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Step-by-step solutions for the 2026-27 NCERT (Latest Edition)

Chapter 6: Application of Derivatives

About this Chapter

In this exercise the derivative tells us where a function is **strictly increasing** and where it is **strictly decreasing**. The key rule: if $f'(x) > 0$ on an interval, f is increasing there; if $f'(x) < 0$, decreasing.

Topics covered: Monotonic functions • First derivative test • Sign analysis • Intervals of monotonicity

Quick Formula Sheet

Increasing on I:
$f'(x) > 0$ for all $x \in I$.

Decreasing on I:
$f'(x) < 0$ for all $x \in I$.

Critical point:
any c with $f'(c) = 0$ or f not differentiable at c.

Exercise 6.2

Q 6.1 **Show that the function given by $f(x) = 3x + 17$ is increasing on $\mathbb{R}$.**

SOLUTION

Concept used. A real-valued, differentiable function f is **strictly increasing** on an interval I if $f'(x) > 0$ for every $x \in I$. To prove monotonicity on $\mathbb{R}$, it is enough to show that $f'(x) > 0$ for all real x.

Step 1. Differentiate $f(x) = 3x + 17$:

$$f'(x) = 3.$$

Step 2. Observe that $f'(x) = 3 > 0$ for every $x \in \mathbb{R}$. Therefore f is strictly increasing throughout $\mathbb{R}$.

Final Answer: $f'(x) = 3 > 0$ on $\mathbb{R}$, so f is strictly increasing on $\mathbb{R}$.

EXPERT'S SOLUTION : *Arjun Mehta, M.Sc Mathematics, IIT Bombay*

Quick reading. A non-constant linear function with positive slope is increasing everywhere. The slope is 3, so done.

Step 1. Slope $= 3 > 0 \Rightarrow$ strictly increasing on $\mathbb{R}$.

Why this matters. For any linear $f(x) = mx + c$, the sign of m alone determines whether f is increasing or decreasing.

> **Final Answer:** f is strictly increasing on $\mathbb{R}$.

Q 6.2 **Show that the function given by $f(x) = e^{2x}$ is increasing on $\mathbb{R}$.**

SOLUTION

Concept used. For an exponential $e^{u(x)}$, the chain rule gives $\frac{d}{dx}\left(e^{u(x)}\right) = u'(x)\, e^{u(x)}$. If $f'(x) > 0$ everywhere, then f is increasing everywhere.

Step 1. Differentiate $f(x) = e^{2x}$:

$$f'(x) = 2\, e^{2x}.$$

Step 2. Since $e^{2x} > 0$ for every real x (the exponential function is always positive) and $2 > 0$, the product $2e^{2x} > 0$ for every real x.

Step 3. Therefore $f'(x) > 0$ on $\mathbb{R}$ and so f is strictly increasing on $\mathbb{R}$.

> **Final Answer:** $f'(x) = 2e^{2x} > 0$ on $\mathbb{R}$, hence f is strictly increasing on $\mathbb{R}$.

♥ **Why This Matters**

The fact that e^x is strictly increasing underpins the entire theory of logarithms (its inverse is $\log x$) and the standard e^{kt} growth/decay models in physics, chemistry and biology.

EXPERT'S SOLUTION : *Krishna Gupta, M.Sc Mathematics, IIT Bombay*

Quick reading. e^{2x} is always positive and its derivative $2e^{2x}$ is also always positive. So e^{2x} is strictly increasing.

Step 1. $f' = 2e^{2x} > 0$ for all x.

Step 2. Hence f is strictly increasing on $\mathbb{R}$.

Why this matters. For any $a > 0$, e^{ax} is strictly increasing on $\mathbb{R}$, while e^{-ax} is strictly decreasing.

Final Answer: f is strictly increasing on $\mathbb{R}$.

Q 6.3 **Show that the function given by $f(x) = \sin x$ is**
(a) increasing in $\left(0, \dfrac{\pi}{2}\right)$
(b) decreasing in $\left(\dfrac{\pi}{2}, \pi\right)$
(c) neither increasing nor decreasing in $(0, \pi)$.

SOLUTION

Concept used. f is increasing on I iff $f'(x) > 0$ on I and decreasing iff $f'(x) < 0$ on I. For $f(x) = \sin x$, $f'(x) = \cos x$.

Step 1. **Part (a).** On $\left(0, \frac{\pi}{2}\right)$, the cosine is positive: $\cos x > 0$. Therefore $f'(x) > 0$ on $\left(0, \frac{\pi}{2}\right)$, so $\sin x$ is strictly increasing on this interval.

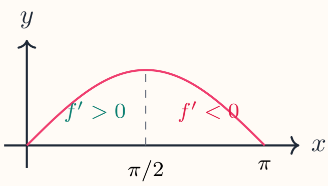
Step 2. **Part (b).** On $\left(\frac{\pi}{2}, \pi\right)$, the cosine is negative: $\cos x < 0$. Therefore $f'(x) < 0$ on $\left(\frac{\pi}{2}, \pi\right)$, so $\sin x$ is strictly decreasing on this interval.

Step 3. **Part (c).** On $(0, \pi)$, $\cos x$ takes both positive values (on $\left(0, \frac{\pi}{2}\right)$) and negative values (on $\left(\frac{\pi}{2}, \pi\right)$). Hence f' does not have a constant sign on $(0, \pi)$, so f is neither increasing nor decreasing on the whole interval.

