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Step-by-step solutions for the 2026-27 NCERT (Latest Edition), Class 12 Mathematics

Chapter 7: Integrals

About this Chapter

Integrals is the gateway to integral calculus for Class 12. Exercise 7.1 trains you to recognise an integrand as the derivative of some standard function and read off its **anti derivative** by inspection. The toolkit here is the table of **standard integrals** (powers of x, sin, cos, $\sec^2$, e^{ax}) combined with the linearity rule

$$\int [k_1 f_1(x) + k_2 f_2(x)]\, dx = k_1 \int f_1(x)\, dx + k_2 \int f_2(x)\, dx.$$

Topics covered: Anti derivatives • Standard integrals • Linearity of integration • Method of inspection

Quick Formula Sheet

Power rule: $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$, $\int \frac{1}{x}\, dx = \log|x| + C$

Trigonometric: $\int \sin x\, dx = -\cos x + C$, $\int \cos x\, dx = \sin x + C$

$\int \sec^2 x\, dx = \tan x + C$, $\int \csc^2 x\, dx = -\cot x + C$

Exponential: $\int e^{ax}\, dx = \frac{e^{ax}}{a} + C$

Linearity: $\int [k_1 f_1 + k_2 f_2]\, dx = k_1 \int f_1\, dx + k_2 \int f_2\, dx$

Exercise 7.1

Q 7.1 **Find an anti derivative (or integral) of $\sin 2x$ by the method of inspection.**

SOLUTION

Concept used. An **anti derivative** (or indefinite integral) of a function $f(x)$ is any function $F(x)$ whose derivative is $f(x)$. In symbols,

$$\int f(x)\,dx = F(x) + C \iff \frac{d}{dx}F(x) = f(x).$$

The *method of inspection* consists in guessing a candidate $F(x)$ from the table of standard derivatives, then differentiating to check. The relevant standard derivative here is

$$\frac{d}{dx}(\cos kx) = -k\sin kx.$$

Step 1. We need $F(x)$ such that $F'(x) = \sin 2x$. By inspection $\cos 2x$ produces a $\sin 2x$ on differentiation (with a sign and a factor 2), so we try

$$F(x) = -\tfrac{1}{2}\cos 2x.$$

Step 2. Differentiate to verify:

$$\frac{d}{dx}\left(-\tfrac{1}{2}\cos 2x\right) = -\tfrac{1}{2}\cdot(-\sin 2x)\cdot 2 = \sin 2x.$$

The derivative equals the integrand, so the guess is correct.

Step 3. Adding an arbitrary constant of integration C gives the general anti derivative.

Final Answer: $\int \sin 2x\,dx = -\dfrac{1}{2}\cos 2x + C$

☞ Exam Tip

For any nonzero constant k, $\int \sin(kx)\,dx = -\dfrac{1}{k}\cos(kx) + C$ and $\int \cos(kx)\,dx = \dfrac{1}{k}\sin(kx) + C$. Remember the reciprocal $1/k$; a common slip is to drop it.

EXPERT'S SOLUTION : *Aarav Sharma, M.Sc Mathematics, IIT Bombay*

Strategic angle. Think of integration as reversing a chain rule. The integrand $\sin 2x$ is the derivative of $\cos 2x$ multiplied by -2 (because the inner function $2x$ has derivative 2). To undo the inner factor we divide by 2 and flip the sign.

Step 1. Write the standard chain-rule identity

$$\frac{d}{dx}\cos(kx) = -k\sin(kx).$$

Set $k = 2$:

$$\frac{d}{dx}\cos(2x) = -2\sin(2x).$$

Step 2. Divide both sides by -2:

$$\frac{d}{dx}\left(-\tfrac{1}{2}\cos 2x\right) = \sin 2x.$$

Step 3. By definition of anti derivative, $-\frac{1}{2}\cos 2x$ is an anti derivative of $\sin 2x$. The general anti derivative is obtained by adding $C \in \mathbb{R}$.

Why this matters. The pattern $\int \sin(kx)\,dx = -\frac{1}{k}\cos(kx) + C$ appears in every Class 12 paper. Internalise the reciprocal factor.

Final Answer: $-\dfrac{1}{2}\cos 2x + C$

Q 7.2 **Find an anti derivative of $\cos 3x$ by the method of inspection.**

SOLUTION

Concept used. The standard derivative $\dfrac{d}{dx}\sin(kx) = k\cos(kx)$ gives, on rearrangement, the standard integral

$$\int \cos(kx)\,dx = \frac{1}{k}\sin(kx) + C.$$

The constant $C \in \mathbb{R}$ accounts for the fact that any two anti derivatives differ by a constant.

Step 1. Try $F(x) = \frac{1}{3}\sin 3x$. Differentiate using the chain rule:

$$F'(x) = \tfrac{1}{3}\cdot\cos 3x \cdot 3 = \cos 3x.$$

Step 2. Since $F'(x) = \cos 3x$ equals the integrand, $F(x)$ is an anti derivative.

Step 3. Add the arbitrary constant C.

