERT'S SOLUTION : *Aditi Banerjee, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Two-step transformation: $\sec^{-1}(1/u) = \cos^{-1}(u)$, then $u = 2x^2 - 1 = \cos 2\theta$.

Step 1. $y = \cos^{-1}(2x^2 - 1) = 2\cos^{-1} x$.

Step 2. $dy/dx = -2/\sqrt{1 - x^2}$.

Final Answer: $-2/\sqrt{1 - x^2}$.

Key Takeaways

- For implicit equations, differentiate both sides w.r.t. x, treat y as $y(x)$, then solve for dy/dx.
- For inverse-trig expressions of standard form, substitute $x = \tan\theta$ or $x = \sin\theta$ to collapse complicated brackets to simple multiples of θ.
- Always check that the substituted multiple-angle lies in the principal range of the outer inverse trig, otherwise add or subtract π correctly.

End of Exercise 5.3