Final Answer: $\sin x$ is increasing on $\left(0, \frac{\pi}{2}\right)$, decreasing on $\left(\frac{\pi}{2}, \pi\right)$, and neither on $(0, \pi)$.

EXPERT'S SOLUTION *: Aanya Kumar, M.Sc Applied Mathematics, IIT Kanpur*

Picture-first. The graph of $\sin x$ on $[0, \pi]$ rises from 0 to 1 on $[0, \pi/2]$, then falls back to 0 on $[\pi/2, \pi]$. The sign of $\cos x$ confirms this.



Step 1. $f' = \cos x$.

Step 2. $\cos x > 0$ on $(0, \pi/2) \Rightarrow f$ increasing.

Step 3. $\cos x < 0$ on $(\pi/2, \pi) \Rightarrow f$ decreasing.

Step 4. Mixed sign on $(0, \pi) \Rightarrow$ neither.

Why this matters. Any change in the sign of f' on an interval is enough to break monotonicity. Detecting the sign-change point is therefore the central skill.

Final Answer: Increasing on $(0, \pi/2)$, decreasing on $(\pi/2, \pi)$, neither on $(0, \pi)$.

Q 6.4 **Find the intervals in which the function f given by $f(x) = 2x^2 - 3x$ is (a) increasing (b) decreasing.**

SOLUTION

Concept used. Differentiate, find where $f'(x) = 0$, and use sign analysis on the resulting intervals.

Step 1. Differentiate $f(x) = 2x^2 - 3x$:

$$f'(x) = 4x - 3.$$

Step 2. Set $f'(x) = 0$:

$$4x - 3 = 0 \;\Rightarrow\; x = \frac{3}{4}.$$

Step 3. Sign of f':

- If $x < 3/4$, then $4x < 3$ so $f'(x) < 0$: f is strictly decreasing.
- If $x > 3/4$, then $4x > 3$ so $f'(x) > 0$: f is strictly increasing.

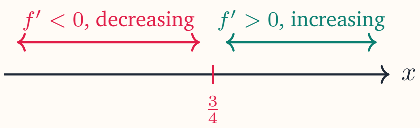
Final Answer: f is decreasing on $\left(-\infty, \frac{3}{4}\right)$ and increasing on $\left(\frac{3}{4}, \infty\right)$.

☞ **Sign-of-derivative test**

On any open interval I, $f' > 0$ everywhere $\Rightarrow$ f is strictly increasing on I; $f' < 0$ everywhere $\Rightarrow$ f is strictly decreasing on I. A single sign-change of f' inside I breaks monotonicity on I.

EXPERT'S SOLUTION *: Sneha Joshi, M.Sc Mathematics, ISI Kolkata*

Quick reading. The derivative $4x - 3$ is a single linear expression; its sign flips at the root $x = 3/4$.



Step 1. Root of f': $x = 3/4$.

Step 2. Left of $3/4$: $f' < 0$, decreasing. Right: $f' > 0$, increasing.

Why this matters. For a quadratic f, the vertex $x = -b/(2a)$ separates the decreasing and increasing branches.

Final Answer: Decreasing on $(-\infty, 3/4)$; increasing on $(3/4, \infty)$.

Q 6.5 **Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (a) increasing (b) decreasing.**

SOLUTION

Concept used. Differentiate, factorise f', and use a sign chart for the intervals between the roots.

Step 1. Differentiate:

$$f'(x) = 6x^2 - 6x - 36 = 6(x^2 - x - 6) = 6(x-3)(x+2).$$

Step 2. Solve $f'(x) = 0$:

$$(x-3)(x+2) = 0 \;\Rightarrow\; x = -2,\ 3.$$

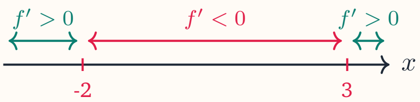
Step 3. Sign chart on the intervals split by $x = -2$ and $x = 3$:

Interval	$(x+2)$	$(x-3)$	$f'(x) = 6(x+2)(x-3)$
$(-\infty, -2)$	$-$	$-$	$+$
$(-2,\ 3)$	$+$	$-$	$-$
$(3,\ \infty)$	$+$	$+$	$+$

Final Answer: f is increasing on $(-\infty, -2) \cup (3, \infty)$ and decreasing on $(-2,\ 3)$.

EXPERT'S SOLUTION : *Pranav Bhat, M.Sc Mathematics, IIT Bombay*

Strategic angle. Cubic with positive leading coefficient $\Rightarrow$ shape "up-down-up". The two critical points sandwich the decreasing middle interval.



Step 1. Factor $f' = 6(x+2)(x-3)$.

Step 2. Both factors negative on $(-\infty, -2)$: $f' > 0$.

Step 3. Mixed signs on $(-2, 3)$: $f' < 0$.

Step 4. Both positive on $(3, \infty)$: $f' > 0$.

Why this matters. A factored derivative is the fastest path to the sign chart in cubics and quartics.

Final Answer: Increasing on $(-\infty, -2) \cup (3, \infty)$; decreasing on $(-2, 3)$.

Q 6.6 **Find the intervals in which the following functions are strictly increasing or decreasing:**

(a) $x^2 + 2x - 5$

(b) $10 - 6x - 2x^2$

(c) $-2x^3 - 9x^2 - 12x + 1$

(d) $6 - 9x - x^2$

(e) $(x + 1)^3(x - 3)^3$.

SOLUTION

Concept used. For each part, compute $f'(x)$, find its zeros, and determine the sign on each resulting subinterval.