Final Answer: $\displaystyle\int \cos 3x\,dx = \frac{1}{3}\sin 3x + C$

EXPERT'S SOLUTION : *Sneha Iyer, M.Sc Applied Mathematics, IIT Kanpur*

Quick reading. Reverse the chain rule. The outer anti derivative of $\cos$ is $\sin$; the inner function $3x$ contributes a 3 that must be cancelled by $1/3$.

Step 1. Start from $\frac{d}{dx}\sin(kx) = k\cos(kx)$. Plug $k = 3$:

$$\frac{d}{dx}\sin 3x = 3\cos 3x.$$

Step 2. Divide both sides by 3:

$$\frac{d}{dx}\left(\tfrac{1}{3}\sin 3x\right) = \cos 3x.$$

Step 3. Hence $\frac{1}{3}\sin 3x$ is an anti derivative; the general one adds C.

> **Final Answer:** $\frac{1}{3}\sin 3x + C$

Q 7.3 Find an anti derivative of e^{2x} by the method of inspection.

SOLUTION

Concept used. For any nonzero constant a, the chain rule gives $\frac{d}{dx}e^{ax} = a\,e^{ax}$, equivalently

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax} + C.$$

Step 1. Try $F(x) = \frac{1}{2}e^{2x}$. By the chain rule,

$$F'(x) = \tfrac{1}{2}\cdot e^{2x}\cdot 2 = e^{2x}.$$

Step 2. This matches the integrand, so F is an anti derivative.

Step 3. The general anti derivative is $F(x) + C$.

> **Final Answer:** $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$

EXPERT'S SOLUTION : *Karan Mehta, B.Tech Engineering Physics, IIT Bombay*

Strategic angle. The exponential is self-replicating under differentiation, so the only correction needed is to undo the inner constant via a $1/a$ factor.

Step 1. Write $\frac{d}{dx}e^{ax} = ae^{ax}$.

Step 2. Setting $a = 2$: $\frac{d}{dx}e^{2x} = 2e^{2x}$.

Step 3. Divide by 2: $\frac{d}{dx}\left(\frac{1}{2}e^{2x}\right) = e^{2x}$. Therefore the anti derivative is $\frac{1}{2}e^{2x} + C$.

Final Answer: $\frac{e^{2x}}{2} + C$

Q 7.4 Find an anti derivative of $(ax+b)^2$ by the method of inspection.

SOLUTION

Concept used. The power rule combined with the chain rule gives, for any constants $a \neq 0$, b and exponent $n \neq -1$,

$$\frac{d}{dx}\left[\frac{(ax+b)^{n+1}}{a(n+1)}\right] = (ax+b)^n.$$

Hence $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C.$

Step 1. Try $F(x) = \frac{(ax+b)^3}{3a}$. Differentiate using the chain rule:

$$F'(x) = \frac{3(ax+b)^2 \cdot a}{3a} = (ax+b)^2.$$

Step 2. This equals the integrand, so $F(x)$ is an anti derivative.

Final Answer: $\int (ax+b)^2\,dx = \frac{(ax+b)^3}{3a} + C$

✗ Common Mistake

A frequent error is to write $(ax+b)^3/3$, forgetting the $\frac{1}{a}$ factor that cancels the inner derivative a. Always divide by the coefficient of x inside the bracket.

EXPERT'S SOLUTION *: Pranav Patel, Ph.D Mathematics, IIT Delhi*

Picture-first. If you expand, $(ax+b)^2 = a^2x^2 + 2abx + b^2$, and the anti derivative is $\frac{a^2x^3}{3} + abx^2 + b^2x + C$. The compact form $\frac{(ax+b)^3}{3a}$ packages the same answer using the chain-rule reversal.

Step 1. Apply the generalised power rule with $n = 2$, replacing x by the linear inner function $ax+b$:

$$\int (ax+b)^2\,dx = \frac{(ax+b)^3}{3}\cdot\frac{1}{a}+C.$$

Step 2. The factor $1/a$ is the inverse of the inner derivative a produced by the chain rule. Verify by differentiating:

$$\frac{d}{dx}\left[\frac{(ax+b)^3}{3a}\right] = \frac{3(ax+b)^2\cdot a}{3a} = (ax+b)^2. ✓$$

Final Answer: $\dfrac{(ax+b)^3}{3a}+C$

Q 7.5 **Find an anti derivative of $\sin 2x - 4e^{3x}$ by the method of inspection.**

SOLUTION

Concept used. **Linearity of integration**: for constants k_1, k_2 and functions f, g,

$$\int [k_1 f(x) + k_2 g(x)]\,dx = k_1\int f(x)\,dx + k_2\int g(x)\,dx.$$

We also use the standard integrals $\int \sin(kx)\,dx = -\frac{1}{k}\cos(kx)+C$ and $\int e^{ax}\,dx = \frac{1}{a}e^{ax}+C$.

Step 1. Split the integrand using linearity:

$$\int (\sin 2x - 4e^{3x})\,dx = \int \sin 2x\,dx - 4\int e^{3x}\,dx.$$

Step 2. Integrate the first piece (Q1 result): $\int \sin 2x\,dx = -\frac{1}{2}\cos 2x$.

