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Step-by-step solutions, alternate methods & exam tips for Class 12 Mathematics

Chapter 5: Continuity and Differentiability

About this Chapter

Exercise 5.4 drills the derivatives of the **exponential function** e^x and the **natural logarithm** $\log x$. Throughout this chapter $\log x$ means $\ln x = \log_e x$. Most problems are direct chain rule applications; one (Q9) uses the quotient rule.

Topics covered: $\frac{d}{dx}e^x$ • $\frac{d}{dx}\log x$ • Chain rule with exp/log • Quotient rule applications

Quick Formula Sheet

Exponential / logarithm derivatives: $\frac{d}{dx}e^x = e^x$, $\frac{d}{dx}\log x = \frac{1}{x}$ $(x > 0)$.

Chain-rule shortcut: $\frac{d}{dx}e^{f(x)} = e^{f(x)}f'(x)$, $\frac{d}{dx}\log f(x) = \frac{f'(x)}{f(x)}$.

Exercise 5.4

Q 5.1 **Differentiate $\frac{e^x}{\sin x}$ with respect to x.**

SOLUTION

Concept used. **Quotient rule**: $(u/v)' = (u'v - uv')/v^2$. Here $u = e^x$, $u' = e^x$, $v = \sin x$, $v' = \cos x$.

Step 1. Apply the quotient rule:

$$\frac{d}{dx}\frac{e^x}{\sin x} = \frac{e^x \sin x - e^x \cos x}{\sin^2 x}.$$

Step 2. Factor e^x:

$$= \frac{e^x(\sin x - \cos x)}{\sin^2 x}.$$

Final Answer: $\dfrac{d}{dx}\left(\dfrac{e^x}{\sin x}\right) = \dfrac{e^x(\sin x - \cos x)}{\sin^2 x},\ x \neq n\pi.$

EXPERT'S SOLUTION : *Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Quick reading. Quotient rule with e^x on top and $\sin x$ on bottom.

Step 1. Numerator after rule: $e^x \sin x - e^x \cos x$.

Step 2. Factor e^x: $e^x(\sin x - \cos x)$.

Step 3. Divide by $\sin^2 x$.

Final Answer: $e^x(\sin x - \cos x)/\sin^2 x$.

Q 5.2 **Differentiate $e^{\sin^{-1} x}$ with respect to x.**

SOLUTION

Concept used. Chain rule. $\dfrac{d}{dx}e^{f(x)} = e^{f(x)} f'(x)$. Here $f(x) = \sin^{-1} x$ with derivative $1/\sqrt{1-x^2}$.

Step 1. Let $y = e^{\sin^{-1} x}$. Then

$$\frac{dy}{dx} = e^{\sin^{-1} x} \cdot \frac{d}{dx}(\sin^{-1} x) = e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1-x^2}}.$$

Final Answer: $\dfrac{dy}{dx} = \dfrac{e^{\sin^{-1} x}}{\sqrt{1-x^2}},\ -1 < x < 1.$

EXPERT'S SOLUTION : *Sneha Iyer, Ph.D Mathematics, IIT Delhi*

Quick reading. Standard e^u chain rule with $u = \sin^{-1} x$.

Step 1. Outer derivative: $e^{\sin^{-1} x}$.

Step 2. Inner derivative: $1/\sqrt{1-x^2}$.

Final Answer: $e^{\sin^{-1} x}/\sqrt{1-x^2}$.

Q 5.3 **Differentiate e^{x^3} with respect to x.**

SOLUTION

Concept used. Chain rule. $\frac{d}{dx}e^{f(x)} = e^{f(x)} f'(x)$ with $f(x) = x^3$, $f'(x) = 3x^2$.

Step 1. Differentiate:

$$\frac{d}{dx}e^{x^3} = e^{x^3} \cdot 3x^2 = 3x^2 e^{x^3}.$$

Final Answer: $3x^2 e^{x^3}$.

EXPERT'S SOLUTION *: Vivaan Gupta, M.Tech CS, IIT Madras*

Quick reading. e^{x^3} times derivative of exponent.

Step 1. Derivative of x^3 is $3x^2$.

Step 2. Multiply: $3x^2 e^{x^3}$.

Final Answer: $3x^2 e^{x^3}$.

Q 5.4 **Differentiate $\sin(\tan^{-1} e^{-x})$ with respect to x.**

SOLUTION

Concept used. Three-layer chain: outer $\sin$, middle $\tan^{-1}$, inner e^{-x}.

Step 1. Let $u = e^{-x}$, $v = \tan^{-1} u$, $y = \sin v$.