(a) $f(x) = x^2 + 2x - 5$.

Step 1. $f'(x) = 2x + 2 = 2(x + 1)$.

Step 2. $f'(x) = 0$ at $x = -1$.

Step 3. $f' > 0$ for $x > -1$ (increasing on $(-1, \infty)$); $f' < 0$ for $x < -1$ (decreasing on $(-\infty, -1)$).

$\Rightarrow$ strictly decreasing on $(-\infty, -1)$, strictly increasing on $(-1, \infty)$.

(b) $f(x) = 10 - 6x - 2x^2$.

Step 1. $f'(x) = -6 - 4x = -2(2x + 3)$.

Step 2. $f'(x) = 0$ at $x = -\dfrac{3}{2}$.

Step 3. $f' > 0$ when $2x + 3 < 0$, i.e. $x < -3/2$ (increasing).

Step 4. $f' < 0$ when $x > -3/2$ (decreasing).

$\Rightarrow$ strictly increasing on $(-\infty, -\frac{3}{2})$, strictly decreasing on $(-\frac{3}{2}, \infty)$.

(c) $f(x) = -2x^3 - 9x^2 - 12x + 1$.

Step 1. $f'(x) = -6x^2 - 18x - 12 = -6(x^2 + 3x + 2) = -6(x + 1)(x + 2)$.

Step 2. $f'(x) = 0$ at $x = -1, -2$.

Step 3. Sign: on $(-\infty, -2)$, $(x + 1)(x + 2) > 0$ so $f' < 0$; on $(-2, -1)$,

$(x+1)(x+2) < 0$ so $f' > 0$; on $(-1,\infty)$, $(x+1)(x+2) > 0$ so $f' < 0$.

$\Rightarrow$ strictly decreasing on $(-\infty,-2)\cup(-1,\infty)$, strictly increasing on $(-2,-1)$.

(d) $f(x) = 6 - 9x - x^2$.

Step 1. $f'(x) = -9 - 2x = -(2x+9)$.

Step 2. $f'(x) = 0$ at $x = -\dfrac{9}{2}$.

Step 3. $f' > 0$ for $x < -9/2$ (increasing); $f' < 0$ for $x > -9/2$ (decreasing).

$\Rightarrow$ strictly increasing on $(-\infty, -\frac{9}{2})$, strictly decreasing on $(-\frac{9}{2}, \infty)$.

(e) $f(x) = (x+1)^3(x-3)^3$.

Step 1. Use the product rule and chain rule:

$$\begin{aligned} f'(x) &= 3(x+1)^2(x-3)^3 + (x+1)^3\cdot 3(x-3)^2 \\ &= 3(x+1)^2(x-3)^2\big[(x-3)+(x+1)\big] \\ &= 3(x+1)^2(x-3)^2(2x-2) \\ &= 6(x+1)^2(x-3)^2(x-1). \end{aligned}$$

Step 2. Now $(x+1)^2 \geq 0$ and $(x-3)^2 \geq 0$ always, with equality only at $x=-1$ and $x=3$. So the sign of f' is the sign of $(x-1)$ everywhere else.

Step 3. $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$, with $f'(-1) = f'(1) = f'(3) = 0$.

$\Rightarrow$ strictly decreasing on $(-\infty, 1)$, strictly increasing on $(1,\infty)$.

Final Answer: (a) Dec $(-\infty,-1)$, inc $(-1,\infty)$. (b) Inc $(-\infty,-3/2)$, dec $(-3/2,\infty)$. (c) Dec $(-\infty,-2)\cup(-1,\infty)$, inc $(-2,-1)$. (d) Inc $(-\infty,-9/2)$, dec $(-9/2,\infty)$. (e) Dec $(-\infty,1)$, inc $(1,\infty)$.

✗ Common Mistake

For part (e), it is tempting to declare f monotonic on $(-\infty,-1)$, $(-1,1)$, $(1,3)$ and $(3,\infty)$ separately because f' has roots at $-1, 1, 3$. But $(x+1)^2$ and $(x-3)^2$ are perfect squares, so they do *not* change sign at $x=-1$ or $x=3$. Only the factor $(x-1)$ changes sign. Always check the multiplicity of each factor in f' before drawing a sign chart.

EXPERT'S SOLUTION *: Aditi Desai, M.Sc Mathematics, IIT Bombay*

Strategic angle. Factor f' as far as possible: squared factors do not affect the sign, only the linear residual factor matters.

Step 1. (a) $f' = 2(x+1)$: turns at -1.

Step 2. (b) $f' = -2(2x+3)$: turns at $-3/2$, sign flips downward.

Step 3. (c) $f' = -6(x+1)(x+2)$: two roots, signs $-, +, -$.

Step 4. (d) $f' = -(2x+9)$: turns at $-9/2$.

Step 5. (e) $f' = 6(x+1)^2(x-3)^2(x-1)$: signs decided only by $(x-1)$ since squared factors are non-negative.

Why this matters. Squared (or any even-power) factors create $f'(c) = 0$ without changing sign, producing points of inflection rather than maxima/minima.

Final Answer: See main solution for the five interval lists.

Q 6.7 **Show that $y = \log(1+x) - \dfrac{2x}{2+x}$, $x > -1$, is an increasing function of x throughout its domain.**

SOLUTION

Concept used. It is enough to show $\dfrac{dy}{dx} \geq 0$ on $(-1, \infty)$, with equality only at isolated points.