Step 3. Integrate the second piece: $\int e^{3x}\,dx = \frac{1}{3}e^{3x}$. Multiply by -4:

$$-4\cdot\tfrac{1}{3}e^{3x} = -\tfrac{4}{3}e^{3x}.$$

Step 4. Combine and add the constant of integration:

$$\int (\sin 2x - 4e^{3x})\,dx = -\tfrac{1}{2}\cos 2x - \tfrac{4}{3}e^{3x}+C.$$

Final Answer: $-\dfrac{1}{2}\cos 2x - \dfrac{4}{3}e^{3x}+C$

EXPERT'S SOLUTION *: Riya Gupta, M.Sc Mathematics, ISI Kolkata*

Strategic angle. Linearity lets us treat each term independently. Manage the chain-rule reciprocal in each term separately, then assemble.

Step 1. Term 1: $\int \sin 2x\, dx$. Use $\int \sin(kx)\, dx = -\frac{1}{k}\cos(kx)$ with $k = 2$ to get $-\frac{1}{2}\cos 2x$.

Step 2. Term 2: $\int -4e^{3x}\, dx = -4 \cdot \frac{1}{3}e^{3x} = -\frac{4}{3}e^{3x}$.

Step 3. Add the two pieces and the constant C.

Why this matters. Most NCERT Q5 type problems combine 2-3 standard forms with a constant multiplier. Handle each in isolation, then sum.

Final Answer: $-\dfrac{1}{2}\cos 2x - \dfrac{4}{3}e^{3x} + C$

Q 7.6 **Find** $\displaystyle\int (4e^{3x} + 1)\, dx$.

SOLUTION

Concept used. Linearity of integration and the standard integrals $\int e^{ax}\, dx = \frac{1}{a}e^{ax} + C$ and $\int 1\, dx = x + C$.

Step 1. Split: $\displaystyle\int (4e^{3x} + 1)\, dx = 4\int e^{3x}\, dx + \int 1\, dx$.

Step 2. Compute $\int e^{3x}\, dx = \frac{1}{3}e^{3x}$, hence $4\int e^{3x}\, dx = \frac{4}{3}e^{3x}$.

Step 3. $\int 1\, dx = x$. Combine: $\frac{4}{3}e^{3x} + x$.

Final Answer: $\displaystyle\int (4e^{3x} + 1)\, dx = \frac{4}{3}e^{3x} + x + C$

EXPERT'S SOLUTION *: Ankit Verma, M.Tech CS, IIT Madras*

Quick reading. Two standard pieces; integrate each, add C.

Step 1. $\int 4e^{3x}\, dx = 4 \cdot \dfrac{e^{3x}}{3} = \dfrac{4}{3}e^{3x}$.

Step 2. $\int 1\, dx = x$.

Step 3. Sum and add the constant.

Final Answer: $\dfrac{4e^{3x}}{3} + x + C$

Q 7.7 **Find** $\displaystyle\int x^2(1 - 1/x^2)\,dx$.

SOLUTION

Concept used. First simplify the integrand algebraically using distributivity: $x^2(1 - x^{-2}) = x^2 - 1$. Then use the power rule $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.

Step 1. Expand the bracket:

$$x^2\left(1 - \frac{1}{x^2}\right) = x^2 \cdot 1 - x^2 \cdot \frac{1}{x^2} = x^2 - 1.$$

Step 2. Integrate term by term using linearity:

$$\int (x^2 - 1)\,dx = \int x^2\,dx - \int 1\,dx.$$

Step 3. Apply the power rule: $\int x^2\,dx = \dfrac{x^3}{3}$ and $\int 1\,dx = x$.

Step 4. Combine and add the integration constant:

$$\int x^2\left(1 - \tfrac{1}{x^2}\right)dx = \tfrac{x^3}{3} - x + C.$$

Final Answer: $\dfrac{x^3}{3} - x + C$

EXPERT'S SOLUTION : *Yash Joshi, M.Sc Mathematics, IIT Bombay*

Structural observation. Always look for algebraic simplification *before* reaching for a method. Here the integrand collapses to a polynomial, which is the simplest case.

Step 1. Distribute: $x^2 - 1$.

Step 2. Use $\int x^n\,dx = x^{n+1}/(n+1)$ with $n = 2$, then linearity for the -1 term.

Step 3. Final: $x^3/3 - x + C$.

Final Answer: $\dfrac{x^3}{3} - x + C$

Q 7.8 **Find** $\int (ax^2 + bx + c)\, dx$**.**

SOLUTION

Concept used. Linearity of integration with the power rule $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.

Step 1. Apply linearity:

$$\int (ax^2 + bx + c)\, dx = a \int x^2\, dx + b \int x\, dx + c \int 1\, dx.$$

Step 2. Compute each: $\int x^2\, dx = x^3/3$, $\int x\, dx = x^2/2$, $\int 1\, dx = x$.