Step 2. Derivatives layer by layer:

$$\frac{dy}{dv} = \cos v = \cos(\tan^{-1} e^{-x}),$$
$$\frac{dv}{du} = \frac{1}{1+u^2} = \frac{1}{1+e^{-2x}},$$
$$\frac{du}{dx} = -e^{-x}.$$

Step 3. Chain rule (three layers):

$$\frac{dy}{dx} = \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+e^{-2x}} \cdot (-e^{-x}) = -\frac{e^{-x}\cos(\tan^{-1} e^{-x})}{1+e^{-2x}}.$$

Final Answer: $\dfrac{dy}{dx} = -\dfrac{e^{-x}\cos(\tan^{-1} e^{-x})}{1+e^{-2x}}$.

EXPERT'S SOLUTION *: Aanya Mehta, Ph.D Pure Mathematics, IISc Bangalore*

Structural observation. Triple chain; differentiate $\sin$, then $\tan^{-1}$, then e^{-x}.

Step 1. $\sin' = \cos(\tan^{-1} e^{-x})$.

Step 2. $\tan^{-1}{}' = 1/(1+e^{-2x})$.

Step 3. $(e^{-x})' = -e^{-x}$.

Step 4. Multiply.

Final Answer: $-e^{-x}\cos(\tan^{-1} e^{-x})/(1+e^{-2x})$.

Q 5.5 **Differentiate** $\log(\cos e^x)$ **with respect to** x.

SOLUTION

Concept used. Three-layer chain: outer $\log$, middle $\cos$, inner e^x.

Step 1. Let $u = e^x$, $v = \cos u$, $y = \log v$.

Step 2. Layer-wise derivatives:

$$\frac{dy}{dv} = \frac{1}{v} = \frac{1}{\cos e^x},$$
$$\frac{dv}{du} = -\sin u = -\sin e^x,$$
$$\frac{du}{dx} = e^x.$$

Step 3. Multiply:

$$\frac{dy}{dx} = \frac{1}{\cos e^x}\cdot(-\sin e^x)\cdot e^x = -e^x\tan(e^x).$$

Final Answer: $\dfrac{dy}{dx} = -e^x\tan(e^x)$ where $\cos e^x \neq 0$.

EXPERT'S SOLUTION : *Priya Singh, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading. Three layers; $\sin / \cos = \tan$ produces a clean answer.

Step 1. Layer 1: $1/\cos e^x$.

Step 2. Layer 2: $-\sin e^x$.

Step 3. Layer 3: e^x.

Step 4. Product: $-e^x \tan e^x$.

Final Answer: $-e^x \tan e^x$.

Q 5.6 Differentiate $e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$ with respect to x.

SOLUTION

Concept used. Sum rule, then chain rule on each e^{x^k}: $\frac{d}{dx}e^{x^k} = e^{x^k} \cdot kx^{k-1}$.

Step 1. Differentiate term-by-term:

$$\begin{aligned}\frac{d}{dx}e^x &= e^x,\\ \frac{d}{dx}e^{x^2} &= 2xe^{x^2},\\ \frac{d}{dx}e^{x^3} &= 3x^2e^{x^3},\\ \frac{d}{dx}e^{x^4} &= 4x^3e^{x^4},\\ \frac{d}{dx}e^{x^5} &= 5x^4e^{x^5}.\end{aligned}$$

Step 2. Add:

$$\frac{dy}{dx} = e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}.$$

Final Answer: $\frac{dy}{dx} = e^x + 2xe^{x^2} + 3x^2e^{x^3} + 4x^3e^{x^4} + 5x^4e^{x^5}$.

EXPERT'S SOLUTION : *Karan Joshi, M.Sc Mathematics, IIT Bombay*

Quick reading. Five separate e^{x^k} derivatives summed.

Step 1. Each e^{x^k} produces $kx^{k-1}e^{x^k}$.

Step 2. Sum five contributions.

Final Answer: $\sum_{k=1}^{5} kx^{k-1}e^{x^k}$.

Q 5.7 Differentiate $\sqrt{e^{\sqrt{x}}}$ with respect to x, $x > 0$.

SOLUTION

Concept used. Simplify first using exponent laws: $\sqrt{e^{\sqrt{x}}} = e^{\sqrt{x}/2}$. Then apply chain rule.

Step 1. Rewrite:

$$y = \sqrt{e^{\sqrt{x}}} = (e^{\sqrt{x}})^{1/2} = e^{\sqrt{x}/2}.$$

Step 2. Differentiate using $\frac{d}{dx}e^u = e^u u'$ with $u = \sqrt{x}/2$, $u' = \frac{1}{2} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{4\sqrt{x}}$:

$$\frac{dy}{dx} = e^{\sqrt{x}/2} \cdot \frac{1}{4\sqrt{x}} = \frac{e^{\sqrt{x}/2}}{4\sqrt{x}} = \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}.$$

Final Answer: $\frac{dy}{dx} = \frac{\sqrt{e^{\sqrt{x}}}}{4\sqrt{x}}$ $(x > 0)$.