Step 1. Differentiate term by term:

$$\frac{d}{dx}\log(1+x) = \frac{1}{1+x}.$$

For the second term, use the quotient rule on $\dfrac{2x}{2+x}$:

$$\frac{d}{dx}\left(\frac{2x}{2+x}\right) = \frac{2(2+x) - 2x(1)}{(2+x)^2} = \frac{4+2x-2x}{(2+x)^2} = \frac{4}{(2+x)^2}.$$

Step 2. Subtract:

$$\frac{dy}{dx} = \frac{1}{1+x} - \frac{4}{(2+x)^2}.$$

Step 3. Bring to a common denominator $(1+x)(2+x)^2$:

$$\frac{dy}{dx} = \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2}.$$

Expand the numerator:

$$(2+x)^2 - 4(1+x) = 4 + 4x + x^2 - 4 - 4x = x^2.$$

Hence

$$\frac{dy}{dx} = \frac{x^2}{(1+x)(2+x)^2}.$$

Step 4. Sign on the domain $x > -1$:

- $x^2 \geq 0$ (zero only at $x = 0$);
- $(2+x)^2 > 0$ since $x > -1$ implies $2 + x > 1 > 0$;
- $1 + x > 0$ since $x > -1$.

Therefore $\frac{dy}{dx} \geq 0$ on $(-1, \infty)$, with equality only at the isolated point $x = 0$. Hence y is (strictly) increasing on its whole domain.

Final Answer: $\frac{dy}{dx} = \frac{x^2}{(1+x)(2+x)^2} \geq 0$ for $x > -1$, so y is increasing on $(-1, \infty)$.

EXPERT'S SOLUTION : *Neha Rao, M.Sc Mathematics, IIT Bombay*

Strategic angle. Combine into a single fraction and simplify: the numerator collapses to x^2, which makes positivity obvious.

Step 1. Compute y' as a single fraction: $\frac{x^2}{(1+x)(2+x)^2}$.

Step 2. All three factors are positive (or non-negative) on $x > -1$.

Step 3. Hence $y' \geq 0$, with equality only at $x = 0$: strictly increasing.

Why this matters. The "show increasing" template is always: compute f', combine, simplify, then read off the sign.

Final Answer: y is strictly increasing on $(-1, \infty)$.

Q 6.8 Find the values of x for which $y = [x(x-2)]^2$ is an increasing function.

SOLUTION

Concept used. First simplify y, then differentiate and factorise y' to locate the sign-changing points.

Step 1. Let $u = x(x-2) = x^2 - 2x$. Then $y = u^2$.

Step 2. By the chain rule, $\frac{dy}{dx} = 2u\frac{du}{dx} = 2(x^2 - 2x)(2x - 2)$.

Step 3. Factor:

$$\frac{dy}{dx} = 2 \cdot x(x-2) \cdot 2(x-1) = 4x(x-1)(x-2).$$

Step 4. Roots of y': $x = 0, 1, 2$.

Step 5. Sign chart for the product $x(x-1)(x-2)$:

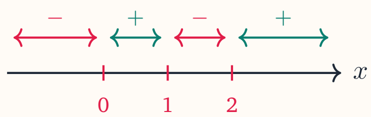
Interval	x	$x-1$	$x-2$	y'
$(-\infty, 0)$	$-$	$-$	$-$	$-$
$(0, 1)$	$+$	$-$	$-$	$+$
$(1, 2)$	$+$	$+$	$-$	$-$
$(2, \infty)$	$+$	$+$	$+$	$+$

So $y' > 0$ on $(0, 1) \cup (2, \infty)$.

Final Answer: y is increasing on $(0, 1) \cup (2, \infty)$.

EXPERT'S SOLUTION *: Diya Kapoor, M.Sc Mathematics, IIT Bombay*

Strategic angle. Square-of-a-product gives derivative with three linear factors; sign chart finishes the job.



sign chart of $y' = 4x(x-1)(x-2)$

Step 1. $y' = 4x(x-1)(x-2)$ vanishes at $0, 1, 2$.

Step 2. Sign alternates: $-, +, -, +$. Increasing where positive: $(0, 1) \cup (2, \infty)$.

Why this matters. For $y = [g(x)]^n$, the derivative is $n[g(x)]^{n-1}g'(x)$; the simplest version of the chain rule.

Final Answer: Increasing on $(0, 1) \cup (2, \infty)$.

Q 6.9 **Prove that $y = \dfrac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ in $\left[0, \dfrac{\pi}{2}\right]$.**

SOLUTION

Concept used. Compute $\dfrac{dy}{d\theta}$ and show it is non-negative on $\left[0, \frac{\pi}{2}\right]$.

Step 1. Differentiate the first term using the quotient rule:

$$\frac{d}{d\theta}\left(\frac{4\sin\theta}{2+\cos\theta}\right) = \frac{4\cos\theta(2+\cos\theta) - 4\sin\theta(-\sin\theta)}{(2+\cos\theta)^2}.$$

The numerator simplifies as

$$4\cos\theta(2+\cos\theta) + 4\sin^2\theta = 8\cos\theta + 4\cos^2\theta + 4\sin^2\theta = 8\cos\theta + 4.$$

(Used $\sin^2\theta + \cos^2\theta = 1$.)