Step 3. Multiply by the respective constants and add:

$$a \cdot \tfrac{x^3}{3} + b \cdot \tfrac{x^2}{2} + c \cdot x = \tfrac{ax^3}{3} + \tfrac{bx^2}{2} + cx.$$

Final Answer: $\displaystyle\int (ax^2 + bx + c)\, dx = \frac{ax^3}{3} + \frac{bx^2}{2} + cx + C$

EXPERT'S SOLUTION *: Aanya Singh, M.Sc Mathematics, IIT Bombay*

Strategic angle. A polynomial integrates monomial-by-monomial. The exponent goes up by 1 and the new term is divided by the new exponent.

Step 1. $ax^2 \to \frac{ax^3}{3}$.

Step 2. $bx \to \frac{bx^2}{2}$.

Step 3. $c \to cx$.

Step 4. Sum and add C.

Final Answer: $\dfrac{ax^3}{3} + \dfrac{bx^2}{2} + cx + C$

Q 7.9 **Find** $\int (2x^2 + e^x)\, dx$.

SOLUTION

Concept used. Linearity, the power rule with $n = 2$, and the standard integral $\int e^x\, dx = e^x + C$ (since $\frac{d}{dx}e^x = e^x$).

Step 1. Split: $\int (2x^2 + e^x)\, dx = 2\int x^2\, dx + \int e^x\, dx$.

Step 2. $\int x^2\, dx = x^3/3$, so $2\int x^2\, dx = \frac{2}{3}x^3$.

Step 3. $\int e^x\, dx = e^x$.

Step 4. Combine: $\frac{2}{3}x^3 + e^x + C$.

Final Answer: $\dfrac{2x^3}{3} + e^x + C$

EXPERT'S SOLUTION *: Diya Kapoor, M.Sc Mathematics, IIT Bombay*

Quick reading. Two standard pieces.

Step 1. Power rule: $\int 2x^2\, dx = \dfrac{2x^3}{3}$.

Step 2. Exponential: $\int e^x\, dx = e^x$.

Step 3. Sum, add C.

Final Answer: $\dfrac{2x^3}{3} + e^x + C$

Q 7.10 **Find** $\int \left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)^2 dx$.

SOLUTION

Concept used. Expand the square using $(a-b)^2 = a^2 - 2ab + b^2$, then apply linearity and the power rule.

Step 1. Expand:

$$\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = (\sqrt{x})^2 - 2\cdot\sqrt{x}\cdot\frac{1}{\sqrt{x}} + \frac{1}{x} = x - 2 + \frac{1}{x}.$$

Step 2. Integrate each term using linearity:

$$\int\left(x-2+\tfrac{1}{x}\right)dx = \int x\,dx - 2\int 1\,dx + \int \tfrac{1}{x}\,dx.$$

Step 3. $\int x\,dx = x^2/2$; $\int 1\,dx = x$; $\int \frac{1}{x}\,dx = \log|x|$.

Step 4. Combine:

$$\frac{x^2}{2} - 2x + \log|x| + C.$$

Final Answer: $\dfrac{x^2}{2} - 2x + \log|x| + C$

✗ Common Mistake

Do not forget the absolute value in $\log|x|$. The function $1/x$ is defined for both positive and negative x, and its anti derivative is $\log|x|$, not $\log x$.

EXPERT'S SOLUTION : *Vivaan Nair, Ph.D Pure Mathematics, IISc Bangalore*

Structural observation. Squaring a binomial that mixes $\sqrt{x}$ and $1/\sqrt{x}$ produces three power-of-x terms; integrate each.

Step 1. $(\sqrt{x} - x^{-1/2})^2 = x - 2 + x^{-1}$.

Step 2. Anti derivatives: $x \to x^2/2$; constant $-2 \to -2x$; $x^{-1} \to \log|x|$.

Step 3. Sum with C.

Final Answer: $\dfrac{x^2}{2} - 2x + \log|x| + C$

Q 7.11 **Find** $\displaystyle\int \frac{x^3 + 5x^2 - 4}{x^2}\,dx.$

SOLUTION

Concept used. Divide each term in the numerator by the denominator (term-by-term division for rational functions where the denominator is a single monomial), then apply linearity and the power rule. We use $\int x^n\,dx = x^{n+1}/(n+1)$ for $n \neq -1$ and $\int x^{-2}\,dx = -x^{-1}$.

Step 1. Term-by-term division:

$$\frac{x^3 + 5x^2 - 4}{x^2} = \frac{x^3}{x^2} + \frac{5x^2}{x^2} - \frac{4}{x^2} = x + 5 - 4x^{-2}.$$

Step 2. Integrate using linearity:

$$\int (x + 5 - 4x^{-2})\, dx = \int x\, dx + 5\int 1\, dx - 4\int x^{-2}\, dx.$$

Step 3. Compute each: $\int x\, dx = x^2/2$; $\int 1\, dx = x$; $\int x^{-2}\, dx = -x^{-1} = -1/x$. Therefore $-4\int x^{-2}\, dx = -4(-1/x) = 4/x$.

Step 4. Combine:

$$\frac{x^2}{2} + 5x + \frac{4}{x} + C.$$

Final Answer: $\frac{x^2}{2} + 5x + \frac{4}{x} + C$

EXPERT'S SOLUTION *: Tara Bhat, M.Sc Mathematics, ISI Kolkata*

Strategic angle. When the denominator is a single power of x, split the fraction immediately and integrate each piece as a power.