☞ **Exam Tip**

Always look for exponent simplifications first. $\sqrt{e^u} = e^{u/2}$ converts a nested square-root and exponential into a single exponential, dramatically simplifying the derivative.

EXPERT'S SOLUTION : *Pranav Kapoor, M.Sc Mathematics, IIT Bombay*

Strategic angle. Exponent simplification before differentiation.

Step 1. $\sqrt{e^{\sqrt{x}}} = e^{\sqrt{x}/2}$.

Step 2. Derivative: $e^{\sqrt{x}/2} \cdot \frac{1}{4\sqrt{x}}$.

Final Answer: $e^{\sqrt{x}/2}/(4\sqrt{x})$.

Q 5.8 Differentiate $\log(\log x)$ with respect to x, $x > 1$.

SOLUTION

Concept used. Chain rule with $\frac{d}{dx}\log u = \frac{1}{u}\cdot u'$.

Step 1. Let $u = \log x$, so $y = \log u$, with $u' = 1/x$.

Step 2. Chain rule:

$$\frac{dy}{dx} = \frac{1}{u}\cdot u' = \frac{1}{\log x}\cdot\frac{1}{x} = \frac{1}{x\log x}.$$

Final Answer: $\frac{dy}{dx} = \frac{1}{x\log x}$ $(x > 1)$.

EXPERT'S SOLUTION *: Tara Pillai, Ph.D Mathematics, IIT Delhi*

Quick reading. Two layers of $\log$ derivatives.

Step 1. Outer: $1/\log x$.

Step 2. Inner: $1/x$.

Step 3. Product: $1/(x\log x)$.

Final Answer: $1/(x\log x)$.

Q 5.9 Differentiate $\frac{\cos x}{\log x}$ with respect to x, $x > 0$.

SOLUTION

Concept used. Quotient rule with $u = \cos x$, $u' = -\sin x$, $v = \log x$, $v' = 1/x$.

Step 1. Apply quotient rule:

$$\frac{dy}{dx} = \frac{u'v - uv'}{v^2} = \frac{(-\sin x)(\log x) - (\cos x)(1/x)}{(\log x)^2}.$$

Step 2. Simplify the numerator:

$$= \frac{-(\sin x)(\log x) - \frac{\cos x}{x}}{(\log x)^2} = -\frac{x(\sin x)(\log x) + \cos x}{x(\log x)^2}.$$

Final Answer: $\frac{dy}{dx} = -\frac{x(\sin x)(\log x) + \cos x}{x(\log x)^2}$.

EXPERT'S SOLUTION : *Krishna Rao, M.Sc Mathematics, IIT Bombay*

Structural observation. Quotient with $\cos/\log$; bring the $1/x$ inside by multiplying numerator and denominator by x.

Step 1. Raw quotient rule output: $((-\sin x)\log x - \cos x/x)/(\log x)^2$.

Step 2. Multiply num and den by x to clear the inner fraction.

> **Final Answer:** $-(x \sin x \log x + \cos x)/(x(\log x)^2)$.

Q 5.10 **Differentiate $\cos(\log x + e^x)$ with respect to x, $x > 0$.**

SOLUTION

Concept used. Chain rule with outer $\cos$ and inner $\log x + e^x$. Derivative of the inner: $1/x + e^x$.

Step 1. Let $u = \log x + e^x$, $y = \cos u$.

Step 2. $\dfrac{du}{dx} = \dfrac{1}{x} + e^x$.

Step 3. Chain rule:

$$\frac{dy}{dx} = -\sin u \cdot \frac{du}{dx} = -\sin(\log x + e^x)\left(\frac{1}{x} + e^x\right).$$

> **Final Answer:** $\dfrac{dy}{dx} = -\left(\dfrac{1}{x} + e^x\right)\sin(\log x + e^x)\ (x > 0)$.

EXPERT'S SOLUTION : *Meera Chatterjee, M.Sc Mathematics, ISI Kolkata*

Quick reading. Standard chain rule with sum-derivative inside.

Step 1. Outer: $-\sin(\log x + e^x)$.

Step 2. Inner derivative: $1/x + e^x$.

Step 3. Multiply.

> **Final Answer:** $-(1/x + e^x)\sin(\log x + e^x)$.

Key Takeaways

- $\dfrac{d}{dx}e^x = e^x$ and $\dfrac{d}{dx}\log x = 1/x$ are the only two new derivatives in this section; everything else follows from chain, product and quotient rules.
- Simplify exponent expressions ($\sqrt{e^u} = e^{u/2}$) before differentiating; it often turns a long chain into a short one.
- For $\dfrac{d}{dx}e^{f(x)}$, the answer always contains an $e^{f(x)}$ factor; for $\dfrac{d}{dx}\log f(x)$, the answer always has a denominator $f(x)$.

End of Exercise 5.4