Step 2. Combine with $\frac{d}{d\theta}(-\theta) = -1$:

$$\frac{dy}{d\theta} = \frac{8\cos\theta + 4}{(2+\cos\theta)^2} - 1 = \frac{8\cos\theta + 4 - (2+\cos\theta)^2}{(2+\cos\theta)^2}.$$

Step 3. Expand the numerator:

$$8\cos\theta + 4 - (4 + 4\cos\theta + \cos^2\theta) = 4\cos\theta - \cos^2\theta = \cos\theta(4-\cos\theta).$$

Therefore

$$\frac{dy}{d\theta} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}.$$

Step 4. Sign on $\left[0, \frac{\pi}{2}\right]$:

- $\cos\theta \geq 0$ on $[0, \pi/2]$;
- $4 - \cos\theta \geq 3 > 0$ since $\cos\theta \leq 1$;
- $(2+\cos\theta)^2 > 0$.

Hence $\frac{dy}{d\theta} \geq 0$ on $[0, \pi/2]$, with equality only at $\theta = \pi/2$. So y is increasing on $[0, \pi/2]$.

Final Answer: $\frac{dy}{d\theta} = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2} \geq 0$ on $[0, \pi/2]$, so y is increasing there.

EXPERT'S SOLUTION : *Tara Singh, Ph.D Mathematics, IIT Delhi*

Strategic angle. The denominator $(2+\cos\theta)^2$ is always positive, so the sign of y' comes from the numerator $\cos\theta(4-\cos\theta)$.

Step 1. Differentiate and simplify: $y' = \frac{\cos\theta(4-\cos\theta)}{(2+\cos\theta)^2}$.

Step 2. On $[0, \pi/2]$: $\cos\theta \geq 0$ and $4 - \cos\theta \geq 3$, both non-negative.

Step 3. Therefore $y' \geq 0$ and y is increasing on $[0, \pi/2]$.

Why this matters. Reducing a messy trigonometric derivative to a product of two clearly signed factors is the standard endgame.

Final Answer: y is increasing on $[0, \pi/2]$.

Q 6.10 **Prove that the logarithmic function is increasing on $(0, \infty)$.**

SOLUTION

Concept used. If $f(x) = \log x$ (natural log), then by the standard derivative

$$f'(x) = \frac{1}{x}.$$

For any $x \in (0, \infty)$, $\frac{1}{x} > 0$.

Step 1. $f(x) = \log x$, $f'(x) = \frac{1}{x}$.

Step 2. For every $x > 0$: $\frac{1}{x} > 0$, so $f'(x) > 0$ on $(0, \infty)$.

Step 3. Therefore $\log x$ is strictly increasing on $(0, \infty)$.

> **Final Answer:** $\log x$ is strictly increasing on $(0, \infty)$ because $(\log x)' = 1/x > 0$ there.

EXPERT'S SOLUTION *: Ananya Patel, M.Sc Mathematics, IIT Bombay*

Quick reading. Derivative of $\log x$ is $1/x$, positive on $(0, \infty)$, hence increasing.

Step 1. $(\log x)' = 1/x > 0$ on $(0, \infty)$.

Why this matters. The same proof works for $\log_b x$ with any base $b > 1$ since $(\log_b x)' = 1/(x \ln b) > 0$.

> **Final Answer:** $\log x$ is strictly increasing on $(0, \infty)$.

Q 6.11 **Prove that the function f given by $f(x) = x^2 - x + 1$ is neither strictly increasing nor decreasing on $(-1, 1)$.**

SOLUTION

Concept used. For f to be strictly monotonic on an interval the derivative f' must keep one sign on the whole interval. Show f' changes sign inside $(-1, 1)$.

Step 1. Differentiate:

$$f'(x) = 2x - 1.$$

Step 2. Solve $f'(x) = 0$: $x = \frac{1}{2}$, which lies inside $(-1, 1)$.

Step 3. Sign of f' on $(-1, 1)$:

- On $\left(-1, \frac{1}{2}\right)$: $2x - 1 < 0$, so $f' < 0$ (decreasing).

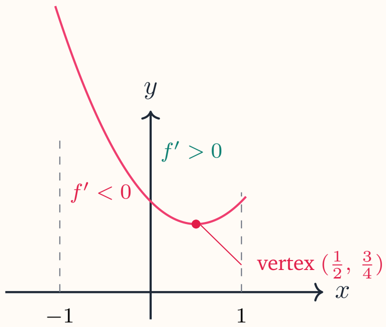
- On $\left(\frac{1}{2}, 1\right)$: $2x - 1 > 0$, so $f' > 0$ (increasing).

Since f' changes sign in $(-1, 1)$, f is neither strictly increasing nor strictly decreasing on $(-1, 1)$.

> **Final Answer:** f decreases on $\left(-1, \frac{1}{2}\right)$ and increases on $\left(\frac{1}{2}, 1\right)$, so it is neither strictly increasing nor strictly decreasing on $(-1, 1)$.

EXPERT'S SOLUTION *: Ishaan Nair, M.Sc Mathematics, IIT Bombay*

Picture-first. A parabola opening upwards has a vertex at the turning point $x = 1/2$. Any interval that straddles the vertex contains both a falling and a rising arm.



Step 1. Vertex of f at $x = 1/2 \in (-1, 1)$.

Step 2. Left of vertex: decreasing; right: increasing.

Step 3. Hence f is non-monotonic on $(-1, 1)$.

Why this matters. Whenever the vertex of a parabola lies strictly inside the test interval, the function cannot be monotonic on that interval.

> **Final Answer:** f is neither strictly increasing nor strictly decreasing on $(-1, 1)$.

Q 6.12 **Which of the following functions are decreasing on $\left(0, \dfrac{\pi}{2}\right)$?**
(A) $\cos x$ **(B)** $\cos 2x$ **(C)** $\cos 3x$ **(D)** $\tan x$.