Step 1. Split: $\frac{x^3 + 5x^2 - 4}{x^2} = x + 5 - 4x^{-2}$.

Step 2. Anti derivatives: $x \to x^2/2$, $5 \to 5x$, $-4x^{-2} \to -4 \cdot \frac{x^{-1}}{-1} = 4/x$.

Step 3. Sum: $\frac{x^2}{2} + 5x + \frac{4}{x} + C$.

Final Answer: $\frac{x^2}{2} + 5x + \frac{4}{x} + C$

Q 7.12 **Find** $\int \frac{x^3 + 3x + 4}{\sqrt{x}}\, dx$.

SOLUTION

Concept used. Term-by-term division and the power rule. Recall $1/\sqrt{x} = x^{-1/2}$ and division of powers: $x^n / x^{1/2} = x^{n-1/2}$.

Step 1. Divide each numerator term by $\sqrt{x} = x^{1/2}$:

$$\frac{x^3 + 3x + 4}{\sqrt{x}} = x^{3-1/2} + 3x^{1-1/2} + 4x^{-1/2} = x^{5/2} + 3x^{1/2} + 4x^{-1/2}.$$

Step 2. Apply the power rule term by term, using $\int x^n\,dx = \frac{x^{n+1}}{n+1}$:

$$\int x^{5/2}\,dx = \frac{x^{7/2}}{7/2} = \frac{2}{7}x^{7/2},$$
$$\int 3x^{1/2}\,dx = 3\cdot\frac{x^{3/2}}{3/2} = 3\cdot\frac{2}{3}x^{3/2} = 2x^{3/2},$$
$$\int 4x^{-1/2}\,dx = 4\cdot\frac{x^{1/2}}{1/2} = 4\cdot 2x^{1/2} = 8x^{1/2}.$$

Step 3. Sum the three pieces and add C:

$$\tfrac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C.$$

Final Answer: $\frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$

EXPERT'S SOLUTION : *Ishaan Reddy, B.Tech CSE, IIT Roorkee*

Quick reading. Convert the surd to a power, split, integrate, simplify.

Step 1. Write $1/\sqrt{x} = x^{-1/2}$, then $\frac{x^3 + 3x + 4}{\sqrt{x}} = x^{5/2} + 3x^{1/2} + 4x^{-1/2}$.

Step 2. Power-rule each term and multiply by the coefficient.

Step 3. Combine: $\frac{2}{7}x^{7/2} + 2x^{3/2} + 8\sqrt{x} + C$.

Final Answer: $\frac{2x^{7/2}}{7} + 2x^{3/2} + 8\sqrt{x} + C$

Q 7.13 **Find** $\int \frac{x^3 - x^2 + x - 1}{x - 1}\,dx$.

SOLUTION

Concept used. Polynomial division (or factorisation) reduces a rational function to a polynomial, after which we apply the power rule. We factor the numerator by grouping.

Step 1. Factor the numerator by grouping:

$$x^3 - x^2 + x - 1 = x^2(x-1) + 1 \cdot (x-1) = (x-1)(x^2+1).$$

Step 2. Cancel the common factor $(x-1)$ from the numerator and denominator (valid for $x \neq 1$):

$$\frac{(x-1)(x^2+1)}{x-1} = x^2 + 1.$$

Step 3. Integrate term by term:

$$\int (x^2+1)\,dx = \int x^2\,dx + \int 1\,dx = \tfrac{x^3}{3} + x.$$

Step 4. Add the constant of integration.

Final Answer: $\dfrac{x^3}{3} + x + C$

♥ **Factoring before integrating**

Looking for an algebraic simplification (factor, cancel, divide) is almost always cheaper than reaching for substitution or partial fractions. Always check whether the numerator factors before applying heavier machinery.

EXPERT'S SOLUTION *: Rohit Desai, M.Sc Mathematics, IIT Bombay*

Structural observation. The numerator $x^3 - x^2 + x - 1$ groups as $x^2(x-1) + (x-1)$, hinting that $(x-1)$ is a factor.

Step 1. Factor: $x^3 - x^2 + x - 1 = (x-1)(x^2+1)$.

Step 2. Divide: the fraction simplifies to $x^2 + 1$.

Step 3. Integrate: $\int (x^2+1)\,dx = x^3/3 + x + C$.

Final Answer: $\dfrac{x^3}{3} + x + C$

Q 7.14 **Find** $\displaystyle\int (1-x)\sqrt{x}\,dx$.

SOLUTION

Concept used. Distribute, then apply the power rule. Recall $\sqrt{x} = x^{1/2}$ and $x \cdot \sqrt{x} = x^{3/2}$.

Step 1. Expand: $(1-x)\sqrt{x} = \sqrt{x} - x\sqrt{x} = x^{1/2} - x^{3/2}$.

Step 2. Integrate using the power rule:

$$\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2},$$
$$\int x^{3/2}\,dx = \frac{x^{5/2}}{5/2} = \frac{2}{5}x^{5/2}.$$

Step 3. Combine: $\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$.