SOLUTION

Concept used. A function g is strictly decreasing on I iff $g'(x) < 0$ throughout I (with equality at isolated points allowed).

Step 1. **(A)** $g(x) = \cos x \Rightarrow g'(x) = -\sin x$. On $\left(0, \frac{\pi}{2}\right)$, $\sin x > 0$, so $g'(x) < 0$. *Decreasing.*

Step 2. **(B)** $g(x) = \cos 2x \Rightarrow g'(x) = -2\sin 2x$. As x varies over $\left(0, \frac{\pi}{2}\right)$, $2x$ varies over $(0, \pi)$, where $\sin 2x > 0$. So $g' < 0$ throughout. *Decreasing*.

Step 3. **(C)** $g(x) = \cos 3x \Rightarrow g'(x) = -3\sin 3x$. As x varies over $\left(0, \frac{\pi}{2}\right)$, $3x$ varies over $\left(0, \frac{3\pi}{2}\right)$. On $(0, \pi)$, $\sin 3x > 0$; on $\left(\pi, \frac{3\pi}{2}\right)$, $\sin 3x < 0$. Hence the sign of g' changes, so g is *not* monotonic. *Not decreasing*.

Step 4. **(D)** $g(x) = \tan x \Rightarrow g'(x) = \sec^2 x > 0$ on $\left(0, \frac{\pi}{2}\right)$. *Strictly increasing, not decreasing*.

Final Answer: Only options (A) and (B) are decreasing on $\left(0, \frac{\pi}{2}\right)$.

EXPERT'S SOLUTION *: Riya Mehta, M.Sc Mathematics, IIT Bombay*

Quick reading. Track the argument range: $x \in (0, \pi/2)$ sends $2x$ into $(0, \pi)$, $3x$ into $(0, 3\pi/2)$. The first stays inside the "$\sin > 0$" zone, the second leaves it.

Step 1. $\cos x$: $g' = -\sin x < 0 \Rightarrow$ decreasing. ✓

Step 2. $\cos 2x$: $g' = -2\sin 2x < 0$ throughout. ✓

Step 3. $\cos 3x$: sign changes $\times$

Step 4. $\tan x$: $\sec^2 x > 0 \Rightarrow$ increasing $\times$

Why this matters. For $\cos kx$, monotonicity over an interval depends on whether kx stays inside a single half-period.

Final Answer: (A) and (B).

Q 6.13 **On which of the following intervals is the function f given by $f(x) = x^{100} + \sin x - 1$ decreasing?**

(A) $(0, 1)$ **(B)** $\left(\frac{\pi}{2}, \pi\right)$ **(C)** $\left(0, \frac{\pi}{2}\right)$ **(D) None of these.**

SOLUTION

Concept used. f is decreasing on an interval iff $f'(x) < 0$ throughout.

Step 1. Differentiate:

$$f'(x) = 100x^{99} + \cos x.$$

Step 2. **(A)** $(0, 1)$**:** For $x \in (0, 1)$, $x^{99} > 0$ so $100x^{99} > 0$; also $\cos x > 0$ since $x < 1 < \pi/2$. Hence $f'(x) > 0$ on $(0, 1)$: *increasing*, not decreasing.

Step 3. **(B)** $(\pi/2, \pi)$**:** For $x > \pi/2 > 1$, $100x^{99} > 100$, while $\cos x \in (-1, 0)$. Thus

$f'(x) \geq 100 + (-1) = 99 > 0$: increasing, not decreasing.

Step 4. **(C)** $(0, \pi/2)$**:** For $x > 0$, $100x^{99} > 0$ and $\cos x > 0$, so $f' > 0$: increasing, not decreasing.

Step 5. Therefore none of (A), (B), (C) gives a decreasing interval.

Final Answer: Option (D), None of these.

EXPERT'S SOLUTION *: Karan Verma, M.Sc Mathematics, IIT Bombay*

Quick reading. $100x^{99}$ grows extremely fast; for $|x| \geq 1$ it dwarfs $|\cos x|$. So f' stays positive on the listed intervals.

Step 1. $f'(x) = 100x^{99} + \cos x$.

Step 2. All listed intervals lie in $x \geq 0$, where $100x^{99} \geq 0$ and $|\cos x| \leq 1$; for $x \geq 1$ the polynomial part dominates.

Step 3. Hence $f' > 0$ on all three intervals, and f is not decreasing on any of them.

Why this matters. Inequality estimates ("the big term beats the bounded term") often prove monotonicity faster than solving $f' = 0$ exactly.

Final Answer: Option (D).

Q 6.14 **For what values of a the function f given by $f(x) = x^2 + ax + 1$ is increasing on $[1, 2]$?**

SOLUTION

Concept used. f is increasing on $[1, 2]$ iff $f'(x) \geq 0$ for all $x \in [1, 2]$.

Step 1. Differentiate:

$$f'(x) = 2x + a.$$

Step 2. Demand $f'(x) \geq 0$ on $[1, 2]$:

$$2x + a \geq 0 \;\Rightarrow\; a \geq -2x \quad \text{for all } x \in [1, 2].$$

Step 3. The right-hand side $-2x$ is largest (least negative) when x is smallest, i.e. at $x = 1$: maximum value is -2. So

$$a \geq -2x_{\min} = -2(1) = -2.$$

Hence $a \geq -2$.

Final Answer: f is increasing on $[1, 2]$ for $a \geq -2$.

☞ **Exam Tip**

For a "for what values of a" type problem, translate the monotonicity condition into an inequality $f'(x) \geq 0$ and minimise f' over the given interval. The minimum value, set ≥ 0, gives the tightest constraint on a.

EXPERT'S SOLUTION : *Pooja Gupta, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Translate "$f' \geq 0$ on $[1, 2]$" into a single inequality at the worst case ($x = 1$).

Step 1. $f'(x) = 2x + a$.

Step 2. On $[1, 2]$, $2x \geq 2$, so $f' \geq 2 + a$.

Step 3. Hence $2 + a \geq 0 \Rightarrow a \geq -2$.

Why this matters. For linear-in-x derivatives, the minimum on an interval is at one endpoint; that single value drives the parameter constraint.

Final Answer: $a \geq -2$.

Q 6.15 **Let I be any interval disjoint from $[-1, 1]$. Prove that the function f given by $f(x) = x + \frac{1}{x}$ is increasing on I.**

SOLUTION

Concept used. If $f'(x) > 0$ on I, then f is increasing on I.

Step 1. Differentiate:

$$f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}.$$

Step 2. Sign of f':

- $x^2 > 0$ always (where f is defined, $x \neq 0$).
- Sign of f' matches sign of $x^2 - 1 = (x - 1)(x + 1)$.
- $x^2 - 1 > 0 \iff |x| > 1 \iff x \in (-\infty, -1) \cup (1, \infty)$.

Step 3. If I is any interval disjoint from $[-1, 1]$, then I lies entirely in $(-\infty, -1)$ or entirely in $(1, \infty)$. In either case $x^2 - 1 > 0$ on I, so $f'(x) > 0$ on I.

Step 4. Therefore f is strictly increasing on I.

Final Answer: $f'(x) = \frac{x^2-1}{x^2} > 0$ on any interval disjoint from $[-1, 1]$, so f is increasing there.

EXPERT'S SOLUTION *: Aditya Iyer, Ph.D Mathematics, IIT Delhi*

Strategic angle. The derivative factors into a single expression $(x^2 - 1)/x^2$ whose sign is decided by $|x|$ vs 1.

Step 1. $f'(x) = 1 - 1/x^2 = (x^2 - 1)/x^2$.

Step 2. $|x| > 1 \Rightarrow x^2 > 1 \Rightarrow f' > 0$.

Step 3. Any I disjoint from $[-1, 1]$ satisfies $|x| > 1 \Rightarrow f' > 0$ on $I \Rightarrow$ increasing.

Why this matters. Combining "$|x| > 1$" with "interval disjoint from $[-1, 1]$" eliminates the need for separate left/right cases.

Final Answer: f is increasing on every interval disjoint from $[-1, 1]$.

Q 6.16 **Prove that the function f given by $f(x) = \log \sin x$ is increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$.**

SOLUTION

Concept used. Use the chain rule: if $f(x) = \log u(x)$ with $u(x) > 0$, then $f'(x) = \frac{u'(x)}{u(x)}$.

Step 1. Here $u(x) = \sin x > 0$ on $(0, \pi)$. So

$$f'(x) = \frac{\cos x}{\sin x} = \cot x.$$

Step 2. Sign of $\cot x$:

- On $(0, \frac{\pi}{2})$, $\sin x > 0$ and $\cos x > 0$, so $\cot x > 0$. Hence $f' > 0$ and f is increasing.
- On $(\frac{\pi}{2}, \pi)$, $\sin x > 0$ but $\cos x < 0$, so $\cot x < 0$. Hence $f' < 0$ and f is decreasing.

Final Answer: f is increasing on $(0, \frac{\pi}{2})$ and decreasing on $(\frac{\pi}{2}, \pi)$.

EXPERT'S SOLUTION : *Sanya Reddy, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading. The derivative is simply $\cot x$, which is positive in Q1 and negative in Q2.

Step 1. $f'(x) = \cot x$.

Step 2. $\cot x > 0$ on $(0, \pi/2)$, $\cot x < 0$ on $(\pi/2, \pi)$.

Why this matters. Logarithmic differentiation collapses $\log \sin x \to \cot x$, a single trigonometric function whose sign chart is well-known.

> **Final Answer:** Increasing on $(0, \pi/2)$, decreasing on $(\pi/2, \pi)$.

Q 6.17 **Prove that the function f given by $f(x) = \log|\cos x|$ is decreasing on $\left(0, \frac{\pi}{2}\right)$ and increasing on $\left(\frac{3\pi}{2}, 2\pi\right)$.**

SOLUTION

Concept used. For $f(x) = \log|\cos x|$, differentiate using the chain rule with $\frac{d}{dx}\log|u| = \frac{u'}{u}$ wherever $u \neq 0$.

Step 1. Let $u(x) = \cos x$. Then

$$f'(x) = \frac{-\sin x}{\cos x} = -\tan x.$$

Step 2. Sign of $-\tan x$:

- On $\left(0, \frac{\pi}{2}\right)$, $\tan x > 0$, so $-\tan x < 0$. Hence $f' < 0$: f is decreasing.
- On $\left(\frac{3\pi}{2}, 2\pi\right)$, $\sin x < 0$ and $\cos x > 0$, so $\tan x = \sin x / \cos x < 0$. Therefore $-\tan x > 0$: $f' > 0$ and f is increasing.

> **Final Answer:** f is decreasing on $\left(0, \frac{\pi}{2}\right)$ and increasing on $\left(\frac{3\pi}{2}, 2\pi\right)$.