Final Answer: $\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$

EXPERT'S SOLUTION : *Meera Banerjee, Ph.D Pure Mathematics, IISc Bangalore*

Quick reading. Two half-integer powers; standard power rule on each.

Step 1. $(1-x)\sqrt{x} = x^{1/2} - x^{3/2}$.

Step 2. Power rule: $x^{1/2} \to \frac{2}{3}x^{3/2}$; $x^{3/2} \to \frac{2}{5}x^{5/2}$.

Step 3. Final: $\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + C$.

Final Answer: $\frac{2x^{3/2}}{3} - \frac{2x^{5/2}}{5} + C$

Q 7.15 **Find** $\int \sqrt{x}\,(3x^2 + 2x + 3)\,dx$.

SOLUTION

Concept used. Distribute $\sqrt{x} = x^{1/2}$ across the polynomial and use the power rule. Recall $x^{1/2} \cdot x^n = x^{n+1/2}$.

Step 1. Distribute:

$$\sqrt{x}(3x^2 + 2x + 3) = 3x^{5/2} + 2x^{3/2} + 3x^{1/2}.$$

Step 2. Integrate each term using $\int x^n\,dx = x^{n+1}/(n+1)$:

$$\int 3x^{5/2}\,dx = 3\cdot\frac{x^{7/2}}{7/2} = \frac{6}{7}x^{7/2},$$
$$\int 2x^{3/2}\,dx = 2\cdot\frac{x^{5/2}}{5/2} = \frac{4}{5}x^{5/2},$$
$$\int 3x^{1/2}\,dx = 3\cdot\frac{x^{3/2}}{3/2} = 2x^{3/2}.$$

Step 3. Combine: $\frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + C$.

Final Answer: $\frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + C$

EXPERT'S SOLUTION *: Dev Chatterjee, B.Tech CSE, IIT Roorkee*

Strategic angle. Multiply through, then power-rule each term independently.

Step 1. $\sqrt{x}\cdot 3x^2 = 3x^{5/2}$; $\sqrt{x}\cdot 2x = 2x^{3/2}$; $\sqrt{x}\cdot 3 = 3x^{1/2}$.

Step 2. Apply $\int x^n\,dx$ with $n = 5/2, 3/2, 1/2$ in turn.

Step 3. Final: $\frac{6}{7}x^{7/2} + \frac{4}{5}x^{5/2} + 2x^{3/2} + C$.

Final Answer: $\frac{6x^{7/2}}{7} + \frac{4x^{5/2}}{5} + 2x^{3/2} + C$

Q 7.16 **Find** $\int (2x - 3\cos x + e^x)\,dx$.

SOLUTION

Concept used. Linearity together with the standard integrals $\int x\,dx = x^2/2$, $\int \cos x\,dx = \sin x$ and $\int e^x\,dx = e^x$.

Step 1. Split using linearity:

$$\int (2x - 3\cos x + e^x)\,dx = 2\int x\,dx - 3\int \cos x\,dx + \int e^x\,dx.$$

Step 2. Compute each: $2\cdot\frac{x^2}{2} = x^2$; $-3\sin x$; e^x.

Step 3. Combine: $x^2 - 3\sin x + e^x + C$.

Final Answer: $x^2 - 3\sin x + e^x + C$

EXPERT'S SOLUTION *: Krishna Rao, M.Sc Mathematics, IIT Bombay*

Quick reading. Three standard pieces.

Step 1. $\int 2x\,dx = x^2$.

Step 2. $\int -3\cos x\,dx = -3\sin x$.

Step 3. $\int e^x\,dx = e^x$.

Step 4. Sum, add C.

Final Answer: $x^2 - 3\sin x + e^x + C$

Q 7.17 **Find** $\displaystyle\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$.

SOLUTION

Concept used. Linearity with the power rule (for x^2 and $\sqrt{x} = x^{1/2}$) and the standard integral $\int \sin x\,dx = -\cos x$.

Step 1. Split: $\int(2x^2 - 3\sin x + 5x^{1/2})\,dx = 2\int x^2\,dx - 3\int \sin x\,dx + 5\int x^{1/2}\,dx$.

Step 2. Compute each: $\int x^2\,dx = x^3/3$, so $2\cdot x^3/3 = \frac{2}{3}x^3$; $\int \sin x\,dx = -\cos x$, so $-3\cdot(-\cos x) = 3\cos x$; $\int x^{1/2}\,dx = \frac{2}{3}x^{3/2}$, so $5\cdot\frac{2}{3}x^{3/2} = \frac{10}{3}x^{3/2}$.

Step 3. Combine: $\frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2} + C$.

Final Answer: $\dfrac{2x^3}{3} + 3\cos x + \dfrac{10}{3}x^{3/2} + C$

EXPERT'S SOLUTION *: Siddharth Pillai, M.Sc Mathematics, IIT Bombay*

Strategic angle. Linearity isolates three independent standard forms; integrate each.

Step 1. $\int 2x^2\,dx = \dfrac{2x^3}{3}$.

Step 2. $\int -3\sin x\,dx = 3\cos x$ (the $-$ from -3 and the $-$ from anti derivative of $\sin$ multiply to $+$).

Step 3. $\int 5\sqrt{x}\,dx = 5 \cdot \frac{2}{3}x^{3/2} = \frac{10}{3}x^{3/2}$.

Final Answer: $\frac{2x^3}{3} + 3\cos x + \frac{10x^{3/2}}{3} + C$

Q 7.18 **Find** $\displaystyle\int \sec x\,(\sec x + \tan x)\,dx$.

SOLUTION

Concept used. Expand the product first, then use the standard derivatives $\frac{d}{dx}\tan x = \sec^2 x$ and $\frac{d}{dx}\sec x = \sec x \tan x$, which give

$$\int \sec^2 x\,dx = \tan x + C, \qquad \int \sec x \tan x\,dx = \sec x + C.$$

Step 1. Distribute: $\sec x(\sec x + \tan x) = \sec^2 x + \sec x \tan x$.

Step 2. Integrate term-by-term:

$$\int (\sec^2 x + \sec x \tan x)\,dx = \int \sec^2 x\,dx + \int \sec x \tan x\,dx = \tan x + \sec x.$$

Step 3. Add the constant: $\tan x + \sec x + C$.

Final Answer: $\tan x + \sec x + C$

EXPERT'S SOLUTION : *Aditya Kumar, Ph.D Mathematics, IIT Delhi*

Quick reading. Two clean standard forms inside one bracket.

Step 1. Expand: $\sec^2 x + \sec x \tan x$.

Step 2. Anti derivatives: $\sec^2 x \to \tan x$; $\sec x \tan x \to \sec x$.

Step 3. Sum: $\tan x + \sec x + C$.

Final Answer: $\sec x + \tan x + C$

Q 7.19 **Find** $\displaystyle\int \frac{\sec^2 x}{\csc^2 x}\,dx$.

SOLUTION

Concept used. Convert $\sec x$ and $\csc x$ to $\sin x$ and $\cos x$, then use the trigonometric identity $\cos 2x = 1 - 2\sin^2 x \Rightarrow 2\sin^2 x = 1 - \cos 2x$.

Step 1. Rewrite using $\sec x = 1/\cos x$ and $\csc x = 1/\sin x$:

$$\frac{\sec^2 x}{\csc^2 x} = \frac{1/\cos^2 x}{1/\sin^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x.$$

Step 2. Use the identity $\tan^2 x = \sec^2 x - 1$ (from $\sin^2 x + \cos^2 x = 1$ divided by $\cos^2 x$):

$$\int \tan^2 x\, dx = \int (\sec^2 x - 1)\, dx.$$

Step 3. Integrate: $\int \sec^2 x\, dx = \tan x$ and $\int 1\, dx = x$, so

$$\int (\sec^2 x - 1)\, dx = \tan x - x + C.$$

Final Answer: $\tan x - x + C$

✗ Common Mistake

Do not try to integrate $\tan^2 x$ directly: there is no elementary anti derivative without first converting to $\sec^2 x - 1$. Always remember the identity $\tan^2 x = \sec^2 x - 1$.

EXPERT'S SOLUTION *: Neha Gupta, M.Sc Mathematics, ISI Kolkata*

Structural observation. The ratio of two squared reciprocal trig functions collapses to $\tan^2 x$, which has a one-line identity that makes it integrable.

Step 1. Simplify: $\dfrac{\sec^2 x}{\csc^2 x} = \tan^2 x$.

Step 2. Identity: $\tan^2 x = \sec^2 x - 1$.

Step 3. Integrate: $\tan x - x + C$.

Final Answer: $\tan x - x + C$

Q 7.20 **Find** $\displaystyle\int \frac{2 - 3\sin x}{\cos^2 x}\, dx.$

SOLUTION

Concept used. Split the fraction by terms in the numerator, then use $1/\cos^2 x = \sec^2 x$ and $\sin x/\cos^2 x = \tan x \sec x$. The relevant anti derivatives are $\int \sec^2 x\, dx = \tan x$ and $\int \sec x \tan x\, dx = \sec x$.

Step 1. Split:

$$\frac{2 - 3\sin x}{\cos^2 x} = \frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x} = 2\sec^2 x - 3\frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = 2\sec^2 x - 3\tan x \sec x.$$

Step 2. Integrate term-by-term using linearity:

$$\int (2\sec^2 x - 3\sec x \tan x)\, dx = 2\int \sec^2 x\, dx - 3\int \sec x \tan x\, dx.$$

Step 3. Evaluate: $2\tan x - 3\sec x$.

Step 4. Add the constant of integration.

Final Answer: $2\tan x - 3\sec x + C$

EXPERT'S SOLUTION *: Priya Sharma, Ph.D Mathematics, IIT Delhi*

Strategic angle. Splitting the fraction by numerator terms is the simplest manoeuvre whenever the denominator is a single (squared) trig function.

Step 1. $\dfrac{2 - 3\sin x}{\cos^2 x} = 2\sec^2 x - 3\tan x \sec x.$

Step 2. Anti derivatives: $\sec^2 x \to \tan x$; $\sec x \tan x \to \sec x$.

Step 3. Combine: $2\tan x - 3\sec x + C$.