EXPERT'S SOLUTION : *Aarav Pillai, Ph.D Pure Mathematics, IISc Bangalore*

Quick reading. The derivative reduces to $-\tan x$. Track where $\tan x$ is positive/negative.

Step 1. $f' = -\tan x$.

Step 2. $\tan x > 0$ on $(0, \pi/2)$: $f' < 0$, decreasing.

Step 3. $\tan x < 0$ on $(3\pi/2, 2\pi)$: $f' > 0$, increasing.

Why this matters. $\log|\cos x|$ behaves the same way on each π-period: monotonic in opposite directions on the two halves.

Final Answer: Decreasing on $(0, \pi/2)$ and increasing on $(3\pi/2, 2\pi)$.

Q 6.18 **Prove that the function given by $f(x) = x^3 - 3x^2 + 3x - 100$ is increasing in $\mathbb{R}$.**

SOLUTION

Concept used. If $f'(x) \geq 0$ on $\mathbb{R}$ (with equality at only isolated points), f is monotonically increasing on $\mathbb{R}$.

Step 1. Differentiate:

$$f'(x) = 3x^2 - 6x + 3.$$

Step 2. Factor: pull out 3 and complete the square:

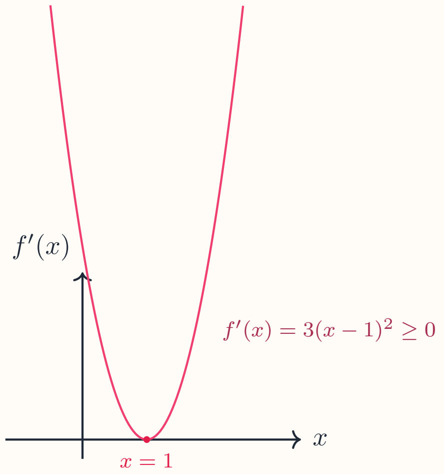
$$f'(x) = 3(x^2 - 2x + 1) = 3(x-1)^2.$$

Step 3. Since $(x-1)^2 \geq 0$ for every real x, we have $f'(x) \geq 0$ on $\mathbb{R}$, with $f'(1) = 0$ being the only zero. Therefore f is monotonically increasing on $\mathbb{R}$.

Final Answer: $f'(x) = 3(x-1)^2 \geq 0$ on $\mathbb{R}$, so f is monotonically increasing.

EXPERT'S SOLUTION : *Yash Chatterjee, M.Sc Mathematics, IIT Bombay*

Quick reading. The derivative is a perfect square (up to a positive multiplier), hence non-negative everywhere.



Step 1. $f'(x) = 3x^2 - 6x + 3 = 3(x-1)^2$.

Step 2. $(x-1)^2 \geq 0$ for all real x.

Step 3. Therefore f is increasing on $\mathbb{R}$ (with horizontal tangent only at $x = 1$).

Why this matters. A cubic with $b^2 - 3ac \leq 0$ (discriminant of f') is monotonic on $\mathbb{R}$, because its derivative cannot change sign.

Final Answer: f is increasing on $\mathbb{R}$.

Q 6.19 **The interval in which $y = x^2e^{-x}$ is increasing is**

(A) $(-\infty, \infty)$ **(B)** $(-2, 0)$ **(C)** $(2, \infty)$ **(D)** $(0, 2)$.

SOLUTION

Concept used. Product rule on $y = x^2e^{-x}$ then sign analysis.

Step 1. Differentiate using the product rule:

$$\frac{dy}{dx} = 2x\,e^{-x} + x^2(-e^{-x}) = e^{-x}(2x - x^2) = -x(x-2)\,e^{-x}.$$

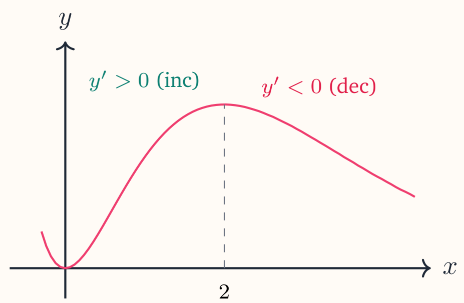
Step 2. Since $e^{-x} > 0$ for every real x, the sign of y' is the sign of $-x(x-2) = x(2-x)$.

Step 3. Solve $-x(x-2) > 0$, i.e. $x(x-2) < 0$, i.e. $0 < x < 2$.

Final Answer: Option (D), y is increasing on $(0, 2)$.

EXPERT'S SOLUTION : *Vivaan Sharma, M.Sc Mathematics, IIT Bombay*

Quick reading. $y' = e^{-x}\,x(2-x)$. The exponential is positive, so the sign of y' follows $x(2-x)$, positive only on $(0, 2)$.



Step 1. $y' = e^{-x}\,x(2-x)$.

Step 2. $x(2-x) > 0 \iff x \in (0, 2)$.

Why this matters. Mixing a polynomial with an always-positive exponential is common in probability and physics; the sign always comes from the polynomial factor.

> **Final Answer:** Option (D), $(0, 2)$.

Key Takeaways

- f is strictly increasing on I iff $f'(x) > 0$ on I (decreasing iff $f'(x) < 0$).
- Critical points are where f' vanishes or fails to exist; sign chart between critical points decides monotonicity.
- Squared factors in f' give $f' = 0$ without sign change (points of inflection).
- Trigonometric monotonicity: track the range of the inner argument as x varies.

End of Exercise 6.2