Final Answer: $2\tan x - 3\sec x + C$

Q 7.21 **The anti derivative of $\sqrt{x} + 1/\sqrt{x}$ equals**

(A) $\frac{1}{3}x^{1/3} + 2x^{1/2} + C$ **(B)** $\frac{2}{3}x^{3/2} + \frac{1}{2}x^2 + C$

(C) $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$ **(D)** $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + C$

SOLUTION

Concept used. Apply the power rule $\int x^n\, dx = \dfrac{x^{n+1}}{n+1}$ with $n = 1/2$ and $n = -1/2$, then compare with the listed options.

Step 1. Write the integrand as $x^{1/2} + x^{-1/2}$.

Step 2. Integrate the first term: $\int x^{1/2}\,dx = \dfrac{x^{3/2}}{3/2} = \dfrac{2}{3}x^{3/2}$.

Step 3. Integrate the second term: $\int x^{-1/2}\,dx = \dfrac{x^{1/2}}{1/2} = 2x^{1/2}$.

Step 4. Combine: $\dfrac{2}{3}x^{3/2} + 2x^{1/2} + C$, which matches option (C).

> **Final Answer:** Option (C): $\dfrac{2}{3}x^{3/2} + 2\sqrt{x} + C$

EXPERT'S SOLUTION : *Ananya Bhat, M.Sc Mathematics, IIT Bombay*

Quick reading. Apply the power rule on each half-integer power.

Step 1. $\int \sqrt{x}\,dx = \dfrac{2}{3}x^{3/2}$.

Step 2. $\int 1/\sqrt{x}\,dx = 2\sqrt{x}$.

Step 3. Sum: $\dfrac{2}{3}x^{3/2} + 2\sqrt{x} + C$, hence (C).

> **Final Answer:** Option (C)

Q 7.22 **If** $\frac{d}{dx}f(x) = 4x^3 - 3/x^4$ **such that** $f(2) = 0$**, then** $f(x)$ **is**

(A) $x^4 + \frac{1}{x^3} - \frac{129}{8}$ **(B)** $x^3 + \frac{1}{x^4} + \frac{129}{8}$

(C) $x^4 + \frac{1}{x^3} + \frac{129}{8}$ **(D)** $x^3 + \frac{1}{x^4} - \frac{129}{8}$

SOLUTION

Concept used. If $\dfrac{df}{dx}$ is given, then $f(x) = \int \dfrac{df}{dx}\,dx + C$. Use the power rule with $n = -4$: $\int x^{-4}\,dx = \dfrac{x^{-3}}{-3} = -\dfrac{1}{3x^3}$. The initial-value condition $f(2) = 0$ pins down C.

Step 1. Integrate the derivative:

$$f(x) = \int \left(4x^3 - \frac{3}{x^4}\right) dx = 4 \cdot \frac{x^4}{4} - 3 \cdot \frac{x^{-3}}{-3} + C = x^4 + \frac{1}{x^3} + C.$$

Step 2. Apply the boundary condition $f(2) = 0$:

$$0 = (2)^4 + \frac{1}{(2)^3} + C = 16 + \frac{1}{8} + C.$$

Step 3. Solve for C:

$$C = -16 - \tfrac{1}{8} = -\tfrac{128+1}{8} = -\tfrac{129}{8}.$$

Step 4. Substitute back:

$$f(x) = x^4 + \frac{1}{x^3} - \frac{129}{8},$$

which matches option (A).

Final Answer: Option (A): $f(x) = x^4 + \dfrac{1}{x^3} - \dfrac{129}{8}$

♥ Initial value problems

Whenever an indefinite-integral problem comes with a boundary condition like $f(a) = b$, the arbitrary constant C becomes uniquely determined. This is the beginning of solving **differential equations** (Chapter 9).

EXPERT'S SOLUTION : *Aditi Verma, M.Sc Mathematics, IIT Bombay*

Strategic angle. Integrate first, then use the condition to fix C.

Step 1. Anti derivative: $4x^3 \to x^4$; $-3x^{-4} \to x^{-3} = 1/x^3$. So $f(x) = x^4 + 1/x^3 + C$.

Step 2. Plug $x = 2$: $f(2) = 16 + 1/8 + C = 129/8 + C$.

Step 3. Set $f(2) = 0$: $C = -129/8$.

Step 4. Final: $f(x) = x^4 + 1/x^3 - 129/8$, option (A).

Final Answer: Option (A)

Key Takeaways

- An anti derivative of $f(x)$ is any function $F(x)$ with $F'(x) = f(x)$; the *indefinite integral* adds an arbitrary constant: $\int f(x)\,dx = F(x) + C$.
- The method of inspection works by guessing $F(x)$ from the table of standard derivatives and verifying via differentiation.
- Linearity: $\int (k_1 f + k_2 g)\,dx = k_1 \int f\,dx + k_2 \int g\,dx$.
- For chain-rule reversals, $\int f(kx)\,dx = \frac{1}{k}F(kx) + C$ where F is the anti derivative of f. The $1/k$ is the most-skipped factor in Class 12 papers.
- An initial condition $f(a) = b$ fixes C uniquely (an initial-value problem).

End of Exercise 7.1